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Evaluating Cost Efficiency in Fuzzy Environment by Using Expected Value

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Abstract. Today, one of the most fundamental issues within the field of industrial and nonindustrial activities is evaluate the costs performance of the units which are associated with industrial and nonindustrial activities. Data envelopment analysis (DEA) is a nonparametric method for evaluating performance. Fuzzy sets theory is a powerful tool for mentioning ambiguous situations. Traditional DEA models cannot work with fuzzy data therefor there is a need for a method which can evaluate this type of activities. Yet, in fuzzy data envelopment analysis, there isn't a powerful method which can evaluate cost efficiency in fuzzy environment. In this paper, a new methods for obtaining cost efficiency measurement with data set of fuzzy numbers in various conditions (variable return to scale and constant return to scale) is suggested. These consist of situations where prices are fuzzy numbers and unknown exactly at each decision making unit (DMU). All offered methods are applied in an assessment project and results are mentioned.

Keywords: Expected Value, Data Envelopment Analysis (DEA), Cost Efficiency, Fuzzy Data.

1. Introduction

Data Envelopment Analysis (DEA) is a method for measuring the efficiency of homogeneous Decision Making Units (DMUs) using mathematical programming problems. The first research on DEA was conducted by Charnes *et al.* (1978) to measure the efficiency of DMUs. This method was phenomenally extended to measure other concepts in economics such as ranking, return to scale, productivity, and cost efficiency. Cost efficiency (CE) evaluates the ability to produce current outputs at minimal cost. The concept of cost efficiency can be traced back to Farrell [9], who originated many of the ideas underlying data envelopment analysis (DEA) (Ariff and Can, 2006). Following Farrell's concept of CE, its estimation requires input and output quantity data as well as exact knowledge of input prices at each decision maker unit (DMU). The first, considers that prices are fixed and known at each DMU and also data set are real or crisp. In this case, the efficiency assessment can follow the approach described by Farrell (Farrell, 1957) and operationalized by Fare *et al.* (1985). Then, they considered that prices fix for each DMU and data sets are Fuzzy numbers. The concept of decision making in fuzzy environment was firstly proposed by Bellman and Zadeh (1970). Fuzzy linear programming problem with fuzzy coefficients was proposed by Negoita (1970). Maleki *et al.* (2000) introduced a linear programming problem with fuzzy variables and proposed a method for its solving. Maleki *et al.* (2000) used a certain ranking function to solve fuzzy linear programming problems. They also introduced a new method for solving linear programming problems with vagueness in constraints using linear ranking function. Jahanshahloo *et al.* (2008) offered a two-phase method for evaluating cost efficiency, which used a ranking function to convert fuzzy coefficients to equal crisp values. The aim of this paper is to measure cost efficiency in fuzzy environment by using expected value concept.Thus, the paper is organized as follows: in the next section, the basic definitions related to DEA and fuzzy theory is reviewed. Section three presents the concept of expected value, briefly. We apply expected value method to measure cost efficiency in fuzzy environment, in the fourth section. Two numerical examples in section 5 illustrate the proposed method. Conclusions are provided in the last section.

2.Preliminaries

In this paper, DEA and fuzzy sets preliminaries are respectively discussed from Bede (2013) and Fang and Li (2013). In this section concept of fuzzy sets, DEA and credibility measure have been presented. We assume that the reader is familiar with linear programming concepts.

Definition 2.1: (Efficiency): Full (100%) efficiency is attained by any DMU if and only if none of its inputs or outputs can be improved without worsening some of its other inputs or outputs.

Definition 2.2: (Relative Efficiency): A DMU is to be rated as fully (100%) efficient on the basis of available evidence if and only if the performances of other DMUs does not show that some of its inputs or outputs can be improved without worsening some of its other inputs or outputs.

Notice that this definition avoids the need for recourse to prices or other assumptions of weights, which are supposed to reflect the relative importance of the different inputs or outputs.

Definition 2.3: If X is a universal set denoted generically by X then a fuzzy set $\tilde{A}$ in X is a set of ordered pairs:

$$\tilde{A} = \{(x,\ \mu_A(x)) | x \in X\}$$

$\mu_A(x)$ is called the membership function or grade of membership.

Definition 2.4: Let $\tilde{A}$ be a fuzzy set, and $\alpha \in [0,\ 1]$. The α-cut of the fuzzy set $\tilde{A}$ is the crisp set $\tilde{A}_\alpha$ given by

$$\tilde{A}_\alpha = \{x \in X : \mu_A(x) \geq \alpha\}$$

Definition 2.5: A fuzzy set $\tilde{M}$ is convex if all α -cuts of $\tilde{M}$ are convex.

Definition 2.6: Let $\tilde{A}$ be a fuzzy set, the height $h(\tilde{A})$ of $\tilde{A}$ is defined as:

$$h(\tilde{A}) = \sup_{x \epsilon \tilde{A}} \mu_{\tilde{A}}(x)$$

If $h(\tilde{A}) = 1$, then the fuzzy set $\tilde{A}$ is called a normal fuzzy set, otherwise it is called subnormal.

Definition 2.7: A fuzzy number $\widetilde{\mathrm{M}}$ is a normal and convex fuzzy set with a piecewise continuous membership function.

Definition 2.8: A fuzzy number $\tilde{\mathrm{M}}$ is called LR if its membership function is defined as follows:

$$\mu_{\tilde{\mathrm{M}}}\left(\mathrm{x}\right)=\begin{cases} l\left(\dfrac{\mathrm{m}-\mathrm{x}}{\alpha}\right) & x\leq m,\alpha>0 \\ R\left(\dfrac{\mathrm{x}-\mathrm{m}}{\beta}\right) & x\geq m,\beta>0 \end{cases}$$

LR Fuzzy number $\tilde{\mathrm{M}}$ is shown as follows:

$$\tilde{\mathrm{M}}=\left(\mathrm{m},\alpha,\beta\right)_{\mathrm{LR}}$$

where m, α, β are the middle, the left and the right width respectively.

Definition 2.9: A triangular fuzzy number is a LR number if the $L(x)$ and $R(x)$ functions are as follows:

$$\mathrm{L}\left(\mathrm{x}\right)=\mathrm{R}\left(\mathrm{x}\right)=\max\left\{0,1-\mid\mathrm{x}\mid\right\}$$

If $\alpha=\beta$, then it is called symmetric triangular fuzzy number.

Let Θ be a nonempty set, and P the power set of Θ. Each element in P is called an event. In order to present an axiomatic definition of credibility, it is necessary to assign to each event A number $\mathrm{Cr}\left\{\mathrm{A}\right\}$ which indicates the credibility that A will occur. In order to ensure that the number $\mathrm{Cr}\left\{\mathrm{A}\right\}$ has certain mathematical properties that we intuitively expect a credibility to have, we accept the following four axioms:

Axiom 1. (Normality) $\mathrm{Cr}\{\Theta\} = 1$.
Axiom 2. (Monotonicity) $\mathrm{Cr}\left\{\mathrm{A}\right\}<\mathrm{Cr}\left\{\mathrm{B}\right\}$ whenever $\mathrm{A}\subset\mathrm{B}$.
Axiom 3. (Self-Duality) $\mathrm{Cr}\left\{\mathrm{A}\right\} + \mathrm{Cr}\{\mathrm{A}^c\} = 1$ for any event A.
Axiom 4. (Maximality) $\mathrm{Cr}\ \{\bigcup_i A_i\} = \sup_i \mathrm{Cr}\{A_i\}$ for any events A_i with $\mathrm{Cr}\{A_i\}<0.5$.

The set function $\mathrm{Cr}\{\}$ is called a credibility measure if it satisfies the normality, monotonicity, self-duality, and maximal axioms.

Definition 2.10: Let Θ be a nonempty set, P the power set of Θ, and $\mathrm{Cr}\{\}$ a credibility measure. Then, the triplet $(\Theta,\ \mathrm{P},\ \mathrm{Cr})$ is called a credibility space.

Definition 2.11: A fuzzy variable is defined as a (measurable) function from a credibility space $(\Theta,\ \mathrm{P},\ \mathrm{Cr})$ to the set of real numbers.

In addition, we can define an n-dimensional fuzzy vector is defined as a function from a credibility space to the set of n-dimensional real vectors.

Now we define operators on fuzzy variables.

The independence of fuzzy variables has been discussed from different angles by many authors. Here we use the condition given by Liu (2004).

Definition 2.12: The fuzzy variables $\xi_1,\dots,\xi_n$ are said to be independent if

$$Cr\{\bigcap_{i=1}^{n}\{\xi_i \in B_i\}\} = min_{1\le i\le n} Cr\left\{\xi_i \in B_i\right\}$$

Definition 2.13: (Extension Principle of Zadeh (Zadeh, 1978)) Let $\xi_1,\dots,\xi_n$ be independent fuzzy variables with membership functions μ_i for each $i=1,\dots,n$, and $f: R^n \to R$ a function. Then the membership function μ of $\xi = f(\xi_1,\dots,\xi_n)$ is following:

$$\mu\left(x\right) = \sup_{x=f(\mathrm{x}_1,\cdots,\ \mathrm{x}_\mathrm{n})} \min_{1\le i\le n} \mu_i\left(x_i\right)$$

By using Extension Principle of Zadeh it is simple to verify following results.

Assume that $\tau = \left(a,b,c\right)$ and $\omega = (e,h,g)$ are two triangular fuzzy numbers therefor the following results hold.

1- Sum of two triangular fuzzy numbers is a triangular fuzzy number.

$$\omega + \tau = \left(e+a, h+b, c+g\right). \text{ (Bede, 2013)}$$

2- Multiplication of two fuzzy numbers $\pi = (\mathrm{e},\mathrm{f},\mathrm{g})$, $\tau = \left(a,b,c\right)$ are defined as follows.

$$\pi \times \tau \approx \left(fb, f\left(b-a\right)+b\left(f-e\right), f\left(c-b\right)+b\left(g-f\right)\right). \text{ (Bede, 2013)}$$

3- Inverse of a fuzzy variable $\tau = (a,b,c)$ is also a fuzzy variable which defined as follows.

Now it is simple to verify that the following function is a credibility measure.

For each real set B we have

$$\mathrm{Cr}\left\{f\left(\xi_1,\ldots,\xi_n\right) \in \mathrm{B}\right\} = \frac{1}{2}\left(\sup_{f\left(x_1,\ldots,x_n\right)\in B} \mu\left(x\right) + 1 - \sup_{f\left(x_1,\ldots,x_n\right)\in B^c} \mu\left(x\right)\right)$$

3.Expected Value

For fuzzy variables, there are many ways to define an expected value operator. Liu (2004) gave the most used definition of expected value operator of fuzzy variable and because of this definition is not only applicable to continuous fuzzy variables but also discrete ones.

Definition 3.1: Let ξ be a fuzzy variable. Then the expected value of ξ is defined by

$$E\left[\xi\right] = \int_0^{+\infty} Cr\left\{\xi \geq r\right\} - \int_{-\infty}^{0} Cr\left\{\xi \leq r\right\}$$

in which at least one of the two integrals is finite.

Now let $\xi = (c,a,b)$ be a triangular fuzzy number therefor by using credibility measure we have the following.

$$Cr\left\{\xi \leq r\right\} = \begin{cases} 1 & if \quad b < r \\ \dfrac{r-c}{2(a-c)} & if \quad c < r \leq a \\ \dfrac{r-2a+b}{2(b-a)} & if \quad a < r \leq b \\ 0 & if \quad r \leq c \end{cases}$$

$$Cr\left\{\xi \geq r\right\} = \begin{cases} 1 & if \;\; r \leq c \\ \dfrac{2a-b-r}{2(a-c)} & if \;\; c < r \leq a \\ \dfrac{b-r}{2(b-a)} & if \; a < r \leq b \\ 0 & if \;\; b < r \end{cases}$$

Then by using the definition of expected value, expected value of a triangular fuzzy number is equal to the following.

$$E\left[\xi\right] = \frac{(c+2a+b)}{4}$$

Remark: By using properties of integral, following result is evident.

$$E\left[\xi\alpha + \beta\zeta\right] = \alpha E\left[\xi\right] + \beta E\left[\zeta\right]$$

For more information, also see (Liu, 2004).

In order to rank fuzzy variables, we say $\xi \geq \eta$ if and only if $E[\xi] \geq \mathrm{E}[\eta]$.

4. Fuzzy Cost Efficiency

One of the most important factors in economic prosperity of each economical institution is having a correct estimate of cost performance of various sections of the institution. By noting to this that estimate the accurate information is not always possible, and there are some errors in estimations, hence for eliminate errors and participate error's proportion in calculation, using information in fuzzy environment is recommended. In this section, first, fuzzy cost efficiency models with triangular fuzzy coefficients are presented and later by using expected value operator, converted to crisp models.

The underpinnings of efficiency measurement back to the work of Debreu (1951). He was the first to define the concept of technical efficiency. The measurement of TE as defined by Farrell (1957) was operationalized and popularized by Charnes *et al.* (1978). In 1978, Charnes *et al.* (1978) developed a procedure for assessing in relative efficiency and inefficiency of decision making units (DMU).

The following model is proposed by fare for evaluate cost efficiency of each DMU in certain environment.

$$\begin{aligned} & min\sum_{i=1}^{m} c_{io}x_{i} \\ & s.\ t.\ \sum_{j=1}^{n} \lambda_{j}x_{ij} \le x_{i}(i=1,...,m) \\ & \sum_{j=1}^{n} \lambda_{j}y_{rj} \ge y_{ro}\,(r=1,...,s) \\ & \lambda_{j} \ge 0\ \ \forall j. \end{aligned} \tag{1}$$

In the model above, c_{io} is the price of input i for the DMU under assessment. x_{i} is a variable that, at the optimal solution, gives the amount of input i to be employed by DMUo. x_{io} is the ith input of the DMU under assessment and y_{ro} is the rth output of the DMU under assessment. Above model is based on constant return to scale (CRS) mood. Following model is based on variable return to scale (VRS) mood.

$$\begin{aligned} & min\sum_{i=1}^{m} c_{io}x_{i} \\ & s.\ t.\ \sum_{j=1}^{n} \lambda_{j}x_{ij} \le x_{i}(i=1,...,m) \\ & \sum_{j=1}^{n} \lambda_{j}y_{rj} \ge y_{ro}\,(r=1,...,s) \\ & \sum_{j=1}^{n} \lambda_{j} = 1 \\ & \lambda_{j} \ge 0\ \ \forall j. \end{aligned} \tag{2}$$

For above models if $\left(x^{*},\lambda^{*}\right)$ be optimal solutions of the DMU under assessment therefor cost efficiency of the DMU is defined as follows:

$$E_{c} = \frac{c_{o}x^{*}}{c_{o}x_{o}} \tag{3}$$

By using an assumption that all data are in fuzzy environment the following models are created. First model is in CRS condition (CRSCE), and second one is in VRS mood (VRSCE).

$$
\begin{aligned}
& min\sum_{i=1}^{m} \tilde{c}_{io} x_i \\
& s.\ t.\ \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} \le x_i (i = 1,..., m) \\
& \sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} \ge \tilde{y}_{ro}\ (r = 1,..., s) \\
& \lambda_j \ge 0\ \ \forall j .
\end{aligned} \tag{4}
$$

and,

$$
\begin{aligned}
& min\sum_{i=1}^{m} \tilde{c}_{io} x_i \\
& s.\ t.\ \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} \le x_i (i = 1,..., m) \\
& \sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} \ge \tilde{y}_{ro}\ (r = 1,..., s) \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \ge 0\ \ \forall j .
\end{aligned} \tag{5}
$$

By using expected value operator, which is mentioned in the past section, on the cost efficiency model in CRS mood, the following fuzzy model is achieved.

$$
\begin{aligned}
& min\sum_{i=1}^{m} \mathrm{E}\left[\tilde{c}_{io}\right] x_i \\
& s.\ t.\ \sum_{j=1}^{n} \lambda_j E\left[\tilde{x}_{ij}\right] \le x_i (i = 1,..., m) \\
& \sum_{j=1}^{n} \lambda_j E\left[\tilde{y}_{rj}\right] \ge E\left[\tilde{y}_{ro}\right] (r = 1,..., s)
\end{aligned} \tag{6}
$$

$$\lambda_j \geq 0 \ \forall j \, .$$

Now assume that all data are triangular fuzzy numbers, therefor following crisp model is achieved.

$$\begin{aligned}
& min\sum_{\mathrm{i=1}}^{\mathrm{m}} \frac{c_{io}^1 + 2c_{io}^2 + \ c_{io}^3}{4} x_i \\
& s.t. \ \ x_i \geq \sum_{j=1}^{n} \lambda_j \frac{x_{ij}^1 + 2x_{ij}^2 + \ x_{ij}^3}{4} \quad (i = 1,\dots,m) \\
& \frac{y_{ro}^1 + 2y_{ro}^2 + \ y_{ro}^3}{4} \leq \sum_{j=1}^{n} \lambda_j \frac{y_{rj}^1 + 2y_{rj}^2 + \ y_{rj}^3}{4} \quad (r = 1,\dots,s) \\
& \lambda_j \geq 0 \ \forall j
\end{aligned} \tag{7}$$

By using expected value operator, which is mentioned in the past section, on the cost efficiency model in VRS mood, the following fuzzy model is achieved.

$$\begin{aligned}
& min\sum_{i=1}^{m} \mathrm{E}\left[\tilde{c}_{io}\right] x_i \\
& s. \ t. \ \sum_{j=1}^{n} \lambda_j E\left[\tilde{x}_{ij}\right] \leq x_i (i = 1,\dots,m) \\
& \sum_{j=1}^{n} \lambda_j E\left[\tilde{y}_{rj}\right] \geq E\left[\tilde{y}_{ro}\right] (r = 1,\dots,s) \\
& \sum_{j=1}^{n} \lambda_j = 1 \qquad \lambda_j \geq 0 \ \forall j
\end{aligned} \tag{8}$$

Now assume that all data are triangular fuzzy numbers, therefor following crisp model is achieved.

$$\begin{aligned}
& min\sum_{\mathrm{i=1}}^{\mathrm{m}} \frac{c_{io}^1 + 2c_{io}^2 + \ c_{io}^3}{4} x_i \\
& s.t. \ \ x_i \geq \sum_{j=1}^{n} \lambda_j (\frac{x_{ij}^1 + 2x_{ij}^2 + \ x_{ij}^3}{4}) \ (i = 1,\dots,m) \\
& \frac{y_{ro}^1 + 2y_{ro}^2 + \ y_{ro}^3}{4} \leq \sum_{j=1}^{n} \lambda_j \frac{y_{rj}^1 + 2y_{rj}^2 + \ y_{rj}^3}{4} \quad (r = 1,\dots,s)
\end{aligned} \tag{9}$$

$$\sum_{j=1}^{n} \lambda_j = 1 \qquad \lambda_j \geq 0 \ \forall j$$

For above models if (x^*, λ^*) be optimal solutions of the DMU under assessment therefor absolute cost efficiency of the DMU is defined as follows:

$$E_{c_o} = \frac{\sum_{i=1}^{m} \left(\frac{c_{io}^1 + 2c_{io}^2 + c_{io}^3}{4} \right) x^*_i}{\sum_{i=1}^{m} \left(\frac{c_{io}^2 (x_{io}^1 + x_{io}^3) + c_{io}^2 x_{io}^2 + x_{io}^2 (c_{io}^1 + c_{io}^3 - x_{io}^3)}{4} \right)} \tag{10}$$

Now consider the following set which is the set of all DMU's absolute cost efficiency.

$$E = \{E_{C_j} \mid j = 1, \ldots, n\}$$

Cost efficiency of DMU_o is defined as follows.

$$E'_{C_o} = \frac{E_{C_o}}{max\{E\}} \tag{11}$$

Theorem 1: Models (7) and (9) have non-empty feasible regions and $0 < E'_{C_o} \leq 1$.

Proof: by setting $\lambda_o = 1 \ \& \forall j \neq 0; \lambda_j = 0 \ \& \ x_i = \frac{x_{io}^1 + 2x_{io}^2 + x_{io}^3}{4}$ there is a feasible solution and therefor, $E \neq \varnothing$. Now assume that $0 \in E$ therefor $\exists \ j; E_{C_j} = 0 \Rightarrow \lambda_j = 0 \Rightarrow y_j = 0$ it is a contradiction with DEA axioms. Since number of DMUs are finite then $E'_{C_o} \leq 1$.

5. Application

In the following Table 1, data of 4 organs of an agriculture organization which mined from that organization's annuals over the years between 1999 and 2008 are listed.

In following Table 1, each organ is considered as a decision making unit. For fuzzification data, average of each element in the period is considered as center and minimum of data considered as lower bund and maximum of data considered as the upper bund of a triangular fuzzy number for each element. In the following table2, information about the consumed cost by each DMU is expressed.

Table 1. Information about 4 Organs

DMU	DMU1	DMU2	DMU3	DMU4
Input1	(2,10,20)	(5,19,23)	(17,21,25)	(13,17,21)
Input2	(18,20,22)	(14,16,19)	(9,12,13)	(14,15,20)
Input3	(13,15,18)	(12,20,44)	(8,12,15)	(9,10,12)
Output1	(4,6,9)	(12,13,15)	(14,16,19)	(9,10,12)
Output2	(14,15,20)	(2,10,20)	(8,12,17)	(4,5,12)

Table 2. Cost

DMU	DMU1	DMU2	DMU3	DMU4
Input1	DMU1	DMU2	DMU3	DMU4
Input2	(12,20,24)	(4,17,23)	(7,11,15)	(3,7,11)
Input3	(8,20,22)	(4,6,9)	(7,9,13)	(4,8,12)

Table 3. Results of Using CRSCE

DMU	Absolute CE	CE
DMU1	1.039146	0.7957
DMU2	0.8076126	0.6184
DMU3	1.305997	1
DMU4	0.9522646	0.7291

Table 4. Results of Using VRSCE

DMU	Absolute CE	CE
DMU1	1.039146	0.7008
DMU2	0.9214609	0.6214
DMU3	1.305997	0.8808
DMU4	1.482777	1

In the above tables, results of assessment the DMU's cost efficiency in various conditions are expressed.

In table 3, results of using constant return to scale cost efficiency model are expressed and observed that DMU 3 is efficient and in table 4, results of using variable return to scale cost efficiency model are expressed and observed that DMU 4 is efficient.

6. Conclusion

Data envelopment analysis is a great method to evaluate performance. It converts assessment problems to mathematical programming problems and especially linear programming problems, which are solvable by regular mathematical programming methods. In today's competitive world, evaluating performance is an important issue, which is participating in every operation. Data envelopment analysis is a powerful tool for evaluating performance of situations. Yet, there is not a powerful method in fuzzy data envelopment analysis, which can evaluate cost efficiency in fuzzy environment. In this paper, a method for evaluating cost efficiency in fuzzy environment in various conditions, have been offered. The offered methods were applied on agriculture organizations organs and results of assessments are listed in the tables.

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