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RESEARCH ARTICLE **Open Access**

Examining the Forex Market Based on Chaos Theory

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Abstract
The Forex market is known as the strongest and richest financial market in the world that has been operating continuously. This market is formed based on the exchange rates of different countries, as well as the prices of oil and gold. The price of currency pairs in the Forex market as the largest market for financial transactions has always been of great importance. The purpose of this article is to study the Forex market based on chaos theory. Its statistical population includes three currency pairs Euro / Dollar, Pound / Dollar and Dollar / Yen. The period of prices of the surveyed currency pairs is from January 2017 to December 2021 and its time range includes daily prices. The results showed that after fitting the model and performing the BDS test, on the GARCH model residues in different dimensions and ε of all 6 groups, which are IID residues and there is no dependence on them. The results of Lyapunov's maximalism test showed that for all dimensions, and all time intervals (n), the value of Lyapunov's exponent is a positive and small number, indicating that the time series of the triple currency pair follows a chaotic process.

Keywords: Forex, Chaos Theory, Euro, Dollar, Pound, Yen

Introduction

With the formation of different currencies as a tool for exchanges between human beings on the one hand and the growth and expansion of cities and countries and consequently the increase of transactions and trade relations around the world on the other hand caused the problem of currency conversion and in other words buy and sell currencies globally.

The foreign exchange market (Forex) is the global market for buying and selling foreign currency, a market in which large banks and financial institutions trade various world currencies such as the dollar, the euro and other international currencies. In other words, it can be said that this market, the largest trading market in the world, is available to traders on a 24-hour basis, with very easy access and simple and fast trading. (Ni et al., 2019).

The Forex market is the largest market in the world. The market's daily trading value is $ 6.6 trillion (according to the latest figures from the International Monetary Fund in 2020), which is a much higher trading volume than the US $ 10 billion turnover. A large part of foreign exchange trading in the Forex market is related to cash and Spot currency trading. These exchanges take place between the US dollar and the four major currencies, which are the British pound, the European euro, the Swiss franc and the yen. These four currencies are traded against the dollar (Li et al, 2021).

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The exchange rate change in the market is about one percent daily (Fatum & Hutchison, 2006). Therefore, traders, especially small and private investors, may not be interested in trading in the foreign exchange market. To solve this problem, the use of leverage credit, which investors can use in the range (1 to 500) equal to their capital in the transaction is considered (Ercen et al, 2022). Of course, it should be noted that the risk of profit or loss of using financial leverage to varying degrees will be borne solely by the trader. Some related statistics indicate that 90 to 95% of speculators lose all their capital in this market in just six months to a year (king et al, 2011).

Reducing the risks of incorrect credit decisions is saving the time, cost and improvement of credit decisions are among the advantages of using credit scoring. Traditionally, the banks and credit institutes have used techniques such as discriminate analysis, probity and logic analysis, logistic analysis for credit scoring. In addition, several methods are used to evaluate investment strategies for portfolio selection problems, examples include funds selection and venture capital investments. Moreover, financial indicators are used in a multi-criteria context also for assessments and creating corporate performance rankings (Modjtahedi & Daneshvar, 2020).

Time series is a set of recorded observations in terms of time of one variable (one variable) or several variables (multivariate). In general, continuous time series data are significant in size and dimension. Chaos theory has been part of scientific research in various fields in recent decades (Shahriar, 2022). For more than two decades, in-depth studies of nonlinear dynamic models have shown that Chaos occurs extensively in engineering and natural systems (Ruihong, 2008).

Financial and monetary markets are very suitable for applying chaos theory. First, theories in financial and monetary economics suggest that monetary variables, such as exchange rates, are random and, as a result, their changes are unpredictable. Second, if the final order in the process of monetary variables is discovered, it will be possible to achieve huge profits (Shakeri et al., 2015).

What has been examined as a fundamental issue by this article is whether using Chaos theory in the period 2017 to 2021, it is possible to make a good prediction of the upward or downward trend of the price of this currency pair?

Literature Review

Several researches have been done on the above subject. Sadeghi, Daneshvar and Madanchi Zaj (2020) in an article entitled "Development of an intelligent method based on fuzzy technical indicators for forecasting and trading the euro-dollar exchange rate" concluded that today the Determining the right strategy for buying or selling in the Forex market is based on forecasting the price trend. Therefore, to select a suitable strategy in Forex, complex meta-heuristic models are used. Also, Shakeri, Homayounifar, Fallahi, Sharabaf Tabrizi (2015) in a study entitled "Study of chaos theory in the price of coins throughout the spring of freedom" concluded that in the analysis of the time series of coin prices is predictable and the assumption of no nonlinear functions in the ARIMA and GARCH pattern residues using the test is rejected. Also, to study the chaotic trend in this time series, Lyapunov Maximum Exponent test has been used. The result of this test shows that the data have a chaotic trend. Therefore, the possibility of nonlinear functions in the coin price time series is accepted and its price predictability is confirmed. Modjtahedi and Daneshvar (2020) in an article entitled " A New Credit Risk System Using Hybrid ELECTRE TRI and NSGA-II Methods" concluded that The proposed method contributes to the literature by utilizing a pair of conflicting objective functions including Type I errors and Type II errors instead of using a single criterion named “classification accuracy” which used frequently in the related works. The proposed bi-objective method is applied to six known credit risk datasets. The NRGA model is used as a benchmark for validation. Computational results indicate outstanding performance of the NSGA-ELECTRE method. Taheri, Shafiee and Evazzadeh Fath (2019) in an article entitled "Investigating the Role of Non-Financial Information Analysis and Risk-Return Analysis along with Financial Information in Increasing the Efficiency of the Stock Portfolio of Banks" concluded that The results showed that the use of non-financial and risk-return analysis information will increase the calculated efficiency of banks portfolio using the Treynor's ratio. All of hypotheses were approved, this means that there is significant difference between mean Treynor's ratio of stock combination based on risk-return information and accounting information, agency theory, management ability and accounting information and intellectual capital. Mousavi Hanjani & Iranban (2019) in an article entitled " The Relationship between Diversification Strategy, Capital Structure and Profitability in Companies Listed in the Stock Exchange by Combining the Data Line and VAR Methods" concluded that The research findings showed that the diversification strategy, capital structure and profitability in the companies accepted in the stock exchange has have a significant relationship. Also, the results of this study showed that diversification strategy has a significant effect on profitability.

Research Methodology

Given the spread of technology in Iranian society and the attractiveness of online trading in both the Iranian stock market and the Forex currency market and the high probability of capital loss by Iranian traders, the scientific purpose of this study was to investigate the price behavior of major currency pairs of Forex market from 01/01/2017 to 12/30/2021 (for 5 fiscal years) to achieve a suitable model using chaos theory (BDS test and Lyapunov Maximum Exponent) to help make a profit and reduce the risk of losing capital. In this research, the data collected from the Metatrader 5 software was loaded into the MATLAB software to perform the chaos test (Lyapanov and BDS), and its findings can be seen as follows.

Chaos Theory Tests

BDS test: The BDS test is an indirect method of testing for nonlinearity. This test is to determine the absence of dependency and to test the residues obtained from nonlinear structures after removing the linear structure from the data that have already been filtered. This test, based on the correlation integral, evaluates the randomness of the process that creates a time series versus the existence of a general correlation in it. This test can be well used to evaluate the existence of a general nonlinear process, including the chaos process, in the observed time series.

The m-dimensional space is defined as follows:

1) history: X_t^1
2) history: $X_t^2 = (X_{t-1} X_t)$
3) history: $X_t^m = (X_{t-m+1} . \dots . X_t$

The BDS test uses the concept of spatial correlation in chaos theory. If X_t (t = 1… T) is assumed to be a definite economic model of the next m and T is the number of observations, then a nonlinear test is performed to identify the dynamic system. According to the theory, all information in a multivariate pattern is surrounded by a univariate time series.

M-history represents a point in the m-dimensional space called the "enclosed dimension". By determining the dimension of m, the vectors are as follows:

4) $X_1^m = (X_1 . X_2 . \dots . X_m)$
5) $X_2^m = (X_2 . X_3 . \dots . X_{m+1})$
6) $X_{T-m}^m = (X_{T-m} . X_{T-m+1} . \dots . X_{T-1})$

If X is a m-dimensional vector, it looks like this:

7) $X_t^m = (X_t . X_{t+1} . \dots . X_{t+m-1})$

The correlation integral measures the correlation between points. In fact, pairs of points (t, s) that are close together in the m-dimensional space are in radius ε of each other.

Relationships 4 to 7 actually seek to show a correlation at points in the enclosed dimension.

If there is a correlation, the following will be done.

The correlation integral for calculating the distance (ε) between two points is as follows:

8) $C_{m.T}(\varepsilon) = (2/T_m^2 - T_m)\sum_t \sum_s I_{t.s;\varepsilon}(\varepsilon - ||X_t^m - X_s^m||) \quad t < s$

Where T_m = T-m + 1, 1 <s <T, 1 <t <T and ‖0‖ the Euclidean distance X_t ^ m and X_s ^ m.

$\boldsymbol{I_{t.s;\varepsilon}}$ is a scale function that takes the values of zero and one.

In fact, BDS statistic is a calculation of the distance between $[\boldsymbol{C_{1.T}(\varepsilon)}]^{\boldsymbol{m}}$ and $\boldsymbol{C_{m.T}(\varepsilon)}$ where the quantity $\{\boldsymbol{C_{m.T}(k)} - [\boldsymbol{C_{1.T}(\varepsilon)}]^{\boldsymbol{m}}\}$has an asymptotic normal distribution with mean zero and variance $\boldsymbol{V_{\varepsilon.m}}$

9) $V_{\varepsilon.m} = 4[K^m + 2\sum_{s=1}^{m-1} K^{m-s}C(\varepsilon)^{2s} + (m-1)^2 C(\varepsilon)^{2m} - m^2 K C(\varepsilon)^{2m-2}]$

$$K = K_\varepsilon = \frac{6}{T_m(T_m-1)(T_m-2)} \sum_{t<s<N} h_{t.s.T;\varepsilon}$$

$$h_{t.s.T;\varepsilon} = \frac{I_{t.s;\varepsilon}I_{s.T;\varepsilon} + I_{t.T;\varepsilon}I_{T.s;\varepsilon} + I_{s.t;\varepsilon}I_{t.T;\varepsilon}}{3}$$

Finally, the BDS statistic is defined as the normalized statistic:

10) $\text{BDS} = W^m = \{C_{m.T}(k) - [C_{1.T}(k)]^m\} \Big/ V_{\varepsilon.m} \Big/ T^{1/2}$

This statistic is also called Wing statistic.

Equations 8, 9 and finally 10 as the final step of the BDS test seeks to prove the existence of chaos in time series data. In case of chaos, number one and in case of absence chaos, number zero will be used.

The method of performing BDS test is that first, the time series linear process is extracted through a model such as ARIMA. The W statistic is then calculated for the model residuals. If the calculated W is significant, the randomness of the time series is rejected, or in other words, the existence of a nonlinear process in the model is confirmed. Otherwise, the test performed will indicate a linear process. The nonlinear nature of the time series process should be determined through additional tests. BDS is often effective on standardized residuals (Brooks et al, 1999). Monte Carlo studies also show that BDS statistics are sensitive to the choice of enclosed dimension and enclosed distance and sample size.

The BDS test will be performed in three stages on the main data, ARIMA model residues and GARCH model residues. Finally, using the ARIMA logo and GARCH model, the model will be presented.

Lyapunov Maximum Exponent test

The most important tool for detecting the presence of sensitivity to initial conditions in a dynamic system is the use of Lyapunov special exponent. In fact, in this method, the mean exponential divergence or convergence of points close to each other but not with the same initial conditions is measured. That is, the positive Lyapunov exponent measures the average exponential divergence of points close to each other but not with the same initial conditions, and the negative Lyapunov exponent measures the average exponential view of the convergence of points close to each other but not with the same initial conditions. Therefore, considering the characteristic of "sensitivity to initial conditions", the positive Lyapunov exponent can be expressed as a definition for a certain system chaos, which is more specifically mentioned in the definition of the maximum Lyapunov exponent. This test can also measure the stability of a dynamic system.

To calculate Lyapanov's exponent, Wolff states that a matrix of memory m vectors must first be formed in order to restructure the variable production process. Matrices have m rows and N - m1 +columns, which are formed using N temporal scalar data. From these matrices, all the pairs of vectors that apply to the following relation are identified.

11) $r_0(m; i.j) = ||X_i - X_j|| < \varepsilon$

Where r0 is the distance between the i-th state with its nearest neighbor and Xi the source state and Xj the nearest neighbor.
In fact, Xj is the point that has the shortest distance from Xi. It is a small positive amount. The above calculation must be performed during n time steps.

12) $r_n(m; i.j) = ||X_{i+n} - X_{j+n}||$

Then the degree of divergence of points close to each other is calculated as follows. If points close to each other, greater than zero, diverge from each other in m-dimensional space, d (m; I, j) will be greater than one.

13) $d_n(m; i.j) = \frac{r_n}{r_0} = \frac{||X_{i+n} - X_{j+n}||}{||X_i - X_j||}$

Finally, Lyapanov's exponent is calculated according to the following equation.

14) $\lambda(m.n) = \frac{1}{N(N-M-1)} \sum log d_n(m; i.j)$

To calculate Lyapanov's exponent, systems whose equations are not known and only their time series are available.

Research Findings

Estimate the existence of chaos in the data

As mentioned, the BDS test is used to investigate the nonlinear trend of the data and the Lyapunov maximum exponent test is used to investigate the chaos of the time series. The BDS test was performed in three stages on the main data, ARIMA model residues and GARCH model residues. To determine the structure of this series, 1,296 observations are divided into 6 groups, in each group of 200 observations, 296 observations are added to the previous group and the last group to determine whether increasing the number of observations increases the predictability of the system. The Lyapunov maximum exponent test is also checked on all data. A positive value of this statistic indicates the existence of a chaotic trend in the system. R and MATLAB software were used to perform the tests.

Table 1.
Root test results of the time series logarithm unit of the EUR / USD currency pair

Number of observations	Variable logarithm level			The first-order difference of the variable logarithm		
	DF statistics	P-value	Result	DF statistics	P-value	Result
200	1.352	0.958	Unreliable	-10.658	0.01	Reliable
400	2.101	0.990	Unreliable	-11752	0.01	Reliable
600	1.825	0.985	Unreliable	-13.954	0.01	Reliable
800	2.116	0.989	Unreliable	-15.245	0.01	Reliable
1000	2.950	0.990	Unreliable	-18.236	0.01	Reliable
1296	1.545	0.972	Unreliable	-10.254	0.01	Reliable

Table 2.
Root test results of the time series logarithm unit of the USD/JPY currency pair

Number of observations	Variable logarithm level			The first-order difference of the variable logarithm		
	DF statistics	P-value	Result	DF statistics	P-value	Result
200	2.452	0.965	Unreliable	-10.658	0.01	Reliable
400	1.856	0.975	Unreliable	-10.752	0.01	Reliable
600	2.236	0.970	Unreliable	-12.956	0.01	Reliable
800	1.985	0.989	Unreliable	-14.235	0.01	Reliable
1000	2.980	0.990	Unreliable	-17.235	0.01	Reliable
1296	1.485	0.968	Unreliable	-10.124	0.01	Reliable

Table 3.
Root test results of the time series logarithm unit of the GBP/USD currency pair

Number of observations	Variable logarithm level			The first-order difference of the variable logarithm		
	DF statistics	P-value	Result	DF statistics	P-value	Result
200	2.236	0.978	Unreliable	-10.248	0.01	Reliable
400	1.985	0.980	Unreliable	-10.752	0.01	Reliable
600	2.980	0.970	Unreliable	-12.956	0.01	Reliable
800	1.985	0.989	Unreliable	-14.235	0.01	Reliable
1000	2.856	0.980	Unreliable	-15.206	0.01	Reliable
1296	1.847	0.952	Unreliable	-10.128	0.01	Reliable

BDS Test

For this test, the reliability of the data is first checked to obtain a stable form of the data. The results of the unit root test are shown in Tables 4 to 6:

Table 4.
Standardized BDS test results on the main data of the EUR / USD pair

ε / Dimensions	2 = m	3 = m	4 = m	5 = m	2 = m	3 = m	4 = m	5 = m
Observations	600 Observations(0.0131=SE)				800 Observations(0.0123=SE)			
ε 0.5	3.036	3.358	3.084	3.786	3.339	3.945	3.809	5.102
ε 1	2.704	2.780	2.825	3.353	3.367	3.501	3.393	4.074
ε 1/5	4.033	3.882	3.779	3.942	4.310	4.097	3.940	4.361
ε 2	5.075	4.578	4.352	4.314	5.822	5.342	5.102	5.041
	1000 Observations(0.0118=SE)				1296 Observations(0.0079=SE)			
ε 0.5	3.789	4.153	3.561	4.506	12.340	13.036	13.572	13.890
ε 1	3.768	3.689	3.291	3.823	9.083	9.281	10.791	11.502
ε 1/5	3.585	3.569	3.367	3.851	12.855	11.533	11.687	11.340
ε 2	5.013	4.742	4.461	4.550	8.826	7.878	7.286	6.732

Table 5.
Standardized BDS test results on the main data of the USD/JPY pair

ε / Dimensions	2 = m	3 = m	4 = m	5 = m	2 = m	3 = m	4 = m	5 = m
Observations	600 Observations(0.0187=SE)				800 Observations(0.0139=SE)			
ε 0.5	3.046	3.458	3.241	3.685	3.365	3.856	3.956	4.564
ε 1	2.895	2.892	2.589	3.124	3.584	3.612	3.421	4.096
ε 1/5	4.125	3.952	3.784	4.124	4.456	4.125	3.956	3.456
ε 2	4.124	4.654	4.452	4.546	5.952	5.456	5.256	5.126
	1000 Observations(0.0121=SE)				1296 Observations(0.0098=SE)			
ε 0.5	3.986	4.254	3.652	4.698	11.364	13.326	13.653	13.954
ε 1	3.854	3754	3.325	3.953	9.124	9.451	10.895	11.687
ε 1/5	3.685	3.457	3.458	3.956	11.754	10.956	11.458	11.215
ε 2	5.123	4.864	4.654	4.550	8.956	7.457	7.356	6.642

Table 6.
Standardized BDS test results on the main data of the GBP/USD pair

ε / Dimensions	2 = m	3 = m	4 = m	5 = m	2 = m	3 = m	4 = m	5 = m
Observations	600 Observations(0.0187=SE)				800 Observations(0.0139=SE)			
ε 0.5	3.369	3.215	3.124	3.985	3.421	3.854	3.745	4.956
ε 1	3.124	3.562	3.754	3.353	3.467	3.621	3.456	4.145
ε 1/5	4.033	3.965	3.879	3.124	4.541	4.254	3.369	4.421
ε 2	4.412	4.548	3.356	4.895	5.786	5.545	5.985	5.245
	1000 Observations(0.0121=SE)				1296 Observations(0.0098=SE)			
ε 0.5	3.321	4.456	4.655	4.512	11.654	10.654	11.546	12.965
ε 1	3.768	3.689	3.291	3.823	10.956	10.784	10.986	11.502
ε 1/5	3.654	3985	4.547	4.369	11.245	12.654	12.985	13.321
ε 2	5.013	4.742	4.461	4.550	9.546	8.132	7.365	6.654

Explanation: SE is the standard deviation of the sample. *: At the 5% level, the null hypothesis is not rejected.

The results of data logarithm reliability were evaluated using Dickey-Fuller test for all 6 groups. The results show that the null hypothesis, which indicates the existence of a single root, is accepted and the data are not reliable in this case. To validate the data, a first-order differentiation is performed. Then, in the first step, BDS test is performed on the main data in different enclosed dimensions. In order to perform this test, a value must be selected for ε, which must be interpreted as bandwidth. This bandwidth is presented as a coefficient of the standard deviation of the sample, which is also consistent with the choice of ε in. The results of this test for all 6 groups are presented in Table (4 to 6).

Null hypothesis states that the data are IID, ie the data are independent and similarly distributed, there is no dependency in them and they are random. The results show that in all dimensions except the first group, in cases marked with *, the null hypothesis is rejected, so the data are not IID and there is a linear or non-linear relationship between them. For this purpose, the test was performed on the ARIMA and GARCH model residues to check the results again by removing the linear and nonlinear dependencies. Before the second stage of the test, the appropriate ARIMA pattern must be fitted. By fitting the ARIMA pattern, linear effects are removed from the pattern and the test is performed on the pattern residues. After fitting the pattern, the unit root test is performed on the residues and the null hypothesis is rejected and the residues do not have a single root. Then BDS test was performed on the residues of the patterns.

The null hypothesis at this stage is that the residues are IID and independent of each other. 1 Hypothesis indicates the existence of dependence on model residues; That is, by removing the linear dependencies, the residues are still dependent. The results for different dimensions in all 6 groups indicate that the null hypothesis is rejected and the model residues are not IID but have nonlinear dependencies.

Table 7.
ARIMA model results for all 6 EUR / USD data sets

Observations	ARIMA	Model
600	(1,1,0)	$r_t = .001 - 0.1554r_{t-1} + \varepsilon_t(0.040)$ $\sigma'_a = .0130 \quad AIC = -3500.12 \quad R^2 = 02.75$
800	(0,1,1)	$r_t = .001 - 0.153\varepsilon_{t-1} + \varepsilon_t(0.032)$ $\sigma'_a = 0.0121 \quad AIC = -4775.15 \quad R^2 = 02.57$

Observations	ARIMA	Model
1000	(0,1,1)	$r_t = .001 - 0.149\varepsilon_{t-1} + \varepsilon_t(0.032)$ $\sigma'_a = .0116 \quad AIC = -6058.47 \quad R^2 = 02.63$
1296	(1,1,0)	$r_t = .0001 - 0.503a_{t-1} + a_t(0.021)$ $\sigma'_a = .097 \quad AIC = -3054.03 \quad R^2 = 25.34\ \%$

Table 8.

ARIMA model results for all 6 USD/JPY data sets

Observations	ARIMA	Model
600	(1,1,0)	$r_t = .011 - 0.1754r_{t-1} + \varepsilon_t(0.060)$ $\sigma'_a = .0150 \quad AIC = -3900.12 \quad R^2 = 01.05$
800	(0,1,1)	$r_t = .003 - 0.263\varepsilon_{t-1} + \varepsilon_t(0.042)$ $\sigma'_a = 0.0141 \quad AIC = -4895.15 \quad R^2 = 03.57$
1000	(0,1,1)	$r_t = .012 - 0.139\varepsilon_{t-1} + \varepsilon_t(0.022)$ $\sigma'_a = .0326 \quad AIC = -5248.47 \quad R^2 = 02.73$
1296	(1,1,0)	$r_t = .0003 - 0.603a_{t-1} + a_t(0.011)$ $\sigma'_a = .067 \quad AIC = -3264.03 \quad R^2 = 26.54\ \%$

Table 9.

ARIMA model results for all 6 GBP/USD data sets

Observations	ARIMA	Model
600	(1,1,0)	$r_t = .001 - 0.1654r_{t-1} + \varepsilon_t(0.035)$ $\sigma'_a = .0221 \quad AIC = -2805.11 \quad R^2 = 01.05$
800	(0,1,1)	$r_t = .0101 - 0.236\varepsilon_{t-1} + \varepsilon_t(0.019)$ $\sigma'_a = 0.0181 \quad AIC = -4915.15 \quad R^2 = 02.57$
1000	(0,1,1)	$r_t = .0261 - 0.189\varepsilon_{t-1} + \varepsilon_t(0.022)$ $\sigma'_a = .0116 \quad AIC = -6058.47 \quad R^2 = 02.63$
1296	(1,1,0)	$r_t = .0001 - 0.593a_{t-1} + a_t(0.028)$ $\sigma'_a = .087 \quad AIC = -3164.03 \quad R^2 = 28.34\ \%$

The third step is the BDS test on GARCH model residues. To estimate the GARCH model, the variance heterogeneity is first investigated. The results of the White test state that the null hypothesis that there is no heterogeneity of variance is rejected and the heterogeneity of the data is confirmed. The effects of ARCH on the mean equation are also examined. The null hypothesis in these tests is the absence of the ARCH effect. The results of Liung Box test and LM-ARCH test reject the null hypothesis and confirm the existence of ARCH effects in the first 4 groups and GARCH effect in the next 2 groups.

Table 10.

Standardized BDS test results on ARIMA model residues EUR / USD currency pair data

ε / Dimensions	2 = m	3 = m	4 = m	5 = m	2 = m	3 = m	4 = m	5 = m
Observations	600 Observations(0.0121=SE)				800 Observations(0.0113=SE)			
ε 0.5	5.896	5.256	5.314	5.865	7.107	7.559	6.695	8.128
ε 1	4.762	4.986	4.325	4.789	6.960	6.458	6.359	7.484
ε 1/5	5.051	4.986	4.662	4.877	6.900	7.013	7.125	7.345
ε 2	5.681	5.229	4.966	5.048	8.011	8.099	8.245	7.785

ε / Dimensions	2 = m	3 = m	4 = m	5 = m	2 = m	3 = m	4 = m	5 = m
	1000 Observations(0.0118=SE)				1296 Observations(0.0999=SE)			
ε 0.5	12.124	12.652	11.856	12.645	18.852	20.126	19.981	19.562
ε 1	11.955	12.428	13.196	14.209	20.422	21.456	19.879	19.971
ε 1/5	12.684	13.150	13.113	12.945	24.512	23.034	19.987	19.254
ε 2	12.050	12.124	11.548	11.124	25.920	23.284	20.173	20.173

Table 11.
Standardized BDS test results on ARIMA model residues USD/JPY currency pair data

ε / Dimensions	2 = m	3 = m	4 = m	5 = m	2=m	3 = m	4 = m	5 = m
Observatio ns	600 Observations(0.0121=SE)				800 Observations(0.0118=SE)			
ε 0.5	5.996	5.466	5.624	5.985	7.287	7.789	6.895	8.658
ε 1	4.854	4.915	4.327	4.989	6.990	6.988	6.489	7.584
ε 1/5	5.151	4.996	4.852	4.957	6.980	7.483	7.655	7.995
ε 2	5.981	5.759	4.986	5.558	8.851	8.529	8.655	7.995
	1000 Observations(0.0108=SE)				1296 Observations(0.0999=SE)			
ε 0.5	11.104	11.992	12.006	12.855	18.952	20.166	19.891	19.692
ε 1	12.955	12.888	13.656	14.289	20.412	21.856	19.589	19.901
ε 1/5	11.644	13.590	13.783	13.025	24.180	23.154	19.157	19.394
ε 2	12.652	12.956	11.658	11.654	25.890	23.414	20.123	20.853

Table 12.
Standardized BDS test results on ARIMA model residues GBP/USD currency pair data

ε / Dimensions	2 = m	3 = m	4 = m	5 = m	2 = m	3 = m	4 = m	5 = m
Observations	600 Observations (0.0138 = SE)				800 Observations(0.0131 =SE)			
ε 0.5	5.896	5.256	5.314	5.865	7.107	7.559	6.695	8.128
ε 1	4.764	4.865	4.325	4.789	6.960	6.458	6.359	7.484
ε 1/5	5.051	4.986	4.662	4.877	6.900	7.013	7.125	7.345
ε 2	5.681	5.229	4.966	5.048	8.011	8.099	8.245	7.785
	1000 Observations(0.0121=SE)				1296 Observations(0.0989=SE)			
ε 0.5	10.245	1.852	11.125	12.456	17.985	18.456	19.254	19.562
ε 1	10.852	11.654	12.965	13.478	19.532	19.125	18.897	18.521
ε 1/5	11.215	11.105	12.568	13.548	22.654	23.034	21.654	19.254
ε 2	12.15	12.65	11.85	11.11	25.85	23.28	22.54	20.17

After fitting the pattern, the BDS test was performed on the GARCH pattern residues. The results of the null hypothesis that the residues are IID and there is no dependence on them and the conditional variance model explains the nonlinear structure in the series are rejected in all 6 groups. In this way, the possibility of a chaotic process in the structure is confirmed.

Lyapunov maximum exponent test

In this research, to calculate Lyapanov's exponent, the "dimension" method has been used, using the Wolf algorithm. Lyapunov's exponent is estimated for 2 to 5 enclosed dimensions and different time intervals (n) . Here, to calculate Lyapanov's exponent, we mean to calculate the largest view, that is, by finding at least one positive exponent, we can say that the system has a chaotic process.

Table 13.

GARCH model results for all 6 EUR / USD data sets

Observations	Model
600	$r_t = 1.396e-03 + a_t.$ $\sigma_t^2 = 1.176e-04 + 0.303a_{t-1}^2$ $(4.910e-4)$ $(9.230e-6)$ (0.105) $AIC = -6.007$ $BIC = -5.985$ $SIC = -6.007$ $HQIC = -5.999$
800	$r_t = 1.449e-03 + a_t.$ $\sigma_t^2 = 9.787e-05 + 0.394a_{t-1}^2$ $(3.921e-4)$ $(7.357e-6)$ (0.114) $AIC = -6.143$ $BIC = -6.125$ $SIC = -6.143$ $HQIC = -6.198$
1000	$r_t = 1.241e-03 + a_t.$ $\sigma_t^2 = 6.526e-050.312a_{t-1}^2$ $(3.124e-4)$ $(7.785e-6)$ $(8.118e-2)$ $(5.843e-2)$ $AIC = -5.786$ $BIC = -5.771$ $SIC = -5.786$ $HQIC = -5.780$
1296	$r_t = 5.601e-03 + a_t.$ σ_t^2 $= 3.402e-05 + 0.840a_{t-1}^2 + 0.540a_{t-1}^2$ $(3.073e-4)$ $(3.205e-6)$ $(7.123e-2)$ $(2.159e-2)$ $AIC = -4.748$ $BIC = -4.753$ $SIC = -4.748$ $HQIC = -4.743$

Table 14.

GARCH model results for all 6 USD/JPY data sets

Observations	Model
600	$r_t = 1.425e-03 + a_t.$ $\sigma_t^2 = 1.658e-04 + 0.548a_{t-1}^2$ $(5.253e-4)$ $(8.125e-6)$ (0.325) $AIC = -5.018$ $BIC = -5.854$ $SIC = -5.155$ $HQIC = -4.890$
800	$r_t = 1.356e-03 + a_t.$ $\sigma_t^2 = 8.652e-05 + 0.654a_{t-1}^2$ $(3.824e-4)$ $(7.657e-6)$ (0.215) $AIC = -6.524$ $BIC = -6.148$ $SIC = -6.254$ $HQIC = -6.215$
1000	$r_t = 1.325e-03 + a_t.$ $\sigma_t^2 = 6.480e-05 + 0.401a_{t-1}^2$ $(3.124e-4)$ $(7.785e-6)$ $(8.118e-2)$ $(5.843e-2)$ $AIC = -5.786$ $BIC = -5.771$ $SIC = -5.786$ $HQIC = -5.780$
1296	$r_t = 5.702e-03 + a_t.$ σ_t^2 $= 3.550e-05 + 0.954a_{t-1}^2 + 0.610a_{t-1}^2$ $(3.473e-4)$ $(3.285e-6)$ $(7.193e-2)$ $(2.189e-2)$ $AIC = -4.601$ $BIC = -4.785$ $SIC = -4.768$ $HQIC = -4.723$

Table 15.

GARCH model results for all 6 GBP/USD data sets

Observations	Model
600	$r_t = 1.215e-03 + a_t.$ σ_t^2 $= 1.340e-04 + 0.425a_{t-1}^2$ $(4.951e-4)$ $(7.125e-6)$ (0.215) $AIC = -5.018$ $BIC = -5.854$ $SIC = -5.155$ $HQIC$ $= -4.512$
800	$r_t = 1.356e-03 + a_t.$ σ_t^2 $= 8.652e-05 + 0.654a_{t-1}^2$ $(3.824e-4)$ $(7.657e-6)$ (0.215) $AIC = -6.524$ $BIC = -6.148$ $SIC = -6.254$ $HQIC$ $= -6.215$
1000	$r_t = 1.428e-03 + a_t.$ σ_t^2 $= 5.362e-05 + 0.258a_{t-1}^2$

	$(3.254e-4)$ $(7.785e-6)$ $(8.118e-2)$ $(5.635e-2)$ $AIC=-5.786$ $BIC=-5.771$ $SIC=-5.786$ $HQIC=-5.780$
1296	$r_t = 5.685e-03 + a_t.$ $\sigma_t^2 = 3.480e-05 + 0.854a_{t-1}^2 + 0.850a_{t-1}^2$ $(4.583e-4)$ $(6.352e-6)$ $(8.254e-2)$ $(2.210e-2)$ $AIC=-4.548$ $BIC=-4.255$ $SIC=-4.858$ $HQIC=-4.823$

The results show that for all the dimensions, and all the time intervals (n), the value of Lyapunov's exponent is a positive and small number, which indicates that the time series of the price of the three currency pairs follows a chaotic process. Lyapunov's exponent calculation was also used. The results of which indicate that the time series has a chaotic process.

Conclusion

Financial and monetary markets are very suitable for applying the theory of Chaos, because first, theories in financial and monetary economics suggest that monetary variables, such as exchange rates, are random and, consequently, their changes are unpredictable. Second, if the final order in the process of monetary variables is discovered, it is possible to achieve huge profits. It is effective in the process of managing the financial risk of the stock portfolio of investment companies. Therefore, understanding the price structure of this product and its modeling has always been the focus of economic research and efforts have been made to investigate the cause of fluctuations and predict it. Systems that are analyzed using chaos theory have nonlinear relationships and their time series are irregular. Economic time series seem to follow a random process and do not seem to be predictable, while these series are not random and will be predictable in the short run. There are various tests for chaos in time series, including the BDS correlation dimension and the Lyapunov maximum exponent. The first-order difference test was used to validate the data. By performing BDS test on the data, the results showed that the data are IID, ie the data are independent and similarly distributed, there is no dependency in them and they are random. The results showed that in all dimensions except the first group, in cases marked with *, the null hypothesis is rejected, so the data are not IID and there is a linear or non-linear relationship between them. The test was performed on ARIMA and GARCH model residues and the results were reviewed again by eliminating linear and nonlinear dependencies. Also, by fitting the ARIMA pattern, linear effects were removed from the pattern and the test was performed on the model residues. After fitting the pattern, a unit root test is performed on the residues and the null hypothesis is rejected and the residues do not have a single root. The results of the White test indicate that the null hypothesis that there is no heterogeneity of variance was rejected and the heterogeneity of the data was confirmed. In this way, the possibility of a chaotic process in the structure is confirmed. The results of this study also showed that the time series data of major currency pairs in the Forex market in the period from the beginning of 2017 to the end of 2021 has a chaotic atmosphere. Using the results, we can look for a model to properly predict the price of the main currency pair. According to the results obtained from this research, it can be suggested to the senior managers active in the stock exchange of Iran and other West Asian countries that by using a more in-depth examination of the time series data available in the stock exchanges of the countries in question, to a suitable horizon of Achieve short-term and medium-term forecasts of possible stock market fluctuations. It is also suggested to use these results in possible fluctuations in the behavior of oil markets against gold.

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