Review Articles

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Hazard functions and conditional probability of earthquake occurrences in major fault zones in Turkey

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Abstract

Among several distribution characteristics, temporal distribution characteristics of earthquakes provide the most crucial information on the temporal patterns of past seismicity. Identification of such patterns is required for seismic hazard, forecast studies and also simulation of future seismicity. The confusion of how to model the past temporal patterns does limit further development:. Though the Poisson model is routinely used in hazard modelling, its validity is often questioned. Furthermore, the question as to which model best represent past temporal patterns of earthquake occurrence is not answered yet. Within this context, in this study, to investigate the interevent time (IET) distribution, two seismically active regions in Turkey are selected where the seismic activity never diminishes and the hazard remains high. These regions, namely the western end of the North Anatolian Fault Zone and East Anatolian Fault Zone are known to produce moderate or large magnitude earthquakes. Four distributions, namely, exponential, gamma, Weibull and lognormal models are tested for how well they fit the earthquake records of the two faults, and importantly, the hazard functions that is instantaneous rate of occurrence of events, and conditional probabilities are also developed for performance evaluation. In the end, it is observed that, each model has flaws in identification of temporal pattern of earthquake occurrences and forecasting earthquakes.

Keywords: *Interevent times, Hazard Functions, Conditional Probability of Earthquakes, North Anatolian Fault, East Anatolian Fault*

1. Introduction

Earthquake forecasting requires identification of temporal distribution characteristics. The IET distribution is frequently used for that purpose since a number of studies emerged in 1970's (Hagiwara 1974, Rikitake 1976). Later, the number of studies are increasingly accelerated and exponential, gamma, Weibull and lognormal models became classic in these studies (Convertito and Faenza, 2014 and references therein). All these four models repeatedly tested with data with varying magnitude ranges, sample sizes, and different regions (Parvez and Ram 1997; Musson et al. 2002; Hasumi 2010; Yazdani and Kewser 2011; Chen et al. 2013; Pasari and Dikshit 2015; Bountzis et al. 2018; Coban and Sayıl 2019).

In almost every study focusing on the identification of IET distribution, different distribution models outperformed the others. Indeed, the best-fitting distribution model seem to vary from region to region (Touati et al. 2009; Naylor et al. 2010) and differs with varying magnitude ranges (Chen et al. 2013) and time period. Bak et al. (2002) and Corral (2003) believed that IET distribution is universal and can be modeled by generalized gamma distribution if the earthquakes are not declustered.

The variation of performances of each model with different data led to the several studies (Parvez and Ram 1997; Musson et al. 2002; Hasumi 2010; Yazdani and Kewser 2011; Chen et al.2013; Pasari and Dikshit 2015; Bountzis et al. 2018; Coban and Sayıl 2019). Every one of these studies reached a different conclusion in terms of identification of the best distribution model. In fact, when the classical and other models are put into use to compute the hazard rates, residual times or the times remaining for an imminent event, seismic hazard computations and simulations, the distribution model's weak and strong sides become more apparent for each different application (Matthews et al. 2002; Pasari and Dikshit 2015; Coban and Sayıl 2019). Indeed, the hazard rates, or the probability of earthquake occurrence at any point in time (t), given that no earthquake has occurred prior to that time, and conditional probabilities of earthquake occurrences are important measure tools to evaluate the performance of these models. Since the implications of the hazard rates and conditional probabilities are more open to physical interpretations, the inconsistencies of the models could be identified easily.

For example, the constant hazard rate of exponential distribution is always subjected to criticisms and objections (Anagnos and Kiremidjiyan 1988; Kuehn et al. 2008) since the earthquake events are claimed to be very random processes, and so unpredictable. Although, the long and unbounded tail of lognormal model that decay more slowly than exponential model, it is always praised thanking to its capablity of producing diverse values of seismic hazard (Sornette and Knopoff 1997). It's this property of lognormal distribution that might be useful in many cases.The hazard function derived from lognormal distribution is criticized since it is not monotonic and subjected to a decay in time eventually. Indeed, Matthews et al. 2002 discussed this decay by associating the fault

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behavior under the circumstances of stress built-up and release cycle.
Generalized gamma distribution became popular when IET distribution is computed without declustering earthquake catalogs (Bak et al. 2002; Corral 2003). The ability in modelling the steep climbing part of IET at its lower values gave the edge to gamma distribution. In some cases, Weibull model is preferred by the researchers since its hazard function is scale invariant (Abaimov et al. 2008). Whereas the inconsistencies of the mentioned model is stressed in Sornette and Knopoff (1997) for certain coefficients. Indeed, all the distribution models have certain ranges of coefficients where they are more likely to reflect the temporal occurrence pattern of earthquakes (Utsu 2002). Out of these ranges, almost all the models fail to reflect the earthquake phenomenon. The mentioned studies pretty much summarize the poor performances of these classical models and their insufficiencies in modeling. In fact, it is when the models were put into practice, the deficiencies of the models become more visible, and only then the models applicability and performance can be evaluated.
Within this context, the North Anatolian Fault Zone (NAFZ) and East Anatolian Fault Zone (EAFZ) are subjected to evaluation in terms of their IET distribution. The selected fault zones are famous for producing large magnitude earthquakes capable of inflicting large-scale damage in nearby cities. Especially NAFZ became worldwide famous after two large earthquakes occurred in 1999. The Izmit earthquake with magnitude of 7.6 and Düzce earthquake with magnitude 7.2 caused large-scale damage throughout Marmara region. Considering the importance of the subject, every bit and piece of information about the seismicity of the region becomes crucial. Knowing that, the past seismic patterns which if identified precisely, allow for better future projections, temporal distribution of earthquakes within the same seismogenic region must be determined. In order to identify the best performing model, the performance of the hazard functions and conditional probabilities are evaluated in addition to the classical methods.

2. Data

Turkey is well known for its seismic activity and large earthquakes which is mostly concentrated in NAFZ and EAFZ. NAFZ is one of the most active fault systems in the world and it has produced 6 large earthquakes since 1939. The number of earthquakes with magnitude greater than 6.0 and the geographical distribution of earthquakes is shown in Fig 1. The subjective selection of the boundaries of NAF and EAF zones is based solely on the concentration of seismicity. The reason why the conjunction of NAFZ and EAFZ is excluded is that the area is governed by several different mechanisms, which if included in any of the selected area, might cause loss of meaning and blurring of the temporal patterns of the seismogenic regions.

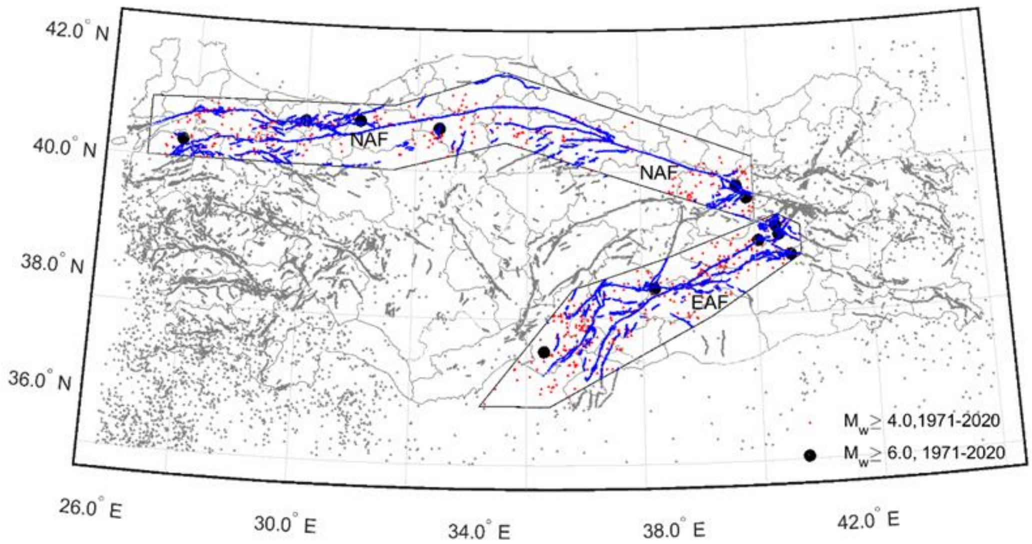

Fig 1. The two Seismically Active Fault Zones, western end of NAFZ and EAFZ with the Turkish seismicity in the background

The earthquakes are gathered from KOERI-RETCM catalog (2020), which offers detailed and compact information on seismic data. For magnitıudes unification, the equations suggested by Ulusay (2004) and Akkar et al. (2010) are used. The declustering is performed by using Gardner and Knophoff (1974) which conservatively remove the aftershocks and foreshocks (Zare et al. 2014). The compltenes analysis is performed by Cao and Gao (2001) and the minimum magnitude threshold is determined as 4.0 starting at 1971.

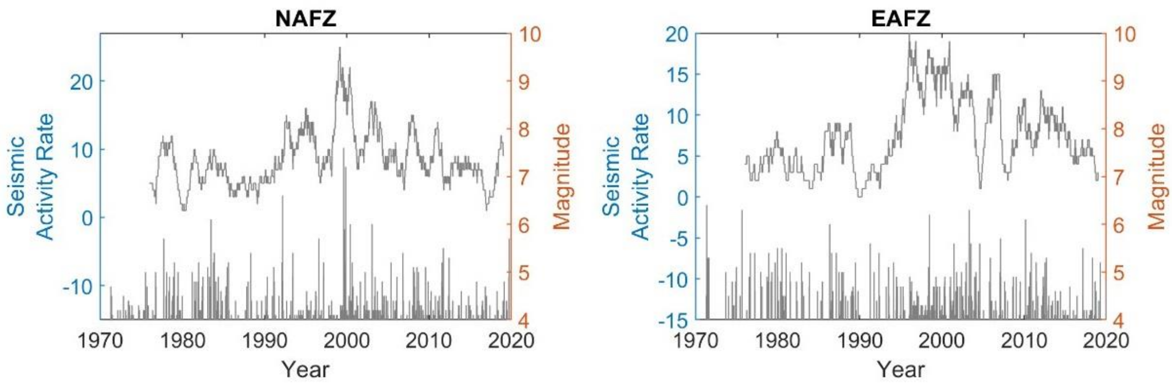

Fig 2. The temporal distribution characteristics in terms of annual earthquake rates and the magnitude-tme distribution of declustered data for NAFZ (left panel) and EAFZ (right panel)

Looking closely to the temporal distribution of magnitudes and earthquake rates in Fig 2, one might easily observe that large magnitude events are always associated with higher rate of earthquake rates. The declustering algorithm, which is a complex subject of investigation (Luen and Stark 2012) is not considered the sole reason for these higher rates since the used algorithm is considered to be the most conservative (Zare et al. 2014). The fluctuations are also considered as natural as they are eventually even out to yield more consistent behavior.

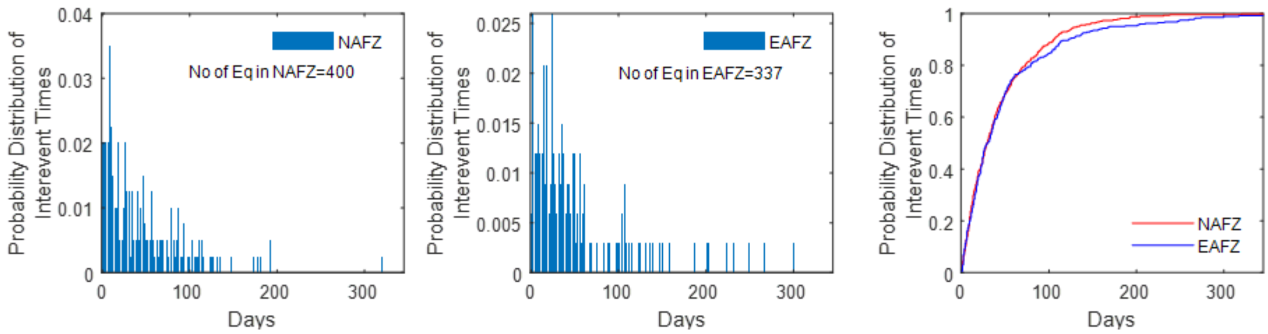

Fig 3. The probability density functions, PDFs, of IETs, *f(t)* (left and middle panels) and the cumulative distribution function, CDF of IETs, *F(t)*, (right panel)

A preliminary analysis of IET distribution of EAFZ and NAFZ yields the probability distribution and cumulative distribution plots as displayed in Fig 3. The ragged shape of probability distribution of the IET's is due to the lack of data at the desired level of precision. It should be noticed that, both NAFZ and EAFZ have double convexity while it is the most striking feature of IET of EAFZ. Additionally, the IET is more condensed in NAFZ while it is more spread out in EAFZ.

3. Method

The evaluation of the models is performed in two phases. In the first phase, the fitting performances of exponential, gamma, Weibull and lognormal models are evaluated by Log Likelihood, Chi Square and Kolmogorov-Smirnov (KS) distance tests. In the second phase, the hazard functions and conditional probabilities of the models are tested for their consistency with the real data and observations (See Table 1 and 2 for equations of models and functions).

Tables 1 and 2, lists the probability and cumulative distribution functions (PDF and CDF) of the tested modes, hazard functions, conditional probability functions the ranges of validity, role of parameters, survivor functions and expected time intervals. Among the listed functions, the pdf is defined as distribution of any variable within the boundaries of the related parameter. In the case of IET, the PDF refers to the distribution of number of IETs normalized by the total number of observed IET values. The CDF refer to the fraction of recurrence times that are shorter than t (See Fig 4 for visualization). In another words, it is cumulative probability of next earthquake that will occur at a time t.

Table 1. The mathematical expression of selected distributions: PDF, CDF, domains of coefficients and input and role of parameters (Utsu 2002; Pasari and Dikshit 2015; Coban and Sayil 2019)

Type of Distribution	PDF,*f(t)*	CDF,*F(t)*	Domain		Role of Parameters
Exponential	$\alpha\exp[-\alpha t]$	$1-\exp[-\alpha t]$	t≥0	$\alpha>0$	α - Scaling
Gamma	$\frac{\beta^{\alpha}}{\Gamma(\alpha)}t^{(\alpha-1)}\exp[-\beta t]$	$1-\frac{\Gamma(\alpha,\beta t)}{\Gamma(\alpha)}$	t>0	$\alpha>0,\beta>0$	α - Shape β -Scaling
Weibull	$\beta\alpha^{\beta}t^{\beta-1}\exp\left[-(\alpha t)^{\beta}\right]$	$1-\exp\left[-(\alpha t)^{\beta}\right]$	t>0	$\alpha>0,\beta>0$	α - Scaling β -Shape
Lognormal	$\frac{1}{\sigma\sqrt{2\pi t}}\exp\left[-\frac{(\log(t)-\mu)^{2}}{2\sigma^{2}}\right]$	$\Phi(t,\mu,\sigma)$	t>0	$-\infty<\mu<\infty$ $\beta>0$	μ - log-Scaling σ -Shape

Incomplete gamma function $\Gamma(\alpha,\beta t)=\int_{\beta t}^{\infty}\exp[-u]u^{\alpha-1}du$, Φ is the error function, $\Phi=\frac{1}{2}\left(1+erf\left[\frac{\log(t)-\mu}{\sigma}\right]\right)$

Table 2. The hazard functions, survivor functions, conditional probabilities and mean arrival times (Utsu 2002; Coban and Sayil 2019)

Type of Distribution	Hazard Function,*h(t)*	SurvivorFuction,*s(t)*	Conditional Probability	Mean Interval Time, *E(t)*
Exponential	α	$\exp[-\alpha t]$	$1-\exp[\alpha t-\alpha(t+\Delta t)]$	$1/\alpha$
Gamma	$\frac{\beta^{\alpha}t^{(\alpha-1)}\exp[-\beta t]}{\Gamma(\alpha)-\Gamma(\alpha,\beta t)}$	$\frac{\Gamma(\alpha,\beta t)}{\Gamma(\alpha)}$	$1-\frac{\Gamma(\alpha,\beta(t+\Delta t)}{\Gamma(\alpha,\beta t)}$	α/β
Weibull	$\alpha\beta(\alpha t)^{\beta-1}$	$\exp\left[-(\alpha t)^{\beta}\right]$	$1-\frac{\exp[-\alpha(t+\Delta t)^{\beta}]}{\exp[-\alpha t^{\beta}]}$	$\alpha^{-1/\beta}\Gamma(1/\beta+1)$
Lognormal	$f(t)/s(t)$	$1-\Phi\left(\frac{\log t-\mu}{\sigma}\right)$	$1-\frac{1-\Phi\left(\frac{\ln(t+\Delta t)-\mu}{\sigma}\right)}{1-\Phi\left(\frac{\ln(t)-\mu}{\sigma}\right)}$	$\exp\left[\mu+\sigma^{2}/2\right]$

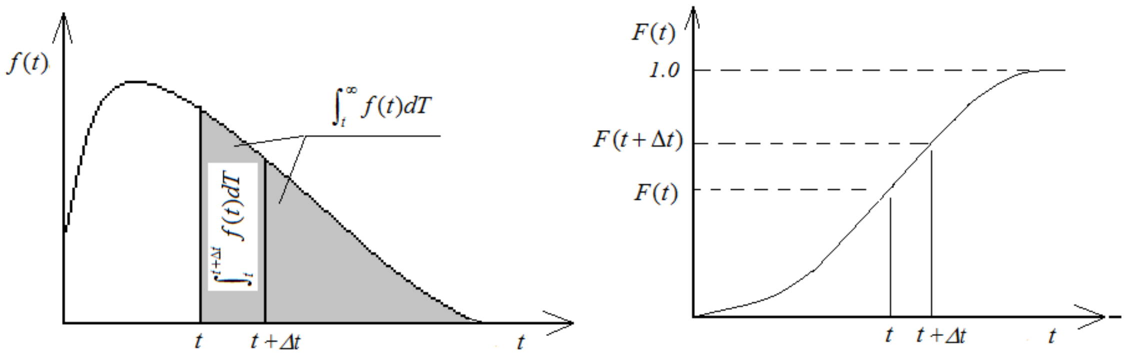

Fig 4. *f(t)* is the probability density function of iet (left panel) the same probability shown on cumulative distribution function (right panel) , the hazard rate, or the probability that an earthquake occurs within □t time from time, *t* is shown in both plots as per equation 3 (Modified from Convertito and Faenza, 2014)

By using probability and cumulative distribution functions, the probability of an event that is going to occur within □*t* time from today is computed as:

$$P(t\le T\le t+\Delta t)=\int_{t}^{t+\Delta t}f(t)dT=F(t+\Delta t)-F(t) \quad (1)$$

Where *f(t)* is the pdf. The probability of no earthquake occurs until time *t* and at least one occurs after time *t* is computed by using the following integral,

$$P(T\ge t)=\int_{t}^{\infty}f(t)dT=1-F(t) \quad (2)$$

Then the hazard function becomes (Convertito and Faenza, 2014),

$$h(t)=\frac{\int_{t}^{t+\Delta t}f(t)dT}{\int_{t}^{\infty}f(t)dT}=\frac{F(t+\Delta t)-F(t)}{1-F(t)}=\frac{f(t)}{1-F(t)} \quad (3)$$

The survivor function is defined as the probability of surviving beyond a time, given the hazard has not occurred yet until that time. In our case, it could be defined as the non-occurrence of earthquakes beyond a certain point time given that no earthquake has occurred until that time. It is mathematically expressed as

$$s(t) = 1 - F(t) = P(T > t) \quad \text{for } t > 0 \tag{4}$$

Hence, the hazard function takes the final form

$$h(t) = P(\Delta t|\,t) = \frac{f(t)}{1 - F(t)} = \frac{f(t)}{s(t)} \tag{5}$$

Using both PDF and CDF of IET, for a specific location and magnitude, the probability of occurrence of an event could be computed given the elapsed time. In fact, even though the timing of a future earthquake is the most important information, the probability of occurrence of a future earthquake is also valuable when the timing cannot be precisely known. For that purpose, the conditional probabilities of earthquake occurrences are computed. Indeed, the conditional probability of the earthquake occurrence given that it has not occurred until time t, is another measure for the evaluation of the IET models (Pasari and Dikshit 2015; Coban and Sayil 2019). The main difference of conditional probability with the values of a hazard function is that, the future time span, which the probability of earthquake occurrence is evaluated, is predetermined. As shown in equation 6, the conditional probability can be defined, as the probability of earthquake occurrence within Δt given it has not occurred before t.

$$P(t|t_{elapsed}) = 1 - \frac{1 - F(t + \Delta t)}{1 - F(t)} = 1 - \frac{s(t + \Delta t)}{s(t)} \tag{6}$$

Both hazard function and conditional probability are a measure that has practical consequences. In other words, the models could be tested for their fitting performance and investigated whether they produce consistent and reliable hazard functions and conditional probabilities. It should also be mentioned that, the ratio of probability of non-occurrence of an earthquake at time $t+\Delta t$ to the probability of non-occurrence of earthquake at time t or the conditional survival rate is complementary to the conditional probability of earthquake occurrence probability.

4. Results

The performance analysis of the models and the tests are performed by built in functions of MATLAB 2020. A built in nonlinear regression is function, *fitnlm*, is employed which uses iterative least square method for the determination of coefficients. The log likelihood of the model is one of the outputs of the built-in function; hence, it automatically produced together with the coefficients. Using the computed and observed values of IET, the chisquare values and KS distances are obtained respectively.

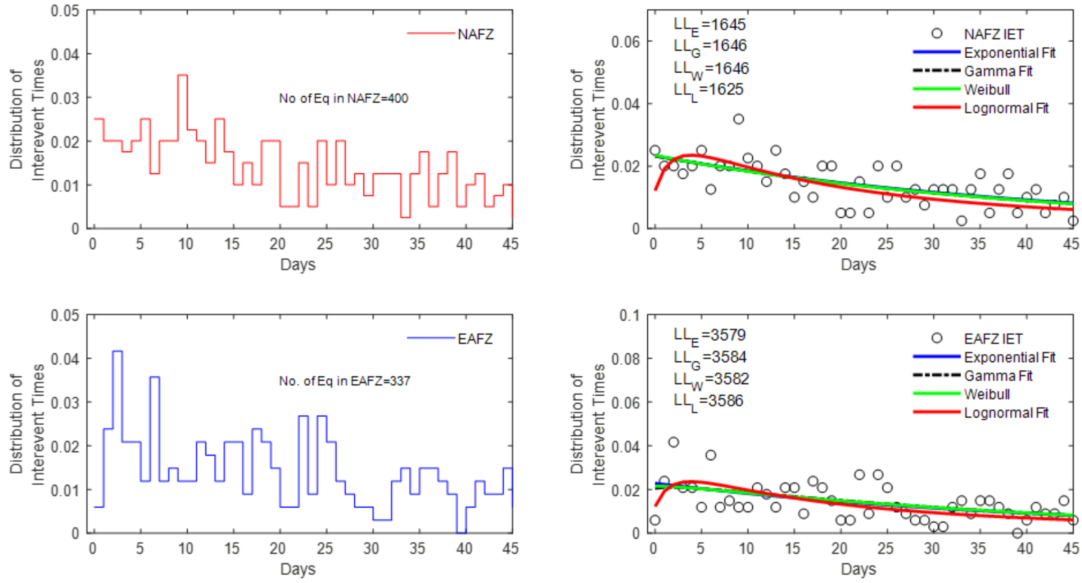

Fig 5. The IET of EAFZ and NAFZ, the exponential, gamma, weibull and lognormal fits to each IET distribution (The exponential, gamma and weibull models almost overlap with each other, Higher LL values indicate better fitting performance)

The stair-plots of NAFZ and EAFZ IET distributions are given in the top left panel and bottom left in Fig 5 while the fitted models are displayed in the right panels for NAFZ and EAFZ respectively. As listed in the plots, the loglikelihood of the models are so close to each other that it is quite difficult to differentiate which model performs best in modeling the IET distributions.

As displayed in the the plots in top panels of the Figure, it could be easily seen that the IET of NAFZ is more of exponential character. Certainly, all four models have capability of modeling such a distribution, whereas none of the models has significant advantages over each other in modeling such data. According to the values in Table 3, the chisquare and KS distance measures are only helpful to eliminate lognormal model whereas it is not precise enough in differentiating the performance of the exponential, gamma and Weibull models. Moreover, KS distance itself is more of an indicator of how much the fitted model deviate from the IET distribution at its maximum value and at times could be misleading. It is at this point, where the hazard function and its implications should be evaluated to distinguish the modeling capability of the distribution functions.

When it comes to the IET of EAFZ, it should be mentioned that, the slight bend at the lower values of IET of EAFZ is the most prominent feature. Here, the lognormal distribution is expected to dominatesince it is efficient in modeling the data with more than single convexity. However, as shown in Table 3, higher chi-square and KS distance values clearly lowers the credibility of the lognormal model, and it is the exponential model, which outperforms the rest in these two measures.

Table 3 gives more detail on the performance measures of the evaluated models. The most striking result in Table 3, is that the shape coefficients of gamma and Weibull, α for gamma and β for Weibull respectively, is very close to unity for NAFZ and slightly depart from unity for EAFZ. This result alone is sufficient to conclude that the IET distribution of NAFZ has more exponential nature than that of IET distribution of EAFZ. Except for lognormal distribution, the mean arrival times are relatively closer for NAFZ. For EAFZ, it is Weibull distribution, which yield lowest mean arrival time with its relatively faster decay than exponential, and gamma distribution. The PDF and CDF of the developed models are given in Fig 6 for better visualization. Here at this point, it should also be reminded that, the coefficients of each models is optimized to model the IET distribution and a slight variation of the coefficients might rescale the model, change the shapes in an unstable pattern and cause large deviations.

Table 3. The coefficients, likelihood tests, and other statistics for EAFZ and NAFZ

		NAFZ					EAFZ				
Type of Dist.	**Par.**	**Mean**	**Std. Error**	**CHI**	**KSD**	**E(t)**	**Mean**	**Std. Error**	**CHI**	**KSD**	**E(t)**
Exp.	α	0.023	0.001	0.037	0.024	42,89	0.023	0.001	0.191	0.046	43,77
Gamma	α	1.003	0.034	0.038	0.025	41,76	1.065	0.032	0.348	0.066	41,09
	β	0.024	0.001				0.026	0.001			
Weibull	α	0.024	0.001	0.037	0.023	42,39	0.024	0.001	0.330	0.064	36,48
	β	0.997	0.024				1.033	0.022			
Logn.	μ	3.455	0.042	1.840	0.100	81,03	3.451	0.035	1.005	0.074	80,70
	σ	1.371	0.035				1.371	0.029			

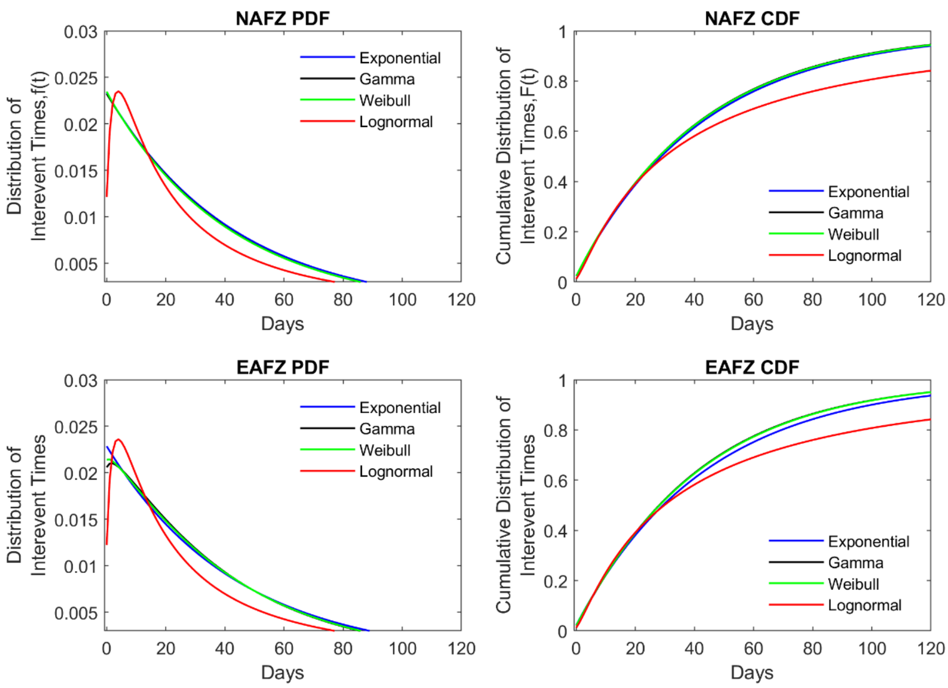

Fig 6. The PDF, CDF models of the exponential, gamma, weibull and lognormal models to NAFZ and EAFZ IET distribution

In summary, according to the various performance measures developed to measure the fitting performances, none of the models can be singled out for its performance. Indeed, contradictory results were obtained, such that models with higher log-likelihoods have lesser performances with measures of chisquare and KS distance and vice versa. Therefore, as stated earlier in the text, additional performance measures are required for better evaluation. For that matter, the hazard functions of each model and conditional probabilities are assessed for their consistency with the observations.

The models with time-dependency allow increase in hazard rate whereas it remains constant for exponential models, which is based on Poisson distribution. In both plots in Fig 7 and 8, for the exponential IET model, the hazard rate largely remains constant or display slight deviation from the constancy regardless of time passed.

The Weibull function always displays exponential character when its scale coefficient (β=1) is unity. It has an increasing hazard rate with increasing time when its β parameters are greater than unity (β>1) (Abaimov et al. 2008) and vice versa. In Figs 7 and 8, both numerical and analytical models of Weibull distribution closely follow exponential distribution.

Gamma distribution has more acceptable hazard functions numerically, whereas the analytical values display a very poor performance, which is hard to justify. The poor performance of Lognormal distribution is hard to explain as the hazard rates fluctuate sharply, which create reverse convexity at the lower values of IET. This observation alone puts the models performance at serious odds with the observations.

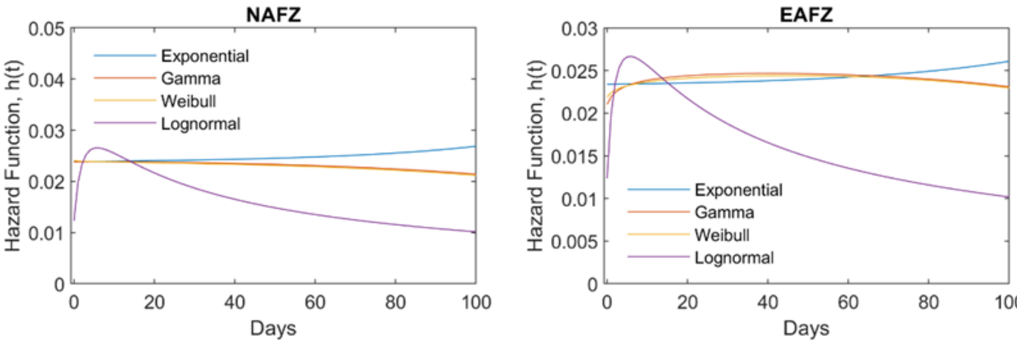

Fig 7. The Numerically computed hazard functions of the fitted models to EAFZ and NAFZ IET distribution

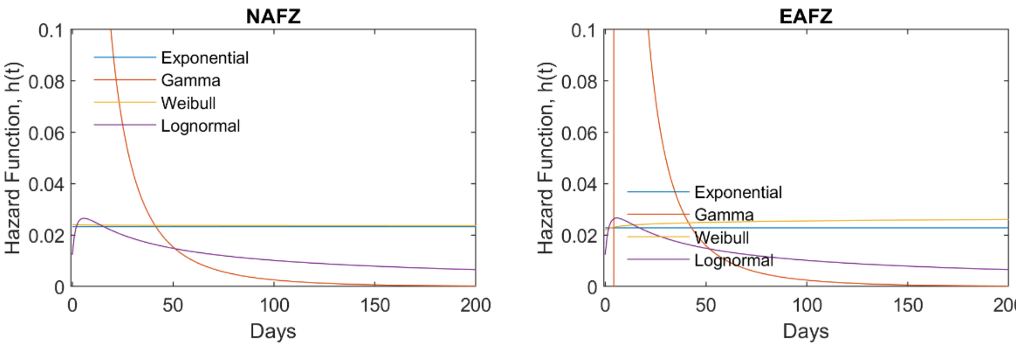

Fig 8. The analytically computed hazard functions of the fitted models to EAFZ and NAFZ IET distribution

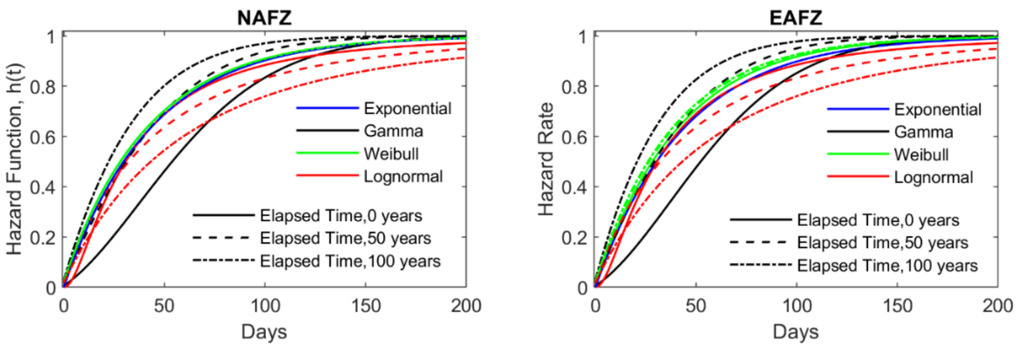

Fig 9. The conditional probability of earthquake occurrence given the elapsed time 0 days (Table 4 and 5)

It is already well known that, as the elapsed time increases the imminence or probability of occurrence of an event increases, or the residual time (waiting time, time remaining to the next event) decreases. Considering this fact alone, there seems to be issues with the hazard functions in Fig 7 and 8, which are developed numerically and analytically respectively.

Conditional probabilities can be used as another performance measure in order to detect and to eliminate the inconsistent and unrealistic models. Fig 9, Tables 2, and 3, offer sufficient information on how the conditional probabilities vary with respect to elapsed time. The inconsistent behavior of lognormal distribution and constant conditional probabilities of exponential distribution are quite apparent according to the Figs 7,8 and 9 and Tables 4 and 5. Only gamma distribution performs according to the expectations based on past observations. Weibull distribution gives almost constant probabilities regardless of elapsed time, which is not surprising as its shape coefficient is too close to 1. The probabilities of Weibull distribution is higher than that of exponential distribution at each elapsed time, and importantly the conditional probability at 0 days is greater than 0. The lognormal distribution display unacceptable conditional probability pattern as the elapsed time increases the probability of earthquake occurrence decreases. For example, given 0 elapsed time, the probability of earthquake occurrence after 100 days is 88.2% whereas given 50 days the same probability is 83%. This feature of lognormal distribution is discussed in Matthews et al. (2002). The decreasing rate of hazard and decreasing conditional probability with time creates doubts and a reason to disqualify the model.

The resultant hazard functions and conditional probabilities given elapsed time of 0, 50 and 100 days are not helping to identify the model with highest performance. Indeed, all these models are deficient in one way or another in modeling the physical phenomenon. The exponential and Weibull models yield unrealistic conditional probabilities and constant hazard rates, while lognormal model totally misses the nature of earthquake occurrence patterns. The gamma distribution also displayed results that cannot be justified as its hazard function is off the charts. The conditional probabilities of gamma function are the sole success of the analysis. In such a situation, instead of looking for a best performing model, one should identify the reasons of failure of these models, which are indispensable part of every study focusing on IET distribution. In our case, as the main reason of failure of modeling the hazard functions and predicting the conditional probabilities, the lack of data is to be blamed first.

Table 4. Conditional probability of earthquake occurrences within NAFZ

	Exponential			Gamma			Weibull			Lognormal		
	Elapsed Time, t			Elapsed Time, t			Elapsed Time, t			Elapsed Time, t		
Δt	**0**	**50**	**100**	**0**	**50**	**100**	**0**	**50**	**100**	**0**	**50**	**100**
1	0	0	0	.005	.018	.028	.024	.024	.024	.0	.023	.017
5	.089	.089	.089	.029	.090	.132	.113	.112	.112	.028	.108	.082
10	.189	.189	.189	.065	.176	.250	.213	.212	.211	.117	.201	.155
20	.358	.358	.358	.149	.336	.446	.380	.378	.378	.318	.352	.281
50	.681	.681	.681	.452	.689	.795	.697	.695	.695	.681	.631	.539
100	.901	.901	.901	.830	.936	.970	.908	.907	.907	.882	.830	.757

Table 5. Conditional probability of earthquake occurrences within EAFZ

	Exponential			Gamma			Weibull			Lognormal		
	Elapsed Time, t			Elapsed Time, t			Elapsed Time, t			Elapsed Time, t		
Δt	**0**	**50**	**100**	**0**	**50**	**100**	**0**	**50**	**100**	**0**	**50**	**100**
1	.0	.0	.0	.005	.019	.030	.021	.025	.025	.001	.023	.017
5	.087	.087	.087	.029	.096	.142	.106	.117	.120	.029	.108	.082
10	.186	.186	.186	.064	.187	.267	.204	.221	.226	.118	.201	.156
20	.352	.352	.352	.151	.654	.471	.374	.394	.401	.319	.353	.282
50	.674	.674	.674	.467	.715	.820	.701	.717	.723	.683	.632	.540
100	.896	.896	.896	.848	.949	.977	.915	.922	.925	.883	.830	.758

The number of sample is always important in any investigation require more precision and less uncertainty. In this case, as well, if the magnitude threshold could be decreased, both uncertainty would decrease hence more reliable functional parameters could be identified. Lesser uncertainty means more reliable future projection and hence more reliable earthquake occurrence probabilities as well.

Indeed, lack of sufficient data might force the models to cover longer IET values, since there would be so many IET's with very high intervals. This in return might cause an advantage for models like lognormal since its slowly decaying, high and unbounded tail is more capable of modeling such data. Indeed, it was the sole reason why this model is advocated at the first place (Davies 1989; Sornette and Knopoff 1997). Hence, as the minimum magnitude is lowered and the number of studied events increases, a totally different aspect of IET would be observed as the IET's with higher values would diminish. In such a scheme, the requirement of lognormal model also diminishes, unless the lower IET values start displaying a skewness.

Among the four classical models, gamma and Weibull distributions always have a lead over exponential distribution, since both distributions have pure exponential form in addition to further capabilities which already ecplained. While, these features give both distributions an edge in modeling IET, due to the sensitivity of the distributions, the resultant model might be far from modeling the real seismic behavior. In other words, the existence of varying shapes might be an advantage for gamma and Weibull models, however, when it is required to model an IET distribution and associated hazard functions and conditional probabilities, the distribution model might fall short in its modeling performance.

5. Conclusion and Discussion

It is already commonly accepted that the earthquakes are related in time, space and magnitude dimensions. However, the inconclusive efforts and the urgent requirements in the field, forces the researchers and practitioners in the field to continue to use Poisson based models in seismic hazard computations, forecast and simulation studies. Indeed, according to the findings in the study, the insignificant differences in likelihood values and higher performances of exponential distribution in chisquare and KS distance tests might be sufficient to convince some in that it might be reasonable to keep favoring the exponential model. However, before putting these models into use, they must be tested with practical applications as well. One of the most practical information that could be offered by these models is the timing of a probable future event. Hence, the hazard rates per unit time and conditional probabilities are computed to assess the performance of the models. The conclusion could be summarized as in the following list:

1) The hazard functions either display an almost constant hazard rates or display an irrelevant pattern of fluctuations. Both type of results is indeed cannot be associated with the reality and contradict the observed seismic patterns.
2) Moreover, it should be mentioned that the shape of the hazard function is strongly dependent on the parameters of the distributions. Moreover, the IET distribution, if include any skews, long tails or deviations, causes unexpected distribution parameters to emerge which could eventually lead to questionable forms of hazard functions.
3) The computation of conditional probabilities at certain number of days given elapsed times of 0, 50 and 100 days gave much needed information on the performance of the distribution models. The already ruled out lognormal model, not surprisingly, performed worst due to the decreased conditional probabilities with the increased elapsed time. Regardless of the elapsed time, unchanging conditional probabilities observed for exponential distribution is not a surprise whereas the reliability of the Weibull model can be questioned if the conditional probabilities of NAFZ is examined. On the other hand, for EAFZ, Weibull distribution yielded probabilities that are more realistic. Gamma distribution is the sole model with conditional probabilities increase given elapsed time. It is indeed, surprising to observe such a result for gamma distribution given that the hazard rates are unrealistic in modeling the observed earthquake hazard rates.

Here it should be mentioned that, the overall variance of the IET distribution could be pointed as the main cause of the questionable results. For such situations, accounting for the hazard rates and conditional probabilities in the development of the distribution functions becomes necessary. Moreover, forcing the models to account for the skew at the lower IET values causes skewness in hazard rates at the lower IET values. This variation at the lower IET values is the reason of loss of meaning since it cannot be associated with the earthquake phenomenon.

Eventually, as an overall assessment, it could be deduced that, such an encounter with inconsistent results in this study, automatically lead to the questioning of the proposed models. These models are developed with a claim that they model a physical phenomenon. If these models, though modeling the PDF and CDF of the IET properly but fail to do so for hazard rates and conditional probabilities, are seriously flawed even for a single case, then precaution must be used before using them. Moreover, as long as the Poisson distribution assumption gives reasonable results in seismic hazard analysis, regardless of its stationary character, and the other models are not capable to model the earthquake phenomenon, then it is reasonable to keep using Poisson distribution in seismic hazard applications as well.

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