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Does Exchange Rate Non-Linear Movements Matter for Analyzing Investment Risk? Evidence from Investing in Iran's Petrochemical Industry

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ABSTRACT

The present study models the risk of investment in the petrochemical industry considering the impacts of exchange rate (US dollar to Iran's Rial) movements using the time series data from November 2008 to March 2019 and ARFIMA-FIGARCH framework. The empirical results prove the existence of the Fractal Market Hypothesis, FMH, and the Long Memory property in both the risk and return of the petrochemical stock index. These findings can be culminated in reaching a reliable and significant model to evaluate the investment risk in the petrochemical industry. In line with this, to analyse the idea whether consider-ing the exchange rate movements matter for assessing the risk management in the petrochemical industry, the effects of exchange rate movements as a crucial source of systematic risk in Iran has been taken into consideration in the pro-cess of modelling the risk of investment in that industry. Our results demon-strate that the exchange rate movements have had a direct and significant ef-fect on the investment risk of that industry so that if, on average, one percent change occurs in the exchange rate, the investment risk in this industry changes by 57% in the same direction.

1 Introduction

In a dramatically ever-changing world, risk management in organizations, services or manufacturing companies, and especially in high-volatile investment areas has become an essential matter. Regarding the significance of this issue, risk management is a process in which managers focus on identifying, measuring, and decision making about risks and monitoring their types to control the existing threats [7]. It is worth noting that the amount of attention to investment risk management and its applications depends on assorted factors like the nature and importance, profitability, job creation at the macroeconomic level, and long term functions of investment field with regarding to relative advantage, strategic importance for the country in the international arena and so forth [24]. On this basis, in Iran's economy, the petrochemical sector, which has perpetually been the attention of the various authorities, could be considered as the most strategic investment area due to the fact that the invest-

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© 2020. All rights reserved. Hosting by IA University of Arak Press ment volume, made in the very area, has been far more than the rest of investment areas ones. Therefore, to have a stable and profitable investment, in the long run, the importance of managing investment risk in the petrochemical industry is unavoidable part of Iran's economy. In essence, as the stylized facts of Iran's economy show, the petrochemical industry with its undeniable relative advantages plays a strategic and significant role through which can help stabilize the labor market (directly or indirectly in the middle and downstream units), boosting the foreign exchange market balance through increasing foreign exchange earnings from exporting the petrochemical products, raising the GDP, national income, and welfare of Iran; consequently, these advantages can considerably improve the economic position of the country in the global economy [36]. Regarding the special status of the petrochemical industry for the economy of Iran, it is noteworthy that this industry not only has had a significant share of the country's exports but based on the reports of Statistical Centre of Iran, the existence of high added value in its manufactured goods have made this industry as the leading investment destination in Iran. In line with this, even after the vast international sanctions imposed by the western countries against Iran's economy, unlike many other industries, the investment attractiveness of the petrochemical industry for domestic and especially international investors has not experienced dramatic changes; as a result, at the times when Iran has been under the sanctions, its foreign exchange market still has had a massive dependence on foreign exchange has earned from exports of petrochemical products [31]. Furthermore, the high value of the petrochemical industry's shares in Tehran's stock market, the almost stable profitability, and the least number of experiencing a collapse during the years of activity can also be considered as other reasons to prove the importance of this industry. Therefore, on the one hand, the existence of such a decisive role, which can reflect the profitability of this industry as an investment destination, has caused the attention of many domestic and international investors be attracted to investing in this industry. On the other hand, since in any investment, high returns are not achieved except by taking significant risks, paying more attention to the source of risks together with assessing, controlling, and managing the different risks of investing seems necessary [34]. To identify the main source of uncertainty and risk in the economy of Iran, it should be first noted that risk, in general, is the degree of probability based on which an investor can make or lose the invested money. More importantly, the magnitude of the general risk in any economy depend on several factors that can be classified as two groups of "Systematic Risk" and "Non-Systematic Risk". While the systematic risks refer to changes that can remarkably affect the entire market and general condition of the economy, the non-systematic risk is unique to a specific group of companies. Although among these types of threats, the impacts of non-systematic risks can be mitigated through hiring specialists to find the roots of problems and provide some feasible and appropriate solutions, using diversification strategy, changing the investment project, and so forth [14]. As many studies proved, the ramifications of systematic risks seem to be more challenging, difficult to manage, and impossible to avoid altogether [8, 10, 18, 23, 30]. On this basis, one of the major sources of the systematic risks in Iran economy is the dramatic movements of the exchange rate, especially in recent years. Some reasons to support why exchange rate movements are one of the most critical sources systematic risk not only for the petrochemical industry but also for most of the other industries can be summarized as follows:

I. The petrochemical industry is a trade-oriented business that the exchange rate risks can severely affect its amount of products export as well as total income from both the standard trade channels and also financial channels.

II. By stimulating many other macroeconomic indicators, exchange rate movements can indirectly influence this industry.

III. The main root of decline in most economic indicators, particularly in recent years, is directly or indirectly related to the international sanctions [12]. In practice, the most central channel, through which the impacts of sanctions pass-through its leading macroeconomic indicators, is the exchange rate one. As a result, if we accept the sanctions as one of the indispensable source of systematic risk in Iran, the exchange rate movement will shoulder the responsibility for transmitting the impacts of sanctions into the different economic indexes.

IV.In the inner realm of Iran's economy, the existence of the excessive growth liquidity, continuous increase in the inflation rate, gradual decline in economic growth, undeniable escalating in unemployment rate etc., can in some way be attributed to international sanctions and its main side effects, which is exchange rate movements [39]. To illustrate the importance of the risk analysis in the field of investment, and analyse the effects of the main factors of systematic risks, e.g. exchange rate movements, on the investment risk in the industry, the primary questions are "how risk of an investment can first be assessed and then be used as a decision- or policy-making tool?", "If a positive or negative change take place in one of the systematic risks' factors, how could it be possible for investors to consider the change in their decision process?", and "Since the nature and main features of different markets are distinctive, how are the determinant and inherent characteristic of the investment destination market taken into account in risk assessment process?". Therefore, applying the statistical advancements and flexible tools to model the investment risk in the field of investing in the petrochemical industry not only can plausibly answer the mentioned questions, but also provide the situation under which all involved economic agents, e.g. managers, macro decision makers, and investors or shareholders, could maximize their profitability and utility. On this basis, to address these concerns, the main contribution of the present study is threefold. First, this is among the first attempts to delve deeply into modelling the investment risk in the petrochemical industry in Iran. Second, to deepen the results of the analysis, the Long Memory property as the main feature of financial markets. Third, the daily time series data of exchange rate (US dollar to Iran's Rial) and petrochemical industry index in Tehran Stock Exchange (TSE) from November 2008 to March 2019 in a ARFIMA-FIGARCH framework is utilised to not only modelling the risk of investing in the petrochemical industry but analyse the impact of exchange rate movements on the risk of investing in that industry as well.

2 Theoretical Frameworks

Petrochemical Industry in Iran: Considering the significant contribution of Iran to the world's oil and gas resources as well as the strategic role of petrochemical industry for oil exporting countries, from one side, the widespread usage of petrochemical products in the production of a vast range of goods, the sale prevention of crude oil, and an added value creation of ten to thirty times the value of raw material from the other side, has made the investment in the petrochemical industry as one of the most attractive investment destinations [17]. Hence, to provide a convincing analysis of the petrochemical industry status, its strengths and weaknesses should be examined from both the macroeconomic and investors perspectives. From the macroeconomic standpoint, according to the reports of Islamic Parliament Research Center of Iran, IPRC, the petrochemical industry has had a share of ap-

proximately 25 percent of the total non-oil exports, and even after the U.S. withdrew from the JCPOA agreement, Joint Comprehensive Plan of Action, on May 8, 2018, this share remains almost constant up to the one year later (May 2019). From the regional dimension, the percentage of Iran's realized supply of petrochemical products from the total supply of all Middle East petrochemical suppliers which was 23 percent in 2017, has experienced a few percent decreases over the years leading up to 2019. In line with this, an examination of the international petrochemical export statistics also indicates that Iran's share of the world's realized petrochemical production capacity was more than 2 percent at the end of 2018 while this share was supposed to be more than 6 percent based on "the 20-Year National Vision of the Islamic Republic of Iran for the dawn of the Solar Calendar Year 1404". However, as another concrete example that can notably present the real status of this industry, it is ranked third in the services provided by the road transport sector after "the agricultural, livestock and food" and "the construction and mineral" sectors during 2018. Paying attention to this ranking along with the fact that the petrochemical industry has had the most added value, more than 40 percent, among all active sectors of the economy shows the substantial contribution of this industry to Iran's GDP, economic growth, and general welfare of society [12]. From the investors viewpoint, given the political and economic conditions of Iran, although there is no doubt that by investing in this industry, investors would take some risks, the plausible returns of the investment should be convincing enough to attract the investors. The cogent reasons behind the attraction of investing in the petrochemical industry for both national and international investors are various: the export-centric production, the perpetual and inexpensive existence of petrochemical feeds, the relative advantages of downstream firms activities, benefiting from investment incentives, tax and customs services exemptions, the presence of trade incentives for exports and imports, the growing domestic markets, accessing to the free waters, the geographical location of the country (along the border with 15 other countries, especially the countries of Central Asia and the South Caucasus, the presence of young), professional, proficient, and cheap labor force, the massive volume of demand in the region and the international community. It will be discovered through deeper thinking that the mentioned reasons have mostly persuaded Chinese, Russian, Indian and South Korean investors together with the national ones to invest in the very business despite the most intense international sanctions imposed by the United States and the Western countries against Iran, and the other restrictions in banking, financial, insurance sectors. Furthermore, by comparing the volume of investments have made in this type of business especially after imposing the sanctions with other types of possible investment in this country, it can be indicated that even if this country is under sanctions, the tendencies and attractiveness of investors to invest in the very industry have constantly been among the top three destinations of investment in the Tehran Stock Exchange. Therefore, the mentioned pieces of evidence and reasoning significantly demonstrate why investing in the petrochemical industry are of particular importance and attraction for domestic and international investors.

Exchange Rate and the Investment Risk of Petrochemical Industry: Empirically, the current conditions of Iran's economy that is subject to many problems such as international sanctions, the foreign exchange market crisis, high inflation and unemployment rates and so forth, have caused the petrochemical industry and its downstream markets to face severe challenges; as a result, the risk of investment in them has changed significantly. In other words, the direct effects and consequences of international sanctions, as the most crucial factor in creating systematic risk, has mainly reflected in the exchange rates and passed through into other markets [39]. Consequently, these pressure, usually, has culminated in unfinished remaining of some contracts with foreign companies, reducing the vol-

ume of production and also the supply of petrochemical products, increasing the risk of investment and subsequently changing the willingness to invest in this industry. Thus, not only will the investors' priorities and the investment's destinations modify, but also the tendency of them will propel towards speculative and non-productive activities to maximize their profits [33]. Continuing such conditions may result in expanding the general economic inefficiency, reducing the domestic production and economic growth, rising unemployment and reducing social welfare; then, the continuation of this situation will even have the potential for economic failure. To sum up, it can be clearly claimed that by controlling the risk of the exchange rate movements, in addition to managing the risk of investment in the affiliated industries such as the petrochemical sector, it is possible to continuously and stably increase the amount of production and profitability; in this way, it can provide sustainable GDP growth and welfare development in the community. Therefore, to control the investment risks especially in the face of such conditions, on the one hand, and to maintain the position of the country's petrochemical industry in the regional petrochemical industries ranking, and reduce the adverse effects of the exchange rate movements, on the other hand, providing a model for managing the investment risk and improving the performance of the strategic policies could be worthwhile.

The Historical Trend of Investment in the Petrochemical Industry: As previously mentioned, since most of the profitable companies in the field of petrochemicals are active in the Tehran Stock Market, in this study the stock index of the petrochemical industry will be used as a benchmark for investment in the petrochemical industry. Regarding the petrochemical stock index graph, from 2008 to 2009, its value had been increased by 83 percent that could be attributed to a variety of reasons like emerging signs of recovery in the global market, rising the crude oil prices, entering the domestic rival markets into a recession period, and growing the variety of financial instruments. During 2009 to 2010, this industry had a 14.3 percent increment. Despite the significant drop in global stock market indices in 2011, the rising trend of the petrochemical stock index had been continued to 2014. However, in the end of 2014, this industry had faced to a great depression owing to imposing international sanctions against Iran and a significant increase in exchange rate volatilities, reducing the global crude oil prices, and augmenting the price of petrochemical feeds. After that, from early 2015 to late 2016 the process was swinging around 20,000 average due to reasons which could be traced in existing a relative stability in the exchange rate market, and most importantly achieving the JCPOA agreement between the P5+1 (i.e. China, United Kingdom, France, United States, Russia, plus Germany), the European Union, and Iran.

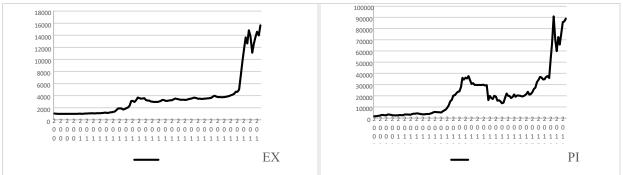


Fig 1: The historical movements of the petrochemical industry stock index and exchange rate **Source:** Official Website of Iran's Islamic Republic Central Bank and Tehran Stock Exchange

With all the matters above, in early 2018, a far increase, from 30000 to just above 92000, which rooted in an unprecedented exchange rate increase due to the U.S withdrew from the JCPOA agreement on May 8, 2018, could be seen followed by another increment, from 20,000 to 30,000, in 2017. Nevertheless, from September to November 2018, an extreme recession, from 92000 to 57000, had been occurred on the ground that Iran's government succeeded in controlling the exchange rate market and reducing it from around 200,000 riyals to roughly 50 percent lower rate. However, since the end of 2018 to March 2019, the sanctions have continued to put strain on different sectors of Iran's economy. To be more precise, the pressure of the imposed sanctions against Iran led the exchange rate to again experience an upward trend which was accompanied by significant volatilities. Although this volatilities causing extensive instabilities in the economy, it was a great opportunity to grow for some export oriented sectors that was not directly under sanctions, e.g. the petrochemical products. As a result, the profit and attractiveness of investment in the petrochemical industries dramatically enhanced so that the petrochemical stock index has experienced an averagely increasing trend.

3 Literature Review

Although there are many studies have examined the impact of exchange rate movements on the risk of investing in different businesses or industries of various stock markets; only a limited number of studies have investigated the risk of investment in the petrochemical industry and the factors that can significantly affect it. It seems that with the imposition of international sanctions and the occurrence of severe exchange rate changes as the most critical systemic risk factors in Iran's economy in recent years, the necessity of conducting new investigations in this field of studies seems to be more than ever. In this regard, some researches that are most relevant to the subject of the present study are categorized into two parts include (i) studies related to risk analysis and management in petrochemical industries, and (ii) researches in line with the effects of systematic risk factors on petrochemical industry performance. These researches are described below.

As the first group of literatures, Tang [34] applies a multilevel Borda model (MLBM) to calculate the quantitative risk value (ERV) and risk priority number (RPN). The results confirm that the MLBM can be utilized in other complex hierarchical structure to solve the comprehensive evaluation problems. In another study, Jaderi [17] through both the traditional RBM and Fuzzy RBM (FRBM) methods analyzes the risk analysis of petrochemical assets failure. The results show that the risk managers and decision makers should customize and priorities their maintenance planning based on the FRBM value for each asset failure. As another example, Mechhoud et al. [23] apply a combination of two analysis methods, i.e. HAZOP (Hazard & Operability) and FMEA (Failure Mode & Effect Analysis). The findings of this analytical tool was risk minimization and dependability enrichment of the system. Finally, Kim et al. [19] research to focus on the status of risk management activities in the Korean petrochemical plants alongside considering the trends of the global market. The second group of literatures, Šimáková [33] investigates the relationship between exchange rate movements and stock market index in the petrochemical industry using Jorion's model. This research provides evidence on the speed and magnitude of the effects of exchange rate movements on the petrochemical industry index. In line with this, Nugroho [28] has depicted a model to analyze and sort the results of some macroeconomic variables on the Total Cost of Petrochemical Industry in Indonesia based on Multiple Regression Equations. The results of the research demonstrate the most influential and significant factors are the rate of inflation, economic growth, oil prices and styrene prices, respectively. In harmony with this study, based on the industry background in the petrochemical industry, Li et al. [21] give a smart factory model framework definition through which its architecture and key technologies can be described. Azadeh et al. [3] have assessed the factors affecting the risk of investment in a petrochemical plant based on a fuzzy cognitive maps method (FCMs). Vijayalakshmi et al. [37] conduct research to identify the primary indicators to measure the sustainability of the petrochemical industry in Malaysia by using the five-tier framework introduced by the Lowell Centre for Sustainable Production. The results show that the majority of the indicators monitored are in line with the compliance, performance and environmental. In compatible with, many other attempts have been made to innovatively develop quantitative risk evaluation methods to obtain quantitative management of different risks and also determine and analyze the impact of effective factors in various industries [see for example 4, 5, 8, 9, 11, 15, 16, 18, 26].

Long Memory Theory: The theory of the long memory includes a strong and wide dependency, autocorrelation, between the observations of time series data. The existence of such a dependency means that if a significant exogenous shock hits the market, the impact of this shock will not disappear quickly; naturally, it will remain in the market's memory and influences decisions of market activists for a specific time period. This period of time can also be accurately calculated through the long memory oriented models. Moreover, the existence of this property in asset returns is consistent with the Fractal Market Hypothesis, FMH, while it is incompatible with the Efficient Market Hypothesis, EMH [24]. The FMH is an alternative theory for the EMH (i.e. the condition based on which by using the historical returns' data, the future asset returns are unpredictable, neither by technical nor fundamental analysis) in the financial markets. The FMH dictates the markets, especially the financial markets, to follow a repeatable and cyclical pattern, which is known as a "fractal pattern" [27]. For instance, unlike the EMH, the FMH framework can provides an explicit description for different behavior of the investors throughout the business cycles in the financial markets such as booms and busts. To be more precise, in a stable economic situation, generally, the number of long term investors and the short term ones are balance, however, if the condition of the economy become unstable, the investors behavior will change from long term to the short term horizons, and this means that the condition of the economy dictate the horizons and market prices to the investors, so that the market will become inefficient and unstable. Unlike the EMH, the FMH can apparently defines the investors' behaviors in all market conditions. The main points, in recognizing and modelling the fractal pattern of the markets, are (i) accurately detecting the length of repeating time, and (ii) figuring out the pattern of the investors' behaviors, which would likely not be exactly the same. Therefore, since the fractal market hypothesis can explain the different behavior of the market, the long memory property, which can affect the behavior of the investors, can only be consistent with the FMH (not the EMH). Hence, based on the long memory and FMH principals, if a time series data has the long memory property, although the behavior of the time series seems like an unpredictable and chaotic process (based on the EMH scenario), by changing the modelling approach from linear to the nonlinear, it will become a predictable process [25]. This theory can be applied in different types of modelling from models related to mean-equations (return) to variance-equations (volatility). On this basis, analyzing the FMH and EMH hypotheses; consequently, considering long-memory features through a non-linear framework, can enhance the accuracy and efficiency of the model base on which the investment risk in petrochemical industry should be extracted.

Long Memory Identifying Tests: The most crucial step in estimating a model that has the long memory property is statistically investigating the return and volatility of its time series data to find out whether the existence of the long memory feature is statistically significant in these series. To this end, there are several techniques, e.g. ACF test, Modified R/S statistic, and GPH test, etc., to identify the existence of this feature (or evaluating the long memory parameter, d) in time series data [1, 20, 22, 38].

Measuring Risk: The measurable potential losses of investment are called risks. In line with this, measuring risks has always been a crucial issue for analysing an investment owing to the fact that it, as an inseparable component in the field of investment, has always been closely related to the investment return. Thus, in recent decades, researchers have introduced various instruments and methods to estimate and analyse the investment risks. In this regard, Markowitz [40], one of the most prominent researchers in the financial fields, has presented a numerical indicator of the investment risk based on which the standard errors of a variable at particular a period could be considered as a representative of the investment risks. After him, other researchers, have applied the variable volatilities in addition to the standard errors as the investment risk index. In other words, despite from being positive or negative amounts, Galatz took any volatilities in any sort of gaining (incomes, profits, returns, and so on) as the numerical representation of investment risks. Moreover, it has been asserted that investment risk involves any phenomenon that could increase the probability of unfulfilled expectations of investors. As a result, every datum that includes the unfulfilled expectations of investors or is close to these amounts could present the investment risks [23]. In the same way, the scientist in this field of study also stated that the probability of a destructive phenomenon affecting the returns of an investment, which is statistically specific and measurable weights of the returns, could demonstrate the investment risks. Ultimately, Shen [32] has contended that the numerical investment risk index should illustrate the differences between the real investment returns and the expected ones. To measure the exact investment risk, as a time series, he has applied the heteroscedasticity series of high-volatile variables which could be estimated by ARCH family models. This way, the effect of other variables on the investment risk can be evaluated. The above mentioned concepts indicate the importance of considering long-term memory properties and combining them with nonlinear regression models to more accurately extract investment risk across different time series data. In other words, disregarding the new achievements in the realm of mathematical finance especially in the modelling sector (such as longterm memory features and nonlinear modelling) and, on the contrary, applying the traditional quantitative models (e.g. general GARCH models) in the risk modelling process can distort the results of both modelling of the risk and also analysing and policy making.

4 ARFIMA-FIGARCH Model

There are generally two methods for estimating the risks, i.e. Quantitative Risk Assessment and Qualitative Risk Assessment, regardless of the risk estimation method [5]. Applying the Quantitative Risk Assessment, which is the most widely used risk assessment method by researchers and experts, has various approaches, among which, using the Conditional Heteroscedasticity type, GARCH Type, models especially for the highly volatile data of the petrochemical stock index have more robust mathematical and analytic background. In addition, considering the main property of financial markets like long memory feature, which will be explained in the following part, is another advantage of this type of models. In such quantitative methods, the mean and variance equation of a high-volatile time series data are estimated simultaneously and, under such conditions, extracting the volatility se-

ries from the GARCH-type models corresponds to the risk of the time series. As stated earlier, in order to model the investment risk in the petrochemical industry, and analyse the effects of exchange rate on it, the ARFIMA-FIGARCH model is used, which, in addition to its significant reliability in modelling the risk of high-volatile data, its capability to consider the essential characteristics of financial markets, long memory feature, is noteworthy. To be more precise, the ARFIMA-FIGARCH framework is a combination of the ARFIMA and FIGARCH models. The ARFIMA (Autoregressive Fractionally Integrated Moving Average) model, which derives from the traditional ARIMA framework, was first suggested by Granger and Joyeux [41] to model the financial time series which are characterized by the long memory processes. The general form of the ARFIMA (p,d,q) model is explained as follows [25]:

$$\varphi(L)(1-L)^d Y_t = \omega(L)\varepsilon_t \tag{1}$$

In which L is the Lag operator, $\varphi(L)(1-L)^d Y_t$ is the Polynomial Autocorrelation, d is the long memory parameter, $\omega(L)\varepsilon_t$ represents the Moving Average Polynomial, and finally Y_t is the mean of y. In that equation, if the long memory parameter, d, is greater than zero and simultaneously less than $0.5 \ (0 \le d \le 0.5)$, there is a positive dependency among the distant observations of the time series that representing the existence of the long memory property. Conversely, if it is between -0.5 and zero, it indicates the presence of a negative dependency among the distant observations which is named as anti-persistence. In the extreme situations, if d equals to zero, it means that the time series has a unit root, and also when d equals to one, the $\varphi(L)(1-L)^d Y_t$ and $\omega(L)\varepsilon_t$ will represent the autoregressive AR and moving average MA polynomials successively. In the line of research it has been introduced the ARCH (Autoregressive Conditional Heteroscedasticity) model; later in 1986, it has been introduced the generalized form of the ARCH, i.e. GARCH model. The Borlerslev's GARCH model is widely using in the financial modelling because of its considerable diversity and high adaptability to the real-world context. As a type of this model, which is significantly consistent with the primary characteristics of financial markets, the FIGHARCH model can be mentioned. The FIGARCH is the extended version of the squared residuals of the ARFIMA model. The general form of this model is presented below:

$$(1-L)^d \varphi(L)\varepsilon_t^2 = \alpha + \psi(L)U_t \tag{2}$$

In which, ε_t^2 contains the serially uncorrelated residuals with zero mean are the integrated conditional variance, ε_t^2 are the squared residuals of the GARCH model, and L is again the Lag operator. Based on the different amount of the long memory parameter, d, various types of GARCH-type models will produce. More precisely, when d equals to zero, the FIGARCH model will reduce to the GARCH, conversely, if d=1, the FIGARCH will reduce to an integrated GARCH or IGARCH, and finally, when 0 < d < 1, the existence of the long memory property will be proved [6].

5 Empirical Results

To achieve the primary purpose of this study, we utilize the daily data of the Petrochemical Industry index in the Tehran Stock Exchange and the Exchange Rate (US dollar to Iran's Rial), i.e. LPI and LEX variables, from November 2008 to March 2019 in a ARFIMA-FIGARCH framework, which

applies the Long Memory analysis to consider the nature and main features of high volatile markets. In what follows, different steps will pass to draw the conclusion.

5.1 Descriptive analysis of the data

To better understanding of the behaviour of petrochemical industry index, as a core variable of the study, and exchange rate movements, as a control variable, before go through the modelling process, it is necessary to analyse the descriptive statistics of these variables (see Table 1 for the details):

Criterion	LPI	LEX
Observations	2736	2736
Mean	11.72	9.71
S.D	3.09	2.57
Skewness	-0.209	0.301
Kurtosis	1.74	1.23
Jarque- Bra	200.48 (0.000)	396.74 (0.000)
Box- Ljung Q(10)	270.47 (0.000)	298.59 (0.000)
McLeod-Li Q2(10)	429.19 (0.000)	363.38 (0.000)
ARCH (10)	16.52 (0.000)	12.57 (0.000)

Table 1: The Descriptive Statistics of the Study Variables

A brief looking at the Table 1 shows that the observations of these indexes during the investigation period is 2736. In this regard, the statistics indicates that the means of LPI and LEX are respectively 11.72 and 9.71, while the standard deviations are 3.09 and 2.57, successively. Through the comparing of these two statistics, the high level of data volatility during the period under investigation can be realized in both cases. These findings are also in compatible with the results of the ARCH and McLeod-Lee statistics that not only prove the existence of conditional heteroscedasticity in the data, but also corroborate the significant serial correlation between the square of the errors for both LPI and LEX variables. On this basis, the existence of nonlinear behaviour between the time series data of the petrochemical index, and exchange rate as well, can be demonstrated. Furthermore, the skewness and kurtosis statistics results along with the results of the Jarque-Bra test indicate that the Probability Density Function of these data are not normal distribution function. the results of the Box-Ljung test show that the null hypothesis of this test, i.e. not existing of a serial correlation between the time series data, is rejected, which means that the data has the potential to be utilized in the modelling process (or the data are not a white noise sequence).

5.2 Stationary Tests

In the next step, the stationarity of the petrochemical index's time series will be assessed through using various tests. It is noteworthy to note that the level of the mentioned variable, LPI, has been nonstationary and all of the results has made on the first difference of this variable, i.e. DLPI (see Table 2 for more details).

Test	Critical Statistic	Accounting Value	Result
ADF ¹	-2.438	-18.326	Stationary
PP ²	-2. 544	-19.651	Stationary
ERS ³	4.149	0.816	Non-Stationary
KPSS ⁴	0.387	0.493	Non-Stationary

Table 2: The Results of Stationarity tests for the DLPI

Generally, if a time series is a stationary process, the results of all stationarity tests will be the same, however, the results of the first difference of the LPI demonstrate that this variable is stationary based on the results of the ADF and PP tests, while the results of the KPSS and ERS tests indicate that the DLPI is non-stationary. As a result, it can be inferred that such condition may have been created due to the existence of the long memory property in that series. There are other tests to accurately assess the presence or absence of this property that will be further discussed.

5.3 ACF Test

Analysing of the Autocorrelation plot can also be consider as another way of detecting the long memory property in the DLPI series.

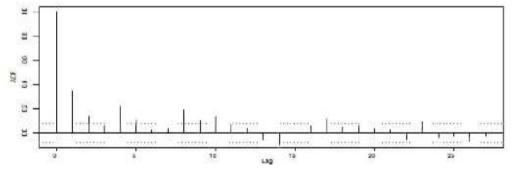


Fig 2: The ACF Graph for the DLPI

As shown in the above figure, the autocorrelation between the different lags of the DLPI has not disappeared even after more than 25 periods; to be more precise, these autocorrelations in this series are declining at a quite slow rate, which can be considered as an existence sign of the long memory.

5.4 Modified R/S Test

Although the ACF test results approximately confirm the presence of long memory process in the DLPI data, using the modified R/S test will help assessing this feature based on a significant statistical test. The results of the modified R/S test are presented in the Table 3. Since the accounting value of the modified R/S test results for DLPI Index is larger than 0.5, the null hypothesis of this test is reject-

¹ Augmented Dickey–Fuller

² Philips-Prone

³ Elliott, Rothenberg and Stock

⁴ Kwiatkowski-Phillips-Schmidt-Shin

ed at the 5% level; consequence, the results confirm the significantly existence of long memory property in the DLPI. Therefore, in the next step, the d parameter will be estimated through the GPH test.

Test	Accounting Value	Probability	Result
Modified R/S	0.737	0.001	H0 is rejected

Table 3: The Modified R/S test results for DLPI Index

5.5 GPH Test and Estimation of the d Parameter

To estimate the long memory oriented models, calculating the d parameter is considered a crucial step. In this study, the value of this parameter is estimated by using the GPH test. This test applies the Log Period gram technique to evaluate the long memory parameter.

Table 4: The results of GPH	I test for petrochemical Inde	ex Based on the NLS Method

Series	d-Parameter	T.Statistics	Probability
DLPI	0.25011	8.921	0.000
LPI	1.20174	13.538	0.000

As shown in Table 4, the long memory parameter's value is non-zero and lower than 0.5. This finding confirms the existence of long memory property in the petrochemical index. Hence, not only is the fractal market hypothesis confirmed in this market, but also using the fractional differencing is needed. Therefore, to assess the efficiency of different models for modelling the risk of investment in the petrochemical index with the long memory feature, in the following sections, we will focus on the estimation and comparing of the models.

5.6 Estimation of the Models

ARFIMA Models: Estimating of the various ARFIMA type models can be made by using different methods such as Approximate Maximum Likelihood (AML), Modified Profile Likelihood (MPL), Exact Maximum Likelihood (EML), and Non Linear Least Square (NLS) (Mensi et al, [24]).

Models	Schwarz-Bayesian information criterion			ARCH Test
Estimation Method	EML	NLS	MPL	AKCH Itsi
ARFIMA(1,0.25,1)	-8.4721	-8.7045	-8.6038	4.123 (0.042)
ARFIMA(1,0.25,2)	-8.5082	-8.8219	-8.6144	4.361 (0.002)
ARFIMA(2,0.25,1)	-8.5238	-8.9388	-8.6237	5.628 (0.000)
ARFIMA(2,0.25,2)	-8.4106	-8.7931	-8.6016	4.297 (0.013)

Table 5: The results of Different ARFIMA Models

* The results of these ARFIMA models have had the lowest information criterion among the alternatives.

In this study, the EML, NLS, and MPL methods have been chosen for calculating of these models that will compare and finalized based on the Schwarz-Bayesian information criterion with the help of Ox-Metrics software (see Table 5 for more details).

Rows	Models	Akaike Information Criterion	Schwarz-Bayesian Information Criterion
1	GARCH	-9.1173	-8.9906
2	EGARCH	-9.1835	-8.9927
3	GJR-GARCH	-9.1841	-9.0064
4	APGARCH	-9.2416	-9.0218
5	IGARCH	-8.9820	-8.7852
6	FIGARCH (BBM)	<u>-9.4184</u>	<u>-9.3019</u>
7	FIGARCH (Chang)	-9.3716	-9.2145

Table 6: Different types of the GARCH Models based on the selected ARFIMA model

Table 7: The results	of the selected GARCH	Model's estimation
	of the beleview of ficeri	niouel b estimation

		Mean Equation	n		
Variables	Coefficient	Standard Er	ror T-Statis	tics Probability	
С	0.18	0.086	2.09	0.045	
d-ARFIMA	0.25	0.028	8.73	0.000	
AR(1)	0.56	0.071	7.86	0.000	
AR(2)	0.27	0.049	5.49	0.000	
MA(1)	-0.43	0.091	-4.71	0.000	
		Variance Equat	ion		
Variables	Coefficient	Standard Er	ror T-Statis	tics Probability	
С	0.39	0.142	2.63	0.012	
d-FIGARCH	0.27	0.035	7.56	0.000	
ARCH	0.49	0.077	6.31	0.000	
GARCH	0.34	0.063	5.32	0.000	
DLEX	0.57	0.065	8.73	0.000	
Log likelihood	1087.32	1087.32 McLe		2.034 (0.979)	
Liang-Box	6.589 (0.4	72)	ARCH	0.1786 (0.997)	

Based on the results of Table 4, it can be argued that the ARFIMA (2,0.25,1) has the lowest Schwarz-Bayesian information criterion score and also has the best performance. As a result, in the next step this type of the ARFIMA with the mentioned order can be applied to estimate the best GARCH model.

FIGARCH Model Estimation: Although the results of Table 4 demonstrate that the ARFIMA (2,0.25,1) model has the best performance based on the Schwarz-Bayesian information criterion, the results of ARCH test in all ARFIMA models prove that the conditional heteroscedasticity effects are confirmed; consequently, to eliminate this problem, using the GARCH-type models can be a convincing solution. The proposed GARCH models, include the non-fractal, fractal, and Integrated oriented GARCH-type models, in Table 6 have been based on the selected ARFIMA model (mean equations), and by comparing the information criteria of the various types of GARCH models, it can be found that the final framework to modelling the risk of investing in the petrochemical industry will be the ARFIMA (2,0.25,1) FIGARCH (BBM) model that has the lowest information criteria among the all models. It is noteworthy that these results prove the existence of the long memory property both in mean and variance equations of the petrochemical index. Moreover, the estimation results of the selected model are presented in the Table 7. According to the results of the estimated model, it should be noted that, firstly, all of the estimated variables are statistically significant at 0.95 level of confidence (except the constants). Secondly, the coefficient of the exchange rate, as the systematic risk representative in the variance equation of the above model, demonstrate that this exogenous variable has had a significant effect on the risk of investment in the petrochemical industry, and also this coefficient has satisfied the theoretical concerns about the relationship between these variables. Finally, all of the diagnostics tests significantly proves the results of the estimation.

6 Conclusions and Implications

Based on the time series data of the exchange rate and petrochemical stock index in the period from November 2008 to March 2019, the present study applies the ARFIMA-FIGARCH framework to model the relationship between the investment risk of the petrochemical index and exchange rate. The results of this study are elaborated below:

The first and most important achievement of this study has been the evaluation and modelling of investment risk. In this regard, assessing the different types of the GARCH models presented in Table 6 confirms that the ARFIMA-FIGARCH(BBM) model has had the best performance in comparison with other competing frameworks (based on the lowest Schwarz-Bayesian information criterion). These findings corroborate the existence of Fractal Market Hypothesis, FMH, and the Long Memory property in both the petrochemical stock return and risk, respectively based on proved long memory in mean by ARFIMA model and variance by FIGARCH(BBM) model. These results have not been unexpected given the inherent characteristics of the stock market and also the price volatility of the DLPI index as a highly productive industry. The most important point of applying the FIGARCH model was that, in addition to confirming the existence of nonlinear behaviour in the petrochemical stock index, using the Fractal Market Hypothesis and also the long memory property has had a significant role in boosting the modelling results. Accordingly, based on the results of estimated model which is presented in Table 7, it is clear that the ARCH and GARCH coefficients as well as the exogenous variable of research, i.e. the exchange rate, are significant. These results are highly reliable with regard to the significance of whole model (based on the diagnostic tests) and the comparing results of the selected model with the rival framework (presented in the Table 6). Therefore, it can be used as a powerful and useful tool for (i) explaining the risk behaviour of investment in the industry, (ii) analysing the impacts of exchange rate movements on the risk of investment in that index, and (iii) making the micro and macro decisions. Furthermore, in analysing the effects of exchange rate as one of the

most important factors of systematic risk and based on the results of estimated coefficients, it should be stated that, these impacts are consistent with empirical evidence in this field (which is consistent with the results of some other studies such as Zolfaghari and Sahabi [39]; Šimáková [33]; Ghosh [13]; Bernhofen and Xu [6]). More precisely, exchange rate movements have had a direct and significant impact on the investment risk of the petrochemical industry so that if, on average, one percent change occurs in the exchange rate, the investment risk in this industry will change by 57% in the same direction. Based on the theoretical concepts and the background of the research, on the one hand, and due to export-centricity of the petrochemical products, the existence of comparative advantage, and the existence of supportive policies for the productions, on the other hand, it seems cogent that the exchange rate movements considerably transmit into the risks of petrochemical investments. In essence, during the research period, the major changes in the exchange rate have been made by the imposing of international sanctions against Iran. The ramifications of this issue have been resulted in establishing a chaotic behaviour in the foreign exchange market as a result of the growth in speculative demand for foreign currency, creating growing problems in the international trade both in terms of physical and financial transactions. Consequently, such conditions have reduced the investment security, and simultaneously enhance the risk of investing in any industry, especially the ones that are export- or import-centric like petrochemical industries. Therefore, the risk of investment in this type of industries would have a significant correlation with the exchange rate movements. It should be noted that these results in the realm of risk management can also be beneficial for investors (who are trying to manage their portfolio in order to achieve greater returns, and to choose and analyze the alternative investment projects, especially in response to exchange rate changes). Similarly, it could be a boon for managers (who want to rapidly receive the signals from the exchange rate changes to optimize their decisions, and also to control and manage the market through adopting the risk-reducing policies, switching to the alternative financing methods, enhancing the exports, identifying and diversifying new target markets, optimizing the production chain, not paying attention to development projects or investing in infrastructure during the turbulent economic conditions). As the limitation of this research, it could be referred to statistics. To be more precise, the source of data used in this article were not the same. In essence, the exchange rate data gather from Iran's Central Bank, while the petrochemical industry index data download from Tehran Stock Exchange official website. Therefore, there were some unmatched data based on dates. However, the researchers had tried to eliminate this problem by sifting unmatched data.

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