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Research Paper

Daily Net Cash Flow Analysis and Forecasting: Transition from Microscopic to Macroscopic Stochastic Equations

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ABSTRACT

The aim of this study is to enhance our understanding of behavior and improve net cash flow forecasting. The research data comprises daily trial balances spanning one year, gathered from 48 bank branches. To achieve this goal, we assessed four distinct models: Geometric Brownian, Arithmetic Brownian, Vasicek, and Modified Square Root, across three levels: microscopic, mesoscopic, and macroscopic. Subsequently, the Geometric Brownian model emerged as the most suitable model at the microscopic level. The findings reveal that the Geometric Brownian Motion model excels in accurately simulating net cash flow, meeting the criteria for mean absolute percentage error. Additionally, net cash flow forecasting for each time series under investigation was conducted over various forecasting horizons, including 7, 14, 21, 30, 60, 90, and 180-day periods, in line with mean absolute percentage error criteria. Another noteworthy outcome of this study is that, based on eight distinct prediction accuracy criteria, the ability of the GBM model to simulate and forecast net cash flow diminishes as the forecasting horizon extends.

1 Introduction

Explaining the cash flow process is one of the most important topics for researchers, economists, and financial analysts, which has created different ways and different perspectives. New mathematical models, regression models, time series models, random steps, and more advanced models were used to analyze cash flow. Cash flow information was used to perform various tests. To measure shortages and to plan for financing, cash flow forecasting seems to be particularly important. In the real world, a cash flow model is an abstract (usually mathematical) representation of cash flow to explain, predict, manage and control cash flow. Cash flow models have become more sophisticated in the past fifty years. By estimating the subject points of the cash flows, the definite and discrete initial models were replaced with probabilistic cash flow models. The probabilistic model turned out to be richer stochastic models over time. Continuous-time stochastic models particularly can describe a wide range of stochastic specifications and properties. Economic activists, investors, and macroeconomic policymakers always pay attention to forecasting issues. The important models are those that are used to simulate and predict

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different economic variables so that using different models can lead to different predictions with different accuracy. In our country, so far, the cash flow is paid less attention despite increasing mathematics research in the financial sciences. In addition, researches in this field have been conducted with a few simple and ordinary models, and it is denied to access models with more complexity. In the present study, a stochastic process and stochastic differential equations are addressed to this issue to take a step towards resolving the above contradiction. The theoretical foundations of the research and mathematical models will be reviewed in the next sections, and then some of the researches related to the present research will be mentioned with their results. After that, items such as research innovations, general research model, method of implementation, application of the model, and collection of data required for research and criteria used to judge the prediction made are mentioned in the research method section. Then, the results and findings will be presented and analyzed. A summary of what has been done in the present research will be finally stated, and practical, and future suggestions will be presented.

2 Literature Review

Maintaining cash or near-cash assets is one of the most important tasks of banks to meet the bank's liquidity needs and to respond to customers' requests to withdraw from their accounts. Therefore, a disturbance in this cash flow has adverse consequences for each bank and the entire banking system, especially if it is due to the systemic connections of banks with each other. The most important element of current assets in financial institutions and banks is Cash. The financial institution is known as a highly liquid financial institution if it has access to cash and reasonable cost over time. On the other hand, insufficient liquidity increases liquidity risk and financial problems. Therefore, banks should closely control cash, which is the amount of money that bank branches keep due to the nature of the banking industry. Excessive cash holdings increase the cost of money, and fewer holdings cannot meet the needs of the bank. Therefore, the bank should borrow large amounts from the market, which leads to liquidity risk. To identify the appropriate options for investing the funds and to profit from the opportunities, the bank should always be ready to estimate the liquidity of its branches in case of forecasting excess liquidity. In addition, to compensate for the lack of liquidity, banks should be able to benefit from suitable options by examining different sources of liquidity and considering the cost of each of these resources in case of forecasting liquidity deficit.

Adequate liquidity of banks extremely depends on the behavior of cash flows in different situations. For managing active liquidity, the time is generally short and includes liquidity during the day. The first days are very important to maintain stability, especially when any liquidity problem occurs. Allocating an appropriate period for liquidity management depends on the nature of the banks' operations. To manage the liquidity, Banks relying on short-term financing mainly focus on very short periods (for periods of 3 to 5 days). Banks should calculate their liquidity situation ideally on a day-to-day basis for this period. On the other hand, banks relying less on short-term markets should actively manage their net cash needs over a longer period, such as the next one to three months [1]. Forecasting daily cash in the bank is the same as the daily net cash flow. All cash documents and accounts of the fund in the cash operations of the bank include Rial fund, foreign currency fund, and Iran Check fund. Therefore, the daily cash balance of these accounts is the net average cash flow of the branch after deducting the balance of these accounts in the previous day that can be used for forecasting. Good and correct forecasting improves short-term investment returns, reduces the cost of issuing commercial securities and other short-term borrowings, reduces unused cash balances, facilitates cash management, and guarantees not to face bankruptcy.

When analysis and statisticians examine forecasting models, the first important point is to consider them over time and identify the appropriate model for the statistical time series of the data. But there are four main reasons for failure (inability to predict correctly) in this regard, the first three reasons are related to the incorrect design of the problem and the fourth reason is related to the characteristics of the statistical model. [12], The four topics mentioned are:

1.Separation of main cash flows. At this stage, we must separate all the cash flows that are not intermittent (intermittent cash flows mean items such as taxes, dividends, etc.). If standard extrapolation and autocorrelation techniques are used to predict daily cash without considering intermittent cash flows, the technique will certainly not be able to predict accurately.

2. Identify components. After removing the intermittent cash flows currents from the daily cash flows, we need to identify its components. In this step, we separate the input and output cash flows. It is often necessary to consider large cash flows in order to predict and track. This is because different cash flows show different statistical characteristics. Especially in patterns related to the days of a month or a week. Because different time combinations do not follow the same statistical patterns.

3.Use of information systems. Due to the large volume of data (large number of cash flows) in forecasting models, information systems must be used.

4. Pattern recognition. Daily cash flows usually have a specific time pattern, for example 1) monthly payment method, bi-weekly, once weekly, 2) day-to-month credits, and 3) internal process procedures and policies such as considering payment flows. On specific days of the week. In forecasting daily net cash flow, first it is necessary to determine the model in terms of days of the month or days of the week, and then the parameters of the model are measured and finally the forecast is made according to the appropriate effect of days of the week and days of the month for a certain time.

Various studies have been conducted to study the behaviors of economic variables that are stochastic diffusion processes have found a significant role in the financial economics literature in a few decades. Diffusion processes are particularly used at the center of financial economics to model exchange rates, interest rates, asset pricing, financial derivatives pricing, risk valuation, optimal portfolio selection, and volatility modeling. Furthermore, stochastic differential equations are used to explain the behavior of economic variables in most models. Stochastic processes are widely used in modeling systems and phenomena that seem to change randomly. The demand for the use of random calculation tools and methods in various fields has had a great rate in the last twenty years. Stochastic differential equations are an important branch of random mathematics and modern finance. There are two types of stochastic nature mathematics in stochastic differential equations. These groups have differentiable or non-differentiable solutions. Each group offers solutions that fundamentally differ from each other. The first group has easier solutions and includes ordinary differential equations with random coefficients, random initial values, random inputs with regular and definite properties, and even a combination of the above. In the second group, equations input is an irregular stochastic process such as Gaussian white noise. These equations are considered as stochastic differential equations and are expressed as equations with Ito random integrals. Such equations do not have a differentiable solution. The origins of the new mathematical foundations of finance go back Louis Bachelier's Theory of Speculation, which was defended at the Sorbonne in 1990. This was, in fact, the birth of a continuous time and, on the other hand, continuous time strategies of risk hedging in finance. From a mathematical point of view, Bachelier 's treatise had a tremendous impact on Kolmogorov's research on continuous time processes in the 1920s, as well as the research of It, the inventor of random calculus in the 1950s.

In the financial literature of several stochastic equations; Arithmetic Brownie Mention (ABM), Geometric Brownie Mention (GBM), vasicek (VC), and Modified Squared Root (MSR) have been noted as notable features of continuous-time random processes. Cash flow models have been used in programs such as cash flow management models, capital project analysis and business evaluation. [18]. Scottish botanist Robert Brown introduced the most famous continuous time stochastic process called Brownian motion in 1827. The Wiener process is also a continuous process named Norbert Wiener, the genius American mathematician (1894-1964). This continuous time stochastic process is also known as the standard Brown movement. Standard Brownian motion (ΔZ) is a special form of Brownian motion in which $\Delta Z = \frac{W_t}{\sigma}$ the standard Brownian motion is the variance of one number. Standard Brownian motion and Wiener process are used in the same sense as described in (1)

$$X_{t+1} = X_t + dz$$
$$dz = \varepsilon\sqrt{dt} \quad \& \quad \varepsilon \sim N(\,0,1)) \tag{1}$$

Arithmetic Brownian motion (AMB): If long-term growth is added to Wiener's motion Mathematically defined as (2).

$$\mathbf{S_{t+1} = S_t + \mu dt + \sigma dz}$$

$$\mathbf{ds = \mu d_t + \sigma dz} \quad \& \quad ds \sim N(\mu dt, \sigma^2 dt) \tag{2}$$

The evolution of arithmetic Brownian motion has two parts.

A: a linear growth with μ rate;

B: A random growth with normal distribution and standard deviation σ. It focuses on changing the value of the variable. In addition, it is known as an additive model because the variable of each period grows by a constant value.

On some days, the branches of banks may suffer a deficit and correct the negative balance, which receives banknotes from the central bank for compensation, given that this model is considered a return to the negative average, this model can be considered.

Geometric Brown Motion (GBM): It is also called exponential Brown Motion which the logarithm of different random values follows a Brownian motion or Wiener process. If s_t follows the Geometric Brownian process, the stochastic differential equation of this model is the formula (3)

$$\frac{ds_t}{s_t} = \mu(s,t)dt + \sigma(s,t)dw_t \tag{3}$$

S_t: Base asset at time t, μ: percentage of thrust and σ: percentage of fluctuations that both are constant. One of the advantages of Geometric brown motion is its simplicity. In this model, parameter estimation is easy and unlike many other models, there is no need for large volumes of data. In addition, the GBM model, while simply considering the random component, can predict Stochastic variables that look like Their construction is facing more problems, do well. [11]. Vasicek Model (VC): Vasicek process is one of the most well-known random return processes used to model the random behavior of short-term interest rates for the first time. It is a type of short-term single-factor rate model that describes interest rate changes according to a type of market risk. Stochastic differential equation of this model is calculated the formula (4)

$$dX_t = \alpha(\theta - X_t)dt + \sigma dw_t \tag{4}$$

w_t: Wiener process, σ: Standard risk, α: average return speed, θ: long-term average of the time series.

Cox- Ingersoll- Ross model (CIR): It is a type of “single factor” model (short-term interest rate model) that describes interest rate changes according to a type of market risk. The differential equation of the model is according to (7).

$$dc_t = a(b - c_t)dt + \sigma\sqrt{c_t}\, dw_t \qquad (5)$$

w_t: the Wiener process, a, b, and σ are its parameters. The parameter a is related to the velocity adjustment relationship, b is the mean and σ is the intensity of the oscillation, and $a(b - c_t)$ is the deflection factor that ensures a return to the average interest rate towards b with a positively adjusted velocity a. Modified Square Root (MSR): For investment value Klumpes and Tippet [8], used the modified square root. They set the optimal investment criteria for a capital project so that its cash flow would evolve in terms of a "modified square root" process. The modified square root trend has similar characteristics to the square root trend of Cox, Ingersoll, and Ross [3]. In addition, it includes the possibility of a negative cash flow.

$$dc_t = (\mu c_t)dt + \sqrt{({k_1}^2 + {k_2}^2 c_t^2)}dw_t \qquad (6)$$

w_t: Wiener process, ${k_2}^2$ ، ${k_1}^2$and μ are its parameters. k_1 = σ is the intensity of the oscillation. ${k_2}^2$, μ= 2μ is the mean.

By parametric estimation and oblique specification of diffusion processes, Tang and Chen [15] estimated diffusion equation coefficients. Estimation of parametric is also gained by using the combination of parametric approach, maximum likelihood, and bootstrap method. The diffusion pattern coefficients are estimated by using the maximum likelihood method, and then the bootstrap approach is used to reduce the skew. From 1963 to 1998, monthly US interest rate data were used to test the proposed method. The results indicate a sloping reduction of the proposed method in univariate and multivariate processes. Using fixed developed parameters, Primbus and Barmish [17] investigated the main model of Geometric Brownian motion. In this study, the process of drift μ and turbulence σ are considered time-dependent and variable. In this regard, the SLS strategy ensures the expected profit of positive trades at any time. In this study, these results are generalized to Geometric Brownian Motion with price dynamics. It allows these two parameters to change continuously with time, and there is no limit to the amount of their change. Corsaro et al. [2] presented an algorithm for pricing Asian options using a framework of random fluctuations and combining the three models of Heston, Levy, and Cox- Ingersoll. This algorithm can be useful in accuracy and time management because it uses algorithms and programming as a parallel strategy in this type of trading. Using the Brownian Geometric model, Handan et al. [7] examined the price of gold in the Malaysian market. The simulated prices for a maximum period of one month are evaluated using Mean Absolute Percentage Error (MAPE). The results show that the model can be used to predict the price of gold for a short period of one month and has high accuracy. Wattatorn and Sombultawee [19] investigated the effect of random fluctuations on option pricing in the Thai market. Their study was based on two models such as Heston and Black Scholes. To evaluate the mentioned models, two squared criteria of Root Mean Square Error (RMSE) and Mean Square Error (MSE) has been used. The results showed that the Heston model covers random fluctuations better and has better.

Sadeghi et al. [11] examined the application of Geometric Brownian Motion in predicting gold prices and exchange rates in the Iranian open market. According to the results, the Geometric Brownian Motion model can simulate prices with high accuracy based on the MAPE criterion. In addition, the other results indicated that the ability of the GBM model to perform simulation decreases by increasing the time horizon. Davallo and Varzideh [4] predicted the total index of the Tehran Stock Exchange by using the Geometric Brownian Motion model and MAPE evaluation criterion. The results showed that the Geometric Brownian Motion model could predict the total index of the Tehran Stock Exchange in a 1-

day time horizon with high accuracy. The accuracy of the values predicted by the model and the model's ability to simulate the index is reduced by increasing the forecast time horizon. The predicted values are still highly accurate up to the 31-day forecast horizon. Fathi Vajarga and et al. [7] investigated oil Price estimating under dynamic economic models using Markov Chain Monte Carlo simulation approach. They attempted to estimate and compare four different models of jump diffusion class combined with stochastic volatility that are based on stochastic differential equations, and their parameters latent variables are estimated by Markov chain Monte Carlo (MCMC) methods. In the Stochastic Volatility with Correlated Jumps (SVCJ) model, volatilities are scholastic, and the term jump is added to both scholastic prices and volatilities. The results of this study showed that this model is more efficient than the others are, as it provides a significantly better fit to the data, and therefore, corrects the shortcomings of the previous models and that it is closer to the actual market prices. Therefore, their estimating model under the Monte Carlo simulation allows an analysis on oil prices during certain times in the periods of tension and shock in the oil market.

Talebi and et.al,.[14] predicted future cash flows. The findings of the research show that both regression and neural network models have the capability of predicting future cash flows. Also, results of neural network models' processes show that a structure with 16 hidden neurons is the best model to predict future cash flows and this proposal neural network model compared with regression model in predicting future cash flows has a better and accurate function. Ghanbari and Goldani [6] investigated Support Vector Regression Parameters Optimization using Golden Sine Algorithm. SVR model have achieved high performance on forecasting problems, however, its performance is highly dependent on the appropriate selection of SVR parameters. In this study, a novel GSA-SVR model based on Golden Sine Algorithm is presented. The performance of the proposed model is compared with eleven other meta-heuristic algorithms on some stocks from NASDAQ. The results indicate that the given model here is capable of optimizing the SVR parameters very well and indeed is one of the best models judged by both prediction performance accuracy and time consumption. The results indicate that the given model here is capable of optimizing the SVR parameters very well and indeed is one of the best models judged by both prediction performance accuracy and time consumption.

3 Methodology

Kotlenze [9] has presented the first accurate achievement of mesoscopic and macroscopic equations from a definite system of microscopic equations in his book entitled “Stochastic Ordinary and Stochastic Partial Differential Equations-Transection from microscopic to macroscopic Equations”. The microscopic equations are presented in the form of a definite system of interconnected nonlinear oscillators for N large particles and infinitely small particles. A partial differential equation describes Stochastic Ordinary Differential Equations (SODE) and macroscopic limit is described by a partial parabolic differential equation. This study uses net cash flow analysis at three levels such as microscopic, mesoscopic, and macroscopic. The primary objective of the study is to consider the net cash flow over time for a single branch at the microscopic level. Stochastic process is considered with jump content and stochastic differential description. This approach examines the unique behavior (branches in this study) and is usually specific to describing such a system and stochastic modeling. At the mesoscopic level, it analyzes from a small system to a large non-linear system in several branches so that the discrete movements of the net cash flow can be ignored. The detailed description of the net cash flow trend of branches of banks can be found through the “Medium” propagation process (approximation) of all jump processes. The main objective of the study at the microscopic level is the individual realization of cash

flows over time. But at the mesoscopic level, the analysis is extended and includes all possible paths that a cash flow trend can follow. The analysis moves from a definite space to a stochastic process where only one realization is possible, where there is the realization of countless (future) researches of a cash flow process. To raise the analysis to the macroscopic level, both the stochastic cash flow of a company and all the companies existing in the study group are considered. This approach considers the condensed behavior of the system and the higher level stochastic process as a common denominator for all branches (institutions). A Fokker-Planck type of Stochastic Partial Differential Equation (SPDE) can describe this system. Each of the equations is based on maintaining the probability principle over time. The diversity of individual cash flows is ignored at the macroscopic level or the macro level of analysis. Here, the average behavior of the cash flow paths for the whole group is important. Therefore, the evolution of the probability density function (unconditional) describes the probabilities of a set that are valid in some cases for describing the stochastic behavior of the cash flow of different branches.
The approach of the present study is as follows:

1. Four continuous-time stochastic processes models are considered as Geometric Brown Motion, Brownian Arithmetic, Vasicek, and Root Modified Square.
2. Each of the models is presented from the general state at the branch level (microscopic)
3. Each model is developed into a continuous-time model at the multi-branch level that includes a combination of net emissions and net mutations (mesoscopic).
4. The developed model will then be added to an equation of all branches (macroscopic).
5. The optimal model is selected at all levels.
6. The final optimal model selected from all levels is finally used to predict cash compensation.

The statistical population of the present study has been used after removing the holidays by using the data of 1-year net cash flow of 48 branches of a certain bank (the bank's letter has been omitted due to the confidentiality of the information). This research investigates both the issue of how long the time horizon can be optimally simulated with high accuracy and how efficient the optimal model approved in step (6) will be for simulating time horizons of different lengths. Net cash flow simulations have been performed for the forecast horizon of 7, 14, 21, 30, 60, and 180 days. It represents respectively one, two and three months. It is predicted to stimulate net cash flow of each of the time series variables under consideration for each of the desired time horizons using the period of estimating the parameters. The prediction has been performed by the criteria for checking the accuracy after performing the simulations related to the mentioned time horizons.

3.1 Model Selection Criteria

In order to evaluate the performance of the model in simulating net cash flow, the projected net cash flow versus its actual values must be examined to determine how close the predictions of the optimal model for net cash flow are to reality. In order to compare the models using the criteria of Mean Absolute Error (MAE), Mean Square Error (MSE) and Root Mean Squared Error (RMSE), at the three levels (microscopic, mesoscopic and macroscopic) are considered as follows:

$$\mathrm{MAE} = \frac{\sum_{i=1}^{n} |y_{model}(i) - y_{actual}(i)|}{n}) \tag{7}$$

$$\mathrm{MSE} = \frac{\sum_{i=1}^{n} (y_{model}(i) - y_{actual}(i))^2}{n} \tag{8}$$

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}\left(y_{model}(i) - y_{actual}(i)\right)^2}{n}} \tag{9}$$

These relationships, n is the number of data for each branch, y_{model} is the net cash flow predicted by the model, and y_{actual} is the actual net cash flow of each branch.

3.2 Criteria Used for Forecasting

In this study, like some other studies (such as Omar & Jaffar, [16]), the criterion of Mean Absolute Percentage Error (MAPE) is used. This criterion is a suitable and understandable criterion for analyzing the proximity of the predicted values to the real values. This criterion considers the effect of the size of the real values.

$$\mathrm{MAPE} = \frac{\sum \left|\frac{y_{tmodel}(i) - y_{actual}(i)}{y_{actual}(i)} \times 100\right|}{n} \tag{10}$$

This regard, n is the number of data for each branch, y_{model} is the net cash flow predicted by the model at time t, and y_{actual} is the actual net cash flow at time t of each branch. In this study, Lawrence et al. [10], states the criterion of judgment of forecast accuracy regarding MAPE as in Table 1.

Table 1: A Criterion of Judgment of Forecast Accuracy

MAPE	Judgment of Forecast Accuracy
<10%	Highly accurate
11% to 20%	Good forecast
21% to 50%	Reasonable forecast
>51%	Inaccurate forecast

From Table 1 it indicates that the smaller the values of MAPE, the forecasting model used is more accurate. Due to the fact that each of the different criteria for predicting accuracy have different characteristics and with the aim of making the research results more reliable and generalizable, out of 8 criteria MSE, MAE, MAPE, RMSE, Median Absolute Percentage Error (MDAPE), Root Median Squared Percentage Error (RMDSPE), Mean Absolute Scaled Error (MASE), Median Absolute Scaled Error (MDASE) are used.

$$\mathrm{MSE} = \frac{\sum_{i=1}^{n}\left(y_{model}(i) - y_{actual}(i)\right)^2}{n}$$

$$\mathrm{MAE}\,\frac{\sum_{i=1}^{n}\left|y_{model}(i) - y_{actual}(i)\right|}{n}$$

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}\left(y_{model}(i) - y_{actual}(i)\right)^2}{n}}$$

$$\mathrm{MAPE} = \frac{\sum \left|\frac{y_{model}(i) - y_{actual}(i)}{y_{actual}(i)} \times 100\right|}{n} \tag{11}$$

$$\mathrm{MDAPE} = \mathrm{Median}\left|\frac{(y_{model}(i) - y_{actual}(i)}{(y_{model}(i)} \times 100\right| \tag{12}$$

$$\mathrm{RMDSPE} = \sqrt{\mathrm{Median}\left(\frac{(y_{model}(i) - y_{actual}(i)}{(y_{model}(i)} \times 100\right)^2} \tag{13}$$

$$\text{MASE=Mean}\left(\left|\frac{(y_{\text{model}}(i)-y_{\text{actual}}(i))}{\frac{1}{n}\sum_{n=2}^{n}(y_{\text{modelt}}(i)-y_{\text{lmodel(t-1)}}(i))}\right|\right) \quad (14)$$

$$\text{MDASE =Median}\left(\left|\frac{(y_{\text{model}}(i)-y_{\text{actual}}(i))}{\frac{1}{n}\sum_{n=2}^{n}(y_{\text{modelt}}(i)-y_{\text{lmodel(t-1)}}(i))}\right|\right) \quad (15)$$

4 Research Findings

4.1 Results of Estimating the Parameters of Stochastic Differential Equations

The ability test of the models has been used for prediction. By moving this part of the data forward, the estimate is repeated. In all models, the maximum likelihood method is used to estimate the parameters. Each of the placed models is examined based on the daily net cash flow. To estimate the parameters of each equation, the amount of data available for each level is divided into two parts. The first part, the majority, is used to estimate the parameters, and the second part is used to test the prediction capacity of the models. Models include: 1. Geometric Brownian Motion, 2. Arithmetic Brownian Motion, 3. Vasicek and 4. Modified Square Root. In addition, three levels of microscopic, mesoscopic, and macroscopic considered for examination. The following table shows the average parameters for each model.

Table 2: Average Estimation of Parameters

Level	Model1	Model2	Model3	Model4
Microscopic	$\mu = 2.31$ $\sigma = 2.18$	$\mu = 2.433$ $\sigma = 2.48$	$\mu = 2.15 \times 10^4$ $\sigma = 7.03$	$\mu = 2.13 \times 10^4$ $K_1 = 3.08$ $K_2 = 6.534 \times 10^4$
Mesoscopic	$\mu = 7.44$ $\sigma = 0.089$	$\mu = 1.08\times10^4$ $\sigma = 2.18$	$\mu = 1.25 \times 10^4$ $\sigma = 1.17$	$\mu = 1.25 \times 10^4$ $K_1 = 1.8$ $K_2 = 4.734 \times 10^4$
Macroscopic	$\mu = 8.45$ $\sigma = 0.19$	$\mu = 3.126\times10^4$ $\sigma = 5.3$	$\mu = 1.35 \times 10^4$ $\sigma = 4.6$	$\mu = 4.24 \times 10^8$ $K_1 = 1.78$ $K_2 = 29120$

4.2 Comparison of Models According to MAE, MSE and RMSE Criteria

In order to compare the predictive power of the models, the criterion of Mean Absolute Error (MAE), Mean Square Error (MSE) and Root Mean Square Error (RMSE) at the three levels are considered as follows**.**

Table 3: Information from Tests

Level	Model	RMSE	MSE	AME
Microscopic	GBM	0.55	0.311	0.806
	ABM	0.89	0.79	1.65
	VC	0.937	0.878	1.72
	MSR	1.0006	1.001	1.94
Mesoscopic	GBM	0.695	0.483	1.014
	ABM	1.11	1.24	2.34
	VC	0.896	0.803	1.63
	MSR	0.973	0.948	1.92
Macroscopic	GBM	0.57	0.33	0.821
	ABM	0.932	0.869	1.69
	VC	1.07	1.15	2.06
	MSR	0.895	0.803	1.65

In the RMSE criterion, estimates with distance and a little further, even in one of the observations and in comparison with the main estimate, the influence criterion will be regarded, and the magnification of this difference will be considered. Theoretically, it shows that the estimated value of a quantity is exactly equal to its true value when this criterion is zero. It is obvious that in MSE, the existing deviation, such as RMSE, has not been adjusted. The limited variation of all the observations in the next view can be the basis of selection of the criterion that is formulated in the criterion MAE.
The number of the model with the lowest estimated value is indicated as the appropriate model in the appropriate column. It means that the model with the smallest value has the least errors. Therefore, it is a more suitable model which confirms the result of the above study of the Geometric Brownie model at the microscopic level. The Geometric Brownie model with parameters with values of 2.31 and 2.18 was confirmed as the optimal model at the microscopic level.

4.3 Cash Flow Forecasting

Table 4 shows the various model accuracy criteria, Geometric Brownian Motion at the microscopic level, described in the previous section of this study for the forecast for actual net cash flow and net cash flow at different time horizons. Each of the calculated criteria represents the amount of prediction error. It means that the predicted value has encountered more errors if the values of these criteria are higher. It is concluded that the strength of the GBM model does not remain constant with the increasing length of the forecast time horizon. They have different results with the increasing forecast time horizon.

Table 4: Horizontal Time Forecast

Horizon (day) / criteria	7	14	21	30	60	90	180
MSE	3.107×10^{16}	4.29×10^{17}	2.84×10^{18}	1.09×10^{18}	3.015×10^{18}	7.72×10^{18}	1.26×10^{18}
RMSE	2.65×10^{8}	6.54×10^{8}	1.68×10^{10}	1.04×10^{9}	1.36×10^{10}	2.77×10^{10}	1.12×10^{9}
MAE	2.31×10^{9}	4.45×10^{9}	8.05×10^{9}	8.35×10^{9}	1.35×10^{10}	7.23×10^{9}	2.83×10^{9}
MAPE	0.130	0.253	0.306	0.58	1	0.548	0.211
MDAPE	7.34	8.73	8.75	9.34	8.89	9.93	8.89
RMSPE	1.35	2.23	3.26	3.92	3.72	3.98	5.56
MASE	1.14	1.78	2.28	3.39	5.32	7.78	8.83
MDASE	1.73	2.26	3.3	5.58	7.73	8.95	9.79

In the table above, the results of the daily net cash flow simulation in the daily time horizons are analyzed. The 7-day MAPE horizon of less than 20% is the highest forecast accuracy in this period due to the table proposed by Lawrence et al. [10]. They believe that predictions with values between 11% and 20% have the appropriate accuracy of predictions. The other criteria of this interval are lower than the other periods.

5 Conclusions

5.1 Results and Discussion

The management of the financial institution intends to apply appropriate liquidity management according to environmental developments and competitive conditions by using various techniques and scientific forecasting models. Nowadays, the use of new and modern tools has been the focus of many academic and professional circles. Because many financial decision variables are stochastic variables and their changes cannot be fully explained by other variables, algebraic mathematics, which deals with deterministic and non- stochastic variables, is less widely used in financial science. For this reason, stochastic differential equations, as one of the branches of mathematics, have received much attention in the financial field; Predicting the flow of criticism is a complex matter that is increasingly welcomed in advanced mathematics, especially stochastic processes, and has become the focus of the present study. The aim of the present study is to gain a new understanding of cash flow behavior and to create an experimental model for cash flow prediction. The focal point of this study is stochastic continuous-time cash flow models. At present these models have been reported relatively sparsely in the literature of finance, and hardly any practical applications are known. Yet, stochastic continuous-time cash flow models, as underpinned by the results of this study, prove to be very useful to describe the rich and diverse nature of trends and fluctuations in cash flow randomness. However, at a price of considerable mathematical and statistical complexity.

Ongoing cash flows are directly related to the evaluation of uncertainty in each of the underlying components. In a continuous stochastic environment, this approach often comes up against explanatory variables of business and finance mathematically and computationally. Explicit answers are often available in continuous formulas. This is the significant and practical advantage of considering the use of continuous-time stochastic differential equations versus discrete models in describing specific economic processes. Therefore, the dependence and sensitivity of the process to the parameters are easily accessible and interpretable (realization of the processes, variance, and distribution values). It rarely shows the application of explicit reactions for the discrete-time model. Therefore, their qualitative discussion is precise and close to reality and can be used in different dimensions of planning, forecasting, and financing needs in different companies.

Substantiated by theoretical considerations and empirical evidence, this study corroborates the suitability of a stochastic differential equation to model continuous-time daily net cash flows. 48 branches of the bank have been used to examine daily net cash for the past year. Three levels (microscopic, mesoscopic, and macroscopic) were considered. At a microscopic level, individual realizations of cash flows over time are the primary object of study. The emphasis is on a detailed specification of cash flow paths of individual branches, including jump processes. By implication the size of the system has to be small otherwise the system becomes unmanageably complex. At a mesoscopic level the focus is expanded to include all possible paths that a cash flow process can follow. The analysis moves from a small-size system to a large-size system in which discrete movements of cash flows can be neglected and the detailed description of cash flow processes of individual branches can be approximated by a diffusion process. The system can be described by a Stochastic Partial Differential Equation ("SPDE"). When elevating the analysis to a macroscopic level, all firms in the ensemble under study are included. Now the emphasis shifts completely to studying the aggregate behavior of the system ignoring cash flow variability of individual branches. A deterministic ordinary differential equation, forms an approximation of the average behavior of the cash flow system. Unquestionably, the microscopic approach describes cash flow processes most accurately, followed by the mesoscopic and then by the macroscopic approach. Four models Geometric Brownian Motion, Arithmetic Brownian Motion, Vasicek and Modified Square Root were used. In order to compare the predictive power of the models from three criteria; MAE, MSE and RMSE that the superiority of the Geometric Brownie model at the microscopic level has been confirmed as a more efficient and closer to reality model to compare the predictive power of the models from three criteria. Then, the forecast of net cash flow at different time intervals was done using this model and at this level, using eight criteria; MAPE, MAE, RMSE, MSE**,** RMSPE, MASE, MDASE and MDAPE. According to the results, the accuracy of the model's prediction over the 7-day horizon is at an appropriate level. The results are consistent with the research of Primbs and Ross

Barmish [17] and Tang and Chen [15], based on the change of time. It changes continuously with time. The results are consistent with Salas Melina's research [13] that nonlinear models are more effective in forecasting cash flow in the real world. The results are also consistent with Van der Burg 's research [18]. In this research, he describes the rich and varied nature of the cash flow trend and its stochastic fluctuations and the validity of a stochastic differential equation to create a continuous-time cash flow model. In terms of efficiency, precision, and time management in a short period, it is also consistent with Corsaro et al.'s [2] research. Based on the approval of the Geometric Brownie model as a stochastic efficient model and efficient at different time horizons for forecasting, Handan et al. [7], Sadeghi et al., [11], Davallo and Varzideh [4] studies are consistent with the present study.

As the relationship between advanced mathematics and finance evolves, and the study is the application of advanced mathematics to finance and accounting. It is suggested to consider different dimensions of stochastic processes as important points. In addition, it is suggested to use other optimization algorithms and machine learning, and other criteria and use other stochastic processes to investigate the behavior and predict cash flow.

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