Adv. Math. Fin. App., 2024, **9**(1), P. 305-319

Advances in Mathematical Finance & Applications

www.amfa.iau-arak.ac.ir

Print ISSN: 2538-5569

Online ISSN: 2645-4610

Doi: 10.22034/AMFA.2022.1966381.1787

Applied-Research Paper

Portfolio Optimization Using Gray Wolf Algorithm and Modified Markowitz Model Based on Co-Garch Modeling

Fahime Jahanian[a], Ahmad Mohammadi[a,*], Seyyed Ali Paytakhti Oskooe[b], Aliasghar Mottaghi[a]

[a] *Department of Accounting, Tabriz Branch, Islamic Azad university, Tabriz, Iran*
[b] *Department of Economic, Tabriz Branch, Islamic Azad university, Tabriz, Iran*

ARTICLE INFO

Article history:

Received 2022-08-27
Accepted 2022-10-23

Keywords:
Portfolio optimization,
Gray wolf algorithm,
Markowitz model,
CO-GARCH

ABSTRACT

Portfolio optimization which means choosing the right stocks based on the highest return and lowest risk, is one of the most effective steps in making optimal investment decisions. Deciding which stock is in a better position compared to other stocks and deserves to be selected and placed in one's investment portfolio and how to allocate capital between these stocks, are complex issues. Theoretically, the issue of choosing a portfolio in the case of minimizing risk in the case of fixed returns can be solved by using mathematical formulas and through a quadratic equation; but in practice and in the real world, due to the large number of choices in capital markets, the mathematical approach used to solve this model, requires extensive calculations and planning. Considering that the behavior of the stock market does not follow a linear pattern, the common linear methods cannot be used and useful in describing this behavior. In this research, portfolio optimization using the gray wolf algorithm and the Markowitz model based on CO-GARCH modeling has been investigated. The statistical population of the current research included the information of 698 companies from the companies admitted to the Tehran Stock Exchange for the period of 2011 to 2020. First, the optimal investment model is presented based on the gray wolf algorithm, and After extracting the optimal model, the efficiency of the gray wolf algorithm is compared with the Markowitz model based on CO-GARCH modeling. The gray wolf algorithm is combined with the modified Markowitz model based on CO-GARCH modeling, which has a better optimal state.

1 Introduction

The financial crisis of 2008, required investment managers to study alternative methods of portfolio construction with a focus on risk distribution optimization. It was believed that portfolio optimization methods based on the standard Markowitz, model are effective in diversifying unsystematic risk [1-5]. But after the recent crises, the financial markets were caught by surprise. In fact, the misjudgment scale created doubts and revealed the gaps in the existing methods in portfolio construction and risk management. Considering the necessity of portfolio diversification and optimization in terms of risk management, in this

* Corresponding author. Tel.: 09143101238
E-mail address: ah.mohammadi.02@gmail.com

© 2024. All rights reserved.
Hosting by IA University of Arak Press

thesis, a new method for building portfolios and optimizing non-systematic risk, using the meta-innovative algorithm based on evolutionary calculations, called GWO, as well as adjusting the classic and standard model Markowitz, is presented based on the risk calculation based on the conditional variances of the GARCH family, with the aim of minimizing the portfolio risk and maximizing the diversity ratio to increase the efficiency of the optimal portfolio; Finally, the statistical results of the presented portfolios are compared with the market performance. The problem of choosing an investment portfolio is one of the classic problems of the financial world, which was first stated by Markowitz [47]. Markowitz's approach seeks to maximize the return of the portfolio and minimize its risk, where usually one component of risk or return is included in the target function and the other is considered as a constraint. One of the solutions mentioned in almost all scientific and experimental studies is to create a stock portfolio through which the existing risks can be properly managed so that the maximum possible profit can be obtained. But the issue raised in this context is the creation of the optimal stock portfolio according to the different goals of investors, which is considered as a complex issue. In a general definition of the problem, the stock portfolio means the lowest risk in the investment according to a certain level of return. forecasts vary greatly due to the inherent high sensitivity of these series to various factors and factors. Portfolio management can quickly become a gamble if the models used to predict future returns are underestimated, and the complexities of sequential changes and their implications for probability are underestimated. Optimizing securities requires knowledge to balance the expected return of the market and the size of the anticipated risk of the investment. The Modern Portfolio Theory (MPT) presented by Markowitz, is an investment theory that emphasizes how risk is an inherent part of portfolio returns and the idea of optimizing the expected return of a portfolio based on an investor's degree of risk tolerance and taking into account It provides market risk. Markowitz's theory for building a basket of eggs shows and explains how investment management plays a simple game of picking a few stocks from the market. The volatility of asset prices in a portfolio is correlated, which is typically the source of market risk [54].

According to capital market theories, the risk of holding a single share is much higher than the risk of a portfolio with minimum correlation. For a limited set of securities in a portfolio, the most common source of share price volatility is market risk, also known as the general factor. Usually, the assets of a portfolio are less volatile if they are separated from market risk; While assets that are exposed to market risks are not only very volatile, but also have large gains or losses. Portfolio risk management has classified risks into two categories: systematic risk and unsystematic risk. Market risk or systematic risk is a risk inherent to the market as a whole and its impact is not specific to a particular industry or sector. While unsystematic risk has almost nothing to do with systematic risk and it is the specific risk of an industry or a sector, and therefore it is considered a suiTable field for the study of identifying new techniques for portfolio risk diversification and optimization [8].

2 Methodology

In this research, a new method for optimizing unsystematic risk by optimally diversifying the assets of a portfolio is presented using the Gray Wolf Algorithm (GWO). In this way, by systematically analyzing the final risk of each share from the collection of shares in possession, the shares that meet the initial conditions for entering the portfolio are selected. In the next step, a gray wolf optimization algorithm is applied with the aim of maximizing portfolio diversification and eliminating unsystematic risk in the portfolio. The proposed model to increase the ability of machine learning includes two well-known portfolio risk management techniques, including risk share [7-11] and maximum diversity ratio [12-15]. The risk budgeting method is used to identify high-risk shares of the available shares. Finally, the findings are reviewed and analyzed by matching with the returns of the main capital market indices such as the total index and 50 active companies, on the one hand, and comparing with the other method of portfolio formation, which is based on volatility modeling by CO-GARCH model and Markov switching. For comparison, the length of the investment horizon, return and risk of both proposed portfolios are considered in terms of optimization and diversification. Finally, according to the results and their comparison, the better performance of the proposed diversification can be used more as a method to create portfolio diversification and risk management using machine algorithms.

2.1 Markowitz Model

Markowitz introduced and developed the concept of diversification of securities. He generally showed how diversification reduces the risk of the investor's stock portfolio. Investors can obtain an efficient portfolio for a given return by minimizing portfolio risk. In addition, the above process can lead to the formation of efficiency portfolios, which are called the efficient frontier of the average variance [16]. To use the Markowitz model, the following data are required:

1) Expected return on share i, E (R_i)

2) The standard deviation of the expected return for share i, which is considered as a measure of the risk of each share, S_i

3) Covariance, as a measure of relationship and correlation between different stock return rates, δ_(i,j)

The reason why a company's stock is a risky asset is that its overall rate of return (weekly, monthly, yearly) is not random. By changing these rates over time, they can be divided into probability distributions and the metrics required for the Markowitz model can be calculated, such as mean, standard deviation, covariance, etc. [17]. The Markowitz model is based on the following assumptions:

Investors are risk averse and expect excess returns, and their marginal utility curve is downward sloping.

Investors choose their portfolio based on expected mean-variance return. Therefore, their indifference curves are a function of expected rate of return and variance. Each investment option is infinitely divisible. Investors have a time horizon and this is the same for all investors. Investors prefer returns higher than a certain level of risk, and on the other hand, investors consider the following two factors in their choice [47].

a) "high expected return" which is a favorable factor.

b) "uncertainty about efficiency" which is an unfavorable factor.

In order to achieve the optimal portfolio selection in the Markowitz method that requires the least variance for a certain level of specific efficiency, the following linear programming model is presented [18]:

$$Min\, Z = \delta_p^2 \quad , \; \delta_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{m} w_i \,.\, w_j . cov(\overline{r_i} \,.\, \overline{r_j}) \tag{1}$$

$$st = \overline{r_p} = \sum_{i=1}^{n} w_i \,.\, \overline{r_i} \tag{2}$$

$$\sum_{i=1}^{n} w_i = 1 \quad , \qquad w_i > 0 \tag{3}$$

w_i: weight corresponding to share i in the portfolio

(r_p)̅: expected portfolio return

r_i: return on share i

δ_p^2: portfolio return variance.

2.2 Gray Wolf Algorithm

The Gray Wolf Algorithm (GWO) is a meta heuristic algorithm inspired by the hieratical hierarchical structure and social behavior of gray wolves when hunting. This algorithm is population-based, has a simple process, and can easily be generalized to large-scale problems. Gray wolves are considered apex predators, at the top of the food chain. Gray wolves prefer to live in a group (pack), each group has an average of 5-12 members. All members of this group have a very strict social dominance hierarchy and have specific duties. In each pack of wolves, there are 4 levels for hunting, which are modeled as a pyramidal structure as shown below.

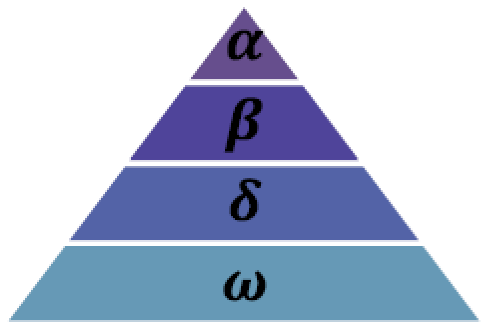

Fig. 1: The Pyramidal Structure of the Gray Wolf Model

The pack leader wolves are called alpha and can be male or female. These wolves dominate the herd. Beta wolves help alpha wolves in the decision making process and are also prone to being chosen instead of them. Delta wolves are lower than beta wolves and include old wolves, hunters, and cubs. Omega wolves are the lowest order in the hierarchy pyramid and have the least rights compared to the rest of the group. After all, they eat and do not participate in the decision-making process. Gray wolf hunting method: In explaining and teaching the gray wolf algorithm, it can be said that this algorithm includes 3 main steps:

1- Observing hunting, tracking and chasing it
2- Approaching, encircling (circling) prey and leading it astray until it stops moving.
3- attack hunting

In the main article describing the algorithm, which was introduced and explained by [49] for the first time in 2014, the hierarchical structure and social behavior of wolves during the hunting process was mathematically modeled and used to design an algorithm for optimization [49]. Optimization is done using alpha, beta and delta wolves. A wolf is assumed to be alpha as the main guide of the algorithm, and a beta and delta wolf also participate, and the rest of the wolves are considered as their followers. Gray wolves have the ability to estimate the position of prey. In the initial search there is no idea about the location of the hunt. It is assumed that alpha, beta, and delta wolves have better basic knowledge about the hunting position (the optimal point of the answer). In the GWO gray wolf optimizer, the most suiTable solution is considered as alpha, and the second and third most suiTable solutions are named beta and delta, respectively. All other solutions are considered omega. In the GWO algorithm, hunting is driven by α β and δ. The solution ω follows these three wolves. When the prey is surrounded by wolves and stops moving, an attack led by the alpha wolf begins. Modeling this process is done by reducing the vector a. Since A is a random vector in the interval [2a,-2a], as a decreases, the vector of coefficients of A also decreases. If $|A|<1$, the alpha wolf will approach the prey (and other wolves) and if $|A|>1$ the wolf will move away from the prey (and other wolves). The gray wolf algorithm requires that all wolves update their position according to the position of alpha, beta, and delta wolves. As mentioned, optimization is guided by alpha, beta and delta, and the fourth group follows these three groups. The modeling of wolf siege behavior uses the following relationships:

$$\vec{D} = \left|\vec{C}.\overrightarrow{X_p}\ (t) - \vec{X}\ (t)\right| \tag{4}$$

$$\vec{X}(t+1) = \overrightarrow{X_p}\ (t) - \vec{A}.\vec{D} \tag{5}$$

In these relationships, the number of iterations, A and C are multiplication vectors, (X_p) is the position vector of the prey and X is the position vector of a wolf. The following relationships are used to calculate vectors A and C:

$$\vec{A} = 2\vec{a}.\vec{r} - \vec{a} \tag{6}$$

$$\vec{C} = 2.\vec{r} \tag{7}$$

In the above relationships, the variable a decreases linearly from 2 to zero during iterations, and r1, r2 are random vectors in the interval [0, 1].

Figure 2 shows the schematic view of the position vectors and their next possible locations:

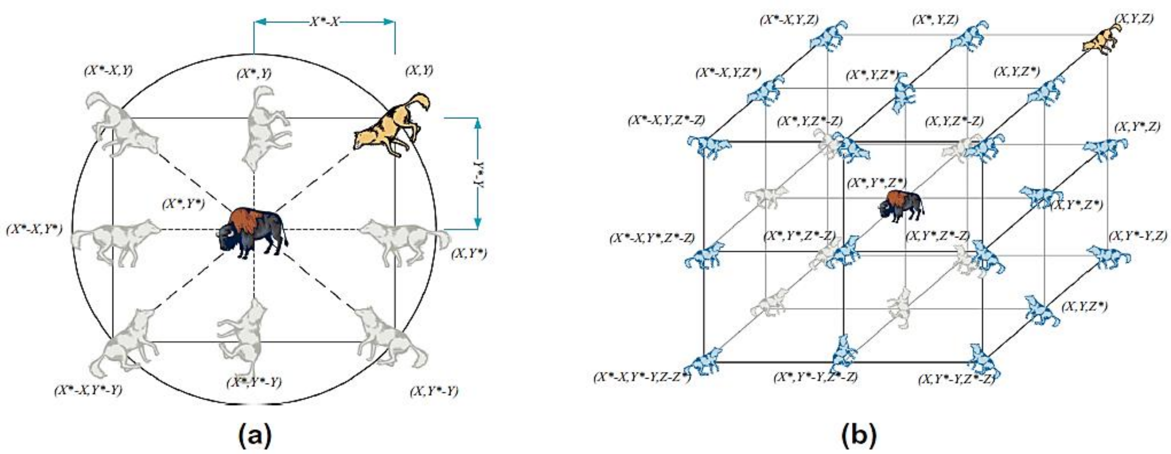

Fig. 2: Position vectors and their next possible locations [49]

The search phase has a process exactly opposite to the attack process: during the search, the wolves move away from each other to track the prey (|A|>1), while after tracking the prey, the wolves approach each other in the attack phase (|A|< 1) This process is called divergence in search - convergence in attack.

Exploration: |A|>1

Exploitation: |A|<1

Role of vector C: vector C is considered to be a creature in nature that hunts close to wolves. The C vector adds weight to the prey and makes it more unattainable for wolves. This vector does not decrease linearly from 2 to zero.

Algorithm order

- The fitness of all answers is calculated and the top three answers are selected as alpha, beta, delta until the end of the algorithm.
- In each iteration, the three best answers (alpha, beta, delta wolves) have the ability to estimate the position of the prey, and they do this in each iteration.

- In each iteration, after determining the position of alpha, beta, delta wolves, the position of the rest of the answers is updated according to them.
- In each iteration, the vector (a) and accordingly A and (C) are updated.
- At the end of the iterations, the alpha wolf position is introduced as the optimal point.

3 Research Method

This research is applied, inductive, descriptive and post-event. Applied research is research that uses the cognitive background and information provided through basic research to meet human needs and improve and optimize tools, methods, objects and models in order to develop welfare and comfort and improve the level of human life. are placed [19-23]. In terms of nature and method, this part can be considered as descriptive research. Descriptive research includes a set of methods that aim to describe the conditions or phenomena under investigation. The implementation of descriptive research can be simply to know more about the existing conditions or to help the decision-making process [24-29].

In the present study, the effectiveness of the new method of portfolio construction and unsystematic risk optimization is tested using a meta-heuristic measurement tool based on evolutionary calculations, known as the Gray Wolf Optimization (GWO) algorithm. In this way, a portfolio formation model is created using machine learning methods to minimize the risk component according to the total assets under consideration and then increase asset diversification by maximizing the maximum diversification ratio. The active stock portfolio management method is done using crowd intelligence as a maximum diversification strategy (MDS) model. The MDS framework is a two-stage model where the first stage involves building the model by learning from historical data and the second stage involves testing and fine-tuning the model to avoid over fitting. In the learning phase, there are three sub-phases: data specification, risk elimination and optimal diversification. Then, by using a set of two well-known techniques known as the maximum diversification ratio and portfolio risk share, the aforementioned strengthening techniques are used to form the stock portfolio [30-33].

The current research uses the strategy of Maximizing Diversification (MDS) proposed by [23] to form an optimal portfolio; Because it has more dimensions than [35] model and considers the risk budget (marginal contribution) (MCn) for each of the n assets of a set (U) of risky assets. In this way, the set of assets U= (X1, X2, ..., Xn) are considered first; Then the asset with the highest share of risk (MCn) is removed for selection in the P portfolio. It is worth noting that P= (W1, W2, …, Wi) and W is the weight of each asset i in the portfolio. Then the cumulative algorithm is applied to determine the wi of each portfolio asset. Problems related to the stochastic nature of time series require optimization, which provides an effective solution to solve complex high-dimensional problems. Due to the capabilities and strong results of the GWO algorithm to solve such problems [49], this algorithm is used to solve the diversification problem. Another part of the current research is determining the optimal portfolio using CO-GARCH volatility modeling. One of the fundamental drawbacks to the Markowitz portfolio model is the use of sample variance as a measure of risk, a variance known as symmetric. Considering the above, and especially with the availability of data, a natural way to take into account the time-based constraints of discrete processes is to use GARCH family models, Kloppelberg et al. He suggested bringing the continuous time model. The continuous-time GARCH (COGARCH) model is a direct analog of the discrete-time GARCH, based on a Lévy process in a background, and generalizes the basic features of the discrete-time GARCH process in a natural way. The statistical population of the current research included the information of 698 companies from the companies admitted to the Tehran Stock Exchange for the period of 2010 to 2019. The analysis of the results for the gray wolf algorithm is done using MATLAB software and to estimate the fluctuations and adjust the Markowitz model based on the CO GARCH model using R software. In general, the GWO algorithm is a two-stage research framework, in which the first stage includes building the model by learning from historical data, and the second stage includes testing the model to avoid over fitting. The learning stage, - includes data determination, risk elimination and optimal diversification. Then, using the desired research techniques, the optimal portfolio is extracted.

Regarding the adjusted Markowitz model, as mentioned, the optimization is applied with the differential evolution algorithm implemented in R software through the DEoptim package of [37]. Each portfolio is rebalanced monthly over an annual period. To diversify and reduce risk, the limit of weight ωi for share i should be between 0.2 and 0.6. [38-41]. In order to compare the performance of the optimal (suggested) portfolios, the comparison test (difference) of the averages of two independent groups is carried out using the Levine test and the Sharpe ratio (as a risk assessment criterion). The Sharpe ratio represents the ratio of the average return earned over the risk-free rate of return. Deducting the risk-free rate from the average return separates the profit associated with risky activities.

4 Findings

Gray wolves have the ability to detect the location of prey and surround them. The usual hunt is under the leadership and guidance of the alpha wolf, beta and delta also participate in the hunt from time to time. However, in an abstract search space, we have no idea about the optimal position (predator. In order to mathematically simulate the hunting behavior of gray wolves, we assume that alpha is the best candidate solution), beta, and delta about the potential position. Hunting has better knowledge. Therefore, we save the first three solutions (the best solutions) obtained so far and force other search agents, including Omega, to update their position based on the position of the best search agents. The following formulas in This relationship is suggested.

$$X(t+1) = X_p(t) - A*D(t) \tag{8}$$

$$D(t) = |C * Xp(t) - X(t)| \qquad t=1,2,\dots, tmax \tag{9}$$

$$A = a(2r_1 - 1) \;, \qquad c = 2r \qquad , \quad r_1 \& r_2 \text{ are random vectors} \in [-1,1] \tag{10}$$

In these equations, t represents the current iteration. A and C are coefficient vectors. (t) (Xp) and X (t) are the position vector of the prey and the position vector of the wolf, respectively. a decreases linearly from 2 to 0. (r_1) and (r_2) are random vectors in (1 and -1). The gray wolf has the ability to identify the hunting place and surround it. The hunt is usually led by the alpha. Beta and Delta may also occasionally participate in hunting. But, in a discrete search space there is no idea about the optimal (prey) location. In order to mathematically simulate gray wolf hunting behavior, it is assumed that alpha is the best answer among the available answers, beta and delta know better about the hunting place. Therefore, the 3 best obtained answers are saved and other search agents (including Omega) are forced to update their position according to the position of the best search agent. Therefore, the following equations are presented.

$$\overrightarrow{D_a} = |\overrightarrow{C_1}.\, \overrightarrow{X_a} - \vec{X}| \tag{11}$$

$$\overrightarrow{D_\beta} = |\overrightarrow{C_2}.\, \overrightarrow{X_\beta} - \vec{X}| \tag{12}$$

$$\overrightarrow{D_\delta} = |\overrightarrow{C_3}.\, \overrightarrow{X_\delta} - \vec{X}| \tag{13}$$

$$\overrightarrow{X_2} = |\overrightarrow{C_\beta} - \overrightarrow{A_2}.\overrightarrow{D_\delta}| \tag{14}$$

$$\overrightarrow{X_2} = |\overrightarrow{C_\delta} - \overrightarrow{A_3}.\overrightarrow{D_\delta}| \tag{15}$$

$$\vec{X}(t+1) = \frac{\overrightarrow{x_1} + \overrightarrow{X_2} + \overrightarrow{X_3}}{3\gamma} \tag{16}$$

To create the optimal function, 100 repetitions with the range (10 and -10) have been used. In this function, an interval of more than 10 and less than -10 is defined. The numbers that are more than 10 and less than -10 violated the defined interval, that is, they are outside the optimal points that must be corrected, and the model is run again after the correction. First, we identify the wrong answers and the correct answers remain:

The numbers that were greater than 10 get the value of 10 and the numbers that were less than -10 get the value of -10. In the GWO algorithm, optimization is done with the help of α-β-δ. ω wolves follow these three wolves.

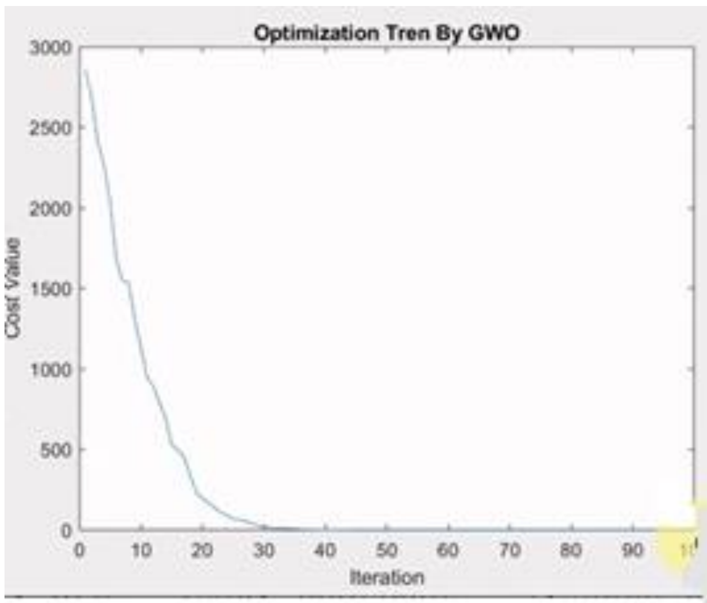

Fig. 3: Optimal cost function

After obtaining the optimal diversification portfolio function, the gray wolf algorithm is executed.

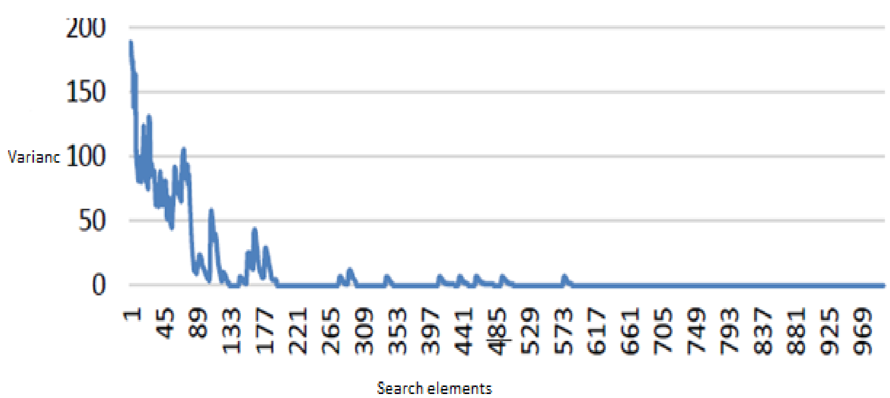

Fig. 4: Diversification in the gray wolf algorithm

The gray wolf algorithm enters the phase of diversification in a specific jump. The reason for this is the presence of powerful operators in the gray wolf algorithm to search the entire space coherently and focus on a specific range to find the best answer. In fact, the gray wolf algorithm has a good speed in finding the optimal solution. The gray wolf algorithm has a very high computing power and has had relative or absolute superiority over other algorithms in almost all the problems that have been used.

$$y_t = \varepsilon_t \sigma_t, \quad \sigma_t^2 = \omega + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{17}$$

where α, β, ω are completely positive. The specification of the COGARCH model consists of two equations similar to the above Equation with a unit source of change driven by the Levy process $(L_t)_t \geq 0$. More precisely, $(G_t)_t \geq 0$ in the COGARCH process is defined in terms of the stochastic differential equation:

$$dG_t = \sigma_t - dL_t, \quad t \geq 0 \tag{18}$$

where the fluctuation process, σ_t, is determined by the following stochastic differential equation:

$$d\sigma_t^2 = (\omega - \gamma\sigma_{t-}^2)dt + \alpha\sigma_{t-}^2 d[L,L]_t^d \sigma_t^2, \quad t > 0 \quad , \omega > 0, \gamma \geq 0, \alpha > 0 \tag{19}$$

The quadratic variance trend $[L, L]t^d$ of parameter L is:

$$[L,L]_t^d = \sum_{0<s\leq t} (\Delta L_s)^2 \qquad , L_t = L_t - L_t - \text{ for } t \geq 0 \text{ with } L_{0-} = 0 \tag{20}$$

And finally, to obtain the stochastic differential solution, the auxiliary Lévy process can be used as described in the following model:

$$\sigma_t^2 = \exp\left(-\frac{x_1 + x_2 + X_3}{3}\right)\left(\omega \int_0^t \exp\left(\frac{x_1 + x_2 + X_3}{3}\right) ds + \sigma_0^2\right), t \geq 0 \tag{21}$$

The latter equation shows that σ_t^2 follows a generalized Ornstein-Uhlenbeck process parameterized by α,γ,ω and driven by the Lévy L process. The returns to the financial system are modeled by increasing the trend of Gt, h = Gt + h − Gt. The Lévy trend is the only source of randomness, and when it jumps, both price and volatility jump at the same time [44]. Finally, the optimization is done with the differential development algorithm implemented in R software through the DEoptim package of [42]. Each portfolio is made monthly over an annual period. For diversity and reduction, the weight ω_i for a value should be between 0.2 and 0.6. Therefore, it is defined as follows:

$$\sigma_t^2 = \exp\left(-\frac{X_1 + X_2 + X_3}{3}\right)\left(\omega \int_{-10}^{10} \exp\left(\frac{X_1 + X_2 + X_3}{3}\right) ds + \sigma_0^2\right) \tag{22}$$

The output of the function is as described in the following Table:

Table 1: Optimal points

Delta	Beta	Beta	Delta
Delta	Beta	Beta	Beta
Beta	Alpha	Alpha	Beta
Beta	Alpha	Alpha	Beta
Beta	Alpha	Alpha	Beta
Delta	Beta	Beta	Beta
Delta	Beta	Beta	Delta

Alpha is the main optimal points and beta is the points after alpha that is chosen as the optimal investment points. Deltas can be selected or not selected. Omega points were completely removed. Alphas are in the center, betas are after alphas and deltas are after betas. For comparison, the comparison test of the averages of two independent groups was used using Levine's test and Sharpe's ratio.

Table 2: Levine's test to compare the difference between the efficiency of Gray Wolf and Markowitz

Mean difference test	f	sig
The difference between the efficiency of gray wolf and Markowitz	34.60	0.000

Since the significance level of Levin's test is 0.000, the assumption of equality of population variance is not confirmed, and the mean equality test is performed with the assumption of inequality of variance.

Table 3: Mean difference test of groups

Mean difference test	t	df	sig	upper limit	lower limit	difference in averages	standard deviation of the difference
The difference between the efficiency of gray wolf and Markowitz	11.375	690.6	0.000	0.0437	0.1092	0.0917	0.0091

Considering that the value of the corresponding significance level is 0.000. Therefore, the assumption of equality of groups is rejected and there is a significant difference between the efficiency of the two groups. And according to the difference between the limit and the minimum, we can conclude that the efficiency of the gray wolf algorithm is higher than the Markowitz ratio.

Table 4: Sharp's criterion for the output of the two methods used

Risk aversion factor	0.000	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900	1.000
Markowitz	0.250	0.280	0.368	0.605	0.759	0.749	1.109	1.000	0.935	0.942	0.823
GWO	0.287	0.281	0.371	0.601	0.738	0.698	1.016	1.046	1.027	0.848	0.751
Markowitz	0.259	0.291	0.340	0.587	0.754	0.776	1.017	1.024	0.885	0.904	0.802
GWO	0.310	0.295	0.366	0.635	0.752	0.697	0.825	1.126	1.158	0.882	0.782
Markowitz	0.277	0.295	0.319	0.615	0.741	0.807	1.017	0.961	0.835	0.910	0.814
GWO	0.299	0.299	0.366	0.671	0.631	0.5	0.780	1.051	1.078	0.818	0.833

Based on the Sharpe ratio, the GWO algorithm performs better than the adjusted Markowitz model based on CO-GARCH modeling.

5 Conclusion

In this research, First, the optimal investment model was presented based on the gray wolf algorithm. After extracting the optimal model, the efficiency of the gray wolf algorithm was compared with the Markowitz model based on the CO-GARCH modeling. The results showed that the gray wolf algorithm has a higher efficiency than the Markowitz model based on the CO-GARCH modeling. Optimization studies have been done using different algorithms. But the result is different in terms of the subject which has a comparative mode. In this research, the gray wolf algorithm is combined with the modified Markowitz model based on CO-GARCH modeling, which has a better optimal state.

References

[1] Ebrahimi, S.B, Jirofti, A., Abdi, M., Optimizing the investment portfolio under the fuzzy credit theory using the average-value model under conditional risk. *Financial knowledge of securities analysis*, 2017; 11(37): 17-27.

[2] Pour Ahmadi, Z., Najafi, A.A., Dynamic optimization of the investment portfolio according to the cost of financial *engineering transactions and securities management*, 2014: 22(6): 146-127.

[3] Dedekhani H., Hojjati Estani S., Presenting a multi-objective planning model for stock selection considering fuzzy value-at-risk: fuzzy credit theory approach. *Financial Engineering and Securities Management*, 2015; 8(32): 239-268.

[4] Ramoz, N., Akbari Aghamshahdi, Z., Atef Dost, A., Choosing the optimal portfolio using consensus planning model in Tehran Stock Exchange. *Financial Management Strategy*, 2019; 8(1): 57-74.

[5] Ramtinnia, S., Atrachyi, R., Stock portfolio optimizer using differential evolution algorithm and conditional risk value approach. *Knowledge of analysis. Securities*, 2017; 11(40): 25-36.

[6] Seifourib Z., Amarib M., Investigation of combined refrigeration systems with microturbine and absorption chiller, *scientific and research journal of energy management*, 2013; 2(3).

[7] Rostami, M.R., Behzadi, A., Higher order moments in stock portfolio optimization in fuzzy environment, *Journal of Financial Engineering and Securities Management*, 2014; 24: 41-61.

[8] Rezaei, As, Falahati, A., and Sohaili, K., Stock portfolio optimization using three-objective particle aggregation algorithm. *Quarterly Journal of Applied Economic Theories*, 2017; 5(4): 31-52.

[9] Humaifar, S., and Roghnian, E., The use of sustainable optimization patterns and ideal planning in the problem of choosing a multi-period investment portfolio, *Financial Engineering and Securities Management*, 2015; (28) 2: 157-163.

[10] Shiri Kahri, A., Dedekhani, H., Khalili, K., Saeedi, P., Comparative study of multi-period multi-objective portfolio optimization model in fuzzy credit environment with different risk criteria. *Financial Management Strategy*, 2016; 5(3): 1-26.

[11] Shahmohammadi, Mohsen, Emami Meibdi, Lili, Zare Mehrjardi, Yahya, Presentation of a hybrid intelligent algorithm based on the mean-variance-skewness fuzzy model for portfolio selection, *International Journal of Industrial Engineering and Production Management*, 2018; 23: 458-448.

[12] Kazemi Mian Gaskeri, M., Yakideh, K., Qolizadeh, M.H., Optimizing the stock portfolio (using the value-at-risk model on cross-functionality), *Financial Management Strategy*, 2016; 5(2): 159-183.

[13] Anagnostopoulos, K. & Mamanis, G. Multiobjective evolutionary lgorithms for complex portfolio optimization problems. *Springer-Verlag*, 2009; 8(3): 259-279.

[14] Bellman,R.E., & Zadeh, L. A., Decision-making in a fuzzy environment. *Management science*, 1970; 17(4): 141.

[15] Bhattacharyya, R., Hossain, S. A., & Kar, S., Fuzzy cross-entropy, mean, variance, skewness models for portfolio selection. *Journal of King Saud University-Computer and Information Sciences*, 2014; 26(1): 79-87.

[16] Beni G, Wang J, Swarm intelligence in cellular robotic systems. In: Dario P, Sandini G, Aebischer P (eds) Robots and biological systems: towards a new bionics? *Springer, Berlin, Heidelberg*, 1993; 703–712.

[17] Bridgewater: the all weather story. https ://www.bridg ewate r.com/resou rces/all-weath er-story .pdf. Accessed Mar 2019.

[18] Cai L, Jin Y, Qi Q, Xu X, a comprehensive study on smart beta strategies in the a-share market. Appl Econ 2018; 50(55): 6024– 6033. doi: 10.1080/00036 846.2018.14891 13.

[19] Cesarone, F., Tardella, F., Equal risk bounding is better than risk parity for portfolio selection. *J Glob Pptimiz*, 2017; 68(2):439–461. doi: 10.1007/s1089 8-016-0477-6 (The Journal of Computational Finance, 1)

[20] Choueifaty Y, Coignard Y, Toward maximum diversification. *J Portfolio Manag*, 2008; 35(1):40–51. doi: 10.3905/ JPM.2008.35.1.40 (Cited by 102)

[21] Choueifaty Y, Froidure T, Reynier J, Properties of the most diversified portfolio. *J Investment Strateg*, doi: 10.2139/ssrn.18954 59.

[22] Chang, T, G., Yang, S, C., Chang, K.G., Portfolio optimization problem different risk measure using genetic algorithm, *Expert system with application*, 2009; 36: 10529-10537.

[23] Chen, L. H., Huang, L., Portfolio optimization of equity mutual funds with fuzzy return rates and risks. *Expert Systems with Applications*, 2009; 36(2): 3720-3727.

[24] Chen, L., Pan, H., Selection of stocks using constrained fuzzy AHP and PROMETHEE. *Advances in information Sciences and Service Sciences (AISS*), 2013; 5(15): 97- 103

[25] Connor, G., Active portfolio management: a quantitative approach to providing superior returns and controlling risk. *Rev Financ Stud*, 2000; 13(4): 1153–1156

[26] Davey, K., Building algorithmic trading systems a trader's journey from data mining to Monte Carlo simulation to live trading, 2014.

[27] Deb, K., Agrawal, S., Pratap, A., Meyarivan, T., A fast elitist non-dominated sorting genetic algorithm for multi-objective optimization: NSGA-II. *Lecture notes in computer science,* 2000; 1917: 849-858.

[28] Dubois, D., Prade, H., The mean value of a fuzzy number. *Fuzzy sets and systems*, 1987; 24(3): 279-300.

[29] Dubois, D., Prade, H., possibility theory: Approach to computerized processing of uncertainty, *plennm* 1988; 4.

[30] Enea, M., Piazza, T. Project selection by constrained fuzzy AHP.Fuzzy Optimization and Decision Making, 2004; 3(1): 39-62.

[31] Fang, Y., Lai, K. K., Wang, S. Y., Portfolio rebalancing model with transaction costs based on fuzzy decision theory. *European Journal of Operational Research*, 2006; 175(2): 879- 893.

[32] Giove, S., Funari, S., Nardelli, C., An interval portfolio selection problem based on regret function. *European Journal of Operational Research*, 2006; 170(1): 253-264.

[33] Gupta, P., Inuiguchi, M., Mehlawat, M. K., Mittal, G., Multiobjective credibilistic portfolio selection model with fuzzy chance-*constraints.Information Sciences*, 2013; 229: 1-17.

[34] Gupta, P., Mehlawat, M. K., Inuiguchi, M., Chandra, S., Fuzzy Portfolio Optimization. Springer-Verlag, Berlin. 2014.

[35] Hao, f. f., Liu, Y. K., Mean-variance model for portfolio selection whit fuzzy random return. *Journal of Applied Mathematics and Computing*, 2009; 30(1-2): 9-38.

[36] Hasuike, T., Katagiri, H., Ishii, H., Portfolio selection problems with random fuzzy variable returns. *In Fuzzy Systems Conference*, 2007, FUZZ-IEEE 2007. IEEE International, 1-6, IEEE.

[37] Heilpern, S., The expected value of a fuzzy number. *Fuzzy sets and Systems*, 1992, 47(1), 81-86.

[38] Jana, P., Roy, T. K., Mazumder, S. K., Multi-objective possibilistic model for portfolio selection with transaction cost. *Journal of Computational and Applied Mathematics*, 2009; 228(1): 188-196.

[39] Kamdem, J. S., Deffo, C. T., Fono, L. A., Moments and semi-moments for fuzzy portfolio selection. Insurance: Mathematics and Economics, 2012; 51(3): 517-530

[40] Kiris, S., Ustun, o., Fuzzy MCDM Approach of Stocks Evaluation and Portfolio Selection*, 24th Mini EURo International Conefernce*, Izmir, Turkey, 2010; 23-26:.330-336.

[41] Kung, J. Y., Chuang, T. N., Ky, C. M., A fuzzy MCDM method to select the best company based on financial report analysis. *InFuzzy Systems (FUZZ), 2011 IEEE International Conference on (2011;*2013-2017)

[42] Kwakernaak, H., Fuzzy random Variables-I. Definitions and theorems. *Information Sciences*, 1978; 15(1):1-29.

[43] Lai, K. K., Wang, S. Y., Xu, J. P., Zhu, S. S., Fang, Y., A class of linear interval programming problems and its application to portfolio selection. *IEEE Transactions on Fuzzy Systems*, 2002;10(6): 698-704.

[44] Li, J., Xu, J. A novel portfolio selection model in a hybrid uncertain environment. *Omega*, 2009; 37(2): 439-449.

[45] Li, X., Qin, Z., Kar, S., Mean-variance-skewness model for portfolio selection with fuzzy returns. *European Journal of Operational Research*, 2010; 202(1): 239-247

[46] Wiley Trading, Hoboken Gupton G, Finger C, Bhatia M., CreditMetrics technical document. J.P. Morgan & Co., New York Investopedia: www.inves toped ia.com (2019). www.inves toped ia.com, www.inves toped ia.com

[47] Markowitz, H., Portfolio selection. *J Finance*, 1952; 7(1):77–91. Doi:10.1111/j.1540-6261.1952.tb015 25.x

[48] Marshall, C., Isolating the systematic and unsystematic components of a single stock's (or portfolio's) standard deviation. *Appl Econ*, 2015; 47(1):1–11. doi:10.1080/00036 846.2014.95965 2 (Cited By 3)

[49] Mirjalili, S., Mirjalili, S., Lewis, A., Grey wolf optimizer. *Adv Eng Softw*, 2014; 69:46–61. doi:10.1016/j.adven gsoft .2013.12.007

[50] Olorunda, O., Engelbrecht, A., Measuring exploration/exploitation in particle swarms using swarm diversity, 2008: 1128–1134. doi:10.1109/CEC.2008.46309 38 (cited by 104).

[51] Pun, C. S., Time-consistent mean-variance portfolio selection with only risky assets. *Economic Modelling*, 2018; 75: 281-292.

[52] Rubin, A., Segal, D., The effects of economic growth on income inequality in the US. *Journal of Macroeconomics*, 2015; 45: 258-273.

[53] Rezaei, S., Vaez-Ghasemi, M., A new Method for Sustainable Portfolio Selection with DEA, *TOPSIS and MIP in Stock exchange*, 2020.

[54] Seltzer, M., Forbes, forbes magazine. www.forbe s.com/sites /forbe sfina nceco uncil /2018/01/11/*premi um-finan ce-risks -andconsi derat ions/. Accessed Mar* 2019

[55] Sharpe, W., The sharpe ratio. *J Portfolio Manag* 1994; 21:1.

[56] Sharpe, W.F., Budgeting and monitoring pension fund risk. Financ Anal J .2002; 58(5):74–86.

[57] S. PGlobal: Us stock index s and p 100 official site 2019.

[58] Stuart, A., Markowitz, H.M., Portfolio selection: efficient diversification of investments. 1959, OR 10:4.

[59] Stoilov, T., Stoilova, K., Vladimirov, M., Explicit Value at Risk Goal Function in Bi-Level Portfolio Problem for Financial Sustainability. *Sustainability*, 2021,13(4): 2315.

[60] Sinha, N., Growth, Inequality and Structural Adjustment: An Empirical Interpretation of the S-Curve for the Indian Economy. In Economic Growth, *Economic Performance and Welfare in South Asia*, Palgrave Macmillan, London 2005; 369-383.

[61] Teiletche, J., Roncalli, T., Maillard, S., The properties of equally-weighted risk contributions portfolios. *In: IDEAS Working Paper Series from RePEc*, 2010.

[62] Shaolin, M., Jiangfeng. W., Zhequan, Y., Yiping, D., Bingheng, L., Thermodynamic analysis of a new combined cooling, heat and power system driven by solid oxide fuel cell based on ammonia–water mixture, *Journal of Power Sources*, 2011; 196: 8463– 8471, doi: 10.1016/j.jpowsour.2011.06.008

[63] Xu, D., Ren, J., Dong, L., Yang, Y., Portfolio selection of renewable energy-powered desalination systems with sustainability perspective: A novel MADM-based framework under data uncertainties. *Journal of Cleaner Production*, 2020; 275:124114. doi: 10.1016/j.jclepro.2020.124114

[64] Wang, J., He, F., Shi, X., Numerical solution of a general interval quadratic programming model for portfolio selection. *PloS one*, 2019, 14(3), e0212913, doi: 10.1371/journal.pone.0212913

[65] Wan P, Alpha and risk factors. *Wiley*, 2015. doi: 10.1002/97811 19057 871.ch11.

[66] Zhu, S.S., Li, D., Sun, X., Portfolio selection with marginal risk control. *J Computat Finance*, 2010. doi: 10.21314 / JCF.2010.213.

[67] Zhou, F., Wang, X., Goh, M., Zhou, L., He, Y., Supplier portfolio of key outsourcing parts selection using a two-stage decision making framework for Chinese domestic auto-maker. *Computers & Industrial Engineering*, 2019; 128: 559-575. doi: 10.1016/j.cie.2018.12.014