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Optimization of Estimates and Comparison of Their Efficiency under Stochastic Methods and Its Application in Financial Models

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ABSTRACT

In this paper, we first introduce stochastic differential equations and provide the definition and basic theories of Monte Carlo and quasi-Monte Carlo methods. We specifically focus on Sobel and Halton sequences. We utilize simulations under these methods to compare their efficiency in obtaining solutions. The results indicate that the approximation achieved by the Sobel sequence is significantly superior to other stochastic methods.
Next, we examine the efficiency of random and quasi-random methods, considering both geometric Brownian movement and the price index of the Tehran stock (equal weight and weight-value). Our findings demonstrate that the quasi-Monte Carlo method outperforms other approaches in terms of efficiency.

1 Introduction and Preliminaries

The study of basic concepts and tools required for stochastic methods, including the calculation and estimation of answers and approximations in financial models, as well as stochastic differential equations, has been extensively explored by numerous authors [1, 7, 9, 11, 15, 14]. Many of these phenomena involve random factors, which necessitates modeling using random differential equations. These random factors manifest in the form of white noise. In order to analyze these equations, they are typically expressed in the form of integrals. However, the integration of white noise cannot be computed using Riemann and Lebesgue integrals. Ito's integral is necessary to address this issue.
In recent years, the most effective and widely used methods for solving these types of simulation equations have involved discrete-time approximation of sample paths using computers. This method relies on discretizing the time frame [0, T] into discrete intervals and generating approximate values for sample paths step by step at these discrete time points. Therefore, this method aims to obtain the best possible answers using random methods based on Markov chain simulation [13]. Monte Carlo simulation

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is a widely used and valuable method that finds applications in various scientific fields, including applied mathematics, biological mathematics, finance, economics, and others [3, 4, 5, 6, 7, 9, 11, 13, 16, 17]. In the following sections, we provide a brief overview of relevant results and definitions related to the theory of stochastic methods. For more detailed explanations, refer to [2, 18]. This paper consists of four sections. The second section covers the literature review and theoretical background of option pricing, including an explanation of the main methods such as Monte Carlo models. In the third section, we outline the methodology employed in our study and present the results of our tests. Finally, the fourth section discusses the results and draws inferences based on them. In the third section, we explained the methodology of our study and we demonstrated the results of our tests, and finally, in the fourth section, we discussed the results and inference regarding them.

1.1 Stochastic Differential Equations with m Independent Noise

Definition 1.1 *(see [15]). Let* $(P.\omega.A)$ *be a probability space and* $d \in \mathbb{N}$. *The random process* $W_t: \Omega \to \mathbb{R}.\, 0 \leq t \leq T$ *is a called the* d*-dimension Brownian motion whenever the following properties are established:*

1. For all $\omega \in \Omega$, then $t \to \ W_t(\omega)$ be a continuous function on $[0.T]$.
2. Each group intervals $\{W(t_0).W(t_1) - W(t_0).W(t_2) - W(t_1).\ldots.W(t_k) - W(t_{k-1})\}$, where $0 \leq t_0 < t_1 < \cdots < t_k = T$ be independent.
3. $W(t) - W(s)$ has a normal distribution with mean 0 and variance $(t-s)I_d$, where 0 is the null vector of order d and I_d is the $d \times d$ identity matrix.

In general, a vector can be considered as a column or a row vector. Consider a m-dimensional wiener process $W = \{W_t: t \in [0.T]\}$ with $W_t^1.W_t^2.\ldots.W_t^m$ components that separate wiener processes. Now consider a d-dimensional vector function $a: [t_0.T] \times \mathbb{R}^d \to \mathbb{R}^d$ and a matrix $d \times m$ in form $b: [t_0.T] \times \mathbb{R}^d \to \mathbb{R}^{d\times m}$ and put the diffusion coefficient $t \in [0.T]$. Now a d-dimensional random differential equation is in form:

$$dX_t = a(t.X_t)dt + b(t.X_t)dW_t.$$

which can also be expressed in the following form:

$$X_t = X_{t_0} + \int_{t_0}^{t_a} a(s.X_s)ds + \int_{t_0}^{t_b} b(s.X_s)dW_s.$$

with the initial value of $X_{t_0} \in \mathbb{R}^d$, in which the integral of Ito and Lebesgue are counted as components.

1.2 Monte Carlo Algorithms and Methods

Nowadays, the use of a variety of simulation methods in different sciences such as economics, finance, risk management, etc. is generalized [3, 4, 6, 7, 13, 18]. Among different simulation methods, Monte Carlo method is more suitable for research and financial calculation than other methods. The tendency to use Monte Carlo methods increases when it is impossible to calculate the exact response using definitive algorithms. Simulation is a method through which a behavior similar to the random component of the designed pattern can be created and thus, it is possible to make the prediction with its degree of reliability calculated. Assuming that we want to set an estimate by Monte Carlo method for parameter λ from the given M community. In this method, we must first find a $T(X)$ estimator, in which X is a random variable with probability density function $f_X(x)$ so that this estimator applies to the following two basic conditions:

1. The estimator T is skewed, i.e. $E(T(X)) = \lambda$,
2. The estimator T estimator has finite variance, i.e. $\mathrm{Var}(T(X)) = \sigma^2 < \infty$.

Then, we consider random variables $X_1 ..... X_n$ distribution f_x which

$$\hat{\theta}_n(X_1 .... X_n) = \frac{1}{n}\sum_{i=1}^{n} T(X_i). \quad (1)$$

Obviously, $E(\hat{\theta}_n) = \theta$ and $\mathrm{Var}(\hat{\theta}_n)$Ì, $= \frac{1}{n}\sigma^2 < \infty$. Therefore $\hat{\theta}_n$ can be considered as a Monte Carlo estimator. This estimator has the following properties (See [2]):

1. $\hat{\theta}_n$ is convergent in probability to θ, i.e. $\lim_{n\to\infty} p(|\hat{\theta}_n - \theta| \geq \varepsilon) = 0$, for all $\varepsilon > 0$,
2. The mean convergence has a second order to the parameter θ, i.e. $\lim_{n\to\infty} E(|\hat{\theta}_n - \theta|^2) = 0$.

Consider the following value:

$$m_M = \frac{1}{M}\sum_{r=1}^{M} X_{\omega_r}. \quad (2)$$

Here $X(\omega_r)$ is the random process sample path of the X random process. According to the central limit (1) it has almost normal distribution with average μ and variance σ^2 in which $\mu = E(X)$ and the big numbers rule states that $\sigma^2 = \mathrm{Var}(X)$ and

$$m_M \to \mu = E(X) \quad \text{as} \quad M \to \infty. \quad (3)$$

Thus (1) can be used for the approximation of $E(X)$. Simply put, the sample path is calculated from X and expresses the mean of the sample as an approximation for the mathematical values of X. We know that for approximation (1) (see [11]) we have:

$$|m_M - E(X)| = O(\frac{1}{\sqrt{M}}). \quad (4)$$

Therefore, the Monte Carlo method is convergent with the $\frac{1}{\sqrt{M}}$ order. In other words, the main idea of Monte Carlo's method is to estimate a certain value by calculating the mathematical values for a large number of independent simulated routes. The basis of this method is based on the law of large numbers and states that the average number of large enough of the samples is almost equal to their average. Monte Carlo's method consists of two steps:

1. We simulate a large number (up) of X samples using random number generators to calculate independent random variables that are also distributed with X.
2. We approximate the average using the equation (1).

In the following, we present a general concept of simulating the values of expectation for an answer from the random differential equation X with a given function g. In general, the approximate error has two parts: statistical error and time discrete error. Estimation of statistical error is related to the central limit issue and estimating the time discrete error from Euler method is directly related to the assumption of the Fymman-Kac theorem.

1.3 The Equation of Random Differentiation and Monte Carlo Method

Consider

$$dX(t) = b(t.X(t))dt + \sigma(t.X(t))dW(t). \quad (5)$$

where $t_0 \leq t \leq T$. We want to calculate the value $E[g(X(T))]$. Monte Carlo's approach to approximation.

$$E(g(X(T))) = \sum_{j=1}^{N} \frac{g(\hat{X}(T.W_j))}{N} \tag{6}$$

It is stated that here $\bar{X}$ is approximately X of the Euler method. Error in Monte Carlo method is as follows

$$\begin{aligned} E(g(X(T))) \quad & -\sum_{j=1}^{N} \frac{g(\hat{X}(T.W_j))}{N} \\ & = E[g(X(T)) - g(\hat{X}(T))] - \sum_{j=1}^{N} \frac{(g(\hat{X}(T.W_j)) - E[g(\hat{X}(T))])}{N}. \end{aligned} \tag{7}$$

On the right is the first part of the time discrete error and the second part is the statistical error.

Statistical Inference of Monte Carlo Estimator

Random integration by Monte Carlo approximation method in form $\theta_n(X) = \frac{1}{n}\sum_{k=1}^{n} \phi(X_k)$ and the exact value

$$\theta = \int_{\Omega} g(x)dx = \int_{\Omega} \frac{g(x)}{f(x)} f(x)dx = E[\phi(X_k)]. \tag{8}$$

Consider $\phi(x) = g(x)/f(x)$ as well as $X_1....,X_n$ a random sample with density function $f(\cdot)$ and $\mathrm{Var}[\phi(X_k)] = \sigma^2$. Because absolute error $|\theta_n - \theta|$ is a random quantity, so it is not possible to find the exact error bound. So the construction of a confidence interval is required, in which we calculate the average and variance of θ_n:

$$E[\theta_n] = E[\frac{1}{n}\sum_{k=1}^{n} \phi(X_k)] = \frac{1}{n}\sum_{k=1}^{n} E[\phi(X_k)] = \frac{1}{n}.n\theta = \theta \tag{9}$$

θ_n is an estimator unbiased so:

$$\begin{aligned} \mathrm{Var}[\theta_n] \quad & = E[(\frac{1}{n}\sum_{k=1}^{n} \phi(X_k) - E[\frac{1}{n}\sum_{k=1}^{n} \phi(X_k)])^2] \\ & = \frac{1}{n^2} E[\sum_{k=1}^{n} (\phi(X_k) - E[\phi(X_k)])^2] = \frac{1}{n^2}\sum_{k=1}^{n} \mathrm{Var}[\phi(X_k)] = \sigma^2/n. \end{aligned} \tag{10}$$

According to Chebyshev's inequality we get

$$P(|\theta_n - \theta| \leq \sigma/\sqrt{\varepsilon n}) \geq 1 - \varepsilon. \tag{11}$$

Therefore $(\theta_n - \sigma/\sqrt{\varepsilon n}.\theta_n + \sigma/\sqrt{\varepsilon n})$ is a confidence interval at the level of $1 - \varepsilon$ for θ, in which the error bound is $\sigma/\sqrt{\theta n}$ and obviously decreases with the increase of n or decrease σ the error. Of course, according to the central limit theorem, a confidence interval at the level of 95% for θ is $(\theta_n - 1/96\sigma/\sqrt{n}.\theta_n + 1/96\sigma/\sqrt{n})$, which is better than the bound obtained through the inequalities of Chebyshev.

Errors in Quasi-Monte Carlo Method

The difference has a major role in determining the boundary of error in quasi- Carlo method. Koksma's inequality [10] in multidimensional case is expressed in the Hardy-Krause's variation as follows:

Definition 1.2 (Variation in the Hardy-Krause's Sense) *For every* $1 \leq k \leq s$ *and* $1 \leq j_1 <\ldots< j_k \leq s$*, define*

$$V(f) = \sum_{k=1}^{s} \sum_{1 \le J_1 \le \ldots \le j_k \le s} V^{(k)}(f, j_1, \ldots, j_k) \quad (12)$$

where

$$V^{(k)}(f.j_{1\ldots.}j_k) = \int_0^1 \ldots \int_0^1 \left| \frac{\partial^s f}{\partial x_1 \ldots \partial x_k} \right| dx_1 \ldots dx_k, \quad (13)$$

This case results in a quasi-Monte Carlo estimation error bound for functions with bounded variations.

Theorem 1.3 *([2, 8, 12]). If f has bounder $V(f)$ changes in the Hardy-Krause definition, we have $p = X_1 \ldots . X_s$ of $[0.1]^s$ for each set*

$$\left| \frac{\sum_{n=1}^{N} f(X_n)}{N} - \int_{[0.1]} f(x) dx \right| \le v(f) D_N^*(p). \quad (14)$$

This inequity states that sequences with low-differences lead to lower errors [8].

Random Numbers

Suppose A is a random variable with a uniform continuous distribution function on the interval (0,1) and $U_1 \ldots . U_n$ random variables are independent and also distributed. If these random variables have real values $u_1 \ldots . u_n$, then these numbers are called random numbers. But whenever we generate numbers by specific mathematical functions and the numbers generated in statistical tests apply, then these numbers are called quasi-random. Sobol, Zarmba, Halton and Fore sequences are some examples of quasi-random number generating sequences (see [2]).

Halton Sequence

The Halton sequence is in high dimension cube $[0.1]^s$. The n'th element of the Halton sequence in $[0.1]^s$ is defined as follows

$$x_n = (\phi_{b_1}(n). \ldots . \phi_{b_s}(n)). \qquad n = 0.1. \ldots . \quad (15)$$

where b_s represents s'th prime number.

1.4 Sobel Sequence

In 1967 the Sobel trail was presented. In the Sobel sequence, the constant value of binary digits is used for all dimensions. So the Sobel sequence is much faster and simpler. This feature produces random numbers with less convergence in high dimensions. To create this sequence, we first write n in binary digits $n = \sum_{i=0}^{M} a_i 2^i$. so that M is the smallest number greater than or equal to $\log_2^n$ and a_i are zero or one values. We consider the q degree primitive polynomial $x^q + c_1 x^{q-1} + \cdots + c_{q-1} x + 1$, where $c_i . i = 1. \ldots . q - 1$ are zero and one and m_i are produced using coefficients c_i as follows

$$m_i = 2c_1 m_{i-1} \oplus 2^2 c_2 m_{i-2} \oplus \cdots \oplus 2^{q-1} c_{q-1} m_{i-q+1} \oplus 2^q c_q m_{i-q}. \quad (16)$$

So $\oplus$ is a bitwise exclusive operator that we have $1 \oplus 0 = 0 \oplus 1 = 1.0 \oplus 0 = 1 \oplus 1 = 0$. The values m_i are odd integers in the interval $[1. 2^i - 1]$, then, generate $v(i)$ with $v(i) = m_i / 2^i$, and finally the elements of the Sobel sequence is produced as follows

$$\phi(n) = a_0 v(1) \oplus a_1 v(2) \oplus \cdots \oplus a_n v(n). \quad (17)$$

Grey code encryption is used to speed up the production of the Sobol sequence, whose algorithm is $\phi(n) = n \oplus n/2$. Now the modified model is $\phi(n+1) = \phi(n) \oplus v(i)$. As the dimension increases, the convergence of the Halton sequence increases and loses its random trend. The Sobol sequence performs better than other quasi-random sequences because it uses base 2 for all dimensions, resulting in

less convergence and higher velocity.

Geometric Brownian Motion (GBM)

As long as Brownian motion can capture negative values, its direct use for stock market pricing is questionable. Therefore, we introduce a non-negative type of Brownian motion called geometric Brownian motion. Geometric brown motion is always positive because the exponential function takes positive values. We define geometric brown motion as follows:

$$S(t) = S_0 e^{X_t} = S_0 e^{\mu t + \sigma W(t)}. \quad (18)$$

where $X(t) = \mu t + \lambda W_t$ is a brownian motion with deviation $S(0) = S_0 > 0$. Get a logarithm of the above relation:

$$X(t) = \ln(s(t)/S_0) = \ln(s(t)) - \ln(S_0) \Rightarrow \ln(s(t)) = \ln(S_0) + X(t) \quad (19)$$

So $\ln(s(t))$ has a normal distribution with mean $\ln(S_0) + \mu t$ and variance $2t\sigma$. Therefore, for every $t. s(t)$ has a normal log distribution. If we put $= \mu + \sigma^2/2\bar{r}. E(s(t)) = e^{\bar{r}t} S_0$. In this case $r << \bar{r}$. Because r is the share growth rate in risk-free conditions, such as investing in banks, and $\bar{r}$ is the share growth rate in risky conditions, such as investing in the stock market. The share growth rate in risky conditions must be much higher than in risk-free conditions in order to encourage investors to invest.

Theorem 1.4 *At constant time t, geometric brownian motion has a normal log distribution with average $ln(S_0) + \mu t$ and variance $t\sigma^2$.*

Stochastic Differential Equation

If a geometric Brownian motion is defined by the following differential equation:

$$dS = rSdt + \sigma SdW. \qquad S(0) = S_0. \quad (20)$$

Then Brownian geometric motion is equal to:

$$S(t) = S_0 \exp((r - \frac{1}{2}\sigma^2)t + \sigma W(t)). \quad (21)$$

At any time, geometric Brownian motion has a normal log distribution with $\ln(S_0) + rt - \sigma\frac{1}{2}t$ and $\sigma^2 t$ parameters. Brownian geometric mean motion is equal to $S_0 \exp(rt)$ and its variance is equal to the following formula:

$$\mathrm{Var}(S(t)) = S_0^2 \exp(2\mu t + \sigma^2 t)\exp(\sigma^2 t). \quad (22)$$

If the main subject of geometric Brownian motion is $S(t) = S_0 \exp(\mu t + \sigma)$, then the formula of its random differential equation is $dS = (\mu + \sigma 2)S(t)dt + \sigma S(t)dW. S(0) = S_0$. At any time, geometric brownian motion has a normal log distribution with $\ln(S_0) + \mu t$ and $2\sigma t$ parameters. Brownian geometric mean motion and its variance are as follows:

$$\mu_{S(t)} = S_0 \exp\left(\mu t + \frac{1}{2}\sigma^2 t\right). \qquad \mathrm{Var}(S(t)) = S_0^2 \exp(2\mu t + \sigma^2 t)\exp(\sigma^2 t). \quad (23)$$

The Relationship Between Monte Carlo and Quasi-Monte Carlo Simulations in Pricing Models

Suppose that common stock $S(t)$ is described by GBM and we have:

$$S(t) = S_0 \exp((r - \sigma^2/2)t + \sigma W(t)). \quad (24)$$

for all $t > 0$. $W(t)$ is Brownian motion. According to the values of the given parameters, the following simulation is due to the Monte Carlo method.

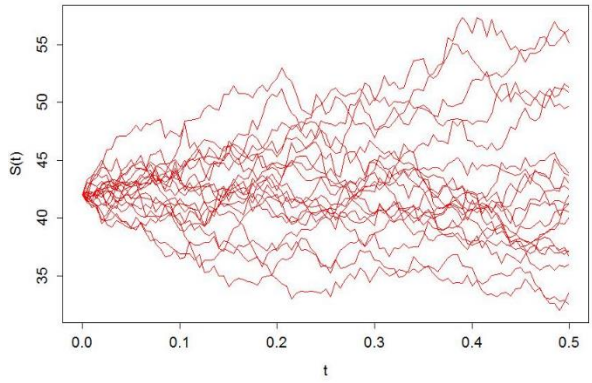

Fig.1: Simulation under Monte Carlo method with $S_0 = 42. r = 0.1. \sigma = 0.2$ and $N = 1000$ over the six-month period

In fact, the main goal is to find the distribution of the final values of $S(T)$, so there is no need to produce sample paths, instead using the fact that $W(T) \sim N[0. T]$ to generate some common prices of $S(T)$ at the time of expiration or final T.

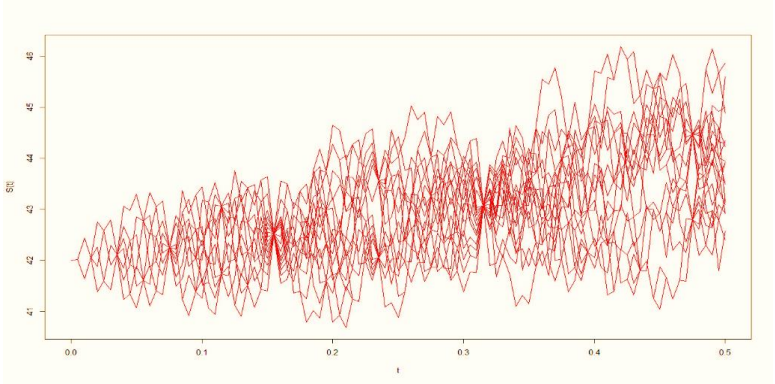

Fig.2: Simulation of possible stock behavior under Sobol sequence with $S_0 = 42. r = 0.1. \sigma = 0.2$ and $N = 1000$

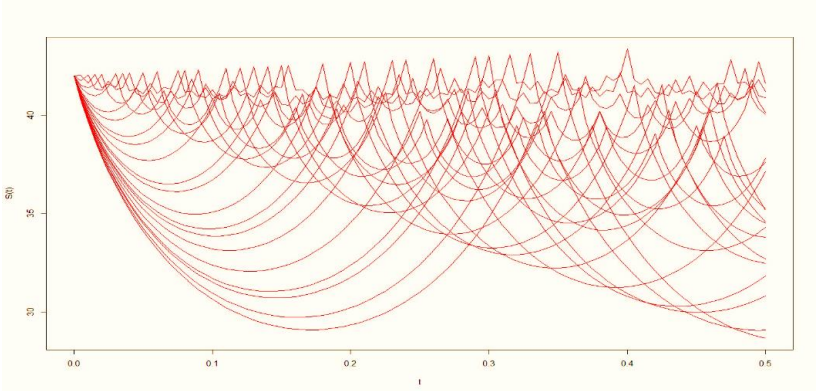

Fig.3: Simulation of possible stock behavior under Halton sequence with $S_0 = 42. r = 0.1. \sigma = 0.2$ and $N = 1000$

In the following, we use classical time series prediction methods and try to predict stock prices using Monte Carlo and quasi-Monte carlo methods and compare these two methods. In the following, we will first mention the terms of the exchange and then compare the efficiency of the two Monte Carlo and quasi-Monte Carlo methods. Although the indices calculated in the world's stock exchanges are very

diverse, this variation is more related to their scope of inclusion than the calculation method. In fact, the methods of calculating the index, especially the price index, are very similar and only have a difference in detail. The most important price indices are the price index (weighted-value), the price index (homogeneity), which is used in the market and the investigation of market volatility as well as the stock price. In this paper, we use relative prediction error (RE) criteria such as mean absolute relative error (MARE), mean square relative error (MSRE), mean absolute error percentage (MAPE) and mean square error percentage (MSPE).

2 Comparison of Monte Carlo and Quasi-Monte Carlo Methods (Sobol and Halton Sequences) from Data Related to Stock Indices

We compare the two models of Markov chain Monte Carlo and quasi-Monte Carlo (Sobol and Halton) from the data related to price index (weight-value) and price index (weight-value) of Tehran Stock Exchange. The following figures show the amount of level changes and efficiency associated with each series (Figure 4 to Figure 7).

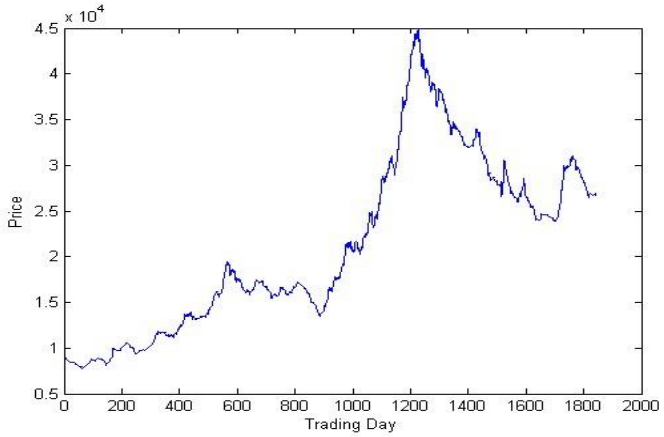

Fig.4: Changes in the levels related to the price index (weight-value) during 1900 days of the Tehran Stock Exchange

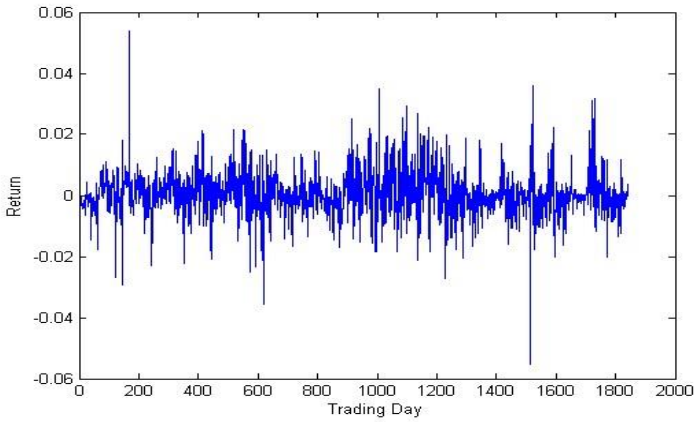

Fig.5: Efficiency related to price index (weight-value) during 1900 days of Tehran Stock Exchange

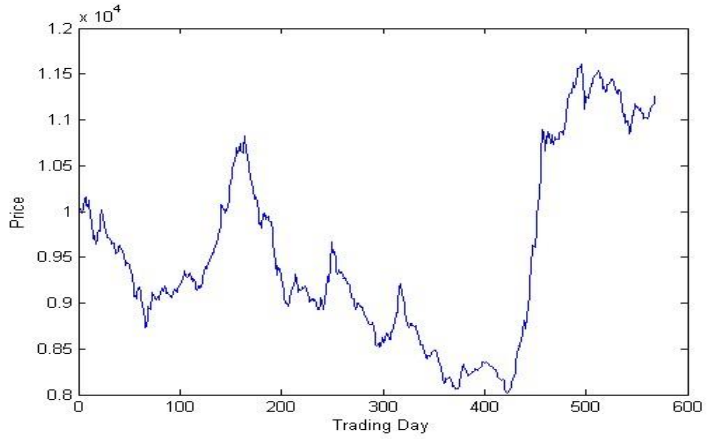

Fig.6: Changes in levels related to the price index (equal weight) during 580 days of the Tehran Stock Exchange

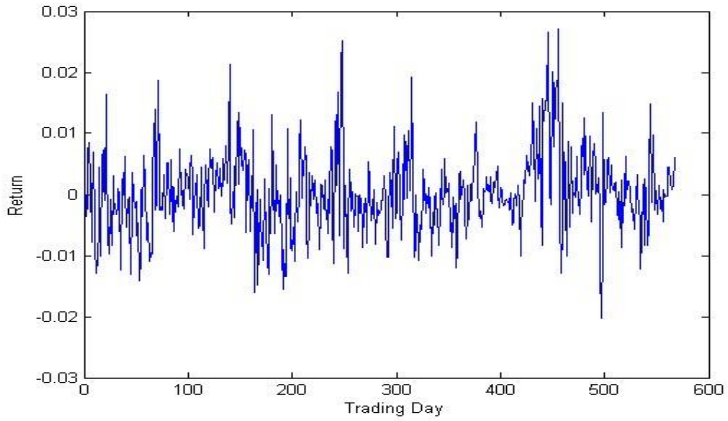

Fig.7: Efficiency related to price index (equal weight) of during 580 days Tehran Stock Exchange

The statistical specifications related to the efficiency of these two-time series are listed in the following table:

Table 1: Statistical Specifications Related to Two Time Series

	Max	Min	Kurtosis	Skewness	Variance	Mean
Price index (weight-value)	0.0540	-0.0551	7.4621	0.2662	$5.8737e-05$	$6.2537e-04$
Price index (equal weight)	0.0270	-0.0203	4.7451	0.6423	$4.2448e-05$	$2.2939e-04$

Carefully in the table above, we find that the kurtosis for the yields is high, and shows that the distribution for these indices is far from normal, and so we use the Monte Carlo and quasi-Monte Carlo methods. We use Monte Carlo, because these methods do not require normal distribution. By performing the Monte Carlo method and using the geometric Brownian motion model for the price index (weight-value) and (equal weight), for 1000 routes and during a period of 20 days, we reached Figure 8 to Figure 13.

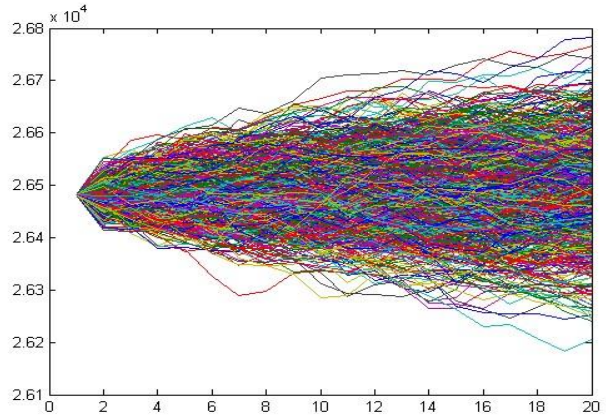

Fig.8: 1000 simulated routes for price index (weight-value) during 20 days using Monte Carlo method (with $S_0 = 26482$)

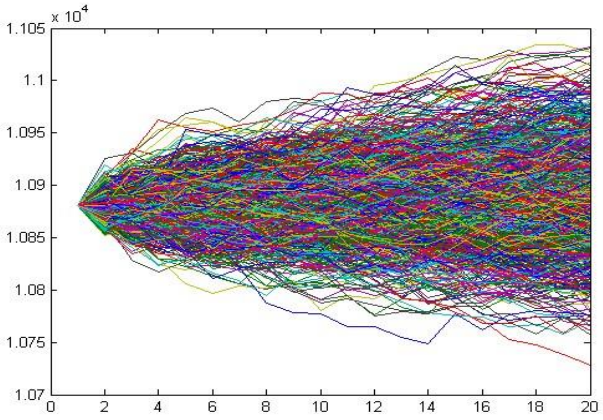

Fig.9: 1000 simulated routes for price index (equal weight) during 20 days using Monte Carlo method (with $S_0 = 10880$)

The above calculations were performed using the Monte Carlo method and the same parameters.

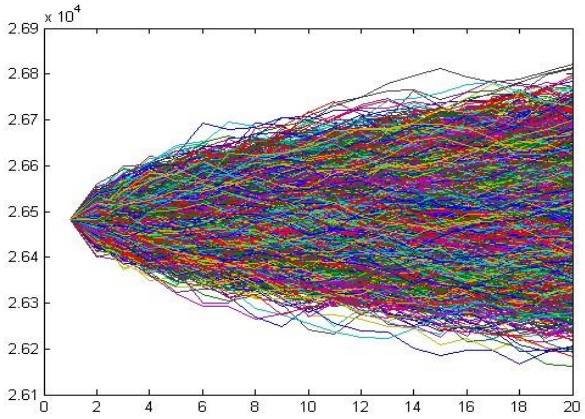

Fig.10: 1000 simulated routes for price index (weight-value) during 20 days using quasi-Monte Carlo method (Sobol sequence) (with $S_0 = 26482$)

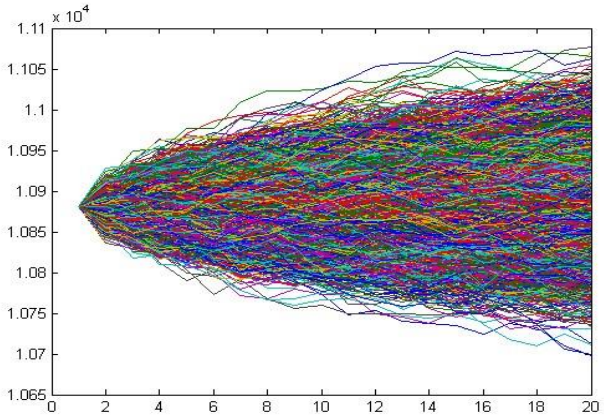

Fig.11: 1000 simulated routes for price index (equal weight) during 20 days with quasi-Monte Carlo method (Sobol sequence) (with $S_0 = 10880$)

The above calculations were performed using the quasi-Monte Carlo method (with the Sobol and Halton sequence, respectively) and the same parameters.

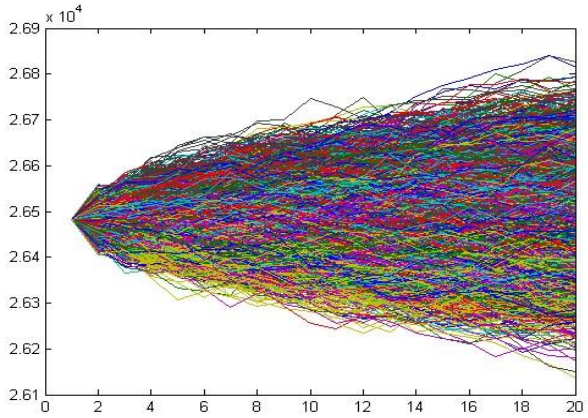

Fig.12: 1000 simulated routes for price index (weight-value) during 20 days using quasi-Monte Carlo method (Halton sequence) (with $S_0 = 26482$)

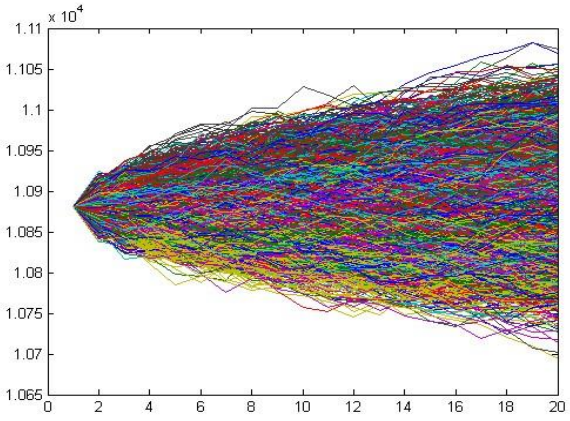

Fig.13: 1000 simulated routes for price index (equal weight) during 20 days with quasi-Monte Carlo method (Halton sequence) (with $S_0 = 10880$)

3 Comparison of Stock Price Forecast Efficiency under Monte Carlo Method quasi-Monte Carlo Method (Sobol and Halton Sequence)

To compare the two methods, we used the evaluation criteria mentioned in the previous sections. The actual value was compared with the simulated value and the highest error of the predicted paths was considered as the evaluation criterion. The results of this evaluation are given in the table below.

Table 2: The results of comparison of Monte Carlo and quasi-Monte Carlo methods in stock price forecasting

	Method	MARE	MSRE	MAPE	MSPE
*Price index (weight-value)	MC	0.0181	3.5327	1.8059	3.5327
	QMC-Sobol	0.0173	$3.2108e-04$	1.7308	3.2108
	QMC-Halton	0.0167	$2.9906e-04$	1.6680	2.9906
Least error (best method)		QMC-Halton	QMC-Halton	QMC-Halton	QMC-Halton
*Price index (equal weight)	MC	0.0246	$6.6286e-04$	2.4639	6.6286
	QMC-Sobol	0.0229	$5.7810e-04$	2.2916	5.7810
	QMC-Halton	0.0241	$6.3423e-04$	2.4130	6.3423
Least error (best method)		QMC-Sobol	QMC-Sobol	QMC-Sobol	QMC-Sobol

As we can see in Table 2, in the price index (weighted-value) the quasi-Monte Carlo method (with The Halton sequence) performed better than the other two methods, but in the price index (homogeneity) the quasi-Monte carlo method (with sobol sequence) was the best method. In both indicators, monte carlo method has performed poorly compared to quasi-Monte Carlo, and this is due to random numbers selected. The results of the best prediction along with the actual trend of the data in Figure 14 and Figure 15 are known.

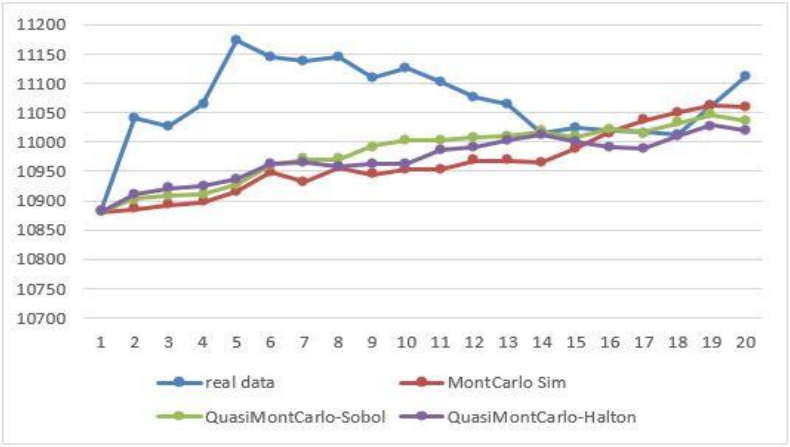

Fig.14: 1000 simulated routes for price index (equal weight) during 20 days with quasi-Monte Carlo method (Sobol sequence) (with $S_0 = 10880$)

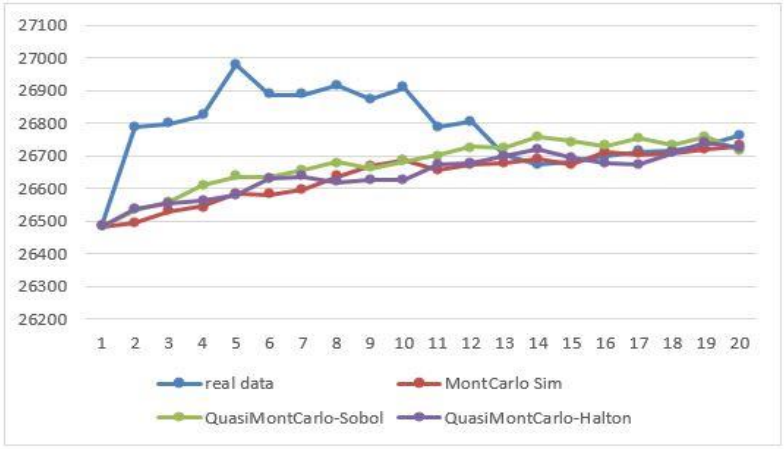

Fig.15: 1000 simulated routes for price index (weight-value) during 20 days using quasi-Monte Carlo method (Halton sequence) (with $S_0 = 26482$)

The above calculations were performed using the quasi-Monte Carlo method (with the Halton sequence) and the same parameters.

4 Conclusion

In this article, the predicted values of the total index of Tehran Stock Exchange are discussed by two methods, Monte Carlo and quasi-Monte Carlo sequences. The quasi-monte Carlo method, or the low difference method, is a method that potentially has a better convergence rate, and the basic idea of this method is to move a random sample in the Monte Carlo method with certain points. The selection criteria for these points are such that the sequence at $[0.1]^s$ has a better uniformity than a random sequence.

As can be seen in the figures, the predictions made by quasi-monte Carlo method and the Sobol sequence are much closer to the actual values of the total stock index than the Monte Carlo method. Especially in certain time intervals, the predictions made by the Sobol sequence are very reliable and according to the results obtained on the comparison of the efficiency of random and quasi-random methods, the geometric Brownian movement and the price index of Tehran stock (equal weight and weight-value), the quasi-Monte Carlo method has better and efficient results than the Monte Method Carlo method and Sobel sequence is the most efficient method among all random and quasi-random methods.

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