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Research Paper

Common Set of Weights Model in the Presence of Non-Homogeneous Outputs

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ABSTRACT

In order to evaluate the companies of the cement industry active in the Tehran Stock Exchange (Iran), it is first necessary to identify the indicators of the industry. Note that some of the indicator values provided by the Exchange Organization to the users may have been lost for any reason. All of the existing models for calculating the efficiency of such units calculate the weight of the indicators related to each unit under evaluation independent of the weight of other units. So, in this study, researchers decided to develop a common set of weight models and propose a model that evaluates efficiency in the presence of heterogeneity decision-making units (DMUs) based on the common set of weight models. Finally, the proposed model evaluates 25 cement industry companies in the presence of heterogeneity DMUs of indicators and the results are being analyzed.

1 Introduction

The economic development of any country is related to the production of cement in that country. Cement is the main building material for the growth of modern housing and infrastructure in any country in need. Among companies producing non-metallic mineral processes in securities, cement production requires a lot of energy, so it is expensive. Therefore, evaluating cement companies active in the Tehran Stock Exchange can be useful. Zatira et al. [17], conducted research on the financial performance of cement companies operating on the Indonesian Stock Exchange. Their main purpose is to analyze the health status of cement companies operating on the Indonesian Stock Exchange. Using the regression model, the indicators concluded that the situation of cement companies in Indonesia is not good enough. Cement production emits hazardous gases such as carbon monoxide (CO), sulfur dioxide (SO) in significant amounts into the air. Ekinci et al. [6], expressed a comprehensive approach based on some strategic level decisions to identify the variables affecting cement production and environmental factors causing air pollution in the region to show their impact on the environment. Using the proposed model, they evaluated the effects of various air quality factors on environmental sustainability. The concept of cementing efficiency is often used to evaluate the effectiveness of supplementary cementitious materials

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(SCM). Cement efficiency coefficient is defined as cement that can be replaced in any quantity (mass). Li et al. [13], individual and composite effects of complementary cementitious properties on the stability properties of Nano-silica (NS) and micro-silica (MS) and carbonation resistance and attractiveness have been studied experimentally. On the other hand, ceramics are close to cement in terms of elegance. Li et al. [12], stated that it has a relatively high reactivity and working cement. To evaluate the cementing performance of ceramic polishing residues (CPR), they calculated different water / cement ratios for testing. By analyzing the results of pressure resistance test, the efficiency coefficient of a cementing efficiency factor (CEF) has been calculated from the efficiency of ceramic polishing residues.

Data envelopment analysis is a technique that uses all the collected observations to measure the efficiency. In fact, in this method, the researcher evaluates the units that are generated simultaneously from multiple inputs and outputs, in contrast to the traditional methods, they optimize each observation by comparing the efficient boundary ([1,4]). In this regard, all organizations need an evaluation system to measure the efficiency of controlled DMUs in order to be aware of the status of the activities of them. One of the most important issues in DEA is the allocation of symmetric and asymmetric weights, among which we can refer to the work of Hatami et al. They stated that weight constraint models, which are often applied to the model due to the number of companies studied compared to the number of inputs and outputs, have potential problems. They have described a weight selection process with their proposed model [7]. One of the most widely used models in data envelopment analysis is the modified slack-based model. There are numerous articles in this field, including an extended model that is able to calculate performance when output is poor and inputs are constant and negative. We can also mention the developed slack-based hybrid model (SBM) and inverse DEA [10,15]. On the other hand, Jahanshahloo et al. have developed a method based on the DEA super-efficient model. Their proposed model is able to rank all efficient units as well. The units under their evaluation include the desired and unfavorable outputs in the production process that are ranked [9].

Measuring efficiency and evaluating performance is one of the most important ways to identify the strengths and weaknesses of the DMUs, which ultimately allow the managers of the DMUs make progress to take steps to improve the strengths and eliminate weaknesses. The loss of some indicators is inevitable, the existence of non-homogeneous indicators have led to the evaluation of performance in the absence of uniformity (non-homogeneous) of DMUs values [16]. In their article, they have addressed the problem of lost data, which refers to the data that produces a specific output unit, but its value is not known for whatever reason. One approach to solve this problem used in their paper is to "create" a value for the lost output (for example, using the average of known values) and to use it to fill in the blanks (lost value) or the value of zero replaces the value lost. They examined a simple case in which units appeared in a two-group cluster. They provided a clear and simple solution for both groups [2]. In this regard, they developed the DEA model for lost outputs, which DMUs consisted of unique subgroups [3]. Heterogeneity was assessed in the input status in measuring the environmental importance of the Chinese provinces, given that not all provincial inputs are of the same nature (non-homogeneous indicators), as well as undesirable outputs are possible to cause any inhomogeneity [11]. To solve this problem, they presented a non-parametric method [14]. Inhomogeneous DMUs were considered in parallel network structures. The non-homogeneous DEA method has been investigated to assess China's sustainable urbanization [5]. A non-homogeneous DEA model was proposed, which examines the problem of homogeneous asynchronous inputs. DMU outputs are intended to measure the environmental importance of China's industrial sectors. The status of outputs and inputs is such that a company may have a lost output and input. They developed the DEA model for non-homogeneous inputs and outputs [8].

The calculation of the relative efficiency of each unit using the basic DEA models has only the basic assumption of non-negativity on the input and output weights. But individual weight differences for each DMU may not be acceptable due to management or due to technical and economic market necessities. To solve this problem, the basic DEA model based on common set of weights (CSW) is presented [18]. Inspired by the ideas of non-homogeneous articles and the common set of weights on model, we intend to propose a model that is developed in the presence of a Non-homogeneous DEA model and is designed based on a common set of weights. In the continuation of this study, we will describe the tools and methods used, which include two parts. In the first part, we will express the DEA model in the presence of non-homogeneous output indicators in the form of an algorithm that is easier for the reader to use. The novelty of the article is explained in the second part. We have developed the DEA model in the presence of non-homogeneous indices so that it has the condition of a common set of weights for the indices. Finally, we will use the proposed model to evaluate the cement companies operating in Tehran Stock Exchange and analyze the results.

2 Tools and Methods

This section consists of two parts. In the first part, the data envelopment analysis model in the presence of non-homogeneous indicators is briefly stated. In the second part, the proposed model of a common set of weights based on non-homogeneous indices is presented.

2.1 Non-Homogeneous Dea Algorithm in Output with Homogeneous Inputs

Suppose n units are assumed to be evaluated so that there are non-homogeneous in the output indicators. The following algorithm can be used to calculate the efficiency of such units.

Step (1): Divide the companies into N_p ($p = 1, \dots, P$), categories or groups based on their output.

Step (2): Specify R_k ($k = 1, \dots, K$), which represents the subset of the outputs so that its members appear as the output of exactly one N_p class of DMUs.

Step (3): Specify L_{N_p}, which represents R_k, which specifies the set of all outputs for each DMU in N_p.

Step (4): Decide to allocate part of the i-th input to each of the output subgroups $L_{N_{pj}}$ for DMU_j with $\alpha_{iR_{kpj}}$ are displayed.

Step (5): Define the efficiency of DMUs in each subgroupR_k.

Step (6): Weight average of the efficiency score of the subgroup to obtain the overall efficiency of the units.

The following model, proposed by Cook et al. [3], has been used to obtain the overall efficiency score of the units.

$$e_o = \mathrm{Max}\Big\{\textstyle\sum_{R_k \in L_{N_{po}}} \sum_{r \in R_k} \mu_r y_{rj_o} \mid \quad \sum_{R_k \in L_{N_{po}}} (\sum_{i \in R_k} \gamma_{iR_kpo} x_{ij_o}) = 1 \quad , \sum_{r \in R_k} \mu_r y_{rj} - \sum_{i \in R_k} \gamma_{iR_kp} x_{ij} \le 0, \forall j \in N_p , R_k \in L_{N_p}, p = 1, \dots, P \ , \ \sum_{R_k \in L_{N_p}} \gamma_{iR_kp} = v_i, \forall i, p = 1, \dots, P, \ v_i a_{iR_kp} \le \gamma_{iR_kp} \le v_i b_{iR_kp}, \forall i, R_k \in L_{N_p}, , p = 1, \dots, P, \ \ \mu_r, v_i, \gamma_{iR_kp} \ge \varepsilon, \forall i, r, R_k \in L_{N_p}, p = 1, \dots, P\Big\} \tag{1}$$

Where x_{ij} and y_{rj} represent the input and output of the units under evaluation, μ_r the weight of the output, and γ_{iR_kp}depend on the percentage of consumption of inputs and weights. A more complete description of it is given in Cook et al. ([3]).

Definition 1: In the evaluation of DMU_o, by solving model (1), if in optimality $e_o = 1$ we say DMU_o is efficient. Otherwise we call DMU_o inefficient.

2.2 The proposed model of common set of weight based on non-homogeneous indicators

The only basic assumption of the above model, in addition to the non-homogeneous outputs, is that the weights are non-negative. But these weights are associated with problems. The following common set of weight models is proposed in the presence of non-homogeneous output.

1-Some input and output indicators may be ignored due to the weight of performance analysis (these input and output indicators have reached zero weight).

2- The model views the decision maker (DM) are not met.

3- The number of factors is relatively high compared to the number of DMUs.

The efficiency of DMU_j in the p category and the output subset R_k is defined as follows:

$$e_{jR_kp} = \frac{\sum_{r\in R_k} \mu_r y_{rj}}{\sum_{i\in R_k} v_i \alpha_{iR_kp} x_{ij}} \quad ; \qquad p = 1, \dots, P, \quad R_k \in L_{N_p} \tag{2}$$

The efficiency of DMU_o in the p^o category and the R_k output subset is defined as follows:

$$e_{j_oR_kp^o} = \frac{\sum_{r\in R_k} \mu_r y_{rj_o}}{\sum_{i\in R_k} v_i \alpha_{iR_kp^o} x_{ij_o}} \quad ; \qquad R_k \in L_{N_p} \tag{3}$$

Consider the efficiency of DMU_j as the convex composition of its R_k categories.

$$e_j = \sum_{R_k\in L_{N_p}} w_{R_kj}\, e_{jR_kp} \tag{4}$$

Where w_{R_kj} the weight of the categories R_k corresponds to DMU_j where $\sum_{R_k\in L_{N_p}} w_{R_kj} = 1$.

Consider steps (1) to (5) of the algorithm provided above. We propose a common set of weight model in the presence of output non-homogeneous as follows:

$$\begin{aligned}
&Max \quad \left\{ \sum_{R_k\in L_{N_1}} w_{R_k1}\, e_{1R_k1}, \dots, \sum_{R_k\in L_{N_p}} w_{R_kn}\, e_{nR_kp} \right\} \\
&s.t. \quad \sum_{R_k\in L_{N_p}} w_{R_kj}\, e_{jR_kp} \le 1;\ \forall j \in N_p, p = 1, \dots, P \\
&\qquad\ e_{jR_kp} \le 1; \quad \forall j \in N_p, R_k \in L_{N_p}, p = 1, \dots, P \\
&\qquad\ \sum_{R_k\in L_{N_p}} \alpha_{iR_kp} = 1; \quad \forall i,\ p = 1, \dots, P \\
&\qquad\ a_{iR_kp} \le \alpha_{iR_kp} \le b_{iR_kp};\ \ \forall i, R_k \in L_{N_p}, p = 1, \dots, P \\
&\qquad\ \mu_r, v_i, \alpha_{iR_kp} \ge 0;\ \forall i, R_k \in L_{N_p}, p = 1, \dots, P
\end{aligned} \tag{5}$$

where $\sum_{R_k\in L_{N_p}} w_{R_kj} = 1$. For each j, w_{R_kj} can be written as $w_{R_kj} = \sum_{i\in R_k} v_i \alpha_{iR_kp^o} x_{ij_o} \Big/ \sum_{R_k\in L_{N_p}} \sum_{i\in R_k} v_i \alpha_{iR_kp^o} x_{ij}$. Therefore, model (5) can be rewritten as follows:

$$
\begin{aligned}
& Min \quad \sum_{j\in N_p}\left(s_j+s'_j\right) \\
& s.t. \quad \frac{\sum_{\mathrm{R_k}\in L_{N_p}}\sum_{\mathrm{r}\in\mathrm{R_k}}\mu_{\mathrm{r}}\mathrm{y}_{\mathrm{rj}}+s_j}{\sum_{\mathrm{R_k}\in L_{N_p}}\sum_{\mathrm{i}\in\mathrm{R_k}}\gamma_{\mathrm{iR_kp}}\mathrm{x}_{\mathrm{ij}}}=1;\ \forall j\in N_p, p=1,\dots,P \qquad (a1) \\
& \qquad \frac{\sum_{\mathrm{r}\in\mathrm{R_k}}\mu_{\mathrm{r}}\mathrm{y}_{\mathrm{rj}}+s'_j}{\sum_{\mathrm{i}\in\mathrm{R_k}}\gamma_{\mathrm{iR_kp}}\mathrm{x}_{\mathrm{ij}}}=1;\ \forall j\in N_p,\mathrm{R_k}\in L_{N_p}, p=1,\dots,P \qquad (\mathrm{a2}) \\
& \qquad \sum_{\mathrm{R_k}\in L_{N_p}}\gamma_{\mathrm{iR_kp}}=v_i;\ \forall i, p=1,\dots,P \\
& \qquad v_i a_{\mathrm{iR_kp}}\le\gamma_{\mathrm{iR_kp}}\le v_i b_{\mathrm{iR_kp}};\ \ \forall \mathrm{I},\mathrm{R_k}\in L_{N_p}, p=1,\dots,P \\
& \qquad \mu_{\mathrm{r}},\gamma_{\mathrm{iR_kp}}\ \ge 0;\ \mathrm{R_k}\in L_{N_p}, p=1,\dots,P
\end{aligned}
\tag{6}
$$

where $v_i\alpha_{\mathrm{iR_kp}}=\gamma_{\mathrm{iR_kp}}$. According to constraint (a2) we can write:

$$
\sum_{\mathrm{r}\in\mathrm{R_k}}\mu_{\mathrm{r}}\mathrm{y}_{\mathrm{rj}}-\sum_{\mathrm{i}\in\mathrm{R_k}}\gamma_{\mathrm{iR_kp}}\mathrm{x}_{\mathrm{ij}}+\ s'_j=0;\ \forall j\in N_p,\mathrm{R_k}\in L_{N_p}, p=1,\dots,P \tag{7}
$$

Hence

$$
\sum_{\mathrm{R_k}\in L_{N_p}}\sum_{\mathrm{r}\in\mathrm{R_k}}\mu_{\mathrm{r}}\mathrm{y}_{\mathrm{rj}}-\sum_{\mathrm{R_k}\in L_{N_p}}\sum_{\mathrm{i}\in\mathrm{R_k}}\gamma_{\mathrm{iR_kp}}\mathrm{x}_{\mathrm{ij}}+\sum_{\mathrm{R_k}\in L_{N_p}}s'_j=0;\ \forall j\in N_p, p=1,\dots,P \tag{8}
$$

And finally, we can write:

$$
\sum_{\mathrm{R_k}\in L_{N_p}}\sum_{\mathrm{r}\in\mathrm{R_k}}\mu_{\mathrm{r}}\mathrm{y}_{\mathrm{rj}}-\sum_{\mathrm{R_k}\in L_{N_p}}\sum_{\mathrm{i}\in\mathrm{R_k}}\gamma_{\mathrm{iR_kp}}\mathrm{x}_{\mathrm{ij}}=-|L_{N_p}|\ s'_j;\ \forall j\in N_p, p=1,\dots,P \tag{9}
$$

Therefore, constraint (a1) will be converted as follows:

$$
-\left|L_{N_p}\right|s'_j+\ s_j=0 \tag{10}
$$

Therefore, model (6) becomes the following linear form.

$$
\begin{aligned}
& Min \quad \sum_{j\in N_p}\left(s_j+s'_j\right) \\
& s.t. \quad -\left|L_{N_p}\right|s'_j+\ s_j=0\ ;\ \ \forall j\in N_p, p=1,\dots,P \\
& \qquad \sum_{\mathrm{r}\in\mathrm{R_k}}\mu_{\mathrm{r}}\mathrm{y}_{\mathrm{rj}}-\sum_{\mathrm{i}\in\mathrm{R_k}}\gamma_{\mathrm{iR_kp}}\mathrm{x}_{\mathrm{ij}}+\ s'_j=0;\ \ \forall j\in N_p,\mathrm{R_k}\in L_{N_p}, p=1,\dots,P \\
& \qquad \sum_{\mathrm{R_k}\in L_{N_p}}\gamma_{\mathrm{iR_kp}}=v_i;\ \forall i, p=1,\dots,P \\
& \qquad v_i a_{\mathrm{iR_kp}}\le\gamma_{\mathrm{iR_kp}}\le v_i b_{\mathrm{iR_kp}};\ \ \forall \mathrm{I},\mathrm{R_k}\in L_{N_p}, p=1,\dots,P \\
& \qquad \mu_{\mathrm{r}}\text{و}v_i,\gamma_{\mathrm{iR_kp}}\ \ge 0;\ \mathrm{R_k}\in L_{N_p}, p=1,\dots,P
\end{aligned}
\tag{11}
$$

Assume $(\mu^*_r, v^*_i, \gamma^*_{\mathrm{iR_kp}}, s^*_j, s'^*_j)$ *is* the optimal answer of model (11). For $\forall i$ و $\forall j\in N_p,\mathrm{R_k}\in L_{N_p}, p=1,\dots,P$, since $\frac{\sum_{\mathrm{R_k}\in L_{N_p}}\sum_{\mathrm{r}\in\mathrm{R_k}}\mu^*_r\mathrm{y}_{\mathrm{rj}}}{\sum_{\mathrm{R_k}\in L_{N_p}}\sum_{\mathrm{i}\in\mathrm{R_k}}\gamma^*_{\mathrm{iR_kp}}\mathrm{x}_{\mathrm{ij}}}+\frac{s^*_j}{\sum_{\mathrm{R_k}\in L_{N_p}}\sum_{\mathrm{i}\in\mathrm{R_k}}\gamma^*_{\mathrm{iR_kp}}\mathrm{x}_{\mathrm{ij}}}=1$.

Therefore, to find the efficiency of DMU_j, we use the following equation.

$$e_j^* = 1 - \frac{s_j^*}{\sum_{\mathrm{R_k} \in L_{N_p}} \sum_{\mathrm{i} \in \mathrm{R_k}} \gamma_{\mathrm{iR_kp}}^* \mathrm{x_{ij}}} \tag{12}$$

Definition 2: Suppose we have $s_o^* = 0$ in the optimal solution of model (11). In this case $DMU_o$is called the Common set of weight efficient in the presence of non-homogeneous outputs.

If model (11) has a multiple solution, assume that $z^*$is the optimal value for the multiple solutions. Obviously, the value of efficiency for other optimal solutions is [a, b] ⊂ (0, 1]. To find the upper bound diffraction DMU_o following model can be used:

$$\begin{aligned}
&Max \quad e_o \\
&s.t. \\
&\sum_{j \in N_p} (s_j + s'_j) = z^* \\
&-\left|L_{N_p}\right| s'_j + s_j = 0 \ ; \ \forall j \in N_p, p = 1, \dots, P \\
&\sum_{\mathrm{r} \in \mathrm{R_k}} \mu_\mathrm{r} \mathrm{y_{rj}} - \sum_{\mathrm{i} \in \mathrm{R_k}} \gamma_{\mathrm{iR_kp}} \mathrm{x_{ij}} + s'_j = 0; \ \forall j \in N_p, \mathrm{R_k} \in L_{N_p}, p = 1, \dots, P \\
&\sum_{\mathrm{R_k} \in L_{N_p}} \gamma_{\mathrm{iR_kp}} = v_i \ ; \ \forall i, p = 1, \dots, P \\
&v_i a_{\mathrm{iR_kp}} \le \gamma_{\mathrm{iR_kp}} \le v_i b_{\mathrm{iR_kp}}; \ \ \forall \mathrm{I}, \mathrm{R_k} \in L_{N_p}, p = 1, \dots, P \\
&\mu_\mathrm{r}, v_i, \gamma_{\mathrm{iR_kp}} \ \ge 0; \ \mathrm{R_k} \in L_{N_p}, p = 1, \dots, P
\end{aligned} \tag{13}$$

3 Research Findings

The economic development of any country is related to the production of cement in that country. Cement is the main building material for the growth of modern housing and infrastructure in any country in need. Among companies producing non-metallic mineral processes in securities, cement production requires a lot of energy, so it is expensive. Therefore, evaluating cement companies active in the Tehran Stock Exchange can be useful. In this section, we evaluate 25 cement companies operating in the Tehran Stock Exchange. The names of the companies are given in Table (1).

Table 1: Names of companies.

Company	number	Company	number	Company	number
Fars Cement	19	Darab Cement	10	Urmia Cement	1
New Fars Cement	20	Dashtestan Cement	11	Isfahan Cement	2
Qaen Cement	21	Sepahan Cement	12	Arta Ardabil Cement	3
Kerman Cement	22	Neyriz white cement	13	Bojnourd Cement	4
Mazandaran Cement	23	Shahroud Cement	14	Behbahan Cement	5
Hormozgan Cement	24	East Cement	15	Tehran Cement	6
Hegmatan Cement	25	Northern Cement	16	Khash Cement	7
		Sufi Cement	17	Caspian Cement	8
		West Cement	18	Khuzestan Cement	9

The indicators studied are as follows:
Input indicators: cost of goods sold (I_1), general and administrative costs (I_2)
Output indicators: income from services and sales (o_1), special profit (o_2), operating profit (loss) (o_3), profit (loss) after tax deduction (o_4).
The values of the indicators collected from the stock exchange organization are given in Table (2).

Table 2: Indicators of cement companies

$\mathbf{DMU_j}$	I1	I2	O1	O2	O3	O4
1	898807	52269	1374359	475552	385929	328205
2	702515	102757	949851	247336	145780	89710
3	549395	69262	889066	339671	271052	255845
4	838787	58065	1207995	369208	263133	240687
5	537529	135363	834548	297019	150111	130760
6	2107872	181227	2540777	432905	138026	5938
7	756148	93159	1036073	279925	186540	180877
8	469838	53678	645607	175769	102128	73820
9	1837692	271525	2536133	698441	426916	273771
10	780288	62970	921787	141499	78529	140594
11	809485	160704	1090089	280604	116528	17002
12	1366018	469757	2165728	799710	181722	40025
13	282325	69542	416887	134562	52202	46702
14	761905	68486	1122363	360458	291972	170527
15	1498478	121654	1765748	267270	190157	22214
16	732267	165528	1045617	313350	136985	204238
17	981748	108250	1465768	484020	221813	109431
18	556443	66010	800377	243934	168241	131068
19	525742	40140	667663	141921	90584	85905
20	918447	360401	1557818	639371	292399	207515
21	510005	44810	709497	199492	154682	184618
22	845702	181435	1307827	462125	280091	447357
23	1001052	175333	1444581	443529	225015	144000
24	1774249	467079	2430082	655833	535532	665352
25	859852	225276	1408879	549027	324046	227384

Suppose the lost outputs (virtual non-homogeneous) $y_{4,20}$ و $y_{2,20}$‘ $y_{1,10}$‘ $y_{4,9}$ و$y_{3,7}$‘ $y_{4,4}$ ‘$y_{1,4}$ (where $y_{r,j}$ indicates the r-th output of DMU_j). We run the algorithm section (2-1) as follows:
The first step, according to the values of the missing indicators is $P = \{1,2,3,4\}$ and
$N_1 = \{y_1, y_2, y_3, y_4\}$, $N_2 = \{y_2, y_3\}$, $N_3 = \{y_1, y_2, y_4\}$, $N_4 = \{y_1, y_3\}$
Step 2: According to the categories of $N_p$specified above, R_k $(k = 1, \dots, 4)$ is as follows.
$R_1 = \{y_1\}$, $R_2 = \{y_2\}$, $R_3 = \{y_3\}$, $R_4 = \{y_4\}$.
Step 3: According to the categories $N_p(p = 1,2,3,4)$ and $R_k(k = 1, \dots, 4)$ specified above, L_{N_p} is determined as follows:
$L_{N_1} = \{R_1, \dots, R_5\}$; $L_{N_2} = \{R_2, R_3\}$; $L_{N_3} = \{R_1, R_2, R_4\}$; $L_{N_4} = \{R_1, R_3\}$;

Step 4: The weights of the allocation of inputs are determined by solving the model, if the expert intends to use the percentages and he wants to produce each output category, then he can introduce and enter the model and the weights assigned to each input to produce each output category. The weights required in this study are as follows:

$$\forall i = 1,2; \begin{cases} \alpha_{iR_1 1}, \alpha_{iR_2 1}, \alpha_{iR_3 1}, \alpha_{iR_4 1} \\ \alpha_{iR_2 2}, \alpha_{iR_3 2} \\ \alpha_{iR_1 3}, \alpha_{iR_2 3}, \alpha_{iR_4 3} \\ \alpha_{iR_1 4}, \alpha_{iR_3 4} \end{cases}$$

Where $\alpha_{iR_k p}$ represents the percentage of input i-th used to generate the R_k output subset of p.

Step 5: Define the efficiency of DMUs in each R_k subgroup.
For example, the definition of efficiencyDMU_3, which is in the categories N١ and $L_{N_2} =$ $\{R_1, R$٢ و$R_3, R_4\}$ will be as follows:

$$e_{R_1}^3 = \frac{\sum_{r\in R_1} u_r y_{r3}}{\sum_i v_i\ \alpha_{iR_1 1} x_{i3}}, \quad e_{R_2}^3 = \frac{\sum_{r\in R_3} u_r y_{r3}}{\sum_i v_i\ \alpha_{iR_2 1} x_{i3}}, \quad e_{R_3}^3 = \frac{\sum_{r\in R_3} u_r y_{r3}}{\sum_i v_i\ \alpha_{iR_3 1} x_{i3}}, \quad e_{R_4}^3 = \frac{\sum_{r\in R_4} u_r y_{r3}}{\sum_i v_i\ \alpha_{iR_4 1} x_{i3}}$$

Step 6: The average weight of the sub-group efficiency score is equal to the overall efficiency of the units. For example, the overall efficiency of DMU_3 is as follows:
$e_3 = w_1 e_{R_1}^3 + w_2 e_{R_2}^3 + w_3 e_{R_3}^3 + w_4\ e_{R_4}^3$
By performing model (1), the efficiency of each unit is calculated, the results of which are given in Table (3) of the column related to the efficiency values.

Table 3: The value of efficiency in the presence of non-homogeneous output indicators

$\mathbf{DMU_j}$	Efficiency score of model (1)	Efficiency score of common set weight
1	1	0.733
2	0.7046	0.854
3	1	0.359
4	0.9756	0.318
5	0.8733	0.487
6	0.5626	0.987
7	1	0.667
8	0.7342	0.751
9	0.7329	0.835
10	0.3256	0.22
11	0.6772	0.651
12	0.8549	1
13	0.7959	0.736
14	0.8766	0.37
15	0.5436	0.8
16	0.7661	0.975
17	0.848	0.879
18	0.8038	0.33
19	0.6582	0.43
20	0.9654	0.986

Table 3: The value of efficiency in the presence of non-homogeneous output indicators

$\mathbf{DMU_j}$	Efficiency score of model (1)	Efficiency score of common set weight
21	0.8236	0.5
22	0.9839	0.903
23	0.7828	0.741
24	0.7723	0.845
25	0.9539	0.981

According to model (1), units with efficiency values equal to one in Table (3) are efficient units. Also according to the second column DMU_1 , DMU_3, DMU_7 are efficient units. By implementing model (11), the value of joint weight efficiency in the presence of non-homogeneous indicators is given as the first column from the right side of Table (3). According to the obtained results, $DMU_{12}$is a common set of weight efficiency in the presence of non-homogeneous output indicators.

Table 4: Correlation between input and output indices with performance value in models (1) and (13).

			Efficiency (1)	Efficiency (13)
Spearman's rho	I1	Correlation Coefficient	-0.093	.532**
		Sig. (2-tailed)	0.658	0.006
		N	25	25
	I2	Correlation Coefficient	-0.009	.749**
		Sig. (2-tailed)	0.965	0.000
		N	25	25
	O1	Correlation Coefficient	-0.014	.613**
		Sig. (2-tailed)	0.948	0.001
		N	25	25
	O2	Correlation Coefficient	0.387	.577**
		Sig. (2-tailed)	0.056	0.003
		N	25	25
	O3	Correlation Coefficient	.558**	0.208
		Sig. (2-tailed)	0.004	0.319
		N	25	25
	O4	Correlation Coefficient	.587**	-0.063
		Sig. (2-tailed)	0.002	0.765
		N	25	25
	Efficiency (1) and (13)	Correlation Coefficient	1.000	1.000
		Sig. (2-tailed)		
		N	25	25

**. Correlation is significant at the 0.01 level (2-tailed).

*. Correlation is significant at the 0.05 level (2-tailed).

To investigate the correlation between input and output indices with the performance value in models (1) and (13), the correlation test in SPSS software environment was used. Due to the abnormality of the data distribution, we used the Spearman correlation test. The results of the correlation coefficient analysis between input and output indices with the efficiency values in models (1) and (13) are specified in Table (4). As it is known, the software output for model (1) does not assume any relationship between the input indicators and the performance value, because the value of the significant coefficient for the inputs has been more than 0.05. Also, there is no relationship between output indicators of special profit and income from services and sales and the amount of efficiency in model (1). However, the output indices of operating profit (loss) and profit (loss) after tax have had a positive and significant relationship with the amount of efficiency in model (1). By performing the correlation test between input and output indices with the efficiency value in model (13), we observed that the reverse of the first case has occurred. Indicators that were not related to the efficiency of model (1) in the first case have a positive and significant relationship with the efficiency of model (13) in the second case, and vice versa, two output indicators of operating profit (loss) and profit (loss) after tax deduction shows no relationship with the amount of efficiency in Model (13).

4 Results of the Discussions

The loss of the values of some indicators in all evaluations seems inevitable. In some cases, missing indicators cannot be calculated for any reason or their values are not known, but in some cases these indicators will be available over time. But when they do not exist, it will have a significant impact on the efficiency of the unit itself and the efficiency of other units. In this article, a real example including 25 cement companies active in Tehran Stock Exchange is considered. Since the weight of all indicators is considered to be the same in the models presented in the presence of non-homogeneous outputs, and this causes the problems mentioned in section (2), it was necessary for the DEA model in the presence of non-homogeneous outputs to consider developing the common set of weight assumption. Therefore, we have proposed a model that solves the problems of previous models and we have used the proposed model to evaluate companies operating in the Tehran Stock Exchange. According to the results of the proposed model, Sepahan Cement, Tehran Cement, Fars No Cement, Hegmatan Cement and North Cement have been ranked first to fifth, respectively. Darab Cement, Behnavard Cement and Gharb Cement are among the last three ranks. For future work, it is proposed to develop the DEA model in the presence of non-homogeneous inputs and outputs based on the common set of weight, in which the proposed model of the present paper will be a special case.

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