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A Fuzzy Random Walk Technique to Forecasting Volatility of Iran Stock Exchange Index

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ABSTRACT

Study of volatility has been considered by the academics and decision makers during two last decades. First since the volatility has been a risk criterion it has been used by many decision makers and activists in capital market. Over the years it has been of more importance because of the effect of volatility on economy and capital markets stability for stocks, bonds, and foreign exchange markets. This research first deals with the evaluation of 8 various models to forecasting volatility of stock index using daily data of Tehran stock exchange. The used models include simple ones such as random walk as well as more complex models like Arch and Garch group. Forecasting volatility index method is developed in this paper. This method is based a random walk using a fuzzy logic approach. This method is used to forecasting volatility of Iran stock exchange index. The proposed method is assessed by comparing other methods such as Moving Average, Random walk… Results show that our proposed method is compatible with existent methods.

1 Introduction

An important issue that researchers and scholars in decision-making and forecasting fields have challenge with is choosing effective variables on decision output and forecasting. So if stock return is can be predicted by good variables and some models can be providing, in fact, more insured condition is provided in capital market which help investment development in financial markets [1].
Volatility relates to the price fluctuations in a period of time which has been considered by many researchers and capital market activists during recent years. However, this question that which method can give the best solution has not had a unique answer and also none of the previous works have not forecasting through combining these methods using fuzzy modeling. It has been attempted in this paper to first implement the present and common models to forecasting the volatility based on Iran stock exchange index (Tepix) and second to design a fuzzy system by random walk and moving average model as the input and the real values as the output to be able to evaluate the accuracy of the volatility

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forecasting using the fuzzy methods in forecasting models. The aim of this research is to obtain a proper estimation of volatility forecasting in Iran stock exchange index using a fuzzy system. Although there is not such possibility to deal on this index because of religious reasons and it is also impossible to have such deals in the future years, such forecasting can be considered from two aspects: First, in case of those ones who try to create a market-based portfolio through their portfolio diversification, the volatility situation can help them greatly. Second, although price movements are not affected by the market index for many trading symbols (especially in short term) but the great fluctuations of market have the same effect on the symbols (especially index) and a proper forecasting of volatility would help have a better comprehension of market risk. Practically, volatility has a major role in documents pricing. Based on the pricing formula of Black Shoulz, the price of European purchasing is a function of volatility. Therefore, the market (the running of this market is a part of the future plan of Tehran stock exchange) can be considered as a place in which the dealers deal the volatility. Additionally, many new models use the volatility as one of major parameters for pricing (especially for transactions). Return series and forecasting of the range can be obtained more accurately through modeling of return rate of the volatility. Volatility amount is one of the most important parameters for process identification in many random processes. As it would be explained in the second part with more details the literature review on this issue would explains various models based on different markets data. In this research we will deal with the study and forecasting of volatility in Iran stock exchange as well as the main index of the market using 8 forecasting models and then we will combine two random walk and moving average models to forecasting the stock exchange fluctuations in the future months.

2 Literature Review

The fast growth of economic and technology in recent years, changed the human's life and faced the modern society complex problems, main characteristic of these problems is existence of criteria or non- homogeneous and incompatible targets such as coast, reliability, operation, safety and productivity [2]. Gaining long term growth and sustainable economic needs optimal equip and allocation of references in level of national economic, and this would not happen without helps of national markets, especially wide and efficient capital market. In a healthy economic, existence of financial efficient system in good distribution has important role. Basically, one of main goal in analysis economic is true forecast of economic variables, so it will be helpful for policy makers in order to obtain true and proportional decision in front of forecast volumes [2]. A special feature of economic forecasting compared to general economic modeling is that we can measure a model's performance by comparing its forecasts to the outcomes when they become available [13]. One of the simplest methods which may be used for fluctuations forecasting is parametric methods but because of their weak performances as research done by [3-4], they have not been considered greatly. It can be said that the presentation of Arch and Garch models was a start for the continuation and attention to the fluctuation rates forecasting. Arch model has been proposed first in 1982 by Engle and then has been finished and developed by researchers as Bera et al. [5]. Garch model has also been proposed in 1986 by Bollerslev and Taylor and has been developed by many others as EGarch (Exponential Garch) which has been introduced by Nelson [6].

Other Garch-based models which have been welcomed are T Garch, Runkle et al. [7]. Akgiary in 1989 has dealt with the properties of stock prices and has also compared Arch and Garch models to EWMA model as well as historical average models and could obtain better results based on the U.S market data [8]. Also, Garch models obtained results based on a research done as England market data that simple models would perform better than models as Exponential Smoothing as well as regression (it is important to mention that they have not used Arch-based models in their research) while Tse (90, Japan market data analysis) and Tse and Tung (92, Singapore market data analysis) gain the results that Exponentially Weighted moving models would give more accurate forecasting than Garch models. West and Cho in1995 have reached a result in a short-term and long-term period that Garch model would give better result. Franses and van Dijk in 1996 have dealt with the study and evaluation of Garch model forecasting against Random walk model for 5 European stock and found that Random walk model performs better (this result has been repeated considering the crisis and failure in 1987[9-10]. Faff and Brailsford in 1996 also announced based on the monthly volatility data of Australian stock index that Garch model as well as Simple Regression model would perform better than others. The important point is that according to the research done by Stoch and Watson in1998 and based on the data received by the U.S. macro economy, Random Walk model showed its performance as the best one among other models [11].

Generally, in case of volatility forecasting there is a complete literature and this topic has taken attention during two last decades by many researchers. Therefore, many econometrics models have been used for studying but it cannot be mentioned that all researchers have reached the agreement on the excellence of one specific model. The better results depend on the market conditions in which research has been done or the situation of that market at that time and conditions. Since there has not been any complete research based on data of Tehran stock exchange it has been attempted here to study and evaluate various methods based on the received data. It should be mentioned that there are various methods to calculate monthly volatility. The first solution introduced by Merton and Perry. The volatility has been obtained as:

$$\delta_T^2 = \sum_{t=1}^{N_T} r_i^2 \tag{1}$$

Akgiary in 1989 proposed another solution as:

$$\delta_T^2 = \sum_{t=1}^{N_T} (r_t - \overline{r_t})^2 [1 + 0.1 \sum_{j=1}^{N_T - 1 - j} \varphi^j] \tag{2}$$

In the above-mentioned relation r_t is the average, φ is the first correlation delay. This is important that both equations emphasize using the square of the daily return rate. Yu [12] used another solution in which the absolute value of the daily return rate has been used. is to use the difference of the highest and lowest daily price which has already been proposed in the literature. This solution gives the better solution but it is much bias. In continue, we will deal with the study and description of the used models. First, 8 common models of literature review have been explained. Then, in the next section, one volatility forecasting model for stock exchange index used through using one fuzzy model and combination of two random walk as well as moving average methods (because of pleasant performance and simplicity).

2.1 Random Walk

Random walk is the simplest model and can be expressed as:

$$\delta^2_{T+1} = \delta_T{}^2 \qquad T=121,0,0,0,162 \qquad (3)$$

Here the forward and backward steps have been considered as zero (p=q=0). Therefore, it is assumed that the best forecasting for the volatility of the next month is the same as the present month.

2.2 Historical Average

In case we assume that the expected moving average is a constant value, the optimized forecasting for the future volatility is as:

$$\sigma^2_{T+1} = 1/T \sum_{T=1}^{T} \sigma^2_T; \qquad T=121,0,0,0,162 \qquad (4)$$

This model has been the most widely used model in the past but the present research shows that the expected amount would change greatly through passing the time.

2.3 Moving Average

According to the historical average model, all previous observations have the same weight. In moving average model, the closer observations have more weight. Two moving average types have been used in this paper as 3-year models as well as 6-year models.

$$1/36 \sum_{j=1}^{36} \sigma^2_{T+1-j} = \sigma^2_{T+1}; \qquad T=121,0,0,0,162 \qquad (5)$$

6-year moving average model would be defined as above and using data for 72 months.

2.4 Simple Regression

Forecasting would be done based on linear simple regression for the period T+1 and based on the volatility of period T as below in which β_i are constant coefficients which are obtained based on historical data:

$$\sigma^2_{T+1} = \beta_1 + \beta_2 \sigma^2_T; \qquad T=121,0,0,0,162 \qquad (6)$$

Two methods are available to estimate parameters: First, in case new data is added, the sample size would be constant as 120 and the last data would be removed. Second, all present data would be used. Therefore, the sample size would be big and bigger. The results of two models are close to each other. In this paper, just results of model with the constant sample volume is reported.

2.5 Exponential Smooth

Exponential smoothing is a simple adaptive forecasting method. Unlike regression method which uses constant methods, forecasting by this method would adjust coefficient based on the previous forecastings. It is expressed as:

$$\hat{\sigma}_{T+1}^2 = \alpha\hat{\sigma}_T^2 + (1-\alpha)\sigma_T^2 \qquad 0<\alpha<1 \quad , \quad T=121,0,0,0,162 \qquad (7)$$

When reverse the relations, we have:

$$\hat{\sigma}_{T+1}^2 = \alpha\sum_{t=1}^{T}(1-\alpha)^t\sigma_{T+1-t}^2 \qquad T=121,0,0,0,162 \qquad (8)$$

Forecasting of $\hat{\sigma}_{T+1}^2$ is the thematic average of the previous amount of $\hat{\sigma}_{T+1-t}^2$ in which weights reduced through passing the time exponentially. In this paper the amount of α would be calculated with the help of function solver in software excel in a way that sum of forecasting root mean squares errors would be minimized. The way of calculating the amount of α has been provided in the attached file.

2.6 Exponential Moving Average (EMA)

This method is a combination of two above-mentioned methods and the forecasting would be as:

$$\hat{\delta}_{T+1}^2 = (1-\alpha)\hat{\delta}_T{}^2 + \alpha 1/L\sum_{j=1}^{l}\hat{\delta}_{T+1-j}^2 \qquad T=121,0,0,0,162 \qquad (9)$$

M=36 and M=72 would be considered for 3-year and 6-year moving averages. The value of α would be forecasting based on the minimization of the sum of root mean square errors and would be obtained based on solver and software excel.

2.7 Arch Model

Arch model which has been first presented by Engle in 1982 is as below [4]:

$$\sigma_n^2 = \gamma v_L + \sum_{i=1}^{m}\alpha_i u_{n-i}^2 \qquad (10)$$

Whereas, v_L is the long-term average of variance and coefficients u and γ would be allocated in way that their sum would be equal 1. Choosing m is a main and important question for implementation. M would be selected based on BIC criterion in this paper. Like regression model, the sample size would be kept constant for this model.

2.8 Garch Model

In most of models which have been dealt with the implementation of Garch model, the amount of m should be selected a big one. This made the calculations complicated. Therefore, Bollerslev in 1986 introduced Garch model (pmq) as [6]:

$$\sigma_n^2 = \gamma v_L + \alpha u_{n-1}^2 + \beta\sigma_{n-1}^2 \qquad (11)$$

In this model determination p and q is an important issue for implementation. Based on the literature review, Grach (1,1) model is the most popular and most widely used one from Garch family. In this paper this model has been applied to be able to compare the model results with other results from other markets which have been studied in the literature review. On the other hand, in order to be able to study the effect of model parameters change, model Garch (3,2) has been used for the forecasting of volatility in Iran stock exchange index. Despite the detailed and complete literature on this issue as it can be seen none of the researches would deal with the volatility forecasting through fuzzy approach or through the combination of two or several models. Therefore, in this paper we will deal with the design of one fuzzy model using the combination of two models as random walk and moving average methods.

3 Proposed Model

According to the high efficiency and the simplicity of calculations by two random walk and moving average models, in this part we use the forecasting values by these two models at the system inputs to present one fuzzy system to forecasting the fluctuations of stock exchange index. Though the term "fuzzy modeling" has not been used so often, fuzzy modeling is the most important issue in fuzzy logic or more widely in fuzzy theory. In fact, we can find the seminal ideas of fuzzy modeling in the early papers of Zadeh. In the fuzzy modeling, the most important problem is the structure identification and parameter identification [14]. To do this, first the real amounts of index fluctuations have been considered as the system output in an interval between 2007 December and 2011 May and then the number of optimized clusters as well as membership degree matrix using GK clustering model have been calculated as [14-15]:

$$S(c)=\sum_{k=1}^{n}\sum_{t=1}^{c}(\mu_{ik})^{m}(\|x_k-v_i\|^2-\|v_i-\bar{x}\|^2), \tag{12}$$

Then the classified outputs would be projected on the inputs (the forecasting done by two random walk and moving average models) and the membership degree matrix would be obtained. In this step MATLAB software has been used for fitting one regular and defined membership function.

In this problem after calculation of S(c) (Fig. 2) for various Cs and m, the number of optimized clusters C=6 would be obtained (amounts of S (c) for different C and m has been provided in the appendix). After doing all above-mentioned processes, the rolls including two inputs and one output that use multi input single output [16] (MISO) (the resulting rolls have been provided in the appendix in Fig. 3 and Fig. 4). After determination of the forecasting amount through random walk and moving average models, and also after doing the above-mentioned process, and doing Mamdani model [17], finally doing Defuzzification process, one scalar number would be obtained as the forecasting amount by fuzzy system.

4 Implementation and Results

Now we will deal with the forecasting of main index fluctuation through data analysis of Iran stock exchange index. There are different indexes in this stock exchange that the oldest one is Tepix which is used here. Tepix is and index which evaluates the data related to the exchangeable symbols values at any time with its equivalent value annually and would announce the situation of Iran stock exchange of that year. Data is available for approximately 14 years. We used the data for the first 10 years (1997 December to 2007 November) to estimate parameters and would be able to estimate and forecasting the data fluctuations of the last 3.5 years (2007 November to 2011 May). Figure 1 shows the fluctuations change of this index during this mentioned 10 years.

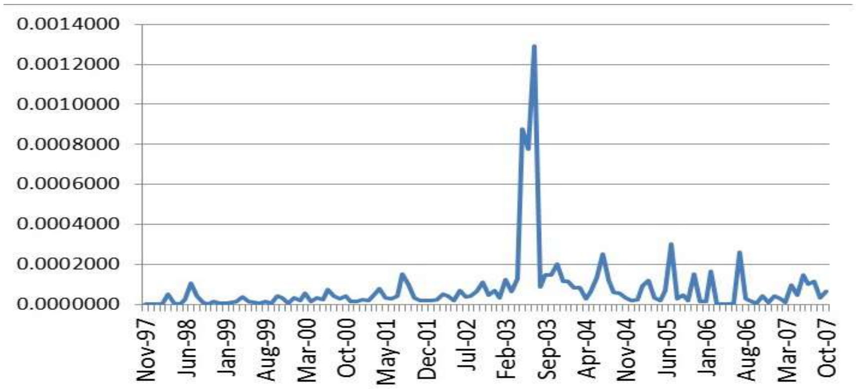

Fig. 1: Diagram of monthly fluctuation change of Iran stock exchange index

In the first part it would dealt with the implementation of model results which have been discussed in the literature review and it has been attempted to analyze these models using the data obtained by Tehran stock exchange one. In the second part it is attempted to present a new model for forecasting with fuzzy approach and through the combination of other models, and finally the results would be compared with other models.

4.1 Implementation of Present Models in the Literature

As it has been mentioned above, 8 different models have been used in this paper. Models Moving average, EWMA, and Garch models have been implemented separately and totally we have 11 models. It should also be mentioned that in case of Exponential smoothing and EWMA models calculation of parameter α has been done through solver and software excel. The forecasting values would be calculated for the values of α and then the sum of square errors would be obtained and finally by the help of this software the value of α would be adjusted in a way the sum of square errors would be minimized. This process would be used for determination of constant coefficients as α, β, and γ in Arch and garch models. In order to determine Arch model parameters (m) the BIC analysis would be used. Finally, the value m=10 would be used for Arch model.
The difference of this paper with other ones in the literature review is first the usage of this for Iran investment market which has been used hardly in the literature. Another difference is the implementation and comparison of the presented models in the literature (simple models such as random walk to complicated models like random volatility).
Other differences of this paper with other present papers in the literature review is the usage of evaluation indexes which has not been dealt completely in the literature and can give examples as Linex which is an Asymmetric index and has higher flexibility in evaluations of forecasting accuracy.
The summery of the results by implementation of these models on data has been provided in the below tables. RSME criterion shows that Garch (3,2) model forecastings the best solution and after that Garch (1,1) model forecastings in a good way. As it has been mentioned in the literature review, Random walk model would give proper solutions despite the simple calculations but its solutions are not

proper for Tepix index of Iran and are not close to the real results. According to the statistic, Exponential smoothing model would be in the last row. Another interesting point about this statistic is the proper performance of Wiener process which is in the fourth rank. In the statistic Theil U just exponential smoothing would act worse than Random walk and its value is more than 1. In this statistic, Garch group have the best performance. Unlike the previous criterion, Arch model has no proper performance.

Table 1: Results and comparison of results of models

	RSME		MAE		Theil-U	
	Value	Rank	Value	Rank	Value	Rank
Random Walk	0.000218898	11	0.000135137	11	1	11
Historical Average	0.000200987	7	0.000117897	2	0.838424374	7
Moving average (M=36)	0.00020951	9	0.00012339	9	0.916065102	9
Moving average (M=72)	0.000194608	4	0.000121353	7	0.790380541	4
Regression	0.00017897	3	0.00011754	1	0.668461617	3
Exponential smoothing	0.000227242	12	0.000149553	12	1.07768369	12
EWMA (M=36)	0.000211586	10	0.000126549	10	0.934303112	10
EWMA (M=72)	0.00019569	5	0.000122204	8	0.799191489	5
Wiener process	0.000197157	6	0.000119082	4	0.811221915	6
Arch	0.000200987	8	0.000117897	3	0.843047928	8
Garch (1,1)	0.000171126	2	0.000120907	6	0.611148365	2
Garch (3,2)	0.000170626	1	0.000120843	5	0.607581591	1

Table 2 has dealt with the study and comparison of models based on statistics Linex which is an asymmetric criterion. More interestingly, according to Linex the ranking of all models is the same and generally group of Garch models have the best performance and Exponential smoothing and Random Walk models have the worst performance. As it has been mentioned above, in loss function Linex for each positive α to underestimate more penalty would be considered and vice versa. According to Iran market data, these models would not show any sensitivity to α change and the final result would not change. In other words, under this condition symmetric and asymmetric criterion would not make a difference and the asymmetric criterion Linex would not create considerable added value.

Table 2: Table of results and results comparison of models implementation

	Linex a=-20		Linex a=-10		Linex a=10		Linex a=20	
	Value	Rank	Value	Rank	Value	Rank	Value	Rank
Random Walk	0.00000958	11	0.00000240	11	0.00000240	11	0.000000959	11
Historical Average	0.00000811	8	0.00000202	8	0.00000202	7	0.000000805	7
Moving Average (M=36)	0.00000881	9	0.00000220	9	0.00000219	9	0.00000875	9
Moving Average (M=72)	0.00000760	4	0.00000190	4	0.00000189	4	0.00000755	4
Regression	0.00000639	3	0.00000160	3	0.00000160	3	0.00000642	3
Exponential smoothing	0.00001036	12	0.00000259	12	0.00000258	12	0.00001029	12
EWMA (M=36)	0.00000898	10	0.00000224	10	0.00000223	10	0.00000892	10
EWMA (M=72)	0.00000768	5	0.00000192	5	0.00000191	5	0.00000764	5
Wiener Process	0.00000775	6	0.00000194	6	0.00000195	6	0.00000780	6
Arch	0.00000805	7	0.00000202	7	0.00000202	8	0.00000811	8
Garch (1,1)	0.00000584	2	0.00000146	2	0.00000147	2	0.00000587	2
Garch (3,2)	0.00000581	1	0.00000145	1	0.00000146	1	0.00000584	1

4.2 Proposed Model Implementation

In order to validate the proposed model, the forecasting values have been obtained for 42 months (2007, November to 2011, March) by three methods of random walk, moving average, and fuzzy model and the value of RMSE criterion would be calculated through three methods (calculations have been presented in excel file). After obtaining the root mean square errors, data of three models as random Walk, Moving Average, and Fuzzy models the following data would be results that by their comparison it would be shown that the value of this criterion is less for the proposed model than two other methods. In other words, the obtained values of the proposed model are closer to the real ones and it proves the more efficiency of the proposed model than others.

Table 3: Table of the forecasting values for three methods

RMSE value	Forecasting method
0.000218898175	Random Walk
0.000194607808	Moving Average
0.000187495625	Proposed fuzzy model through combination of two above-mentioned models

5 Discussion and Conclusions

During the last years forecasting of volatility is one of the most important research fields in finance markets. Because of many applications of the volatility, various models have been presented for forecasting. In this paper we dealt with the study and testing of 8 different models and used them based on 10-year data of the main index of Iran stock exchange (Tepix) as well as for the forecasting of the next 3.5 years. Then we designed one fuzzy system to forecasting fluctuations of stock exchange index using the forecasting values by two Random Walk and Moving Average (because of high efficiency and simplicity) models as the inputs and real values as the outputs. According to the above-mentioned points it seems that the following items are proper for the future researches. Since random processes models have been welcomed greatly, it seems that the study and evaluation of other forecastinging models like SV models and their comparisons with the present methods are proper for researches. Additionally, the forecasting values of other models can be considered as the fuzzy system input and other models like Logical or Hybrid ones can be used.

Appendix

Table 4: Input and output data

Month	Input1: RW	Input2: MA (72)	Output
2007, November	0.00007	0.000108	0.00012
2007, December	0.00012	0.000110	0.00005
2008, January	0.00005	0.000111	0.00006
2008, February	0.00006	0.000112	0.00003
2008, March	0.00003	0.000113	0.00002
2008, April	0.00002	0.000114	0.00004
2008, May	0.00004	0.000115	0.00022
2008, June	0.00022	0.000117	0.00062
2008, July	0.00062	0.000117	0.00007
2008, August	0.00007	0.000118	0.00006
2008, September	0.00006	0.000119	0.00015
2008, October	0.00015	0.000120	0.00062
2008, November	0.00062	0.000120	0.00014
2008, December	0.00014	0.000121	0.00003
2009, January	0.00003	0.000122	0.00002
2009, February	0.00002	0.000123	0.00009
2009, March	0.00009	0.000123	0.00006
2009, April	0.00006	0.000124	0.00010
2009, May	0.00010	0.000124	0.00007
2009, June	0.00007	0.000114	0.00006
2009, July	0.00006	0.000104	0.00015
2009, August	0.00015	0.000088	0.00076
2009, September	0.00076	0.000088	0.00010
2009, October	0.00010	0.000087	0.00010
2009, November	0.00010	0.000086	0.00019
2009, December	0.00019	0.000085	0.00008
2010, January	0.00008	0.000084	0.00004
2010, February	0.00004	0.000084	0.00009
2010, March	0.00009	0.000084	0.00027
2010, April	0.00027	0.000084	0.00008
2010, May	0.00008	0.000085	0.00007
2010, June	0.00007	0.000085	0.00010
2010, July	0.00010	0.000084	0.00026
2010, August	0.00026	0.000082	0.00035
2010, September	0.00035	0.000082	0.00029
2010, October	0.00029	0.000082	0.00006
2010, November	0.00006	0.000082	0.00006
2010, December	0.00006	0.000083	0.00016
2011, January	0.00016	0.000084	0.00023
2011, February	0.00023	0.000085	0.00031
2011, March	0.00031	0.000084	0.00046
2011, April	0.00046	0.000084	0.00046

Table 5: S(c) values

C, m	2	3	4	5	6
1.6	-6.06E-27	-9.97E-27	-2.21E-26	-2.22E-26	-2.22E-26
1.7	-6.23E-27	-1.04E-26	-2.21E-26	-2.21E-26	-2.21E-26
1.8	-6.37E-27	-1.10E-26	-2.20E-26	-2.20E-26	-2.20E-26
1.9	-6.48E-27	-1.21E-26	-2.19E-26	-2.19E-26	-2.19E-26
2	-6.57E-27	-1.40E-26	-2.17E-26	-2.17E-26	-2.16E-26
2.1	-6.64E-27	-1.57E-26	-2.15E-26	-2.13E-26	-2.13E-26
2.2	-6.69E-27	-1.65E-26	-2.11E-26	-2.09E-26	-2.08E-26
2.3	-6.73E-27	-1.69E-26	-2.06E-26	-2.03E-26	-2.01E-26
2.4	-6.75E-27	-1.71E-26	-2.00E-26	-1.96E-26	-1.94E-26
2.5	-6.77E-27	-1.72E-26	-1.92E-26	-1.87E-26	-1.84E-26
2.6	-6.77E-27	-1.71E-26	-1.84E-26	-1.78E-26	-1.74E-26
2.7	-6.7E-27	-1.69E-26	-1.75E-26	-1.67E-26	-1.62E-26
2.8	-6.77E-27	-1.67E-26	-1.79E-26	-1.56E-26	-1.49E-26
2.9	-6.77E-27	-1.64E-26	-1.70E-26	-1.44E-26	-1.36E-26
3	-6.77E-27	-1.60E-26	-1.61E-26	-1.31E-26	-1.23E-26

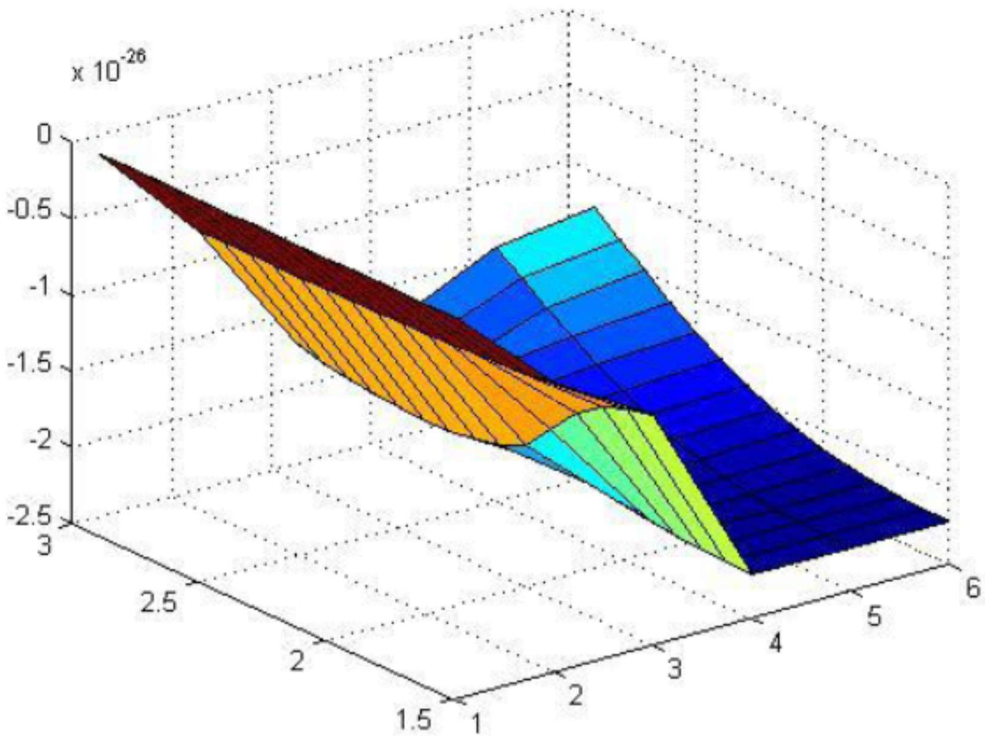

Fig. 2: Diagram of S(c) values

Table 6: Membership function matrix for outputs

	C1	C2	C3	C4	C5	C6
1	6.30E-07	2.24E-09	2.82E-08	9.98E-01	2.01E-05	2.29E-03
2	4.14E-07	1.43E-09	1.82E-08	9.98E-01	1.37E-05	1.72E-03
3	1.96E-08	5.68E-11	7.69E-10	1.00E+00	8.02E-07	1.81E-04
4	4.28E-09	1.17E-11	1.61E-10	1.00E+00	1.89E-07	5.37E-05
5	2.91E-09	7.86E-12	1.09E-10	1.00E+00	1.31E-07	3.91E-05
6	2.17E-09	5.80E-12	8.07E-11	1.00E+00	9.88E-08	3.07E-05
7	1.99E-12	4.04E-15	6.16E-14	1.00E+00	1.32E-10	1.58E-07
8	6.92E-11	1.34E-13	2.07E-12	1.00E+00	4.94E-09	7.92E-06
9	1.28E-10	2.45E-13	3.80E-12	1.00E+00	9.27E-09	1.60E-05
10	1.46E-09	2.63E-12	4.17E-11	1.00E+00	1.15E-07	2.90E-04
11	1.83E-09	3.29E-12	5.23E-11	1.00E+00	1.46E-07	3.87E-04
12	8.70E-09	1.49E-11	2.40E-10	9.97E-01	7.48E-07	2.87E-03
13	1.33E-08	2.24E-11	3.63E-10	9.95E-01	1.17E-06	5.09E-03
14	3.50E-08	5.65E-11	9.29E-10	9.80E-01	3.29E-06	1.99E-02
15	7.24E-08	1.13E-10	1.88E-09	9.40E-01	7.20E-06	5.99E-02
16	1.27E-07	1.92E-10	3.22E-09	8.45E-01	1.34E-05	1.55E-01
17	1.16E-07	1.51E-10	2.66E-09	8.38E-02	1.54E-05	9.16E-01
18	7.50E-08	9.52E-11	1.69E-09	3.66E-02	1.05E-05	9.63E-01
19	1.13E-09	1.19E-12	2.24E-11	7.11E-05	2.19E-07	1.00E+00
20	2.58E-10	2.62E-13	4.98E-12	1.11E-05	5.43E-08	1.00E+00
21	1.34E-12	1.08E-15	2.19E-14	8.69E-09	4.48E-10	1.00E+00
22	3.90E-12	3.10E-15	6.34E-14	2.32E-08	1.34E-09	1.00E+00
23	3.58E-11	2.75E-14	5.69E-13	1.69E-07	1.32E-08	1.00E+00
24	1.35E-10	1.02E-13	2.12E-12	5.38E-07	5.29E-08	1.00E+00
25	2.61E-07	1.46E-10	3.32E-09	1.79E-04	2.03E-04	1.00E+00
26	8.05E-05	2.41E-08	6.50E-07	3.13E-03	4.31E-03	5.65E-01
27	2.04E-05	3.74E-09	1.15E-07	1.42E-04	9.91E-01	9.07E-03
28	1.87E-05	3.38E-09	1.04E-07	1.24E-04	9.92E-01	7.72E-03
29	3.57E-06	5.01E-10	1.65E-08	1.09E-05	1.00E+00	4.74E-04
30	5.31E-08	2.18E-12	9.49E-11	7.00E-09	1.00E+00	1.17E-07
31	1.66E-02	1.00E-07	6.32E-06	5.43E-05	9.83E-01	0.000495
32	1.23E-01	3.71E-07	2.62E-05	1.28E-04	8.76E-01	0.001035
33	9.94E-01	8.81E-08	9.65E-06	7.09E-06	5.60E-03	4.11E-05
34	9.99E-01	3.16E-08	3.81E-06	1.96E-06	1.16E-03	1.08E-05
35	1.00E+00	2.54E-14	5.14E-12	4.58E-13	9.20E-11	2.05E-12
36	1.00E+00	6.00E-10	1.86E-07	4.80E-09	5.61E-07	1.91E-08
37	9.83E-01	1.27E-05	1.58E-02	1.55E-05	0.000723	4.98E-05
38	7.43E-13	1.46E-13	1.00E+00	1.61E-15	2.22E-14	3.75E-15
39	8.71E-13	2.47E-13	1.00E+00	2.13E-15	2.83E-14	4.91E-15
40	4.69E-06	9.99E-01	0.000597	1.05E-07	6.25E-07	1.91E-07
41	1.81E-06	0.999795289	2.03E-04	4.26E-04	2.49E-07	7.72E-08
42	2.59E-05	9.99E-01	4.78E-04	1.54E-06	6.10E-06	2.47E-06

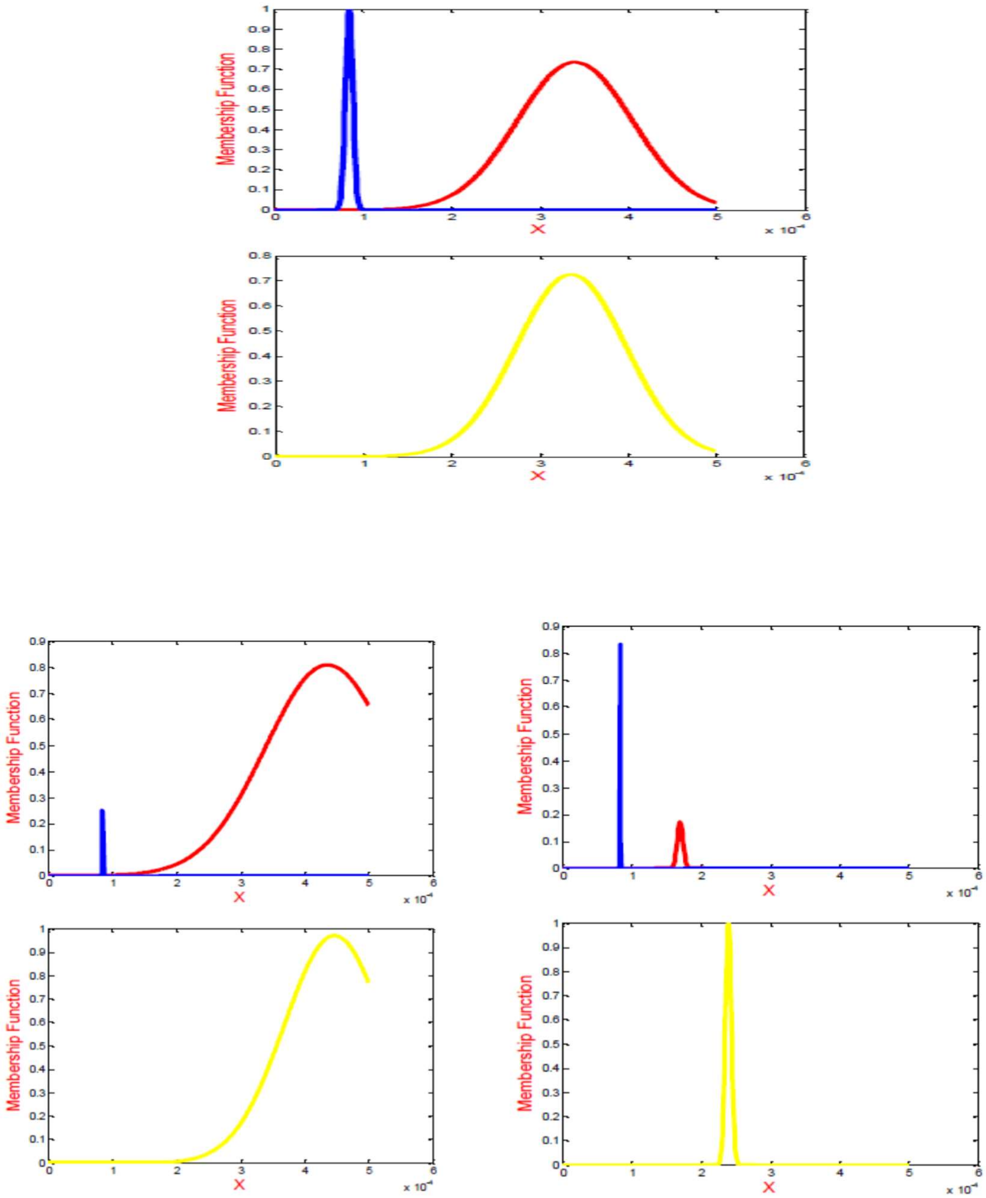

Fig. 3: Rules (Part 1)

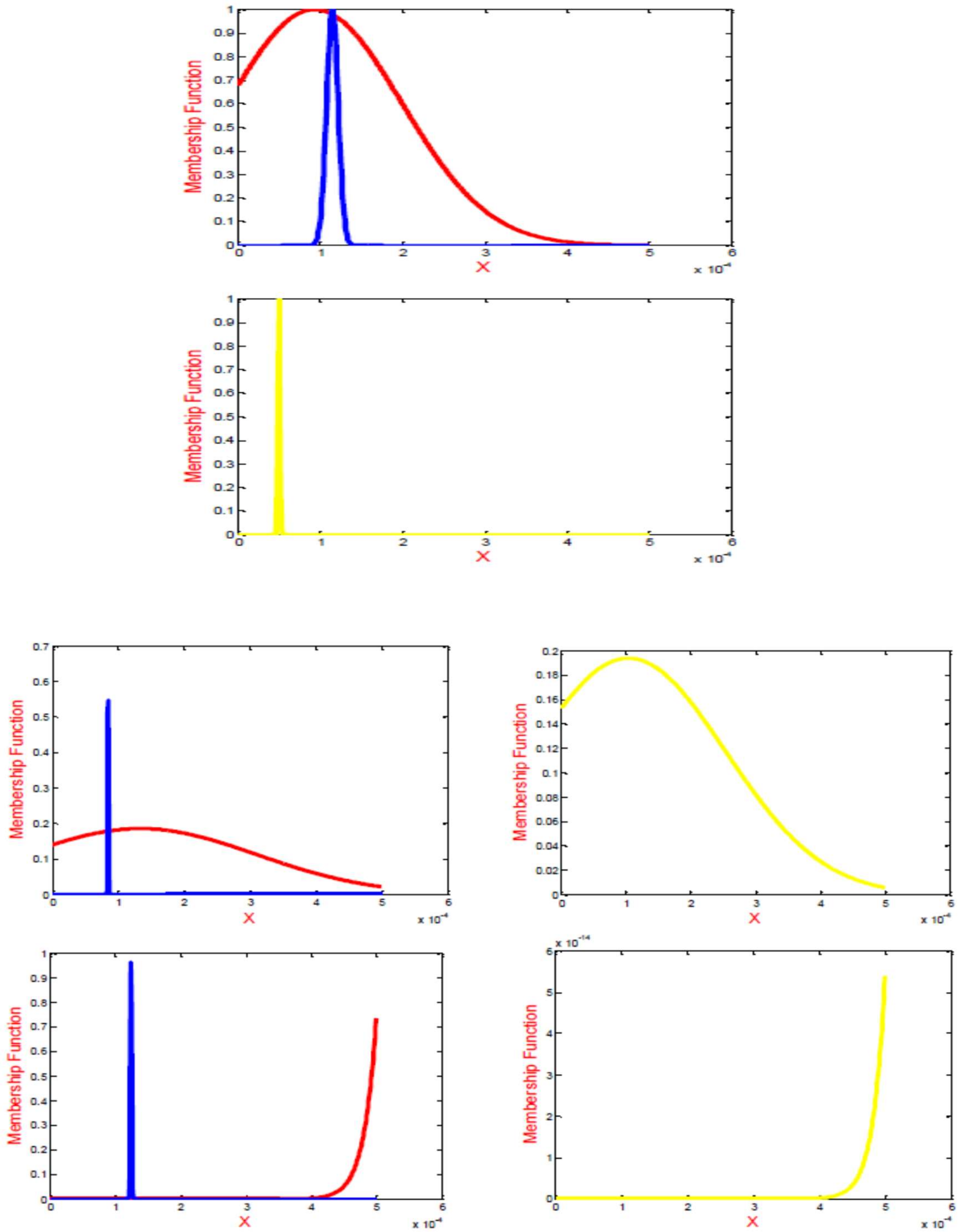

Fig. 4: Rules (Part 2)

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