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Application of Mathematics in Financial Management

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ABSTRACT

The Time Value of Money is an important concept in Financial Management. The Time Value of Money includes the concepts of future value and discounted value or present value. In the present article, the basic notions are described and their applications in the field of investment are presented in the mathematical terms by using some useful theorems. Then, the applications of some well-known problems with the proof such as mortgage loan problem, investment in bond and an individual who plans to retire in certain years who plan for investment for its future life. We also presented the application of calculus such as limit, derivative and integration in financial management.

1 Introduction

We first define the basic concept of calculus as follow [1]: **(i).** constant: A quantity which does not change during any set of mathematical operations is called constant. There are two types of constants, Absolute constant and arbitrary constant. **(ii).** Variable: A quantity which changes during any set of mathematical operations is called variable. There are two types of variables, Independent and Dependent. **(iii)**. Intervals: The of real numbers lying between two real numbers is called intervals: If a and b are two real numbers then, we say: Closed Interval: [a,b]; Open Interval: (a,b) ; Semi-Closed: [a,b); Semi-Opened: (a,b]. **(iv).** Function: If two variables x and y are related such a way that, for every value of x there is a definite value of y, then y is said to be a function of x. We denote this function by the symbol: y = f(x). There are many types of functions such as: Explicit, Implicit, Parametric, Inverse, Algebraic, Transcendental, Trigonometric, Exponential, Logarithmic, Hyperbolic and many more. **(v).** Limit of a function: Let A(t) be any accumulated function of the independent time variable t. Then if there exists a quantity L such that the difference of L and A(t) may be made to differ by less than any assignable quantity however small by taking t approaches sufficiently near its assigned value a, the function A(t) is said to have the limit L at x = a. This fact can be denoted by the following notation: $\lim_{x\to a} A(t) = L$. **(vi).**Continuity of a function: A function A(t) is said to be continuous at the point x = a, if $\lim_{x\to a} A(t) = A(a)$. (vii). Differentiability of a function: If A(t) be a function of t, then, $\lim_{h\to 0} \frac{A(x+h)-A(x)}{h}$ is called the differential coefficient of A(t) and it is denoted by $A'(t)$,

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provided this limit exists. **(vii).** Finally, integration is the is the reverse process of differentiation that is if $\frac{d}{dt}A(t) = a(t)$, then $\int A(t)\,dt = a(t)$, which reads as integration of function $A(t)$ with respect to t is $a(t)$.

The Financial Management (FM) is a combination of two words "Finance" and "Management". Finance means to arrange payments for it or in other words Finance means the study of money, its nature, creation and behaviour, it is true that one needs money to make money, therefore finance is a lifeblood of business enterprise and there must be a continuous flow of funds in and out of a business enterprise. Money makes the wheels of business run smoothly. Sound plans, efficient production system and excellent marketing network are all hampered in the absence of an adequate and timely supply of funds. Efficient management of business is closely linked with efficient management of its finance [6,9]. Hence, we conclude that FM is concerned with planning, directing, monitoring, organizing and controlling monetary resources of an organization. Thus, FM is simply deals with management of money. As an integral part of overall Management, FM is not a totally independent area. FM is related with the other disciplines such as Economics, Accounting, Production, Marketing and Quantitative methods. Importance of FM cannot be over-emphasized. It is, indeed the key to successful business operations. Without proper administration of finance, no business enterprise can reach its potentials for growth and success. The important of FM is all about planning, investment, funding the investment, monitoring expanses against budget and managing gains from the investment. FM means management of all matters related to an organization's finance. The importance of FM is to describe some of the tanks that is involved

1. Taking care not to over-invest in fixed assets.
2. Balancing cash-outflow with cash-inflow.
3. Ensuring that there is a sufficient level of short term working capitals.
4. Increasing gross profit by setting the correct pricing for the products or services.
5. Tax planning that will minimize the taxes a business has to pay.

The scope of FM has undergone changes over the year. Under the middle of this century, its scope was limited to procurement of funds under major events in the life of the enterprise such as promotion, expansion, mergers etc. In modern times, the FM includes besides procurement of funds, the three different kinds of decisions as investment, financing, and dividend. Efficient FM requires the existence of some objective or goals because judgement as to whether or not a financial decision is efficient must be made in the light of some objective. Although various objectives are possible but we assume two objectives of FM which is very important and these are Profit Maximisation and Wealth Maximisation. In this article, we focus only on the investment management, which is one of the application of time value of money. Now, investment means an asset or item that is purchased with the hope that it will generate income or appreciate in the future. In economic sense, an investment is purchase of goods that are not consumed today but are used in the future to create wealth. In finance, an investment is a monetary asset purchased with the idea that the asset will provide income in the future or appreciate and be sold at a higher price. There are various types of investment available in the market in which one can invest the money, for example Stock, Bonds, Mutual funds, Deposits in Banks, Deposit in Post Office, Real Estate, Gold and many others.

2 Time Value of Money

Time Value of Money (TVM) is an important concept in Financial Management [7,8]. It can be used to compare investment alternatives and to solve problems involving loans, mortgages, leases, savings, and Annuities. Time value of money is the concept that the value of a Rupee to be received in future is less than the value of a Rupee on hand today or Money has time value, a rupee today is more valuable than a year hence. One reason is that money received today can be invested which generating more money. Another reason is that when a person opts to receive a sum of money in future rather than today, he is effectively lending the money and there are risks involved in lending such as default risk and inflation. Default risk arises when the borrower does not pay the money back to the lender. Inflation is the rise in general level of prices. A key concept of TVM is that a single sum of money or a series of equal, evenly-spaced payments or receipts promised in the future can be converted to an equivalent value today. Conversely, you can determine the value to which a single sum or a series of future payments will grow to at some future date. The components of TVM are as follows:

1. Time period (n): can be any time interval such as year, half a year, and month. Each period should have equal time interval. Zero period represents a starting point.

2. Payments (P): are the transactions that refer to a business receiving money for a good or service.

3. Interest (r): Interest is charge against use of money paid by the borrower to the lender in addition to the actual money lent. We study generally two types of interest, Simple and Compound Interest.

4. Present Value (PV): When a future payment or series of payments are discounted at the given rate of interest up to the present date to reflect the time value of money, the resulting value is called present value. We study generally, Present Value of a Single Sum of Money and Present Value of an Annuity.

5. Future Value (FV): Future value is amount that is obtained by enhancing the value of a present payment or a series of payments at the given rate of interest to reflect the time value of money. We study generally, Future Value of a Single Sum of Money and Future Value of an Annuity.

6. Cash Flow (CF): The term cash flow refers literally to the flow, or movement of cash funds into or out of a business/individuals. There are two types of cash flow: (i) Cash outflow: an amount that you pay, cost to you, or spending amount (has a negative sign or negative cash flow) and (ii) Cash inflow: an amount that you receive or savings amount (has a positive sign or positive cash flow).

7. Discounted cash flow (DCF): The discounted cash flow is an application of the time value of money concept—the idea that money to be received or paid at some time in the future has less value, today, than an equal amount actually received or paid today.

8. Time Line (TL): The time line is a very useful tool for an analysis of the time value of money because it provides a visual for setting up the problem. It is simply a straight line that shows cash flow, its timing, and interest rate. A time line consists of the following components: (i) Time periods (ii) Interest rate and (iii) Cash flow.

9. Cash Flow TimeLine (CFTL): A cash flow timeline can be a useful tool for visualizing and identifying cash flows over time. A cash flow timeline is a horizontal line with up-arrows that represent cash inflows and down-arrows to indicate cash outflows. The down arrow at Time 0 represents an investment today (PV); the up-arrow n periods in the future represents FV, the future value (or compounded value) of the investment. For example, a cash flow stream of □ 100 at the end of each of

next five year at the rate of 10% can be represented on a time line as follows:
10. Amortization is an accounting term that refers to the process of allocating the cost of an intangible asset over a period of time. It also refers to the repayment of loan principal over time. Mortgage is a legal document between a mortgagor and a mortgagee that establishes a home and/or property as security for a home loan.

3 Techniques of Time Value of Money

In modern finance, time value of money concepts plays a central role in decision support and planning [7,8]. When investment projections or business case results extend more than a year into the future, professionals trained in finance usually want to see cash flows presented in two forms, with discounting and without discounting. That is there are two techniques for adjusting time value of money. They are: (i) Compounding Techniques / Future value techniques and (ii) Discounting Techniques / Present value techniques [2].
Let us consider an investment situation in which there are two times : say "now ($t_0 = 0$)" and "later ($t_1 = 1$). An amount of money invested at time t_0 is denoted by $A = A(0)$and its value at time $t_1 = 1$ is denoted by $A_1 = A(t_1)$. In general, the function $A(t)$ represent the total accumulated value of an investment at time . The Interest rate is a percentage measure of interest, the cost of money, which accumulates to the lender. Simple interest is a basic formula for calculating how much interest to apply to a principal balance or Simple interest is calculated only on the initial amount that you invested. The compound interest is the addition of interest to the principal sum of a loan or deposit, or in other words, interest on interest. In this concept, the interest earned on initial principal amount becomes a part of the principal at the end of the compounding period. An annuity is a sequence of payments, usually equal in size and made at equal intervals of time. The examples are premiums of life insurance, monthly deposits in bank, instalment loan payments etc. The types of annuities are, Ordinary annuity (Immediate annuity) is an annuity where the first payment of which is made at the end of first payment interval, Annuity Due is annuity where the first payment of which is made at the beginning of the first payment interval and Perpetuity is an annuity whose continue forever.

4 Main Results

The Theorems below give the formula for simple interest, compound interest, annuities and its accumulated amount:

Theorem 1. If the initial amount $A(0)$ is invested for t years at interest rate r percent per year then,

$Simple\ Interest(I) = A(t) - A(0) = A(0) \times r \times t$ and the accumulated Amount at the end of t years is given by $A(t) = A(0)(1 + rt)$.

Theorem 2. If the initial amount $A(0)$ is invested for t years at interest rate r percent per year then, $Compound\ Interest(I) = A(t) - A(0)$ and the accumulated amount at the end of t years is given by

$A(t) = A(0)(1 + r)^t$.

Proof.

Suppose the initial amount of $\$\boldsymbol{A(0)}$ is deposited at a rate of interest $\boldsymbol{r}$% per year, then the compounding amount at the end of First year:
Principal : A(0)
Interest for the year : $A(0) \times r$
Principal at the end : $A(0) + A(0) \times r = A(0)(1 + r)$.
At the end of Second year:
Principal : $A(0)(1 + r)$
Interest for the year : $A(0)\,(1 + r) \times r$
Principal at the end : $A(0) \times (1 + r) + A(0) \times\ r \times\ (1 + r) = A(0) \times (1 + r)^2$.
Generalizing the above procedure for t years, we get
$A(t) = A(0) \times (1 + r)^t$
and the formula for calculating the compound interest are
$Compound\ Interest\ (CI) = A(t) - A(0)$

Notes:

(i) Remember, If the transaction is to be made for less than one year, that is time may be given in days, weeks and months but in formula time t represents the number of years, hence the time given which is not in year must be converted into the year as follows:

$(a)\ \ X\ days = \frac{X}{365}\ years\ \ (b)\ X\ weeks = \frac{X}{52}\ years\ and\ \ (c)\ X\ months = \frac{X}{12}\ years.$

(ii). In the above Theorem 2, the term $(1 + r)^t$ is called Future Value Interest Factor (FVIF), which is denoted by $FVIF_{r,n} = (1 + r)^n$. We can directly get the value of FVIF for certain combinations of period and interest rate from the Table provided. For example, the value of $FVIF_{r,n}$ for various combinations of $r\ and\ n$ are as follows:

Table 1: The Value of FVIF for Certain Combinations

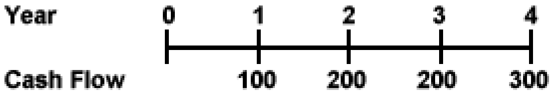
t/r	6%	8%	10%
2	1.124	1.166	1.210
4	1.262	1.360	1.464
6	1.419	1.587	1.772
8	1.594	1.851	2.144

(iii). Interest can be compounded monthly, quarterly and half yearly. If compounding is quarterly, annual interest rate is to be divided by 4 and the number of years is to be multiplied by 4. If compounding is monthly, annual interest rate is to be divided by 12 and the number of years is to be multiplied by 12.

Theorem 3. The present value $A(0)$ and future value $A(t)$ at an interest rate r % for t years is

$$A(0) = \frac{A(t)}{(1+r)^t} \quad and \quad A(t) = A(0)(1+r)^t$$

For example, the Present Value and Future value of the following cash flow stream



given that the interest rate is 10% is calculated as follow:

$$A(0) = \frac{100}{(1+.10)^1} + \frac{200}{(1+.10)^2} + \frac{200}{(1+.10)^3} + \frac{300}{(1+.10)^4} = \text{⍰}\, 611.37$$

and

$$A(4) = 100(1+.10)^3 + 200(1+.10)^2 + 200(1+.10)^1 + 300 = \$895.10$$

Theorem 4. The future value of an ordinary annuity that makes n constant payments \$P at the end of each unit time interval is:

$$FV_{ordinary \quad annuity} = P\left[\frac{(1+r)^n - 1}{r}\right]$$

where r is the constant interest rate per period.
Corollary: If there are m compounding per year with nominal yearly interest rate i , then the future value of ordinary annuity at time t years is:

$$FV_{ordinary \quad annuity} = P\left[\frac{\left(1+\frac{i}{m}\right)^{mt} - 1}{\frac{i}{m}}\right]$$

Proof.
Let us consider an annuity of n payments of \$P each with the interest rate per period is r and the first payment is made one period from now or at the end of the first payment interval.
Now,
Amount of the 1st payment : $P(1+r)^{n-1}$
Amount of the 2nd payment : $P(1+r)^{n-2}$
Amount of the 3rd payment : $P(1+r)^{n-3}$
.
.
.
Amount of the nth payment : P
Adding all these amount in the reverse order and denoting the amount of this annuity by

$FV_{ordinary\ \ annuity}$, we have

$$FV_{ordinary\ \ annuity} = P + P(1+r) + P(1+r)^2 + \cdots + P(1+r)^{n-1}$$

$$= P(1 + (1+r) + (1+r)^2 + \cdots + (1+r)^{n-1}$$

$$= P\left[\frac{(1+r)^n - 1}{1+r-1}\right]$$

Therefore, we have,

$$FV_{ordinary\ \ annuity} = P\left[\frac{(1+r)^n - 1}{r}\right]$$

Hence the theorem is proved.

Similarly, we can prove below theorems.

Theorem 5. The present value of an ordinary annuity that makes n constant payments \$P at the end of each unit time interval is:

$$PV_{ordinary\ \ annuity} = P\left[\frac{1-(1+r)^{-n}}{r}\right]$$

where r is the constant interest rate per period.

Corollary: If there are m compounding per year with nominal yearly interest rate i, then the present value of ordinary annuity at time t years is:

$$PV_{ordinary\ \ annuity} = P\left[\frac{1-\left(1+\frac{i}{m}\right)^{-mt}}{\frac{i}{m}}\right]$$

Theorem 6. The future value of an annuity due that makes n constant payments \$P at the beginning of each unit time interval is:

$$FV_{annuity\ \ due} = P(1+r)\left[\frac{(1+r)^n - 1}{r}\right]$$

Where r is the constant interest rate per period.

Corollary: If there are m compounding per year with nominal yearly interest rate, then the future value of annuity due at time t years is:

$$FV_{annuity\ \ due} = P\left(1+\frac{i}{m}\right)\left[\frac{\left(1+\frac{i}{m}\right)^{mt} - 1}{\frac{i}{m}}\right]$$

Theorem 7. The present value of an annuity due that makes n constant payments \$P at the beginning of each unit time interval is:

$$PV_{annuity\ \ due} = P(1+r)\left[\frac{1-(1+r)^{-n}}{r}\right]$$

where r is the constant interest rate per period.

Corollary: If there are m compounding per year with nominal yearly interest rate, then the present value of annuity due at time t years is:

$$PV_{annuity\ \ due} = P(1+r)\left[\frac{1-\left(1+\frac{i}{m}\right)^{-mt}}{\frac{i}{m}}\right]$$

Theorem 8. The present value of a perpetuity immediate in which payments of ₹ P are made at the end of each time interval, and the per period interest is r is:

$$PV_{perpetuity \quad due} = \frac{P}{r}$$

and present value of perpetuity due in which the payments are made at the beginning of each period is:

$$PV_{perpetuity \quad due} = \frac{P(1+r)}{r}$$

Theorem 9. Suppose that one takes a mortgage loan for the amount L that is to be paid back over n months with equal payments A at the end of each month, compounded monthly. Then

$$The\ monthly\ instalment = A = \frac{L \times r \times (1+r)^n}{(1+r)^n - 1},$$

$$The\ remaining\ amount\ after\ jth\ instalment = R_j = \frac{L \times \left(\alpha^n - \alpha^j\right)}{\alpha^n - 1},$$

$$Interest\ compount\ in\ jth\ instalment = I_j = \frac{L \times \left(\alpha^n - \alpha^{j-1}\right)}{\alpha^n - 1} \times (\alpha - 1)$$

and

$$The\ principal\ component\ in\ jth\ instalment = P_j = \frac{L \times \left(\alpha^{j-1}\right)(\alpha - 1)}{\alpha^n - 1}$$

where, $j = 0,1,2,3, \dots, n$.

Solution:

We know that the present value of these cash flow stream towards n equal monthly instalment of A at the interest rate $r$per month compounded monthly is L. Thus we have

$$L = \frac{A}{(1+r)} + \frac{A}{(1+r)^2} + \frac{A}{(1+r)^3} \pm \cdots + \frac{A}{(1+r)^n}$$

$$= \frac{A}{(1+r)}\left[1 + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} \pm \cdots + \frac{1}{(1+r)^{n-1}}\right]$$

$$= \frac{A}{(1+r)}\left[\frac{1 - \frac{1}{(1+r)^n}}{1 - \frac{1}{(1+r)}}\right]$$

$$= \frac{A}{r}\left[1 - \frac{1}{(1+r)^n}\right]$$

$$= \frac{A}{r}\left[\frac{(1+r)^n - 1}{(1+r)^n}\right]$$

Therefore, we have

$$A = \frac{L \times r \times (1+r)^n}{(1+r)^n - 1}$$

Which is required relation of A in terms of L , n , r.

(b) Let R_j denote the remaining amount of principal after the payment at the end of month $(j = 0,1,2, \dots, n)$. Here it must be noted that $R_0 = L$ and $R_n = 0$. Since R_j being remaining amount after j^{th} instalment is paid. Therefore the future value or principal after next month is R_{j+1} that is $R_{j+1}\ (1+r)$. Therefore, we have

$R_{j+1} = R_{j+1}(1+r) - A$, $j = 0,1,2,3, \dots, n$ is the principal amount after $(j+1)^{th}$ instalment is paid.
$R_1 = (1+r)R_0 - A = \alpha L - A \quad (\alpha = 1+r, R_0 = L)$
$R_2 = (1+r)R_1 - A = \alpha^2 L - A(\alpha+1)$
$R_3 = (1+r)R_2 - A = \alpha^3 L - A(\alpha^2+\alpha+1)$
In general, for $j = 0,1,2, \dots, n$, we obtain
$R_j = \alpha^j L - A(1+\alpha+\cdots+\alpha^{j-1})$
$= \alpha^j L - A\frac{\alpha^j - 1}{\alpha - 1})$
$= \alpha^j L - L\,\alpha^n(\frac{\alpha^j - 1}{\alpha^n - 1})$
$= L\,(\frac{\alpha^n - \alpha^j}{\alpha^n - 1})$
Let I_j and P_j denotes the amounts of the payments at the end of month j that are for interest and for principal reduction respectively, then since R_{j-1} we owed at the end of the previous month, we have
$I_j = r\,R_{j-1} = L(\alpha-1)\,(\frac{\alpha^n-\alpha^j}{\alpha^n-1})$ and $P_j = A - I_j = \frac{L(\alpha-1)}{\alpha^n-1}\left[\alpha^n - (\alpha^n - \alpha^{j-1}\right] = \frac{L(\alpha-1)}{\alpha^n-1}\,\alpha^{j-1}$

5 Applications

Some examples confirm the results.

Numerical Example 1: Calculation of Simple and Compound Interest

(i). Sanju borrowed \$3,000 from a bank at an interest rate of 12% per year for a 2-year period. How much interest does he have to pay the bank at the end of 2 years?
Solution: $Interest(I) = 3000 \times 12\% \times 2 = 3000 \times \frac{12}{100} \times 2 = \$720.$
That is, Sanju has to pay the bank \$720 at the end of 2 years.

(ii). Sanju deposited \$10000 in a saving bank account which pays 8% simple interest. He makes two more deposits of \$15000 each, the first at the end of 3 months and the second in 6 months. How much will be in the account at the end of the year, if he makes no other deposits and no withdrawals during this time?

Solution:
$A(1) = 10000(1 + 0.08 \times 1) = 10800$
$A(2) = 15000\left(1 + 0.08 \times \frac{3}{12}\right) = 15300$
$A(3) = 15000\left(1 + 0.08 \times \frac{6}{12}\right) = 15600$
Therefore total amount at the end of the year is $A(1) + A(2) + (93) = \$41700$

(iii). Sanju and Sons invest \$10000, \$20000, \$30000, \$40000 and \$50000 at the end of each year.

Calculate the compound value at the end of the 5th year, compounded annually, when the interest charges is 5% per annum.

Table 2: Solution

End of the year	Amount Deposited	Number of years compounded	Compounded interest Factor ($FVIF_{r,n}$)	Future value
(1)	(2)	(3)	(4)	(2) x (4)
1	10000	4	1.216	12160
2	20000	3	1.158	23160
3	30000	2	1.103	33090
4	40000	1	1.050	42000
5	50000	0	1.000	50000
Accumulated Amount at the end of 5th year is				160410

(iii). A trust fund has invested \$30,000 in two different types of bonds which pays 5% and 7% interest respectively. Determine how much amount is invested in each type of bond if the trust obtains an annual total interest of \$1600. The above problem can be solve as follows:

Let the money invested in 5% bond be \$X.

Therefore, money invested in 7% bond is $\$(30000 - \mathrm{X})$. Now, interest on 5% bond$= P \times r \times t = X \times \frac{5}{100} \times 1 = \frac{5X}{100}$ and interest on 7% bond$= P \times r \times t = (30000 - X) \times \frac{7}{100} \times 1$. According to given condition, we have $\frac{5X}{100} + (30000 - X) \times \frac{7}{100} \times 1 = 1600$. Therefore, $X = 25000$. Hence, money invested in 5% bond is \$ 25000 and money invested in 7% bond is \$5000

(iv). A man invests in a bank \$x on the 1st day of 1966. In the subsequent years on the 1st January he deposits money double that of money deposited in the previous year after withdrawing the interest only on the same day. It was found that balance in his account on 2nd January 1966 was = \$2046. Find the amount he deposited on 1st January 1966. The above problem can be solved as follow:

Let the money invested on 1st January 1966 be \$x
the money invested on 1st January 1967 be \$2x
the money invested on 1st January 1968 be \$4x
the money invested on 1st January 1966 be \$8x
.
.
.
the money invested on 1st January 1975 be \$512x
Therefore, we are given that sum of all these deposits is equal to 2046
$\Rightarrow x + 2x + 4x + \cdots + 512x = 2046$

$\Rightarrow x(2^0 + 2^1 + 2^2 + \cdots + 2^9 = 2046 \Rightarrow x\left(\frac{2^{10}-1}{2-1}\right) = 2046 \Rightarrow x = 2$

Therefore the amount he deposited on 1st January 1966 is $2

Numerical Example 2:

If the loan is for 100000 to be paid back over 360 months at a nominal yearly interest of 9% compounded monthly. (a) Find the payment of the loan (b) The remaining amount of principle and the interest after the payment of first instalment.

Solution:

Here,$L = 100000\ , n = 360, r = \frac{9}{100\times 12}$. Let the monthly payment will be A. Now,

$$A = \frac{L \times (1+r)^n \times r}{(1+r)^n - 1} = \frac{100000 \times (1.0075)^{360} \times (0.0075)}{(1.0075)^{360} - 1} = 8046.2252$$

$$R_j = \frac{L \times \left(\alpha^n - \alpha^j\right)}{\alpha^n - 1}$$

Therefore,

$$R_1 = \frac{L \times (\alpha^{360} - \alpha)}{\alpha^{360} - 1} = \frac{100000(14.7306)}{14.7306 - 1} = \frac{13723100}{13.7306} = 999453.7748$$

$$I_j = \frac{L \times \left(\alpha^n - \alpha^{j-1}\right)}{\alpha^n - 1} \times (\alpha - 1)$$

Therefore,

$$I_1 = \frac{L \times (\alpha^n - \alpha^0)}{\alpha^n - 1} \times (\alpha - 1) = \frac{100000(14.7306 - 1)}{(14.7306 - 1)} \times (0.0075) = 7500$$

Finally,

$$P_j = \frac{L \times \left(\alpha^{j-1}\right)(\alpha - 1)}{\alpha^n - 1}$$

Therefore,

$$P_1 = \frac{L \times (\alpha^0)(\alpha - 1)}{\alpha^n - 1} = \frac{100000(0.0075) \times 1}{14.7306 - 1} = 546.2252$$

Verification:

1. $A_1 = P_1 + I_1 = 546.2252 + 7500 = 8046.2252 \approx 8046.23.$
2. $L - R_1 = 100000 - 999453.7748 = 546.2252 = P_1$

Table 3: Mortgage Loan Amortization Schedule

PERIOD	EMI	INTEREST	PRINCIPAL	BALANCE
0				1000000.00
1	8046.23	7500	546.23	999453.77
2	8046.23	7495.90	550.32	998903.45
3	8046.23	7491.78	554.45	998349.00

Table 3: Continue

PERIOD	EMI	INTEREST	PRINCIPAL	BALANCE
4	8046.23	7487.62	558.45	997790.39
5	8046.23	7483.43	558.61	997790.39
. . .	. . .	. . .	. . .	. . .
358	8046.23	178.36	7867.87	15913.21
359	8046.23	119.35	7926.88	7986.33
360	8046.23	59.90	7986.33	00
Total	896641.42	1896641.42	1000000	

Loan Amount: Rs. 1000000
Number of EMIs: 360
Annual Interest Rate: 0.0900
Monthly Interest Rate:.00750
Monthly EMI Payment: Rs. 8046.23
Total Loan Amount Payable: Rs. 2896641.42
Total Interest Payable: Rs. 1896641.42

Numerical Example 3: An individual who plans to retire in 20 years has decided to put an amount A in the bank at the beginning of each of the next 240 months, after which she will withdraw 1000 at the beginning of each of the following 360 months. Assuming a nominal yearly interest rate 6% compounded monthly, how far does A need to be?
Solution:
Suppose A is invested at the beginning of each month foe next 240 months, then the future value of these stream flow is given as follow:

$$FV_{240} = A(1+r) + A(1+r)^2 + \cdots + A(1+r)^{240}$$

$$= A(1+r)\left[\frac{(1+r)^{240}-1}{r}\right]$$

Now she withdraws 1000 at the beginning of each of the following next 360 months, then the present value of these stream flow is as follow:

$$PV_{360} = 1000 + \frac{1000}{1+r} + \frac{1000}{(1+r)^2} + \cdots + \frac{1000}{(1+r)^{359}}$$

$$= 1000\left[\frac{1-\left(\frac{1}{1+r}\right)^{360}}{1-\left(\frac{1}{1+r}\right)}\right] = \frac{1000 \times (1+r)((1+r)^{360}-1)}{(1+r)^{360} \times r}$$

As, per our problem , we must have $FV_{240} = PV_{360}$.

$$A(1+r)\left[\frac{(1+r)^{240}-1}{r}\right] = \frac{1000 \times (1+r)((1+r)^{360}-1)}{(1+r)^{360} \times r}$$

Therefore, $r = 6\%$, we have

$$A(1+0.06)\left[\frac{(1+0.06)^{240}-1}{0.06}\right]=\frac{1000\ \times\ (1+0.06)((1+0.06)^{360}-1)}{(1+0.06)^{360}\times 0.06}$$

Hence, we get

$A=360.9901\ \approx 361$

The concept of limit of a function can be applied to derive the formula for finding the accumulated amount in continuously compounding, which we have in the form of the following Theorem 10. The concepts of differentiability and integration can be applied to obtain the formula for force of interest, which we have put in the form of Theorem 11.

Theorem 10. If the principal amount P is invested for t years at an interest rate r% and the interest rate is compounded continuously m times a year, then the accumulated amount at the end of t years is given by $A(t)=Pe^{rt}$.

Proof. When interest is so computed that the interest per year get larger and larger as the numbers of compounding periods increasing continuously that is $m\longrightarrow\infty$, we say that interest is compounded continuously and in this case, we have

$$A(t)=P\left(1+\frac{r}{m}\right)^{mt}$$

Taking logarithm on both side we get

$$logA(t)=log\left[P\left(1+\frac{r}{m}\right)^{mt}\right]$$

$$logA(t)=\left[logP+log\left(1+\frac{r}{m}\right)^{mt}\right]=logP+mtlog\left(1+\frac{r}{m}\right)$$

Taking limit $m\to\infty$ on both side , we get

$$\lim_{m\to\infty}l\,ogA(t)=\lim_{m\to\infty}logP+\lim_{m\to\infty}mtlog\left(1+\frac{r}{m}\right)$$

Therefore, we get

$$\lim_{m\to\infty}l\,ogA(t)=logP+t\lim_{m\to\infty}log\left(1+\frac{r}{m}\right)^{m}=logP+loge^{rt}$$

$$log\lim_{m\to\infty}A(t)=logPe^{rt}$$

Taking exponential on both side we get

$$A(t)=Pe^{rt}$$

Hence theorem is proved.

Theorem 12. If the derivative of accumulated function $A(t)$ is $A^{'}(t),\ t\geq 0$ satisfying $\delta(t)=\frac{A^{'}(t)}{A(t)}$ then $\delta(t)$ is called the force of interest for the investment and the accumulated function $A(t)$ is given by $A(t)=A(0)\ e^{\int_0^t\delta(s)ds}$.

Proof. We have $\delta(t)=\frac{A^{'}(t)}{A(t)}$

Taking integration on both side with respect to s, we get

$$\int_0^t \delta(s)ds = \int_0^t \frac{A^{'}(s)}{A(s)} ds = (logA(s))_0^t = logA(t) - logA(0) = log\left(\frac{A(t)}{A(0)}\right)$$

Thus taking exponential both side we get

$A(t) = A(0)\, e^{\int_0^t \delta(s)ds}$

Hence theorem is proved.

6 Conclusions

In this paper, the basic notions of finance and with their application in the field of investment were presented in the mathematical terms. Then, the notions were described by using some theorems. Also, the application of calculus (limit, derivative and integration) in financial management were presented.

References

[1] Rudin, W., *Principles of Mathematical Analysis*, McGraw-Hill, 1976.

[2] Baseri, S., Hakaki, A., *Analysis of Financial Leverage, Operating Leverage and Capital Venture Effect on Tobin's Q Ratio of Investment and Holding Companies Listed in Tehran Stock Exchange*, Advances in Mathematical Finance and Applications, 2018, **3**(1), P.91-96. Doi: 10.22034/amfa.2018.539137

[3] Kevin, J.H., *Introduction to Financial Mathematics*, CRC Press, A Chapman and Hall Book, 2015.

[4] Mc-Menamin, J., *Financial Management; An introduction*, Oxford University Press, 2000.

[5] Sheldon, M.R., An elementary introduction to Mathematical Finance, Cambridge University press, 2003.

[6] Izadikhah, M., *Improving the Banks Shareholder Long Term Values by Using Data Envelopment Analysis Model'*, Advances in Mathematical Finance and Applications, 2018, **3**(2), P. 27-41. Doi: 10.22034/amfa.2018.5 40829

[7] López-Díaz, M.C., López-Díaz, M., Martínez-Fernández, S., *A stochastic order for the analysis of investments affected by the time value of money*, Insurance: Mathematics and Economics, 2018, **83**, P.75-82, Doi:10.1016/j.insmatheco.2018.08.001.

[8] Hill, J., *Time Value of Money: Interest, Bonds, Money Market Funds,* Academic Press, 2018, P.157-176, Doi:10.1016/B978-0-12-813497-9.00007-X.

[9] Eshaghi Gordji, M., Askari, G., *Hyper-Rational Choice and Economic Behaviour*, Advances in Mathematical Finance and Applications, 2018, **3**(3), P.69-76. Doi: 10.22034/amfa.2018.544950