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Economic Appraisal of Investment Projects in Solar Energy under Uncertainty via Fuzzy Real Option Approach

(Case Study: a 2-MW Photovoltaic Plant in South of Isfahan, Iran)

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ABSTRACT

Investment in renewable energies especially solar energies is encountered with numerous uncertainties considering the increased dynamism in economic and financial conditions and makes investment in this field irreversible to a large extent, paying attention to modern methods of economic appraisal of such investments is highly important. A framework is provided in the current study in order to employ the real option theory in evaluation of photovoltaic plants comparing with traditional methods. To this end, first, uncertainty factors of these plants in Isfahan province (one of highly susceptible regions in Iran) are identified from the view point of experts and impact factor of each one on interests and expenses of the above plant will be evaluated in order to insert these parameters in the form of fuzzy numbers in the model for better coverage of uncertainty. Then, the project under study is evaluated through both traditional methods and fuzzy real option approach with the help of Black-Scholes model and the results are compared. The results disclosed that investment value in these plants is increased if real expansion and abandonment options are considered. As a result, the real option theory has a higher adequacy than the traditional methods for evaluation of projects.

1 Introduction

Humans will encounter with two great crises in a near future: environmental pollution due to ignition of fossil fuels and the ever-increasing acceleration to bring to an end such resources. Today, political and economic crises and the problems such as limitations of fossil fuel reserves, environmental concerns, overcrowding, economic growth and coefficient of utility have obliged the scholars to find appropriate strategies including the use of renewable energy sources to solve the difficulties of energy in the world. Among renewable energies, electric energy generation from the unlimited source of sun has a special status in all countries especially in countries that are located on the solar belt of the world such as Asian countries including Iran. In the agenda of the government of the Islamic Republic of Iran, a special attention is paid to these projects in Photovoltaic Plants with different capacities and consumptions. The most important advantages of solar energy than other renewable sources are the possibility of use in a wider scope, the possibility to install with desirable capacity and low maintenance costs. Solar energies safe environmentally and is associated well with various cultural condi-

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tions. However, there are several economic and technical restrictions which have made some problems for investment in such energies [25]. The nature of investment in solar energy projects, investors are faced with high uncertainties. Increased dynamism of the conditions governing economic and financial activities in this field, thus, requires a suitable and efficient analytical method which does not have the deficiencies of common methods of discounted cash flows (DCF) such as stationary. Therefore, considerable advances have recently been obtained in the scope of different energies to develop methods of decreasing investment risk in the form of real options theory (ROT). Given the nature of energy projects (fossil and renewable energies), the use of real options approach is in priority than the traditional methods of capital budgeting. The main reasons are that [25]:

1. Many investments in energy are irreversible if they are not successful.
2. If managers have suitable flexibility and managerial authority in the use of production, demand and price in a positive direction desirably, probability of uncertainty in the future can increase the project value.

Generally speaking, assessment methods of projects are divided into two classes of traditional and modern methods. Discounted cash flows (DCF) are one of the widely used approaches in financial estimations and present several financial indexes. Each index is proposed as an assessment tool and they belong to the group of traditional methods in the division of financial evaluation methods. Net present value (NPV), internal rate of return (IRR) and payback period (PP) are the most important of these techniques. Traditional methods are based on creating a fixed image of future events and lose the required adequacy in dealing with high uncertainty and high complexity of the environment, economic parameters and investment. Justifiability of investment to the governing uncertainty over the project environment, management flexibility in decision-makings and finally additional value of it are not considered in these methods. Hence, the traditional method lacks the required flexibility for complicated situations with high uncertainty because of being stationary and does not have dynamism to deal with complex situations. Increased dynamism of the governing conditions over economic and financial activities and decision making for investment projects require an effective and efficient analytical method which does not have the deficiencies of traditional methods such as being stationary. In respond to modern needs, it is nearly three decades that financial assessment of projects has been considered from the viewpoint of real options theory (ROT) and proposes a modern thought with regard to evaluation of investment projects. Unlike traditional methods, various paths are considered for decision making in real options approach which provides adequate flexibility for management decision-makings in uncertain environments. Generally, the advantage of this approach to other traditional methods is appeared when strategic decisions of investment are analyzed under uncertainty conditions. Several studies have proved that real options approach enhances the project value because it considers flexibility for project evaluation [7]. In other words, option in investment enhances value of investment so that:

Investment value = net present value of investment + real options value

Thus, real option can be led to motivations for entering into plans which have not had any justification before [6]. Real options theory is adopted from financial options theory that was proposed by Black, Scholes and Merton in 1973 for the first time tried to evaluate financial tools referred to as financial options to decrease investment risk. The most important division for financial options is to divide them into two groups of put option and call option. The former actually gives its holder the right (rather than obligation) to buy the underlying asset with specified price on a specified date or prior to it. Likewise, put option gives its holder the right to sell the underlying asset with specified price on a specified date or prior to it. The price stated on the contract is called “strike price” or “exercise price”, and specified date is “expiration date” or “option maturity”. Call or put options are divided into European and American option. The former can be just exercised on maturity date, while the latter can be exercised any time before maturity date or on maturity date. Option contracts on stock are seen as cash options, because buy or sell of asset with agreed price can be undertaken immediately when option is exercised [1]. Option is the right and not the commitment to buy (sell) an asset with a certain price in a certain date. The asset base in financial option contracts is usually the stock while a real option is the right of adopting investment decisions about real assets. However, this option does not create any commitment and this right includes the right of postponing, creating, transferring and changing of situation and the like [9]. Financial options have maturity and the option has no credit after that time. These options allow the investor to exercise his/her option any time before maturity. They are referred to as American options. The options that are exercised just at the maturity are known as European options. From the viewpoint of experts of financial evaluation of projects, valuation of American options is more important than that of European options and many recognized real options are equal to American options [26]. Real options under uncertainty conditions can be justified and whatever uncertainty is higher, value of options or flexibility is higher. In other words, value of flexibility has a direct relationship with degree of uncertainty of variables following the project value. This is led to difference between the real option viewpoint and the traditional viewpoint in encountering with uncertainty. Types of common flexibility in investment projects are: option to hesitate in investment or option to defer, abandoned option, expansion option and compound option that is a combination of all types of other options [11].The key advantage and value of real option analysis is to integrate managerial flexibility into the valuation process and thereby assist in making the best decisions [33]. As it was mentioned earlier, investment projects related to establishment of renewable electricity plants require high and often irreversible investment costs with high uncertainty. Due to all these features, real option theory is an appropriate approach in evaluation of projects. Thus, this research is based on the application of real option approach in evaluation of power plant projects using fuzzy logic to better cover the uncertainties and increasing the flexibility of investment decisions.

Fuzzy logic is a mathematical approach to increase the flexibility of decisions and reduce uncertainty and ambiguity. Today, the use of fuzzy logic has been widely used in financial studies. For an example Kalantari et al. [16] developed a mathematical model for performance-based budgeting and combine it with rolling budget for increased flexibility. Their model has been designed by Chebyshev's goal programming technique with fuzzy approach and reduced .68% of the total refinery's budget compared with the actual budgets from gas refineries of Iran for 2016. Given the above issues, the major questions in this paper are as follows: Can real option approach be a suitable method for decision-making? Can real option approach be introduced as a supplementary criterion for the discounted cash flows model? The main purpose of this paper is generally to criticize and explore common methods of evaluation and introduction of an optimal method for decision-making in investment projects through a new approach known as real option theory.

2 Theoretical Fundamentals And Research Background

Real options are based on financial options. However, the nature of real options involves permanent, fixed or immovable assets. In contrast to financial options, real options are not tradable e.g. the factory owner cannot sell the right to extend his factory to another party; he can only make this decision [33].

Myers was the first person who compared financial option with real investments and concluded that option pricing theory is used for real assets and non-financial investments in 1977. He employed the expression of real options for the first time to distinguish between real assets options and marketable financial options which was accepted in academic circles and the market. From his viewpoint, these options can be evaluated like financial options [28]. Since then, several studies have been conducted about the use of these methods.

Real option is a systematic approach that is based on decision-making under uncertain and complicated conditions in which determining of expectations of future changes plays a major role by considering the existing uncertainties [37]. In this approach, valuation of assets such as physical and financial and costing of plans and economic projects are performed by means of options theory, economic analysis, operations research, decision theory, statistics and econometrics modeling in a dynamic decision-making space as well as uncertain commercial environments in the form of strategic investment decision-making [17].

Role of real option is very vital in the below cases [37]:

- Decision-making about investment under high uncertainty conditions
- valuation of the proposed strategic decisions
- optimization of strategic investment decisions with the help of various paths
- determining an appropriate time to enter or exit an investment
- management of current opportunities and development of strategic decision-making opportunities in the future

Real options under uncertainty conditions can be justified and whatever uncertainty is higher, value of options or flexibility is higher. In other words, value of flexibility has a direct relationship with degree of uncertainty of variables following the project value. This is led to difference between the real option viewpoint and the traditional viewpoint in encountering with uncertainty. Types of common flexibility in investment projects are: option to hesitate in investment or option to defer, abandoned option, expansion option and compound option that is a combination of all types of other options [11]. Totally, it is appropriate to use real option when investment is irreversible and there is high uncertainty about the future. The use of real option approach is appropriate when value of asset base has high uncertainty and management has high flexibility to change lifecycle of the option and be able to implement the option at a suitable time [20]. In recent years, a considerable growth has been occurred about the use of real option models especially in energy sector.

For example Venetsanos, et al. [35] used real option model to evaluate renewable wind power generation projects. They, first, identified the uncertainty related to energy resources and then, selected an option proportional to the project and used Black-Schols model to valuate it. They compared the results with net present value and found that value of option in the project under study was positive while net present value of the project was negative. Kjarland [19] used real option theory to evaluate investment opportunities related to hydroelectric power generation in Norway and concluded that there is a relationship between electricity price level and optimal scheduling of investment in hydroelectric plants. He employed the developed framework by Dixit and Pindyck. Munoz, et al. [27] developed a model to evaluate investment in electricity sector using wind energy. They used a random model for the parameters affected by discounted cash flow and real option approach to evaluate probabilities related to investment, expectation or abandonment of the project. Collan, et al. [5] proposed a fuzzy model to evaluate real option in Finland University by emphasizing the income method. According to this model, the use of fuzzy logic and numbers can help evaluate this option in industries considerably. Kahraman and Ucal [15] solved Black-Schols model through the simple real option approach and also its fuzzy approach for investment in an oil field and reported a considerable difference between real option in simple state and its fuzzy state. They showed a deep link between real option theory and fuzzy logic. Since then, fuzzy studies have been developed in research and development projects, information technology and oil and gas. Lei and Fan [21] employed real option model in foreign investment decisions in the oil industry in China. To this end, they developed the real option model and pointed out that how an investor can make decision about exchange rate and the investment environment under uncertainties of oil price. Martinez and Mutale [23-24] showed that expected profits in projects that are evaluated via real option approach are more than when they are evaluated with other methods. Likewise, they developed an advanced real option approach for the projects of renewable energy generation and tested their method in a case study about renewable energies. Then introduced all types of renewable energies and evaluated investment in such energies via bi-nominal, Black-Schols and simulation models and separated each type of energy. He showed in brief that real option theory increases the value of different options in investment and flexibility in investment decisions considerably. Luiz, et al. [22] used real option approach for economic appraisal of wind farms in Spain. They considered the electricity price and government subsides as the functions of a random process and valuated an American option through Mont Carlo simulation and optional bi-nominal tree. They identified uncertainty factors such as government subsides, electricity price fluctuation and expiry date of options as the most important indefinite parameters that were effective on real option in wind farms in Spain. Sheen [32] employed fuzzy numbers for real option valuation under uncertainty conditions for wind plants in Taiwan. He inserted the uncertainty variables in fuzzy form into Black-Schols model and evaluated the real option and concluded that fuzzy logic can cover the uncertainty of investment environment of these projects well and provide more value than valuation in unfuzzy states. Xian, et al. [36] proposed a new model based on real option approach to evaluate investment in carbon capture and storage (CCS) projects under multiple uncertainty conditions with the help of tri-nominal tree model commissioned by the Energy and Environmental Development Research Center in China (as an applied and helpful model in CCS pricing). Chanwoong, et al. [4] combined real option models and system dynamics and studied the complex relations between investors and policy-makers of photovoltaic plants in South Korea under uncertainty conditions. They suggested a method to optimize government financial subsidies and omit unnecessary subsidies. They valuated the expansion option of these plants via Black-Schols model and showed that this value is more than net present value of the project. Osanlu et al. [31] calculated least squares of option to defer in their survey by considering the electricity price as a random variable and by means of simulation method. They showed that real option approach increases value of the wind plant under study. Kim et al. [18] evaluated photovoltaic projects in South Korea through real option approach under uncertainty conditions. They valuated abandonment option of these plants using bi-nominal tree and concluded that considering managerial options in evaluation of such projects will be led to more attractive results in evaluation and encourage the investors to invest despite numerous uncertainties in the future. The applied scope of research in Iran includes mostly the oil and gas sector, petrochemicals, power generation plants, different mines and IT projects through bi-nominal and tri-nominal trees, Black-Scholes model and simulation methods which are employed in comparison with the traditional DCF methods. All methods have emphasized the prominent role of such options in heavy and risky investments in the above scopes under uncertainty and ambiguous conditions as well as increased flexibility in decision-makings.

3 Proposed Methodology

An investment project in a 2-MW photovoltaic plant located in south of Isfahan province was evaluated via traditional methods and real option approach and the results were compared in the current study. Hence, this study is descriptive from objective aspect and qualitative from methodological aspect. As decision-making in this study includes uncertainty conditions, first, uncertainty factors of the investment project should be identified and importance factor of each one is determined. Then, evaluation of the project through traditional methods and finally value of real options are calculated. Therefore, the research will be performed in three stages.

3.1 Stage One: Identification of the Existing Uncertainty Factors in a 2-MW Photovoltaic Plant

In this stage, an opinion poll about the existing uncertainty factors in the project was carried out. The statistical population included the experts of solar energies with at least two features: 1) familiarity with effective technical and environmental problems on electricity generation in photovoltaic plants, and 2) familiarity with economic appraisal methods of investment projects.

The statistical sample in this stage was selected using non-probability purposive sampling in which selection is based on accessibility, existence of a logical proportionality between the sample and research needs and scientific and specialized proportionality of the sample members with the research topic. The reasons for this selection are: 1- specialization of the research topic 2- confining the research topic to people who have the proportional awareness in this regard and 3- necessity of theoretical compatibility with the research topic for members of the statistical sample [34].

Since the sample size in empirical studies should at least be equal to 15 [14], thus 30 experts who were familiar with exploitation of solar energies across the country were selected as the research sample for interview and data collection and the research tool was questionnaire.

Content validity of the questionnaire was examined with the help of Lawshe's Content Validity Ratio (CVR). Opinion polls were used as the input of fuzzy Delphi analytical hierarchy process (FDAHP). The purpose is to determine importance factor of each factor and contingency rate of factors (as a reliability criterion). Then pair wise comparison matrix corresponding to each factor is created for each expert separately.

To prepare the fuzzy pair wise comparison matrix for all factors, the obtained opinions are considered directly and below relations are used according to Liu and Chen's method in 2007 by assuming the fuzzy triangular membership function for these numbers [2]:

$$a_{ij} = \left(\alpha_{ij}, \delta_{ij}, \gamma_{ij}\right) \tag{1}$$

Where

$$a_{ij} = Min\left(\beta_{ijk}\right), \delta_{ij} = Min\left(\prod_{k=1}^{n} \beta_{ijk}\right)^{\frac{1}{n}}, \gamma_{ij} = Max\left(\beta_{ijk}\right), \quad k = 1,2,3,\dots,n$$

In the above relations, $\boldsymbol{\beta_{ijk}}$ shows relative importance of factor ***i*** on factor ***j*** from the view point of expert ***k***. It is clear that components of the fuzzy number are defined in a way so that $\boldsymbol{\alpha_{ij}} \leq \boldsymbol{\delta_{ij}} \leq \boldsymbol{\gamma_{ij}}$. These components change in the interval$\left[\frac{1}{9}, 9\right]$. The fuzzy pair wise comparison matrix is constructed as below:

$$\tilde{A} = \left[\tilde{a}_{ij}\right], \quad \tilde{a}_{ij} \times \tilde{a}_{ji} \approx 1 \quad \forall i, j = 1,2,3,\dots,n$$

$$\tilde{A} = \begin{bmatrix} (1,1,1) & \cdots & \left(\alpha_{1j}, \delta_{1j}, \gamma_{1j}\right) & \cdots & \left(\alpha_{1n}, \delta_{1n}, \gamma_{1n}\right) \\ \vdots & & \vdots & & \vdots \\ \left(\frac{1}{\gamma_{1j}}, \frac{1}{\delta_{1j}}, \frac{1}{\alpha_{1j}}\right) & \cdots & (1,1,1) & \cdots & \left(\alpha_{2n}, \delta_{2n}, \gamma_{2n}\right) \\ \vdots & & \vdots & & \vdots \\ \left(\frac{1}{\gamma_{1n}}, \frac{1}{\delta_{1n}}, \frac{1}{\alpha_{1n}}\right) & \cdots & \left(\frac{1}{\gamma_{2n}}, \frac{1}{\delta_{2n}}, \frac{1}{\alpha_{2n}}\right) & \cdots & (1,1,1) \end{bmatrix} \tag{2}$$

In order to calculate contingency rate (as an index for validity of the questionnaire) the above fuzzy inverse matrix should first be divided into two matrices: middle limit matrix $(m)$and high limit and low limit geometrical mean matrix(g). Then, the contingency rate is calculated according to the below relation given Gogus and Boucher's method [13]:

$$CI^m = \frac{\lambda_{max}^m - N}{N-1}, \;\; CR^m = \frac{CI^m}{RI^m} \qquad , \qquad CI^g = \frac{\lambda_{max}^g - N}{N-1}, \;\; CR^g = \frac{CI^g}{RI^g} \tag{3}$$

Where, CI is the inconsistency index, λ_{max}^m is the highest Eigen value of matrix m that is equal to mean $(m \times W_m)/W_m$, $\lambda_{max}^g$is the highest Eigen value of matrix g that is equal to mean, $\left(g \times W_g\right)/W_g$, N is number of factors, RI is random contingency rate whose value is selected from the table of random contingency rates and is the contingency rate that must be less than 10% for both matrices m

and g ; otherwise (even for one matrix) the questionnaire should be given again to the respondents for revision. In the next step, the fuzzy inverse matrix is used and relative weight of factors is calculated

$$\tilde{Z}_i = [\tilde{a}_{ij} \otimes \dots \otimes \tilde{a}_{in}] \tag{4}$$

$$\widetilde{W}_i = \tilde{Z}_i \oslash (\tilde{Z}_i \oplus \dots \oplus \tilde{Z}_n) \tag{5}$$

$\widetilde{W}_i$ is a row vector that shows fuzzy weight of factor i . Finally, geometrical mean of factors according to the below relation is used to calculate non-fuzzy weight of factors:

$$W_i = \left(\prod_{j=1}^{3} \widetilde{W}_{ij}\right)^{\frac{1}{3}} \tag{6}$$

3.2 Stage Two: Evaluation of Traditional Indexes (DCF)

For the economic appraisal of this plant via traditional indexes (DCF), the powerful Ret Screen software for decision-making in renewable energies is applied. This software can be used by all people freely as the clean energy project analysis software by the government of Canada and as one part of the need of countries to use an integrated approach for climatic changes and reduction of pollution. It helps the decision-makers explore practicality of renewable energy projects (solar, wind, wave, water, earth, heat energy, etc.), energy productivity and simultaneous generation of electricity and heat technically and financially in a rapid manner and with low cost. Access to information banks of weather, hydrology, NASA's data as well as maps of energy sources in the world, simple use in Excel environment and translation of it into 36 languages for the use of two third of world population are the unique characteristics of this software. One of the advantages of this software for economic appraisal of projects is that it simplifies evaluation of various steps of a project for decision-making. financial position worksheet in this software with input parameters such as the avoided cost of energy, discount rate, loan value, inflation rate and so on and the computed output parameters such as internal rate of return, payback period, net present value, saving due to reduction of pollutants make it possible for the project decision-makers to investigate different financial parameters.

All technical appraisal steps (calculation of number of panels, estimation of solar radiation under different climatic conditions, computation of cash flows and the plant expenses, estimation of the profits of decreased emission of greenhouse gases, etc.) and economic appraisal steps (discounted cash flows, calculation of evaluation indexes and their sensitivity analysis) will be performed for the photovoltaic plant under study.

3.3 Stage Three: Real Option Valuation And Sensitivity Analysis

Most attempts that have already been performed for real options valuation (ROV) are equal to proposing numerical methods to estimate value of American options. Generally, numerical methods to estimate value of American and European options can be divided into three total classes [12]:

1- Solving partial differential equations and valuation of option via finite differential

2- Black- Schols Closed form

3- lattice Model (Binomial & Trinomial)

4- Mont-Carlo simulation Methods

Black-Schols equation may seem proper for the analyses related to real option, because it is widely used in real option valuation and is easy to use.

Black-Schols formula is as below [3]:

$$ROV = S.N(d_1) - Xe^{-r.T}.N(d_2) \qquad \text{For call option} \quad (7)$$

$$ROV = Xe^{-r.T}.N(d_2) - S.N(d_1) \qquad \text{For put option} \quad (8)$$

$$d_1 = \frac{Ln\left(\frac{S}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \quad , \qquad d_2 = d_1 - \sigma\sqrt{T} \quad (9)$$

In these relations, where ROV denotes the real option value, S is the present value of asset base, X is stock price option or agreed price, r is the compound annual short-term interest rate, T is the maturity date of option, $N(d\ )$is normal density distribution function and σ is fluctuation of the asset base price.

An important assumption about real option valuation models is lack of arbitrage opportunities; i.e. it is not possible to buy an asset in efficient markets with one price and sell it simultaneously with a higher price. Professional investors buy the assets hypothetically and sell it rapidly and close the price gaps. Thus, creating arbitrage opportunities is rare. The critics' argument is that lack of arbitrage opportunities about real assets is impossible, because they are not liquidated as financial assets.

Hence, option pricing models are not suitable for real options valuation. Definite denial of these models to solve the problems of real options is incorrect; thus, lack of an arbitrage opportunity is just a limitation for the model that can be overcome easily via suitable adjustments. For instance, it is possible to overcome the conditions of lack of arbitrage opportunities via risk-free interest rate more than the real discount rate in discounted cash flows related to value of asset base. In this case, value of option is decreased but it will be more conservative [20-3]. Now, parameters of the two models will be equated according to Table 1

Table 1: Equation of parameters of Black-Schols model and real option value

Parameter	Real option value model	Black-Schols model
S	present value of project cash flows	present value of asset base
X	present value of initial investment	stock option price or agreed price
$r = r_f$	risk-free interest rate	compound annual short-term interest rate
T	time period for decision-making	maturity
σ	fluctuation of expected cash flows	variability of asset base price

If we want to use these relations by considering triangular fuzzy numbers, we have will:

$$\tilde{S} = (S_1, S_2, S_3) \quad , \quad \tilde{X} = (X_1, X_2, X_3)$$

Thus, for fuzzy call option we have [32]:

$$FROV = \tilde{S}.N(d_1) - \tilde{X}e^{-\tilde{r}_f T}.N(d_2)$$
$$= \big(S_1.N(d_1) - X_3 e^{-r_3 T}.N(d_2), S_2.N(d_1) - X_2 e^{-r_2 T}.N(d_2), S_3.N(d_1) - X_1 e^{-r_1 T}.N(d_2)\big) \quad (10)$$

And for fuzzy put option, we have:

$$FROV = \tilde{K}e^{-\tilde{r}_f T}.N(d_2) - \tilde{S}.N(d_1) = \big(K_1 e^{-r_3 T}.N(d_2) - S_3.N(d_1), K_2 e^{-r_2 T}.N(d_2) - S_2.N(d_1), K_3 e^{-r_1 T}.N(d_2) - S_1.N(d_1)\big) \quad (11)$$

In these relations, S is current value of fuzzy expected cash flows, X is current value of fuzzy investment costs, K is fuzzy salvage value, r_f is fuzzy risk-free interest rate, T is the time period for decision-making, $N(d\,)$ is normal density distribution function and $\sigma_{\tilde{S}}$ is fluctuation of cash flows or standard deviation of growth rate of cash flows. Also, we will have:

$$\tilde{d}_1 = \frac{Ln\left(\frac{E(\tilde{S})}{E(\tilde{X})}\right) + \left(\tilde{r}_f + \frac{1}{2}\sigma_{\tilde{S}}^2\right)T}{\sigma_{\tilde{S}}\sqrt{T}} \quad , \quad \tilde{d}_2 = \tilde{d}_1 - \sigma_{\tilde{S}}\sqrt{T} \quad (12)$$

Where $E(\tilde{S}) = \frac{S_1+S_2+S_3}{3}$ is the mathematical expectation or mean fuzzy value of cash flows, $E(\tilde{X}) = \frac{X_1+X_2+X_3}{3}$ is the mathematical expectation or mean fuzzy value of cash flows and $\sigma_{\tilde{S}}$ is standard deviation of growth rate of cash flows according to the below relation:

$$\sigma_{\tilde{S}} = \left(\sqrt{\frac{1}{18}(S_1^2 + S_2^2 + S_3^2 - S_1S_2 - S_2S_3 - S_3S_1)}\right) / E(\tilde{S}) \quad (13)$$

In such circumstances, we will have:

$$ROV = E(FROV) = \frac{ROV_1 + ROV_2 + ROV_3}{3} \quad (14)$$

Finally, investment value is calculated through the below relation:

$$Investment\ value = V_{without\ Option} + V_{with\ Option} = NPV + ROV \qquad (15)$$

4 Analysis And Findings

4.1 Identification of the Existing Uncertainty Factors in a 2-MW Photovoltaic Plant Project

In this step, preliminary interviews were carried out with the experts and a questionnaire was prepared. Unnecessary factors were omitted after gaining primary responses. Content validity was confirmed with the help of Lawshe's Ratio (CVR) and the final questionnaire containing 13 different uncertainty factors was designed according to Table 2.

Table 2: Opinion poll form of the effective uncertainty factors on investment in solar plants

Types of uncertainties related to the project	Row	Uncertainty factors in establishment of solar plants
Technical uncertainties	1	degree of annual solar radiation on flat and fixed panels (ASR)
	2	average solar radiation per day (SRD)
	3	number of sunny days in a year (NSD)
	4	Transparency of air (air pollution) (TA)
	5	cleanliness of surfaces (CS)
	6	air temperature and other climatic conditions like rainfall (AC)
	7	efficiency of photovoltaic modules (EPM)
Market-related uncertainties	8	market interest rate for investment in solar power(r)
	9	market inflation rate (*f*)
	10	tariff of guaranteed solar power purchase by the government grid (T)
	11	periodic operating and maintenance costs (OMC)
	12	exchange rate fluctuation (effective on expenses of buying equipments from foreign countries) (V)
Structural uncertainty	13	governmental incentives and supports (subsidies and tax exemptions) (S)

The final questionnaire was sent for 36 experts. They were asked to give their opinion about the importance of each factor as follows:

not important	less important	mid importance	important	very important
1	3	5	7	9

30 completed questionnaires were received and used as the inputs of fuzzy Delphi analytical hierarchy process.

Table 3: Contingency rate

Result: Contingency rate is acceptable	$CI^g = 0.02408$	$CI^m = 0.0108$
	$RI^g = 0.46910$	$RI^m = 1.4555$
	$CR^g = 0.05133 < 10\%$	$CR^m = 0.0074 < 10\%$
Source: researcher's calculations		

In fuzzy Delphi method, the required information is obtained in the form of natural language from experts and is analyzed in fuzzy form. In this method, all opinions are covered in one exploration phase, there is no need to spend much time and expense and ambiguity of the process will be omitted. The purpose is to determine importance factor and contingency rate of factors (as a reliability index). Weighting of parameters in hierarchical analysis is performed based on their two-by-two comparison in the form of a pair wise comparison matrix [2]. Then the corresponding pair wise comparison matrix to each factor in viewpoint of different experts is created separately for each expert. In order to prepare this matrix for all factors, the obtained opinions are considered directly. Findings of this method for the prepared questionnaire in this research are as Table 3 and Table 4:

Table 4: Fuzzy and non-fuzzy weight of the effective uncertainty factors on investment in solar plants

Uncertainty factors	$\widetilde{W}_i$	W_i	Coefficient	Priority
ASR	(0.025782, 0.090293, 0.302891)	0.089007	8.90%	3
SRD	(0.026992, 0.090155, 0.302891)	0.090331	9.03%	2
NSD	(0.027820, 0.086836, 0.310833)	0.090893	9.09%	1
TA	(0.023575, 0.077854, 0.291404)	0.081173	8.12%	4
CS	(0.022988, 0.073212, 0.267790)	0.07667	7.67%	7
AC	(0.014625, 0.067779, 0.251604)	0.062947	6.29%	13
EPM	(0.023124, 0.074027, 0.272845)	0.077587	7.76%	6
r	(0.016130, 0.072743, 0.278522)	0.06888	6.89%	11
f	(0.014062, 0.070142, 0.252382)	0.062906	6.29%	12
T	(0.018233, 0.077634, 0.291404)	0.074439	7.44%	8
OMC	(0.018624, 0.070389, 0.273189)	0.071014	7.10%	9
V	(0.016120, 0.069991, 0.303559)	0.069965	7.00%	10
S	(0.023715, 0.078945, 0.279999)	0.080631	8.06%	5
Source: researcher's calculations		1.0	100%	--

Since all identified uncertainty factors are effective on either the obtained earnings from power generation or expenses of power generation and generally net present value of the plant (as a criterion to accept or reject the investment), we will insert the above factors in the valuation model using fuzzy logic when real options of the plant are valuated.

4.2 Evaluation of Traditional Indexes (DCF)

The plant under study is a 2-MW photovoltaic plant for solar power generation that is located in south of Isfahan province. This province is a suitable region for using the solar source to supply energy because it has numerous sunny days (239 days in a year). In this survey, Ret Screen software was employed for calculation of degree of received solar energy and capacity factor of the plant, estimation of primary and periodical expenses, and computation of reduction of greenhouse gases and financial evaluation. Given the selected situation for the plant and technical characteristics of equipments, estimation of annual produced energy, capacity factor and the electricity exported to the grid in the area under study based on climatic information of NASA are performed summarily in Tables 5. A summary of sum of expenses, primary expenses before exploitation as well as annual and maintenance costs after exploitation for the plant are shown in Table 6.

Table 5: Geographic coordinates of the place and Technical characteristics of the plant

<table>
<tr><td rowspan="2">Station:</td><td>Lat</td><td>Lon</td><td>Elevation (m)</td><td>Solar tracking mode</td><td>Fixed</td></tr>
<tr><td>32.5</td><td>51.7</td><td>1550</td><td>land</td><td>40000 m²</td></tr>
<tr><td colspan="3">Azimuth 180.53</td><td>Tilt 31.99</td><td>Panel(Number)</td><td>8000</td></tr>
<tr><td colspan="6">Capacity factor of the plant and the electricity exported to the grid in the area under study</td></tr>
<tr><td colspan="3">Electricity exported to grid</td><td>3545 MW/h</td><td>Capacity factor</td><td>%20.2</td></tr>
<tr><td colspan="6">Received solar energy in the area under study (south of Isfahan in year 2017)</td></tr>
<tr><td rowspan="2">Annual</td><td>Air temp</td><td>Relative humidity</td><td>Atmospheric pressure</td><td>Wind speed</td><td>Earth temp</td></tr>
<tr><td>16.2 °C</td><td>35.6%</td><td>81.1 K Pa</td><td>2.6 m/s</td><td>16.4 °C</td></tr>
<tr><td colspan="6">Estimation of annual produced energy in the area under study in year 2017</td></tr>
<tr><td rowspan="2">Annual</td><td colspan="2">Daily solar radiation - horizontal</td><td colspan="2">Daily solar radiation - tilted</td><td>Electricity exported to grid</td></tr>
<tr><td colspan="2">5.34 kWh/m²/d</td><td colspan="2">5.86 kWh/m²/d</td><td>3545.3 MW/h</td></tr>
<tr><td colspan="6">Source: calculations of Ret Screen software</td></tr>
</table>

Table 6: Summary of investment costs

<table>
<tr><td>Annual and maintenance costs after exploitation</td><td>Primary expenses before exploitation</td></tr>
<tr><td>IRR 2048 million</td><td>IRR 91568 million</td></tr>
<tr><td colspan="2">supplying the required capital</td></tr>
<tr><td>60% government participation*</td><td>40% capital provided by the applicant</td></tr>
<tr><td colspan="2">*According to Article 84 of the Fifth Development Plan of the Islamic Republic of Iran (2011-2017), National Development Fund pays facilities through the related banks (for developed plans with 16 or %18 rate and less developed plans with %12 rate).</td></tr>
<tr><td colspan="2">Source: researcher's findings</td></tr>
</table>

Table 7: Summary of financial assumptions of the project

Row	**Subject**	**Degree**	**Unit**
1	annual inflation rate[1]	9.6	percentage
2	tariff for guaranteed power purchase in the base year[2]	4900	Rial
3	increasing rate of electricity purchase price	0	percentage
4	degree of accessibility of the generator	97	percentage
5	nominal power of generator	2	mega watt
6	effective rate of income tax[3]	0	percentage
7	fuel price increase	0	percentage
8	project age	20	year
9	salvage value (book value)	75	billion Rial
10	salvage value that is 60% of book value	45	billion Rial
11	accumulated depreciation[4]	16.847	billion Rial
12	nominal interest rate or expected profit rate[5]	16	percentage

1.According to the latest news of the Central Bank of Iran and based on the report of the International Monetary Fund, inflation rate in twelve months ended to May 2017 in comparison with the same time in 2016 has been reported equal to 9.6%.
2.According to Article 133 in the Fifth Development Plan approved by board of ministers in 2016, the base rate of guaranteed power purchase from the plants liable to Article 4 of this instruction is IRR 4900 for 10 years.
3. Article 132 of tax rules the manufacturing sector of the plant exempts from income tax (Ministry of Energy, 2017).
4. According to Articles 149 and 150 of direct taxes in Iran, depreciation rate of solar plants is 1% and the depreciation method is descending.

5. According to the report in July 2016 by the Central Bank of Iran, the calculated profit rate for the clients should not be less than 16 percent in a year (Central Bank of Iran, 2016).
Source: researcher's findings

A summary of financial assumptions of the project is displayed in Table 7.

Table 8: Annual cash flows of the project

Year	Pre-tax cash flows	After-tax cash flows	Cumulative cash flows
0	-36,627,200,000	-36,627,200,000	-36,627,200,000
1	-113,889,849	-113,889,849	-36,741,089,849
2	-329,380,354	-329,380,354	-37,070,470,203
3	-565,557,949	-565,557,949	-37,636,028,152
4	-824,408,592	-824,408,592	-38,460,436,743
5	-1,108,108,897	-1,108,108,897	-39,568,545,640
6	13,822,068,170	13,822,068,170	-25,746,477,470
7	13,481,282,824	13,481,282,824	-12,265,194,646
8	13,107,782,086	13,107,782,086	842,587,440
9	12,698,425,276	12,698,425,276	13,541,012,716
10	12,249,770,213	12,249,770,213	25,790,782,928
11	11,758,044,263	11,758,044,263	37,548,827,192
12	11,219,112,623	11,219,112,623	48,767,939,814
13	10,628,443,545	10,628,443,545	59,396,383,359
14	9,981,070,235	9,981,070,235	69,377,453,595
15	9,271,549,088	9,271,549,088	78,649,002,683
16	8,493,913,911	8,493,913,911	87,142,916,593
17	7,641,625,756	7,641,625,756	94,784,542,350
18	6,707,517,939	6,707,517,939	101,492,060,289
19	5,683,735,772	5,683,735,772	107,175,796,061
20	286,026,151,034	286,026,151,034	393,201,947,095
Source: calculations of Ret Screen software			

For more development of using renewable energies, the tariff for guaranteed power purchase from renewable and clean plants was announced by Ministry of Energy in 2016. By assuming that 60% of initial investment for setup costs of the plant is obtained through facilities with 12% rate and 5-year maturity, the following results will be obtained according to Table 7. Based on software calculations, annual pre-tax cash flows after-tax cash flows are according to Table 8. Results of calculating the traditional standards are as the 9 table. The result is that this photovoltaic plant with the production capacity of 2 MW solar power which has been established 20 years ago in south of Isfahan is an economic project from the view of traditional indexes of economic appraisal with positive net present value, higher internal rate of return than the expected profit rate of 16% and ratio of profits to expenses and the profitability index greater than 1. Besides, total investment expenses will be recycled after 7/9 years. Therefore, it can be stated that using renewable energies with fossil fuels reduction can be a suitable strategy.

Table 9: Results after granting the facilities in initial investment

	Unit	Assumptions
12	%	interest rate of loan
60	%	participation percentage
5	year	debt period
15,241,112,601	IRR in year	annual loan payment
17.6	%	internal rate of return (IRR)
7.9	year	payback period (PP)
7,126,808,512	IRR	net present value (NPV)
1.195	-	profitability index (PI)
1.19	-	cost benefit ratio
Source: calculations of Ret Screen software		

4.3 Real option valuation with the help of Black-Schols closed form

As it was mentioned earlier, Black-Schols model is appropriate for European real option valuation and a suitable adjustment must be used in order to employ it in American options. For instance, discount with risk-free interest rate greater than real discount rate can be a suitable strategy and although it reduces the option value but it reaches to a more conservative value. Discount rate in each period is the expected return rate higher than risk-free interest rate per one unit risk. For better judgment, real discount rate should be calculated from the below relation through its deflation [10]:

$$r = \frac{1+i}{1+f} - 1 \quad (16)$$

Real discount rate of this plant based on the published information by the Central Bank of Iran is calculated according to the below relation given the expected nominal interest rate 16% and inflation rate 9/6% for the first half of year 2017:[1]

$$\text{Real discount rate (r)} = \frac{1+nominal\ interst\ rate}{1+inflation\ rate} - 1 = \frac{1+16\%}{1+9.6\%} - 1 = 0.0584 \approx 6\%$$

It is noteworthy that this rate has been used by Ret Screen software for discount based on principles of engineering economics. There are various methods to calculate risk-free interest rate. According to the modern financial theory in valuation of fixed income assets for countries where the securities market is not advanced especially when valuation of securities is not possible via arbitrage pricing

[1].According to the report by International Monetary Fund, gross domestic product rate of Iran reached 7.4% in 2016 due to increasing of oil production. This rate has been predicted equal to 6.6% in the first half of 2017 but it will decrease to 3.3% in the second half of that year. This is while inflation rate in the same time will have an ascending order. According to this report, the inflation rate in 2016 reached 9% and in the first half of year 2017 it reached 9.6%. It is predicted that this rate will increase temporarily to more than 11.9% at the end of 2017 because of high growth of liquidity and inflation effects arising from the recent increase of exchange rate (source: Central Bank of Iran, periodic reports of the International Monetary Fund from Iran's economy).[30]

theory, the best method of calculating risk-free interest rate is the below method in which f is the inflation rate and G is gross domestic product rate [8].

$$r_f = (1 + f)(1 + G) - 1 \quad (17)$$

Considering the nature of Iran's economy, this relation can be used to estimate risk-free interest rate. Thus, risk-free interest rate (given the domestic growth rate 6/6% in the first half of year 2017) is calculated as below:

$$r_f = (1 + \%9.6)(1 + \%6.6) - 1 = \%16.8$$

For the analysis of options valuation via fuzzy numbers, first, fuzzy value of the input parameters of the model should be estimated as follows.

In the identification stage of uncertainty factors, it was stated that these factors are effective on expenses and earnings of the plant and create net present value of cash flows of the project. According to calculations of Ret Screen software in the second stage, the definite value of present value of cash flows is equal to:

$$\tilde{S} = (39.379,43.754,48.129)$$

Hence, present value of investment costs of the project is definitely equal to IRR 91.568 billion. Given 60% participation of the government, the provided capital by the applicant was equal to IRR 36/627 billion. Considering 5% changes of foreign exchange rate in Iran during the eleventh government, its fuzzy value is equal to:

$$\tilde{X}_1 = (34.795,36.627,38,495)\ ,\ \tilde{X}_2 = (86.898,91.568,96.238)$$

Now, by considering such information, we calculate real option value via Black-Schols model through the two following scenarios. Our major assumption is to consider option period equal to $T = 5$ years. This number is based on the table of annual cash flows of the project and because these flows are negative in five primary years (based on calculations of Ret Screen software).

Now each scenario is valuated by means of Black-Schols model based on Sheen's approach [32]. So, according to relations (10) to (17), we have:

The first scenario: project development by assuming double cash flows and production capacity (from 2 to 4 MW). $T = 5$ and $r_f = \%16.8$:

1- E_1: with government support and 60% participation in investment with low-interest bank facilities of 12%.
2- E_2: without government support and participation in investment.

The results in Table 10 show that paying attention to expansion option by assuming double activities and production capacity of the photovoltaic plant in Isfahan province increases investment value. If this expansion is accompanied by government supports and participation in investment through low-interest facilities equal to 12% and 60% of development costs, option value will be equal to IRR 71.69 billion while without support and participation in investment, it will be equal to IRR 48.14 billion. This reveals high importance and vital role of the thirteenth uncertainty factor, i.e. government incentives in investment in these plants in Iran. Value of expansion option of the plant by means of Black-Schols closed form is higher than the obtained net present value from traditional methods.

Thus, investment value in this plant is equal to 78.82 in the first state and 55.27 in the second state using relation (15).

Table 10: Expansion option valuation based on assumptions of the first scenario (billion IRR)

Scenario 1	**The required capital to develop** $\tilde{X}$	**Double present value of fuzzy cash flows** $2\tilde{S}$	**Value of fuzzy expansion option** $FROV$	$E(FROV)$	NPV	**Investment value**
E_1	(34.795,36.627,38,495)	(78.757,87.508,96.258)	(62.138,71.696,81.237)	71.69	7.13	78.82
E_2	(86.898,91.568,96.238)	(78.757,87.508,96.258)	(32.21,47.978,59.243)	48.14	7.13	55.27
Source: researcher's calculations						

The second scenario: project abandonment. $T = 5$ and $r_f = \%16.8$:

1- A_1: Abandonment value equal to book value of assets. ($K = 75$).
2- A_2: Abandonment value equal to book value of assets plus the remaining value of cash flows. ($K = 75 + 30 = 105$).

Reviewing the calculations in Table 11 shows that abandonment of the plant to another sector is highly dependent on its abandonment value. If abandonment value and exiting the project is equal to book value of assets of the plant at the time of abandonment, i.e. salvage value is considered equal to IRR 75 billion; abandonment will not be economical, because it will have a negative value of 11.375. But in the event that abandonment value is considered equal to salvage value of assets at the end of useful life of the plant plus the value of durability of the plant activities equal to IRR 30 billion (or present value of remaining cash flows at the abandonment time), then abandonment option value will become equal to IRR 1.615 billion. According to the obtained results in this table, investment value based on relation (15) is estimated equal to IRR -2.245 billion in the first state and IRR 8.745 billion in the second state.

Table 11: Abandonment option valuation based on assumptions of the second scenario (billion IRR)

Scenario 2	**Present value of fuzzy cash flows** $\tilde{S}$	**Value of fuzzy abandonment option** $FROV$	$E(FROV)$	NPV	**Invest-ment val-ue**
A_1	(39.379,43.754,48.129)	$(-15.752, -11.377, -7.0)$	-11.375	7.13	-4.245
A_2	(39.379,43.754,48.129)	(0.0258,1.615,3.20)	+1.6150	7.13	+8.745
Source: researcher's calculations					

Generally, ignoring real options will be led to losing managers' flexibility in decision-making and ignorance of elasticity in changing previous decisions of management when conditions are changing. Investment projects are not usually stable after selection and beginning of investment in them and managers sometimes can create changes by having access to new information and conditions and be effective on future cash flow of investment projects. This freedom of action of managers is hidden inside the investment projects. Therefore, indisputable acceptance of traditional methods of discounted cash flows will be led to ignorance of these options [6].

4.4 Sensitivity Analysis of Parameters in Real Option Model

Variables such as present value of cash flows, the remaining time to option expiry, investment value (or salvage value of assets), fluctuation of present value of cash flows and risk-free interest rate are effective on options value.

Hence, proposing suitable strategies in the options market requires that changing of option value with regard to change in the above variables is calculated precisely. Change in option value per one unit change in each variable is referred to as sensitivity.

Sensitivity analysis of the parameters in option model is usually performed through Greeks letters. The most important letters are displayed in Table12 [29]:

Table 12: Greeks letters for sensitivity analysis of model of real options valuation

Greeks Letters	Formula	Sign for Call	Sign for Put
Delta	$\Delta = \frac{\partial ROV}{\partial S} = \frac{change\ in\ option\ value}{change\ in\ value\ of\ cash\ flows}$	+	-
Theta	$\theta = \frac{\partial ROV}{\partial t} = \frac{change\ in\ option\ value}{change\ in\ expiration\ time}$	+	+ or -
Vega	$\nu = \frac{\partial ROV}{\partial \sigma_S} = \frac{change\ in\ option\ value}{change\ in\ fluctuation}$	+	+
Rho	$\rho = \frac{\partial ROV}{\partial r_f} = \frac{change\ in\ option\ value}{change\ in\ risk-free\ interest\ rate}$	+	-
Source: (Nabavi and Ghasemi, [29])			

Results of calculations related to sensitivity analysis for different scenarios are according to Table 13.

Delta is a standard which measures changes of option value due to changes in present value of cash flows. Range of delta for put option (abandonment) is between 0 and -1 and for call option (expansion) is between 0 and 1. Calculation of delta for the scenarios under study shows that one percent change in present value of cash flows is led to 0.9988 changes in expansion option value in scenario E_1 and 0.9433 changes in real option value in scenario E_2. Similarly, negative delta or abandonment option shows that by one percent change in present value of cash flows, we will have - 0.213 change in abandonment value in scenario A_1 and -0.3984 changes in scenario A_2, that is, it has an inverse effect and there will be a reduction. Theta is ratio of change of option value given passing of time under stability conditions of other factors. Theta value is usually negative for option value, i.e., when the remaining time to maturity (T) is reduced and it is assumed that other factors are stable, option value is decreased [29]. The results of calculating this parameter for scenarios E_1 and E_2 (negative theta) show decreased value of expansion option given reduction of the remaining time to maturity and increased abandonment value of scenarios A_1 and A_2 given increasing of the remaining time (positive theta). Vega measures ratio of change of real option value to fluctuation of present value of cash flows. Whatever value of Vega is higher, the investor expects more changes in his/her investment value. This expectation for the investor of the photovoltaic plant under study will be higher by scenarios A_2, A_1, E_2, and E_1 respectively, because the option Vega is increased when fluctuation is enhanced. With one percent change in fluctuation of cash flows, the highest option value for scenarios A_2, A_1, and E_2 will be obtained with Vega equal to 0.3777, 0.2847 and 0.2232 respectively and the lowest option value will be obtained in scenario $E_1$with Vega equal to 0.0076.

Table 13: Results of calculating Greeks letters for different scenarios under study

Greeks letters	scenario E_1	scenario E_2	scenario A_1	scenario A_2
Δ	0.9988	0.9433	-0.213	-0.3984
θ	-0.0073	-0.0168	0.0043	0.0105
ν	0.0076	0.2232	0.2847	0.3777
ρ	0.7845	1.6418	-0.7013	-1.4595
Source: researcher's calculations via Black-Schols Calculator Online .www.fintools.com				

Rho shows ratio of changes in option value due to one percent change in risk-free interest rate. Whatever the interest rate is higher, value of expansion option (call option) will be higher because sign of the parameter is positive. When the risk-free interest rate is increased, value of Rho parameter of abandonment (put) option is reduced because of the negative sign of Rho. This means high sensitivity of investment to partial changes in interest rate [29].

The calculated Rho value shows with one percent change in risk-free interest rate, expansion option value for scenarios E_1 and E_2 will be increased to 0.7845 and 1.6418 and abandonment option value for scenariosA_1 and A_2 will be reduced to 0.7013 and 1.4595 respectively.

5 Discussion and Conclusion

This survey explored economic appraisal of a photovoltaic plant in south of Isfahan under uncertainty conditions. First, the governing uncertainty factors over establishment and investment conditions in this plant were identified from experts' viewpoint. Importance factor and role of each factor in influencing the profits and expenses or cash flows of the project was calculated as a common standard in economic appraisal. In the next step, the plant under study was evaluated based on traditional methods with the help of Ret Screen software. By considering the fuzzy numbers for profits and expense variables of the plant that are highly affected by uncertainty factors, real expansion and abandonment options were valuated via Black-Schols closed form. The findings are consistent with studies of Colan et al. [5], Kahraman and Uçal [15], Martinez et al. [23-24] and Sheen [32].

The results of expansion option valuation of the plant disclosed that whatever the incentives, supports and participation of the government sector in offering subsidies and low-interest facilities (as an indefinite and sensitive factor) are increased, expansion option will be enhanced and thus, investment value in these plants will be increased and numerous attractions for investors will be provided in this scope. As a result, the application of the real option theory in capital budgeting leads to Decision-making about investment under high uncertainty conditions and management of current opportunities and development of strategic decision-making opportunities in the future.

Abandonment option valuation showed that this option in the above plant is highly affected by abandonment value at the time of exiting the investment as an indefinite factor that is full of sensitivity.

Whatever this value is higher; abandonment option and also investment value will be higher. The result is that the use of real option theory in economic appraisal of projects leads to optimization of strategic investment decisions with the help of various paths of decision and determining an appropriate time to enter or exit an investment.

The obtained results of sensitivity analysis of model parameters had a considerable and supplementary achievement. In other words, present value of cash flows, the time remaining to expiration, fluctuation of cash flows and risk-free interest rate are to a large extent effective on option value of the project, and this leads to the valuation of the strategic decisions in the framework of financial justifiability. These results can be observed in Table 14:

Table 14: Summary of sensitivity analysis of model parameters to real option value

Type of real option	**Fluctuation**		**Time remaining to expiration**		**Present value of cash flows**		**Risk-free interest rate**	
	Decrease	**Increase**	**Decrease**	**Increase**	**Decrease**	**Increase**	**Decrease**	**Increase**
expansion option	+	-	+	-	+	-	+	-
abandonment option	+	-	+	-	-	+	-	+

Considering the above analyses and several advantages of real options model, it is suggested to use it in exploring the investment projects that are conducted under highly uncertainty conditions. This can help more accurate exploration of investment projects and their valuation and eliminate restrictions of traditional methods like stationary, ignoring flexibility and inattention to uncertainty in cash flows of project and the risk arising from it.

As it was mentioned earlier, some projects have negative present value after being discounted with the intended rate of investor due to their risky nature. However, they will gain a positive value and become justifiable because of considering the real option and the positive value that is added to the present value. Real option, thus, can be led to motivations to enter the projects which have not been previously justifiable.

There are various methods for real option valuation. In this survey, Black-Schols closed form was employed with some adjustments. It is suggested to use different simulation models which show higher efficiency in using random processes for fuzzy valuation of real options in photovoltaic plants with high uncertainty.

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