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A comment on "Supply chain DEA: production possibility set and performance evaluation model"

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Abstract

In a recent paper in this journal, Yang et al. [Feng Yang, Dexiang Wu, Liang Liang, Gongbing Bi & Desheng Dash Wu (2009), supply chain DEA:production possibility set and performance evaluation model] defined two types of supply chain production possibility set which were proved to be equivalent to each other. They also proposed a new model for evaluating supply chains. There are, however, some shortcomings in their paper. In the current paper, we correct the model, the theorems, and their proofs.
Keywords: Supply chain, Dea model, performance evaluation model

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1. Introduction

Yang et al. [1] defined two types of supply chain production possibility set.

On the basis of the production possibility sets, they proposed a model that can find the most efficient supply chain by improving inefficient subsystems (supply chain members). But the two production possibility sets introduced are not equivalent, and Feng et al.'s proposed model is incorrect. The proofs of some theorems are also not correct. This paper deals with these shortcomings and resolves the problems.

2. Comments

2.1

By Theorem 1 in Yang et al. [1],

$$T_{sc\_p} \equiv T_{sc\_sp}$$

and

$$T_{sc\_sp} \subseteq T_{sc\_p}.$$

But we show by a counter-example that

$$T_{sc\_p} \nsubseteq T_{sc\_sp}.$$

Counterexample

$$S_3 + M_4 \in T_{sc\_p} \text{ but } S_3 + M_4 \notin T_{sc\_sp}.$$

It is obvious that $S_3 + M_4 \in T_{sc\_p}$. By putting $S_3 + M_4$ in T_{sc_sp} we obtain the system

$$\begin{cases} 1.666\lambda_3 + 2\lambda_4 \le 2 \\ 2\lambda_3 + 3\lambda_4 \ge 2 \\ 5\lambda_3 + 6\lambda_4 \ge 5 \\ 2\lambda_3 + 2\lambda_4 \le 2 \\ 5\lambda_3 + 6\lambda_4 \le 4 \\ 10\lambda_3 + 8.33\lambda_4 \ge 6.67 \\ \lambda_3, \lambda_4 \ge 0 \end{cases}$$

This system is inconsistent, so $S_3 + M_4 \notin T_{sc\_sp}$. Furthermore, to prove

$$T_{sc\_p} \subseteq T_{sc\_sp}$$

in Theorem 1, Liang et al. argued:

For $\forall(\tilde{X}, \tilde{Y}) \in T_{sc\_p}$ Suppose that $(\tilde{X}, \tilde{Y}) \notin T_{sc\_sp}$. It is obvious that $\exists\tilde{\lambda} \ge 0$ such that $\sum_{j=1}^{N} \tilde{\lambda}_j X_j \theta_{sj}^* = \tilde{X}$ and $\sum_{j=1}^{N} \tilde{\lambda}_j Y_j / \theta_{Mj}^* < \tilde{Y}$. However, the line of argument below should have been followed in the above-mentioned proof.

By contradiction, suppose that

$$\exists(\tilde{X}, \tilde{Y}): \quad \begin{aligned} &(\tilde{X}, \tilde{Y}) \in T_{sc\_p} \\ &(\tilde{X}, \tilde{Y}) \notin T_{sc\_sp} \end{aligned}$$

$$(\tilde{X}, \tilde{Y}) \notin T_{sc-sp} \Rightarrow \nexists\lambda, \begin{aligned} &\textstyle\sum_{j=1}^{N} \lambda_j \theta_{sj}^* X_j \le \tilde{X} \\ &\textstyle\sum_{j=1}^{N} \lambda_j I_j \ge \tilde{I} \\ &\textstyle\sum_{j=1}^{N} \lambda_j I_j \le \tilde{I} \\ &\textstyle\sum_{j=1}^{N} \lambda_j Y_j / \theta_{Mj}^* \ge \tilde{Y} \\ &\lambda_j \ge o, \quad j = 1, \dots, N \end{aligned}$$

This means for any $\lambda_j \geq 0$ at least one of the above constraints is not satisfied. However, the fact that one of the constraints, say

$$\sum_{j=1}^{N} \lambda_j Y_j / \theta_{Mj}^* \geq \tilde{Y},$$

does not hold does not mean that

$$\sum_{j=1}^{N} \lambda_j Y_j / \theta_{Mj}^* < \tilde{Y}, \sum_{j=1}^{N} \lambda_j \theta_{Sj}^* X_j = \tilde{X} \qquad 2.2$$

In Model (9)

$$\begin{aligned} \theta_d &= \min \theta \\ \text{s.t} \quad & \sum_{j=1}^{N} \lambda_j X_j^* \leq \theta X_d \\ & \sum_{j=1}^{N} \lambda_j Y_j^* \leq Y_d \\ & (X_j^*, Y_j^*) \in T_{sc\_sp} \\ & \lambda_j \geq 0 \quad j = 1, \ldots, N \end{aligned}$$

In the above model if $\sum_{j=1}^{N} \lambda_j Y_j^* \leq Y_d$, the optimal solution will be zero by setting $\theta = 0$ and $\lambda_j = 0 (j = 1, \ldots, N)$. However, by setting

$$\sum_{j=1}^{N} \lambda_j Y_j^* \geq Y_d, (X_d^*, Y_d^*) \in T_{sc\_sp}$$

and with the data in Table (6), the optimal value θ_j^* will be obtained, as reported in the sixth column of Table (7). Therefore, in all the theorems that use this model, the constraint ($\sum_{j=1}^{N} \lambda_j Y_j^* \leq Y_d$) must be in the form $\geq$ and $(X_d^*, Y_d^*) \in T_{sc\_sp}$.

2.3

By Theorem 2, we have

$$\theta_d^* \leq \theta_d^{CCR},$$

where

$$\theta_d^{CCR} = \max\ \theta$$

$$\text{s.t} \qquad \sum_{j=1}^{N} \lambda_j X_j \le \theta X_d$$

$$\sum_{j=1}^{N} \lambda_j Y_j \le Y_d \qquad (11)$$

$$\lambda_j \ge 0 \quad j = 1, \dots, N$$

In Model (11) above, by setting $\theta = \infty$ and $\lambda_j = 0$ the optimal value ∞ is obtained. If we consider $\sum_{j=1}^{N} \lambda_j Y_j \ge Y_d$, by setting $\theta = \infty$ and $\lambda_j \ne 0$ we will obtain∞ as the optimal value again. However, if the problem is one of minimization and the constraints are in the form $\le$ then the optimal value will be zero. Therefore, Model (11) must be transformed to Model below.

$$\theta_d^{CCR} = \min \theta$$

$$\text{s.t} \qquad \sum_{j=1}^{N} \lambda_j X_j \le \theta X_d$$

$$\sum_{j=1}^{N} \lambda_j Y_j \ge Y_d$$

$$\lambda_j \ge 0 \quad j = 1, \dots, N$$

Thus, Theorem 2 and its proof will be as follows.

$\theta_d^* \le \theta_d^{CCR}$ where

$$\theta_d^{CCR} = \min \theta$$

$$\text{s.t} \qquad \sum_{j=1}^{N} \lambda_j X_j \le \theta X_d$$

$$\sum_{j=1}^{N} \lambda_j Y_j \ge Y_d$$

$$\lambda_j \ge 0, \quad j = 1, \dots, N$$

Suppose that

$$T_{sc} = \left\{ \binom{X}{Y} \middle| \sum_{j=1}^{N} \lambda_j X_j \le X \right.$$

$$\sum_{j=1}^{N} \lambda_j I_j \ge I$$

$$\sum_{j=1}^{N} \lambda_j I_j \le I$$

$$\sum_{j=1}^{N} \lambda_j Y_j \geq Y$$

$$\lambda \geq O\}$$

First we show $T_{sc} \subseteq T_{sc\_sp}$. Assume$(\bar{X}, \bar{Y}) \in T_{sc}$; then since $\theta_{Sj}^* \leq 1, \theta_{Mj}^* \leq 1$, we have

$$\sum_{j=1}^{N} \bar{\lambda}_j X_j \theta_{Sj}^* \leq \sum_{j=1}^{N} \bar{\lambda}_j X_j \leq \bar{X}$$

$$\sum_{j=1}^{N} \bar{\lambda}_j Y_j / \theta_{Mj}^* \geq \sum_{j=1}^{N} \bar{\lambda}_j Y_j \geq \bar{Y}$$

$$\Rightarrow (\bar{X}, \bar{Y}) \in T_{sc\_sp}$$

Consider the following two models:

$$\begin{aligned} & min\theta \\ \text{s.t} \quad & \sum_{j=1}^{N} \lambda_j X_j \leq \theta X_d && \text{(I)} \\ & \sum_{j=1}^{N} \lambda_j I_j \geq I && \text{(c)} \\ & \sum_{j=1}^{N} \lambda_j I_j \leq I && \text{(d)} \\ & \sum_{j=1}^{N} \lambda_j Y_j \geq Y_d \\ & \lambda \geq 0 \end{aligned}$$

$$\begin{aligned} & min\theta \\ \text{s.t} \quad & \sum_{j=1}^{N} \lambda_j X_j \leq \theta X_d \\ & \sum_{j=1}^{N} \lambda_j Y_j \geq Y_d && \text{(II)} \\ & \lambda \geq 0 \end{aligned}$$

Obviously, any solution for problem (I) is a solution for problem(II).

We claim that constraints (c) and (d) are redundant. Suppose that $(\bar{\lambda},\bar{\theta})$ is a solution for problem II. Let$\sum_{j=1}^{N}\bar{\lambda}_j\, I_j = I$, then

$$\sum_{j=1}^{N}\bar{\lambda}_j\, X_j \leq \bar{\theta} X_d$$
$$\sum_{j=1}^{N}\bar{\lambda}_j\, I_j = I$$
$$\sum_{j=1}^{N}\bar{\lambda}_j Y_j \geq Y_d$$
$$\bar{\lambda} \geq 0$$

So $(\bar{\lambda},\bar{\theta})$ is a solution for problem I.

A similar argument can be made regarding the redundancy of the corresponding constraints in the following Models.

$$\begin{aligned}
& min\theta \\
\text{s.t}\quad & \sum_{j=1}^{N}\lambda_j X_j^* \leq \theta X_d \\
& \sum_{j=1}^{N}\lambda_j I_j \geq I \\
& \sum_{j=1}^{N}\lambda_j I_j \leq I \\
& \sum_{j=1}^{N}\lambda_j Y_j^* \geq Y_d \\
& \lambda_j \geq 0, \quad j = 1, \dots, N \\
& (X_d^*, Y_d^*) \in T_{sc_sp}
\end{aligned}$$

And

$$\begin{aligned}
& \text{Min}\theta \\
\text{s.t}\quad & \sum_{j=1}^{N}\lambda_j X_j^* \leq \theta X_d \\
& \sum_{j=1}^{N}\lambda_j Y_j^* \geq Y_d \\
& \lambda_j \geq 0, \qquad j = 1, \dots, N \\
& (X_d^*, Y_d^*) \in T_{sc_sp}
\end{aligned}$$

Therefore, in order to compare the optimal values of these two models we will have:

$$\begin{aligned} & min\theta \\ \text{s.t} \quad & \sum_{j=1}^{N} \lambda_j X_j^* \leq \theta X_d \\ & \sum_{j=1}^{N} \lambda_j Y_j^* \geq Y_d \\ & (X_d^*, Y_d^*) \in T_{sc\_sp} \\ & \lambda_j \geq 0, \quad j = 1, \ldots, N \end{aligned}$$

Above model can be rewritten as the following program:

$$\begin{aligned} & \theta_d = \min \theta \\ \text{s.t} \quad & \sum_{j=1}^{N} \lambda_j X_j^* \leq \theta X_d \\ & \sum_{j=1}^{N} \lambda_j Y_j^* \geq Y_d \\ & \sum_{j=1}^{N} \bar{\lambda}_j X_j \theta_{Sj}^* \leq X_d^* \\ & \sum_{j=1}^{N} \bar{\lambda}_j I_j \geq I_d^* \\ & \sum_{j=1}^{N} \bar{\lambda}_j I_j \leq I_d^* \\ & \sum_{j=1}^{N} \bar{\lambda}_j \, Y_j / \theta_{Mj}^* \geq Y_d^* \\ & \lambda_j, \bar{\lambda}_j \geq 0 \quad j=1,\ldots,N \end{aligned} \qquad \text{(III)}$$

$$\begin{aligned} & min\theta \\ \text{s.t} \quad & \sum_{j=1}^{N} \lambda_j X_j \leq \theta X_d \\ & \sum_{j=1}^{N} \lambda_j Y_j \geq Y_d \\ & \lambda_j \geq O, \quad j = 1, \ldots, N \end{aligned} \qquad \text{(IV)}$$

Let(θ^*, λ^*) is a solution for problem (IV). Since$T_{sc} \subseteq T_{sc_sp}$,then

$$(\theta^*, \lambda^*, \bar{\lambda}) \quad \bar{\lambda}_j = \begin{cases} 0 & j = 1, \dots, N \quad j \neq d \\ 1 & j = d \end{cases}$$

is a solution for problem (III). Suppose $\theta_{IV}^*$is the optimal value of problem IV and θ_{III}^* is the optimal value of problem III . Then

$$\theta_{IV}^* \geq \theta_{III}^*$$

where

$$\theta_{IV}^* = \theta_d^{CCR}, \theta_{III}^* = \theta_d^*$$

Therefore

$$\theta_d^* \leq \theta_d^{CCR}$$

and the proof is completed. □

References

[1] F. Yang, D. Wu, L. Liang, G. Bi, D. D. Wu, Supply chain DEA:production possibility set and performance evaluation model.Journal of Ann oper Res, DOI:10.1007/s10479-008-0511-2 (2009).