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The Most Efficient Unit in Data Envelopment Analysis Using Genetic Algorithm

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Abstract

Data Envelopment Analysis (DEA) is a useful and practical to evaluate the performance based on multiple inputs and multiple outputs. DEA can determine efficient and non-efficient decision-making units (DMUs) through performance calculations. The most efficient unit is the DMU that is ranked among the most efficient units. However, there is no need to rank all units in order to obtain the most efficient decision-making unit. In this paper, the efficient unit hyperplane is selected with the help of the genetic algorithm of all the efficient units, which is the total distance between all the units on the other side of this hyperplane is the maximum amount possible with this hyperplane.

Key words: Data Envelopment Analysis, Genetic Algorithm, Hyperplane, Efficient Unit

1 Introduction

Data Envelopment Analysis (DEA) is actually to obtain the efficiency using inputs and outputs, which can determine efficient and inefficient units with the help of performance calculations such that the efficiency for efficient DMUs is one and less than one for inefficient DMUs [1]. In some cases, the ranking of the decision-making units are not important for decision-makers, and they are only looking to obtain the most efficient DMUs, so that they can directly achieve the most efficient DMU from all existing DMUs.
In a number of papers regarding this subject, several steps were taken to find the most efficient DMU, [2,3], which later examined the problems of these models and other models were announced by the researchers to resolve them. Some of the problems that can be mentioned:
1. Solving the model took place in two steps and could not directly reach the final answer.
2. Many unnecessary conjunctions were used in the models.
Other models were proposed to solve the problems that could eliminate unnecessary conjunctions and solve the problem in two phases [5].
Therefore, in this paper, it has been tried to provide a model that avoids unnecessary conjunctions, and most importantly, maximizes the distance between the other DMUs of an efficient DMU [6].

2 Finding an efficient DMU using the genetic algorithm

Amin and Toloe model [7] is based on the CCR model and is not effective for situations in which the DMU returns to a different scale. This model is suggested as follows:

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$M^* = \min M$

$s.t. M - dj \geq 0 j = 1, 2, \ldots, n,$

$\sum_{i=1}^{m} w_i x_{ij} \leq 1 j = 1, 2, \ldots, n,$

$\sum_{r=1}^{s} u_r y_{rj} - u_0 - \sum_{i=1}^{m} w_i x_{ij} + d_j - \beta_j = 0 j = 1, 2, \ldots, n \qquad (1)$

$\sum_{j=1}^{n} d_j = n - 1,$

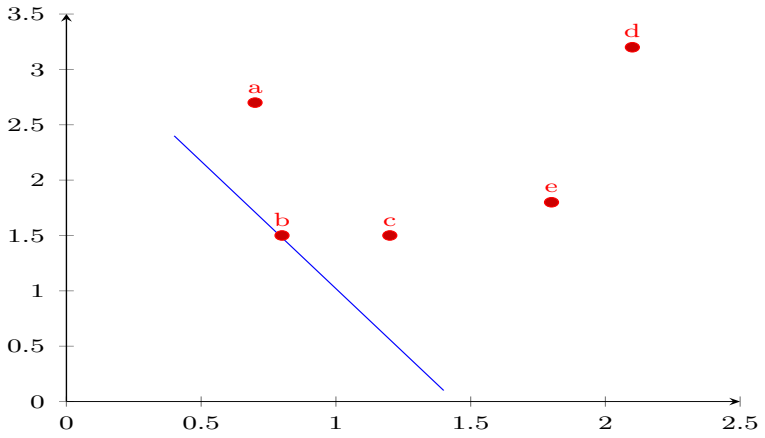
$0 \leq \beta_j \leq 1, d_j \varepsilon \{0, 1\} j = 1, 2, \ldots, n,$

$M, u_0 free,$

$w_i \geq \epsilon^* \ , \ i = 1, 2, \ldots, m,$

$u_r \geq \epsilon^*, r = 1, 2, \ldots, s$

DMU_j is the most efficient BCC if and only if $d_j = 0$. Similar to Amin and Toloe the main idea of the model (1) is attempting to find the most efficient DMU, but in situations where the return to scale is varied. Adding the u_0 variable improves the modeling capability of situations in which DMUs return to the variable scale, which means that the most efficient BCC is obtained using the model (1). $\sum_{j=1}^{n} d_j = n - 1$, constraint shows the forces among all DMUs for the most efficient BCC unit.
Now, we intend to calculate the most efficient DMU using the BCC model as the input, as follows:



The most efficient DMU is the unit that the total distance of all the units on the other side of the hyperplane to this hyperplane is maximum.
By coding the BCC model in the MATLAB software, the efficiency calculates n input and m output as well as u_0, V, U for inputs and outputs. Then, the inputs and outputs of the efficient DMUs in the output are obtained.
To obtain the most efficient DMU, the distance between each DMU and hyperplane is calculated. If the sum of the DMU distances to the hyperplane is the highest, the DMU whose hyperplane is plotted is the most efficient DMU. The distance will be obtained from the following equation:
In this regard, the values of the variables of each DMU are applied. To get the most efficient DMU, the following model is used (2):

$$\text{Max } \sum_{i=1,i\neq p}^{n} \frac{|UY_i - VX_i + u_0|}{||U,V||_2}$$

$$\text{s.t } UY_p - VX_p + u_0 = 0 \qquad (2)$$

$$UY_i - VX_i + u_0 \leq 0$$

$$0 \leq U, V$$

$$u_0 free$$

In this model, each efficient DMU appears only once in the constraint equal to zero, which cannot be observed in the objective function and other constraints. Other efficient DMU_s are used in smaller units.

3 Applicable Example

A numerical example is used to illustrate the applicability of the proposed model. A hypothetical data set containing 3 inputs and 2 outputs are used in Table 1:

Table 1: data with 3 inputs and 2 outputs

inputs		outputs			DMU
Y_2	Y_1	X_3	X_2	X_1	
28	40	68	61	22	1
83	71	26	86	29	2
59	69	9	75	44	3
37	28	90	76	43	4
21	77	27	83	62	5
7	102	86	32	47	6
69	53	68	50	83	7
24	34	40	100	65	8
65	69	23	75	47	9
45	37	88	57	68	10
11	27	28	7	71	11
36	31	63	95	64	12

46	72	36	65	65	13
14	88	86	45	23	14
100	95	14	45	80	15
97	53	31	44	101	16
59	71	75	99	39	17
99	56	18	74	42	18
53	41	90	49	59	19
102	50	79	10	26	20
53	38	47	68	78	21
63	23	98	47	11	22
53	79	71	101	24	23
46	22	65	29	53	24
12	90	81	100	80	25
75	82	5	38	54	26
28	84	46	73	50	27
52	10	73	58	13	28
65	83	15	82	37	29
64	24	7	81	31	30

Of these data, there are 13 effective DMUs whose Y, X values are shown in Table 2:

Table 2: X, Y values for efficient DMUs

inputs		outputs			Df-DMU
Y_2	Y_1	X_3	X_2	X_1	
83	71	26	86	29	1
7	102	86	32	47	2
11	27	28	7	71	3
14	88	86	45	23	4
100	95	14	45	80	5
99	56	18	74	42	6
102	50	79	10	26	7
63	23	98	47	11	8
53	79	71	101	24	9
75	82	5	38	54	10
52	10	73	58	13	11
65	83	15	82	37	12
64	24	7	81	31	13

We must calculate model (2) for each DMU, since the number of efficient DMUs is high, for example, for the first DMU:
"formul"
The sub-distance values of effective DMUs are from the hyperplane:

Table 3: Distance values from the cloud page

DMU	1	2	3	4	5	6
distance	0.4524	0.2607	0.1707	0.1018	0.3092	1.6199

Table 3: Continuation Of Table 3

DMU	7	8	9	10	11	12	13
distance	0.0975	0.1808	7.7232	0.0998	0.3644	0.3386	0.1724

According to this table, DMU 9 is the most efficient DMU, with a distance of 7.7232 from the hyperplane.

Conclusion

Identifying the most efficient decision making unit for decision makers is an important point. In this paper, it has been tried to provide a model using the genetic algorithm that can be applied to select the efficient unit hyperplane by solving only one model and, with the least constraints so that the total distance of all the units on the other side of the hyperplane to this hyperplane is maximum.

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