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Measurement of profit inefficiency in presence of interval data using the directional distance function

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Abstract

In many applied programs in real-life problems, both physical inputs and outputs are heterogeneous which in this case the efficient cost and income model can not apply to evaluate the cost and income of related turnover. So, a measurement based on the directional value of profit was presented which we have developed it in this paper and have computed it for interval data. In fact, we have measured the inefficiency of cost in presence of interval data using the directional distance function which is mostly meaningful for those companies that their essential behavioral goals are maximizing the profit with least ambiguity. To this end, considering some branches of Tejarat bank in Iran, the efficiency of profit in presence of interval data is computed by means of the distance directional function.

Key words: Measurement of profit, DMU, Interval data.
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1 Introduction

Chambers et al (1998, 1996) [4], [5], were those who presented some methods for experimental implementing CE and RE measurement in DEA for the first time. Since then, measurement of costs and revenues explored in many studies such as Cooper et al (1996) [7], Tone and Sahoo (2005) [16], Jahanshahloo et al (2008) [12] and Sahoo et al (2012) [14]. Both CE and RE models which were presented by Farr et al (1985) [5] not only require input and output data but also the price in each of the companies. This model can just limitedly be used in real applications when defect is presented in market. Economic theories suggest that those companies which benefit from exclusive power should operate different prices in case of existence of heterogeneity in productivity of inputs. This is empirically valid as the slope of supply curve in purchasing decisions of companies is also upward. These observations show that the common unit price which is preserved as a necessary and sufficient condition for Pareto productivity in competitive markets is studied by Cerci et al (2006) [13]. Also, the CE measurement which was developed by Farr et al (1985) [9] can be of limited value in real applied problems even if the inputs are homogeneous.

As it was implied by Dyson and Kamanhoo (2005) [3], measurement of CE only indicates the technical inefficiency or ineffectiveness of allocation while it does not represent the income inefficiency. So, to overcome to this problem, they presented the comprehensive CE measurement which includes both input factor and income inefficiency.

In many real-life problems, the input and output data are uncertain and because of this reason they represent the average price instead of the total one whilst analyzing based on the average price can distort the allocation efficiency measurement as it was proved by Fukuyama and Weber (2008) [11]. Hence, when inputs and outputs

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are heterogeneous the Tone's CE and RE models (2002) [15] which are regulated in a space of input-cost and output-income should be used. Fukuyama and Weber (2004) [10] as well as Farr et al (2006) [8] developed the CE model using the directional function in DEA so that they could present the distance input-cost directional function (DICDF) which was a step toward measuring the technical directional efficiency. Measuring the cost by means of the distance directional function covers the unit invariability which is presented by Cooper et al (1999) [6], and the strong uniformity by Russell et al (1999) [2] as well as the Cooper et al (1996) [6]. DCE model is organized based on the assumption of homogeneity of physical output and heterogeneity of input. Similarly, DRE is developed based on the assumption of heterogeneity of output and homogeneity of physical inputs. However, when both physical inputs and outputs are heterogeneous, the provided DCE and DRE models can not apply for measuring the cost and income related turnover. Taking this point into account, Sahoo et al (2004) [1] presented the inefficiency based on the directional value of profit which can be used in case of those companies that are intended to maximize profit with less ambiguity. Uncertainty, inaccuracy or incompleteness of data can affect the assessing of profit and in most cases profit is tangible to swings of data. In addition to heterogeneity, input and output data can be probabilistic, interval, or ordinal. In this paper we concentrate on uncertain input and output data which are in other words are interval data and the inefficiency of profit is presented via using distance directional function in presence of this type of data.

2 Measurement of efficiency of total profit

Assume that there exist n units under assessing which each of them uses m inputs for s output function. Let c and r to be the price vectors of input and output corresponding to DMUs respectively such that $c \geq 0, r \geq 0, c \neq 0, r \neq 0$. The following model measures the total profit:

$$
\begin{aligned}
Max \quad & \mathit{\Phi} - \mathit{\Theta} \\
S.t \quad & \lambda \geq 0 \\
& \mathit{\Phi}[r^t y_0] \leq r^t Y \lambda \\
& \mathit{\Theta}[c^t x_0] \geq c^t X \lambda
\end{aligned}
\tag{2.1}
$$

where the index 0 determines the under assessing DMU. The objective function of above model maximizes profit through maximizing the total revenue ($\mathit{\Phi}$) and minimizing the cost (Θ) for the given price vector $(r^t, c^t) = p^t$.

Model (2.1) is developed and efficiency of total profit of DMU with n different price vectors is measured and the following model is given:

$$
\begin{aligned}
Max \quad & \Phi - \Theta \\
& \Phi[r^t y_0] \leq {r_j}^t Y \lambda \quad j = 1, ..., n \\
& \Theta[c^t x_0] \geq {c_j}^t X \lambda \quad j = 1, ..., n \\
& \lambda \geq 0
\end{aligned}
\tag{2.2}
$$

Although the objective function of model (2.2) is similar to model (2.1) but the second one considers all the price vectors ${p_j}^t = ({r_j}^t, {c_j}^t)$.

Point: DMU_0 is total profit efficient if $\mathit{\Phi} - \theta$ holds in model (2.2).

They show that for the optimal solution $(\mathit{\Phi}^*, \theta^*, \lambda^*)$ of model (2.2), $\mathit{\Phi}^* - \theta^* \geq 0$ always holds. Note that the model (2.2) is a constant scale turnover which considers the maximum profit equal to zero and so the restriction $1.\lambda = 1$ is added to this model in order to characterize turnover in terms of variable scale.

In many real-life applications both physical inputs and outputs are heterogeneous so that the measurement of efficiency of the presented cost and income can not apply to evaluate the cost and income of related turnover. As a solution to this problem, a directional measurement of profit is provided which of course requires further developments for being suitable for maximizing profit, minimizing cost or minimizing revenue purposes. Consider the following maximizing profit problem based on the value T_{XY}^{DEA}:

$$
\begin{aligned}
K_0^* = Max \sum_{r=1}^{s} \frac{g_r^+}{G_r^+} + \beta_r^+ + \sum_{i=1}^{m} \frac{g_i^-}{G^-} + \beta_i^- \qquad (2.3) \\
s.t. \sum_{j=1}^{n} \lambda_j \bar{x}_{ij} \le \bar{x}_{i0} - \beta_i^- g_i^- \quad i = 1, ..., m \\
\sum_{j=1}^{n} \lambda_j \bar{y}_{rj} \ge \bar{y}_{r0} + \beta_r^+ g_r^+ \quad r = 1, ..., s \\
\sum_{j=1}^{n} \lambda_j = 1 \\
\lambda_j \ge 0
\end{aligned}
$$

If $K_0^* = 0$ then DMU_0 gives turnover in terms of profit. If $K_0^* > 0$ then DMU_0 has no turnover. Model (2.3) can be measured for the following direction vectors:

(1) $g_i^- = 1$, $g_r^+ = 1 \qquad i = 1, ..., m, r = 1, ..., s$

(2) $g_i^- = \bar{x}_{i0}$, $g_r^+ = \bar{y}_{r0} \quad i = 1, ..., m, r = 1, ..., s$

(3) $g_i^- = \underset{1 \le j \le n}{Max}\{\bar{x}_{ij}\}$, $g_r^+ = \underset{1 \le j \le n}{Max}\{\bar{y}_{rj}\}$
$i = 1, ..., m, r = 1, ..., s$

(4) $g_i^- = \bar{x}_{i0} - \underset{1 \le j \le n}{Min}\{\bar{x}_{ij}\}$, $g_r^+ = \underset{1 \le j \le n}{Max}\{\bar{y}_{rj}\} - \bar{y}_{r0}$
$i = 1, ..., m, r = 1, ..., s$

(5) $g_i^- = \underset{1 \le j \le n}{Max}\{\bar{x}_{ij}\} - \underset{1 \le j \le n}{Min}\{\bar{x}_{ij}\}$, $g_r^+ = \underset{1 \le j \le n}{Max}\{\bar{y}_{rj}\}\underset{1 \le j \le n}{Min}\{\bar{y}_{rj}\}$
$i = 1, ..., m, r = 1, ..., s$

Measurement of inefficiency of profit in presence of interval data: In many applied programs in real-life problems not only the physical inputs and outputs are heterogeneous but also probably inexact which vary in a certain interval. X_{ij}^L and X_{ij}^U are the lower and upper bounds of i^{th} input of DMU_j, respectively. Y_{ij}^L and Y_{ij}^U are the lower and upper bounds of rth output of DMU_j, respectively. Namely, $X_{ij}^L \leq X_{ij} \leq X_{ij}^U$ and $Y_{ij}^L \leq X_{ij} \leq Y_{ij}^U$. Note that $X_{ij}^L \leq X_{ij}^U$ and $Y_{ij}^L \leq Y_{ij}^U$. If $X_{ij}^L = X_{ij}^U$ then we can conclude that the i^{th} input of DMU_j has a specified amount. The problems of interval data relates to the amount of parameters in the intervals. Hence, measuring the directional profit cannot be applied. In order to fix this problem, measuring the inefficiency of directional profit in presence of interval data is presented throughout this paper which is mostly usable for those companies that their essential behavioral goals are maximizing the profit with least ambiguity. Consequently, the problem of maximizing profit based on T_{XY}^{DEA} and in presence of interval data is as follows:

$$
\begin{aligned}
\tilde{K}_0^* = Max \sum_{r=1}^{s} \frac{g_r^+}{G_r^+} + \beta_r^+ + \sum_{i=1}^{m} \frac{g_i^-}{G^-} + \beta_i^- \qquad\qquad & (2.4)\\
s.t. \sum_{\substack{j=1\\ j\neq 0}}^{n} \lambda_j \tilde{\tilde{x}}_{ij} + \lambda_0 \tilde{\tilde{x}}_{i0} \leq \tilde{\tilde{x}}_{i0} - \beta_i^- g_i^- \qquad & i = 1, ..., m\\
\sum_{\substack{j=1\\ j\neq 0}}^{n} \lambda_j \tilde{\tilde{y}}_{rj} + \lambda_0 \tilde{\tilde{y}}_{r0} \geq \tilde{\tilde{y}}_{r0} + \beta_r^+ g_r^+ \qquad & r = 1, ..., m\\
\sum_{j=1}^{n} \lambda_j = 1 \qquad & j = 1, ..., n\\
\lambda_j \geq 0 \qquad & j = 1, ..., n
\end{aligned}
$$

Where $G^+ = \sum_{r=1}^{s} g_r^+$, $G^- = \sum_{i=1}^{m} g_i^- \beta_i{}^-$ and β_i^- is the rate of improvement of ith cost input and β_i^+ is the rate of improvement of r^{th} income output of DMU_j.

The following models are presented for computing the lower and upper bounds of model (2.4):

$$\tilde{K}_0^* = Max \sum_{r=1}^{s} \frac{g_r^+}{G_r^+}\beta_r^+ + \sum_{i=1}^{m} \frac{g_i^-}{G^-}\beta_i^- \tag{2.5}$$
$$\begin{aligned} s.t. \sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j \bar{x}_{ij}^L + \lambda_0 \bar{x}_{i0}^U &\leq \bar{X}_{i0}^U - \beta_i{}^- g_i{}^- && i = 1, ..., m \\ \sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j \bar{y}_{rj}^U + \lambda_0 \bar{y}_{r0}^L &\geq \bar{y}_{r0}^L + \beta_r^+ g_r^+ && r = 1, ..., m \\ \sum_{j=1}^{n} \lambda_j &= 1 \\ \sum_{j=1}^{n} \lambda_j &= 1 && j = 1, ..., n \\ \lambda_j &\geq 0 && j = 1, ..., n \end{aligned}$$

$$\tilde{K}_0^* = Max \sum_{r=1}^{s} \frac{g_r^+}{G_r^+}\beta_r^+ + \sum_{i=1}^{m} \frac{g_i^-}{G^-}\beta_i^- \tag{2.6}$$
$$\begin{aligned} s.t. \sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j \bar{x}_{ij}^U + \lambda_0 \bar{x}_{i0}^L &\leq \bar{X}_{i0}^L - \beta_i^- g_i^- && i = 1, ..., m \\ \sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j \bar{y}_{rj}^L + \lambda_0 \bar{y}_{r0}^U &\geq \bar{y}_{r0}^U + \beta_r^+ g_r^+ && r = 1, ..., m \\ \sum_{j=1}^{n} \lambda_j &= 1 \\ \sum_{j=1}^{n} \lambda_j &= 1 && j = 1, ..., n \\ \lambda_j &\geq 0 && j = 1, ..., n \end{aligned}$$

The relations (2.5) and (2.6) can in fact be measured for the following direction vectors which all of the DMU_s are in their best case while writing the directions:

(1) $g_i^- = 1, g_r^+ = 1 \qquad i = 1, ..., m, r = 1, ..., s$

(2) $g_i^- = \bar{x}_{i0}^L, g_r^+ = \bar{y}_{r0}^U \qquad i = 1, ..., m, r = 1, ..., s$

(3) $g_i^- = \underset{1 \le j \le n}{Max}\{\bar{x}_{ij}^L\}, g_r^+ = \underset{1 \le j \le n}{Max}\{\bar{y}_{rj}^U\} \qquad i = 1, ..., m, r = 1, ..., s$

(4) $g_i^- = \bar{x}_{i0}^L \underset{1 \le j \le n}{Min}\{\bar{x}_{ij}^L\}, g_r^+ = \underset{1 \le j \le n}{Max}\{\bar{y}_{rj}^U\} - \bar{y}_{r0}^U$
$i = 1, ..., m, r = 1, ..., s$

(5) $g_i^- = \underset{1 \le j \le n}{Max}\{\bar{x}_{ij}^L\} - \underset{1 \le j \le n}{Min}\{\bar{x}_{ij}^L\}, g_r^+ = \underset{1 \le j \le n}{Max}\{\bar{y}_{rj}^U\} - \underset{1 \le j \le n}{Min}\{\bar{y}_{rj}^U\}$
$i = 1, ..., m, r = 1, ..., s$

Now we show that the measurement of inefficiency of directional profit is obtained in an interval consisted of lower and upper bounds.

Theorem 2.1 *If $K_0^{*L}, K_0^{*U}, \tilde{K}_0^*$ be the optimal values of the models* (2.4), (2.5) *and* (2.6) *respectively, then $K_0^{*L} \le K_0^* \le K_0^{*U}$.*

Proof. First we show that $\tilde{K}_0^* \le K_0^{*U}$. Let $(\lambda^1, \beta^{-1}, \beta^{+1})$ be the optimal solution of model (2.4). It is proved that this is also the feasible solution of model (2.5). We have:

$$\begin{aligned}
\sum_{\substack{j=1 \\ j \ne 0}}^{n} \lambda_j^1 \bar{x}_{ij}^L + \lambda_0^1 \bar{X}_{i0}^U &\le \sum_{\substack{j=1 \\ j \ne 0}}^{n} {\lambda_j}^1 \tilde{\bar{X}}_{ij} + {\lambda_0}^1 \bar{X}_{i0}^U + \lambda_0^1 \tilde{\bar{X}}_{i0} - {\lambda_0}^1 \tilde{\bar{X}}_{i0} \\
&\le \tilde{\bar{X}}_{i0} - {\beta_i^{-1}} g_i^- + {\lambda_0}^1 \bar{x}_{i0}^U - \lambda_0^1 \tilde{\bar{X}}_{i0} + \bar{X}_{i0}^U - \bar{X}_{i0}^U \\
&= \bar{X}_{i0}^U - {\beta_i^{-1}} g_i^- + \tilde{\bar{X}}_{i0} + \lambda_0^1 \bar{x}_{i0}^U - \lambda_0^1 \tilde{\bar{X}}_{i0} - \bar{x}_{i0}^U \\
&= \bar{x}_{i0}^U - \beta_i^- g_i^- + (1 - \lambda_0^1)(\tilde{\bar{x}}_{i0} - \bar{x}_{i0}^U)
\end{aligned}$$

As $\sum_{j=1}^{n} \lambda_j = 1$ so $(1 - \lambda_0^1)$ is nonnegative and since $\bar{x}_{i0}^L \le \tilde{\bar{X}}_{i0} \le \bar{X}_{i0}^U$ then the result of $\tilde{\bar{x}}_{i0} - \bar{x}_{i0}^U$ would be negative so that it makes the multiple $(1 - \lambda_0^1)(\tilde{\bar{x}}_{i0} - \bar{x}_{i0}^U)$ negative as well. Hence

$$\sum_{\substack{j=1\\j\neq 0}}^{n} \lambda_j{}^1 \bar{x}_{ij}^L + \lambda_0^1 \bar{X}_{i0}^U \leq \bar{X}_{i0}^U - \beta_i^{-1} g_i^-$$

Now we consider the second restriction:

$$\begin{aligned}
\sum_{\substack{j=1\\j\neq 0}}^{n} \lambda_j^1 \bar{y}_{rj}^U + \lambda_0^1 \bar{y}_{r0}^L &\geq \sum_{\substack{j=1\\j\neq 0}}^{n} \lambda_j^1 \tilde{\bar{y}}_{rj} + \lambda_0^1 \bar{y}_{r0}^L + \lambda_0^1 \tilde{\bar{y}}_{r0} - \lambda_0^1 \tilde{\bar{y}}_{r0} \\
&\geq \tilde{\bar{y}}_{r0} + \beta_r^{+1} g_r^+ + \lambda_0^1 \bar{y}_{r0}^L - \lambda_0^1 \tilde{\bar{y}}_{r0} \\
&= \bar{y}_{r0}^L + \beta_r^{+1} g_r^+ + \lambda_0^1 \bar{y}_{r0}^L - \lambda_0^1 \tilde{\bar{y}}_{r0} + \tilde{\bar{y}}_{r0} - \tilde{\bar{y}}_{r0}^L \\
&= \bar{y}_{r0}^L + \beta_r^{+1} g_r^+ + \lambda_0^1 \bar{y}_{r0}^L - \lambda_0^1 \tilde{\bar{y}}_{r0} + \tilde{\bar{y}}_{r0} - \tilde{\bar{y}}_{r0}^L \\
&= \bar{y}_{r0}^L + \beta_r^{+1} g_r^+ + (1 - \lambda_0^1)(\tilde{\bar{y}}_{r0} - \tilde{\bar{y}}_{r0}^L)
\end{aligned}$$

As $\sum_{j=1}^{n} \lambda_i = 1$ so $(1 - \lambda_0^1)$ 4 is nonnegative and since $\lambda_{rj}^L \leq \tilde{\bar{y}}_{rj} \leq \bar{\lambda}_{rj}^L$ then the result of $\tilde{\bar{y}}_{r0} - \bar{\lambda}_{r0}^L$ would be nonnegative which makes the result of multiple $(1 - \lambda_0^1)(\tilde{\bar{y}}_{r0} - \lambda_{r0}^L)$ nonnegative as well. So

$$\sum_{\substack{j=1\\j\neq 0}}^{n} \lambda_j^1 \bar{y}_{rj}^U + \lambda_0^1 \bar{y}_{r0}^L \geq \bar{y}_{r0}^U + \beta_r^{+1} g_r^+$$

Hence the optimal solution $\tilde{K_0}^*$ holds in $\tilde{K_0}^* \leq \tilde{K_0}^{*U}$ and similarly it would be proved that $\tilde{K_0}^{*L} \leq \tilde{K_0}^*$

According to the efficiency of units' profits, three following sets are introduced:

$K^{++} = \left\{ DMU_j | K_j^{*U} = 0 \right\}$
$K^{+} = \left\{ DMU_j | K_j^{*L} = 0 \right\}$
$K^{-} = \left\{ DMU_j | K_j^{*U} > 0 \right\}$

Since $K_0^{*L} \leq \tilde{K_0}^* \leq K_0^{*U}$ then if $K_j^{*U} = 0$ it can be concluded that DMU_j is profit efficient in its best and worst case and that it belongs to K^{++} set. Also, if $K_j^{*L} = 0$ then it can be concluded that DMU_j is profit efficient in its best case and contains in the k^+ set. If $K_j^{*L} > 0$ then the amount of inefficiency is positive and DMU_j is not profit efficient and contains in the K^- set.

3 Experimental example

Now we compute the inefficiency of profit in presence of interval data by means of distance directional function and over 20 different branches of Tejarat bank in Iran which each branch uses 3 inputs for producing 5 outputs. Table (1) shows these inputs and outputs.

Inputs	outputs
Payable portion	Sum of four main deposits
Personnel	Other deposits
Unnecessary loans	Loan
	Profit
	cost

Table 1
of inputs and outputs

Interval inputs and outputs for DMU_0 are recorded in tables (2) and (3).

DMU_j	x_{1j}^L	x_{1j}^U	x_{2j}^L	x_{2j}^U	x_{3j}^L	x_{3j}^U
1	5007.37	9613.37	36.29	36.86	87243	87243
2	2926.81	5961.55	18.8	2016	9945	12120
3	8732.7	17752.5	25.74	27.17	47575	50013
4	945.93	1966.39	20.81	22.54	19292	19753
5	8487.07	17521.66	14.16	14.8	3428	3911
6	13759.35	27359.36	19.46	19.46	13929	15657
7	587.69	1205.47	27.29	27.48	27827	29005
8	4646.39	9559.61	24.52	25.07	9070	9983
9	1554.29	3427.89	20.47	21.59	412036	413902
10	17528.31	36297.54	14.84	15.05	8638	10229
11	2444.34	4955.78	20.42	20.54	500	937
12	7303.27	14178.11	22.87	23.19	16148	21353
13	9852.15	19742.89	18.47	21.83	17163	17290
14	4540.75	9312.24	22.83	23.96	17918	17964
15	3039.58	6304.01	39.32	39.86	51582	55136
16	6585.81	13453.58	25.57	26.52	20975	23992
17	4209.18	8603.79	27.59	27.95	41960	43103
18	1015.52	2037.82	13.63	13.93	18641	19354
19	5800.38	11875.39	27.12	27.26	19500	19569
20	1445.68	2922.15	28.96	28.96	31700	32061

Table 2
of inputs- Data of 20 branches of Tejarat bank

DMU_j	y^L_{1j}	y^U_{1j}	y^L_{2j}	y^U_{2j}	y^L_{3j}	y^U_{3j}	y^L_{4j}	y^U_{4j}	y^L_{5j}	y^U_{5j}
1	2696995	3126798	263643	382545	1675519	1853365	108634.76	125740.28	965.97	5769.33
2	340377	440355	95978	117659	377309	390203	32396.65	37836.56	304.67	749.4
3	1027546	1061260	37911	503089	1233548	1822028	96842.33	108080.01	2285.03	3174
4	1145235	1213541	229646	268460	468520	542101	32362.8	39273.37	207.98	510.93
5	390902	395241	4924	12136	129751	142873	12662.71	14165.44	63.32	92.3
6	988115	1087392	74133	111324	507502	574355	53591.3	72257.28	480.16	869.52
7	144906	165818	180530	180617	288513	323721	40507.97	45847.48	176.58	370.81
8	408163	416416	405396	486431	1044221	1071812	56260.09	73948.09	4654.71	5882.53
9	335070	410427	337971	449336	1584722	1802942	176436.81	189006.12	560.26	2506.67
10	700842	768593	14378	15192	2290745	2573512	662725.21	791463.08	58.89	86.86
11	641680	606338	114183	241081	1579961	2285070	17527.58	20773.91	1070.81	2283.08
12	453170	481943	27198	29553	245726	275717	35757.83	42790.14	375.07	559.85
13	553167	574989	21298	23043	425886	431815	45652.24	50255.75	438.43	836.82
14	309670	342598	20168	26172	124188	126930	8143.79	11948.04	936.62	1468.45
15	286149	317186	149183	270708	787959	810088	106798.63	111962.3	1203.79	4335.24
16	321435	347848	66169	80453	360880	379488	89971.47	165524.22	200.36	399.8
17	618105	835839	244250	404579	9136507	9136507	33036.79	41826.51	2781.24	4555.42
18	248125	320974	3063	6330	26687	29173	9525.6	10877.78	240.04	274.7
19	640890	679916	490508	684372	2946797	3985900	66097.16	95329.87	961.56	1914.25
20	119948	120208	14943	17495	297674	308012	21991.53	27934.19	282.73	471.22

Table 3
of outputs- Data of 20 branches of Tejarat bank

The amount of inefficiency of profit in presence of interval data is presented in table (4) by means of distance directional function.

DMU_j	K_0^{*L}	K_0^{*U}	classification
1	0	4.32	K^+
2	0	0.99	K^+
3	0	7.29	K^+
4	0	13.59	K^+
5	0.01	1.26	K^-
6	0	32.45	K^+
7	0	5.44	K^+
8	0	35.17	K^+
9	0	65.8	K^+
10	0	153.25	K^+
11	0	8.88	K^+
12	0	0.98	K^+
13	0	0.99	K^+
14	0.01	0.99	K^-
15	0	143.15	K^+
16	0	0.98	K^+
17	0	807.84	K^+
18	0.03	0.99	K^-
19	0	38.78	K^+
20	0	0.96	K^+

Table 4
amount of inefficiency of profit and classification of DMU_s.

According to table (3) none of the DMU_s in this example contains in K^{++} class. This means that no DMU is profit efficient in its best or worst case.
DMU_5, DMU_{14}, DMU_{18} belong to K^- class and are not profit

efficient. Other DMU_s belong to K^+ class and are profit efficient in their best case.

4 Conclusion

Based on the existed interval inputs and outputs, none of the $DMUs$ are profit efficient in their best or worst case and other ones except DMU_5,$DMU_1 4$, $DMU_1 8$ belongs to K^+ class which means that they are profit efficient in their best cases. DMU_5, $DMU_1 4$, $DMU_1 8$ are not profit efficient even in their best cases and are inefficient which have no turnover in terms of profit.

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