Theory of Approximation and Applications

Vol. 12, No.1, (2018), 11-27

A Chance-Constrained DEA model with random input and output data: Considering maintenance groups of Iranian Aluminum Company

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Received 15 June 2017; accepted 22 January 2018

Abstract

In this paper, we use an input oriented chance-constrained DEA model with random inputs and outputs. A super-efficiency model with chance constraints is used for ranking. However, for convenience in calculations a non-linear deterministic equivalent model is obtained to solve the models. The non-linear model is converted into a model with quadratic constraints to solve the non-linear deterministic model. Finally, data related to twenty-eight maintenance groups of Iranian Aluminum Company (IRALCO) is used to demonstrate the applicability of the used Models in this paper.

Key words: Data Envelopment Analysis, Chance constraints, Random variables, Quadratic constraints, Super-efficiency.

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1 Introduction

Data Envelopment Analysis (DEA) is a nonparametric method based on mathematical programming for evaluating the efficiency firms or Decision Making Unit (DMU) using deterministic inputs and outputs. In real-world situations, the observed values of the input and output data are sometimes inexact, incomplete or vague. These kinds of uncertainty data can be represented as stochastic data. The stochastic data in DEA can be explained by random variables. Several methods have been associated with the random data in DEA.

The Iranian Aluminum Company as the first manufacturer of aluminum in Iran and Middle East is located in the arak city. The company's products in various lines of production is done 24 hours a day so if a simple failure occur in machines imposed huge losses to the company. In order to technical planning, usually unit evaluated maintenance groups, but this assessment is not a good evaluation, so in this study this is done by using stochastic DEA models.

Data envelopment analysis (DEA) was emerged in 1978 by Charnes, Cooper, and Rhodes and the first DEA model was called CCR model [10]. On the basis of the original CCR model, Banker, Charnes and Cooper developed a variable returns to scale variation in 1984 was called BCC model [9]. DEA has been widely used for evaluating the performance and measuring the relative efficiency of a group of firms or Decision-Making Units (DMUs) that uses multiple inputs and multiple outputs ([5,6,15]). After solving the DEA models, DMUS are converted into two categories: efficient and inefficient units. However, one of the problems those researchers faced with how to rank efficient DMUs. So, Andersen and Petersen [2] introduced super-efficiency models to rank efficient Decision Making Units in 1993.

Many studies have been done in this area after the introduction of early DEA models ([4,21,22]). Mehrabian et al. [24], developed a super-efficiency model to rank efficient units obtained by their own model. Tone [25] developed a super-efficiency measure based on the slack based measure introduced in the literature. Ranking papers using super-efficiency models up to 2002 have been reviewed in Adler et al. [1]. Furthermore, Khodabakhshi [17,18] developed a super-efficiency measure based on, improved outputs, input relaxation model introduced in Jahanshahloo and Khodabakhshi [36]. Li et al. [19], also, provided a super-efficiency model to rank efficient units obtained by the CCR model. The conventional DEA is deterministic, does not require prior weights or explicit specification of the functional relationships between the outputs and inputs and assumes that inputs and outputs are measured precisely ([13,14,16]).

However, the uncertainties inherent in the real-life performance measurement problems inhibit using deterministic DEA models in practice. Therefore stochastic data envelopment analysis models were developed. Stochastic input and output variations into DEA have been studied by, for example, Cooper et al. [11,12], Land et al. [30], and Olesen and Petersen [31], Morita and Seiford [20], Khodabakhshi and Asgharian [34], Khodabakhshi [17,18]. Several papers have been published on stochastic DEA in different Journals.

In what follows, some of these applications are reported. Khodabakhshi [17] proposed an output oriented super-efficiency measure in stochastic data envelopment analysis to evaluate the efficiency Iranian electricity distribution companies. In their own study the concept of chance constrained programming approaches is used to develop output oriented super-efficiency model in stochastic data envelopment analysis. Khodabakhshi et al. [18] proposed an input-oriented super efficiency measure in stochastic data envelopment analysis to evaluate chief executive officers of US public banks and thrifts. Azadi and Farzipoor Saen [3] proposed a chance-constrained DEA method to assist the decision makers to identify the most appropriate third-party reverse logistics providers in the presence of both dual-role factors and stochastic data. Hosseinzadeh Lotfi et al. [26] presented the stochastic centralized resource allocation method for allocating centralized resources in the presence of stochastic inputs and outputs.

Tavana et al. [23] proposed a new chance-constrained DEA model with bi-random input and output data for evaluating the efficiency of the DMUs in a stochastic environment that is more suitable and less restrictive in some real-life problems. There are various types of uncertainties in real-life problems. Random phenomenon is one class of uncertainty that has been studied exhaustively. Another class of uncertainty involves twofold uncertain variables that we use random data.

In this paper, we use a chance-constrained DEA model with random inputs and outputs for evaluating the efficiency of the DMUs in a stochastic environment. In addition, a super efficiency approach is developed in which the constraints are treated as random events. A deterministic equivalent model is formulated, which is non-linear. We convert the non-linear model into a model with quadratic constraints to solve the nonlinear deterministic model. We use a real example to show the efficacy of the new model proposed in this study.

The remainder of the paper is organized as follows. In the next section, Input-oriented models in classic DEA are introduced. In Section 3, the input-oriented models are developed in stochastic data envelopment analysis, and its deterministic equivalent is also obtained. Furthermore, it is shown that the deterministic equivalent of the stochastic model can be converted to a quadratic program. In Section 4, we evaluated maintenance groups of IRALCO With the use of stochastic models. Section 5 concludes the paper.

2 Preliminaries

We assume there are **n** homogeneous DMUs such that all the DMUs use **m** inputs x_{ij} $(i = 1, \ldots, m)$ to produce **s**outputs y_{rj} $(r = 1, \ldots, s)$. We also assume that these cases are non-negative deterministic elements [32]. One of the early models used to evaluate DMUs efficiency is the input-oriented CCR model introduced by Charnes, Cooper, and Rhodes [10]. This model which evaluates DMUo is as follows:

$$
\begin{aligned}
& min\ \theta \\
& s.t. \\
& \qquad \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io},\ i = 1, \ldots, m, \\
& \qquad \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \quad r = 1, \ldots, s, \\
& \qquad \lambda_j \geq 0, \qquad\qquad j = 1, \ldots, n.
\end{aligned}
\tag{2.1}
$$

we have the following definition:

Definition 2.1 *(Efficiency)DMU_o is efficient when in optimal solution(s):*

1. $\theta_o^* = 1$
2. All slack variables are zero in alternative optimal solution .

Excluding the column vector correspond to DMUo from the LP coefficients matrix of model (2.1), input-oriented super-efficiency model introduced by Andersen and Petersen [?] is defined as follows:

$$
\begin{aligned}
&\theta_o^{AP} = min\ \theta \\
&s.t. \\
&\qquad \sum_{j=1, j\neq o}^{n} \lambda_j x_{ij} \leq \theta x_{io},\ i = 1, \ldots, m, \\
&\qquad \sum_{j=1, j\neq o}^{n} \lambda_j y_{rj} \geq y_{ro}, \quad r = 1, \ldots, s, \\
&\qquad \lambda_j \geq 0, \qquad\qquad j = 1, \ldots, n,\ j \neq o.
\end{aligned}
\tag{2.2}
$$

Efficient DMUs have super-efficiency score greater than or equal to 1, while inefficient DMUs have super-efficiency score less than 1.

Note that following cases maybe occur in the calculation of CCR and AP models:

- $\theta^{CCR} = \theta^{AP} < 1$: This occurs when the unit under assessment is inefficient.
- $\theta^{CCR} = \theta^{AP} = 1$: This occurs when the unit under assessment is non-apical efficient.
- θ^{CCR} =1, $\theta^{AP} > 1$: This occurs when the unit under assessment is vertex efficient.
- θ^{CCR} =1, Model AP is infeasible : This occurs when AP model is Impossible.

we can solve the super-efficiency model for ranking efficient units without solving the CCR model. The super-efficiency scores of the DMUs obtained by the model (2.2) can then be ranked in a descending order.

Table 1
The random inputs with normally distributed.

DMU	maintenance groups	Input1	Input2	Input3	Input4
1	General Electric	N(10.58 , 7.90)	N(15.75 , 19.48)	N(11.25 , 9.84)	N(12.83 , 0.70)
2	Power anode production	N(16.5 , 13)	N(24.83 , 28.88)	N(13.75 , 8.93)	N(18.83 , 0.15)
3	instrumentation	N(2.33 , 042)	N(26.92 , 59.54)	N(29.5 , 71.18)	N(7.83 , 0.15)
4	Old Mechanical resuscitation	N(46.33 , 117.70)	N(14 , 10.55)	N(32.42 , 55.90)	N(39.33 , 5.33)
5	New Mechanical resuscitation	N(13.42 , 15.17)	N(2.58 , 0.45)	N(8.75 , 5.48)	N(41 , 0.36)
6	Mechanical morphology	N(54.67 , 108.42)	N(14.5 , 7.91)	N(6.92 , 1.90)	N(26.17 , 0.70)
7	Mechanical anode production	N(13.25 , 11.11)	N(17.75 , 18.39)	N(13.25 , 11.11)	N(20.58 , 0.27)
8	Mechanical anode baking	N(22.25 , 22.02)	N(20.92 , 20.08)	N(18.67 , 15.88)	N(23.25 , 0.57)
9	Turning	N(30.67 , 97.88)	N(21.08 , 45.36)	N(12.08 , 15.54)	N(11.67 , 0.24)
10	Welding	N(20.67 , 24.24)	N(14.67 , 12.79)	N(19.17 , 20.33)	N(15.67 , 0.24)
11	Auto shop	N(13.08 , 15.36)	N(10.75 , 11.11)	N(5.83 , 4.15)	N(14.58 , 0.27)
12	General Mechanic	N(35.17 , 23.61)	N(26.33 , 13.15)	N(26.33 , 13.15)	N(18.5 , 0.27)
13	Public services	N(24.92 , 50.08)	N(20 , 31.09)	N(9.17 , 7.06)	N(14.83 , 5.79)
14	Fire poured	N(1.58 , 0.45)	N(5.25 , 5.11)	N(0.83 , 0.15)	N(8.5 , 0.27)
15	Cooking Fire	N(9.17 , 31.61)	N(8.83 , 3.06)	N(1 , 0)	N(19.67 , 0.24)
16	Repair of buildings	N(18 , 15.27)	N(13.33 , 8.42)	N(7.75 , 2.75)	N(17.83 , 4.70)
17	Pipe installations	N(7.58 , 6.99)	N(19.92 , 44.27)	N(10.25 , 10.93)	N(10 , 1.82)
18	Compressor	N(6.08 , 2.63)	N(4.5 , 1.55)	N(6.08 , 2.63)	N(11.33 , 2.06)
19	Gas Facility	N(0.83 , 0.33)	N(3.33 , 4.24)	N(0.83 , 0.33)	N(7.67 , 0.24)
20	Old Power resuscitation	N(3.25 , 0.39)	N(3.58 , 0.81)	N(56.33 , 179.88)	N(17.33 , 0.24)
21	New Power resuscitation	N(2.25 , 0.20)	N(3.25 , 0.20)	N(15.67 , 8.97)	N(15.5 , 0.27)
22	Repair of boilers	N(9.83 , 7.97)	N(1.25 , 0.75)	N(2.08 , 1.36)	N(5.67 , 0.42)
23	Repairs supers	N(4.08 , 2.08)	N(1.5 , 0.27)	N(1.5 , 0.27)	N(8.92 , 0.45)
24	Power shed	N(5.5 , 2.64)	N(7.33 , 5.15)	N(23.75 , 54.02)	N(14.5 , 0.45)
25	Windings	N(12.67, 7.15)	N(37.92 , 66.08)	N(12.67 , 7.15)	N(12.83 , 0.88)
26	Mechanical nailing	N(4.33 , 2.24)	N(4.33 , 2.24)	N(10.75 , 17.48)	N(12.42 , 0.27)
27	Power nailing	N(5.67 , 10.97)	N(3.33 , 3.52)	N(4.83 , 7.79)	N(9.92 , 0.08)
28	Installations line 6	N(3.08 , 4.81)	N(3.08 , 4.81)	N(4.08 , 7.90)	N(3.83 , 0.15)

Table 2
The random outputs with normally distributed.

Row	DMU	Output1	Output2	Output3	Output4
1	General Electric	N(14.25 , 19.84)	N(26.25 , 69.30)	N(16.58 , 26.27)	N(1.52 , 0.11)
2	Power anode production	N(18.83 , 26.52)	N(25.08 , 46.45)	N(18.83 , 26.52)	N(1.14 , 0.03)
3	instrumentation	N(3.92 , 0.99)	N(31.5 , 56.82)	N(30.33 , 52.61)	N(1.15 , 0.02)
4	Old Mechanical resuscitation	N(55.83 , 203.42)	N(16.67 , 18.24)	N(39 , 100.18)	N(1.22 , 0.05)
5	New Mechanical resuscitation	N(14.92 , 10.08)	N(2.67 , 0.42)	N(9.42 , 3.90)	N(1.14 , 0.05)
6	Mechanical morphology	N(73.58 , 363.17)	N(17.33 , 20.79)	N(11.25 , 9.30)	N(1.33 , 0.04)
7	Mechanical anode production	N(17.08 , 14.45)	N(19.58 , 19.72)	N(12.33 , 7.88)	N(1.1 , 0.01)
8	Mechanical anode baking	N(31.08 , 69.90)	N(33 , 78.73)	N(33 , 78.73)	N(1.57 , 0.09)
9	Turning	N(44.92 , 222.27)	N(31.75 , 113.84)	N(16.92 , 30.63)	N(1.52 , 0.23)
10	Welding	N(26.92 , 20.99)	N(17.83 , 9.06)	N(26 , 20.91)	N(1.34 , 0.07)
11	Auto shop	N(12.92 , 14.81)	N(10.5 , 10.09)	N(6.58 , 3.90)	N(1.05 , 0.10)
12	General Mechanic	N(45.25 , 24.75)	N(34 , 14.55)	N(34 , 14.55)	N(1.29 , 0.03)
13	Public services	N(24.58 , 72.99)	N(20.08 , 49.17)	N(11.33 , 15.70)	N(1.08 , 0.15)
14	Fire poured	N(1.58 , 0.27)	N(5.42 , 2.81)	N(0.83 , 0.15)	N(1.23 , 0.57)
15	Cooking Fire	N(9.75 , 2.02)	N(9.17 , 1.61)	N(1 , 0)	N(1.08 , 0.09)
16	Repair of buildings	N(20.42 , 56.81)	N(15.75 , 34.20)	N(9 , 10.91)	N(1.16 , 0.13)
17	Pipe installations	N(7.75 , 5.84)	N(22.58 , 49.72)	N(10.17 , 10.33)	N(1.09 , 0.03)
18	Compressor	N(9.92 , 13.54)	N(7.08 , 6.63)	N(10.33 , 15.88)	N(1.65 , 0.41)
19	Gas Facility	N(0.67 , 0.42)	N(3.08 , 4.81)	N(0.67 , 0.42)	N(1.31 , 2.25)
20	Old Power resuscitation	N(3.08 , 0.45)	N(5.17 , 1.42)	N(64.92 , 186.99)	N(1.17 , 0.03)
21	New Power resuscitation	N(3.33 , 0.79)	N(2.42 , 0.27)	N(17.08 , 15.17)	N(1.11 , 0.06)
22	Repair of boilers	N(10.5 , 7.55)	N(1.33 , 0.79)	N(3.33 , 4.79)	N(1.90 , 1.78)
23	Repairs supers	N(4.58 , 16.99)	N(1.67 , 2.61)	N(1.33 , 1.52)	N(1.13 , 1.25)
24	Power shed	N(7.42 , 6.27)	N(7.42 , 6.27)	N(33.5 , 129.18)	N(1.32 , 0.09)
25	Windings	N(16.33 , 15.70)	N(47.17 , 131.61)	N(22.33 , 28.61)	N(1.37 , 0.07)
26	Mechanical nailing	N(4.33 , 3.70)	N(4.33 , 3.70)	N(12.08 , 28.45)	N(1.08 , 0.03)
27	Power nailing	N(6.25 , 7.11)	N(3.83 , 3.24)	N(5.17 , 5.24)	N(1.17 , 0.06)
28	Installations line 6	N(3.67 , 1.33)	N(3 , 0.73)	N(5.42 , 2.45)	N(2.31 , 11.78)

3 Stochastic models

3.0.1 Deterministic equivalent

Now in this part we show how to obtain the θ_o^* from deterministic equivalent of the stochastic model (**??**). From the first constraint in model (**??**) we have :

$$P(\sum_{j=1}^{n}\lambda_j\tilde{x}_{ij} \leq \ \theta_o\tilde{x}_{io}) \ \geq 1-\alpha, \qquad i=1,\ldots,m, \tag{3.1}$$

then:

$$P(-\sum_{j=1}^{n}\lambda_j\tilde{x}_{ij}+\theta_o\tilde{x}_{io} \ \leq 0) \leq \alpha, \qquad i=1,\ldots,m, \tag{3.2}$$

above equation is equivalent:

$$P(\frac{(-\sum_{j=1}^{n}\lambda_j\tilde{x}_{ij}+\theta_o\tilde{x}_{io}+(\sum_{j=1}^{n}\lambda_j x_{ij}-\theta_o x_{io}))}{\omega_i} \leq \frac{\sum_{j=1}^{n}\lambda_j x_{ij}-\theta_o x_{io}}{\omega_i}) \leq \ \alpha \qquad i=1,\ldots,m, \tag{3.3}$$

where

$$\begin{aligned}(\omega_i)^2 = &\sum_{j\neq o}\sum_{k\neq o}\lambda_j\lambda_k cov(\tilde{x}_{ij},\tilde{x}_{ik})\\ &+2(\lambda_o-\theta_o)\sum_{j\neq o}\lambda_j cov(\tilde{x}_{ij},\tilde{x}_{io})\\ &+(\lambda_o-\theta_o)^2 var(\tilde{x}_{io}),\\ &i=1,\ldots,m,\end{aligned} \tag{3.4}$$

to obtain the deterministic equivalent of (3.3)and(3.4) we write :

$$Z_i = \frac{-\sum_{j=1}^{n}\lambda_j\tilde{x}_{ij}+\theta_o\tilde{x}_{io}+(\sum_{j=1}^{n}\lambda_j x_{ij}-\theta_o x_{io})}{\omega_i} \quad ,i=1,\ldots,m, \tag{3.5}$$

If we assume the input and output to be normally distributed ,then Z_i is also normally distributed,with mean zero and unit variance,since Z_i is normally distributed. The deterministic equivalent of (3.3) is as:

$$\frac{\sum_{j=1}^{n}\lambda_j x_{ij}-\theta_o x_{io}}{\omega_i} \leq \ \Phi^{-1}(\alpha) \quad ,i=1,\ldots,m, \tag{3.6}$$

Where(3.6), represents the normal cumulative distribution function and Φ^{-1} is its inverse. since, by the similar manner like (3.6) we can obtain the deterministic equivalent for other constraint of model (3.4).
Hence,by using the aforesaid relations, the deterministic equivalent of (3.4) can be represented as:

$$\begin{aligned}
&min\ \theta_o\\
&s.t.\\
&\quad \sum_{j=1}^{n}\lambda_j x_{ij}-\Phi^{-1}(\alpha)\omega_i+s_i^- =\theta_o x_{io} \qquad ,i=1,\ldots,m,\\
&\quad -\sum_{j=1}^{n}\lambda_j y_{rj}+y_{ro}-\Phi^{-1}(\alpha)\omega_r+s_r^+=0\ ,r=1,\ldots,s,\\
&\quad s_i^-,s_r^+,\lambda_j \geq \ 0
\end{aligned} \tag{3.7}$$

where

$$\begin{aligned}
(\omega_i)^2 \ &=\sum_{j\neq o}\sum_{k\neq o}\lambda_j\lambda_k cov(\tilde{x}_{ij},\tilde{x}_{ik})\\
&\quad +2(\lambda_0-\theta_0)\sum_{j\neq o}\lambda_j cov(\tilde{x}_{ij},\tilde{x}_{io})+(\lambda_0-\theta_0)^2 var(\tilde{x}_{io})\\
(\omega_r)^2 \ &=\sum_{j\neq o}\sum_{k\neq o}\lambda_j\lambda_k cov(\tilde{y}_{rj},\tilde{y}_{rk})\\
&\quad +2(\lambda_o-1)\sum_{j\neq o}\lambda_j cov(\tilde{y}_{rj},\tilde{y}_{ro})+(\lambda_o-1)^2 var(\tilde{y}_{ro})
\end{aligned} \tag{3.8}$$

By using model(3.3), the proposed stochastic super-efficiency model can be obtained as:

$$
\begin{aligned}
& min\ \theta_o \\
& s.t. \\
& \quad P(\sum_{j=1, j\neq o}^{n} \lambda_j \tilde{x}_{ij} \leq \ \theta_o \tilde{x}_{io}) \geq 1-\alpha, i=1,\ldots,m, \\
& \quad P(\sum_{j=1, j\neq o}^{n} \lambda_j \tilde{y}_{rj} \geq \ \tilde{y}_{ro}) \geq 1-\alpha, r=1,\ldots,s, \\
& \quad \lambda \geq 0
\end{aligned}
\tag{3.9}
$$

where α is a predetermined number between 0 and 1 which specifies the significance level and P means "*probability*", DMU_o is stochastic super-efficiency at significance level α if the optimal value of the objective function is greater than 1.

3.1.1 Deterministic equivalent

By the similar manner we can obtain the deterministic equivalent of stochastic super-efficiency as follows:

$$
\begin{aligned}
& min\ \theta_o \\
& s.t. \\
& \quad \sum_{j=1, j\neq o}^{n} \lambda_j x_{ij} - \Phi^{-1}(\alpha)\omega_i + s_i^- = \theta_o x_{io}, i=1,\ldots,m, \\
& \quad -\sum_{j=1, j\neq o}^{n} \lambda_j y_{rj} + y_{ro} - \Phi^{-1}(\alpha)\omega_r + s_r^+ = 0, r=1,\ldots,s, \\
& \quad s_i^-, s_r^+, \lambda_j \geq \ 0
\end{aligned}
\tag{3.10}
$$

where

$$
\begin{aligned}
(\omega_i)^2 &= \sum_{j\neq O}\sum_{k\neq O}\lambda_j\lambda_k cov(\tilde{x}_{ij},\tilde{x}_{ik}) \\
&\quad -2\theta_0\sum_{j\neq O}\lambda_j cov(\tilde{x}_{ij},\tilde{x}_{io})+\theta_0^2 var(\tilde{x}_{io}) \\
(\omega_r)^2 &= \sum_{j\neq O}\sum_{k\neq O}\lambda_j\lambda_k cov(\tilde{y}_{rj},\tilde{y}_{rk}) \\
&\quad -2\sum_{j\neq O}\lambda_j cov(\tilde{y}_{rj},\tilde{y}_{ro})+var(\tilde{y}_{ro})
\end{aligned}
\tag{3.11}
$$

4 Application

As an empirical research, use some actual data of Iranian Aluminium Company (IRALCO) maintenance groups in the year 2015. The first Producer of Aluminium in Iran that established in 1972, is Located at 5^{th} km of Arak – Tehran Rd. The area of the plant is 232 hectares and annual production capacity is 120000 TPY consisting of different pure ingots in the shapes of T-bar, 1000Lb, Casting alloys, Billets with different size, Slab and E.C ingots.

The company products in various lines of production is done 24 hours a day, so if a simple failure occur in machines imposed huge losses to the company. So, maintenance groups of IRALCO play an important role [28,27]. The IRALCO have 30 maintenance groups. But, we only examine groups that are active and work According to plan. Thus reducing the number of maintenance groups to 28. The groups are divided into two categories: general and workshops. According to Survey, the most frequently used inputs are operating costs, number of employees, kinds of orders. The most widely useful outputs are also proportion of completion and kind of finished order. The cost data usually is not available. In this study, we select eight variables: four inputs and four outputs. The first input is order type 1 (Time required to perform is more than 50 hours). The second input is order type 2 (Time required to do is 20 to 50 hours), the third is order type 3 (Time required to perform is less than 20 hours) and the last input is number of employees. The outputs include: finished order type 1, finished order type 2, finished order type 3 and proportion of completion (the finished orders to the requested orders).

Table 3
The stochastic efficiency and super-efficiency scores for different α values.

DMU	α=0.5			α=0.6			α=0.7		
	SES	SSES	R	SES	SSES	R	SES	SSES	R
1	1	1.199	11	1	1.034	11	0.88	0.88	9
2	0.796	0.796	23	0.704	0.704	21	0.618	0.618	20
3	1	3.828	2	1	3.36	2	1	2.922	2
4	0.921	0.921	19	0.818	0.818	18	0.719	0.719	17
5	0.739	0.739	26	0.651	0.651	24	0.57	0.57	23
6	1	2.647	5	1	2.356	4	1	2.074	4
7	0.836	0.836	21	0.744	0.744	19	0.657	0.657	18
8	1	1.122	12	0.999	0.999	12	0.88	0.88	9
9	1	1.614	9	1	1.456	8	1	1.291	7
10	0.942	0.942	18	0.86	0.86	17	0.781	0.781	16
11	0.718	0.718	28	0.612	0.612	27	0.515	0.515	26
12	0.956	0.956	16	0.908	0.908	16	0.859	0.859	11
13	0.761	0.761	25	0.648	0.648	25	0.543	0.543	24
14	1	1.512	10	1	1.249	9	1	1.023	8
15	1	1.845	7	1	1.779	6	1	1.709	5
16	0.802	0.802	22	0.689	0.689	22	0.581	0.581	22
17	0.858	0.858	20	0.734	0.734	20	0.62	0.62	19
18	1	1.122	12	0.952	0.952	13	0.793	0.793	14
19	1	2.027	6	1	1.225	10	0.594	0.594	21
20	1	3.426	3	1	3.087	3	1	2.758	3
21	1	1.058	15	0.942	0.942	15	0.83	0.83	12
22	1	13.251	1	1	6.581	1	1	3.328	1
23	0.949	0.949	17	0.658	0.658	23	0.395	0.395	28
24	1	1.122	12	0.952	0.952	13	0.795	0.795	13
25	1	1.63	8	1	1.465	7	1	1.313	6
26	0.786	0.786	24	0.644	0.644	26	0.517	0.517	25
27	0.725	0.725	27	0.588	0.588	28	0.47	0.47	27
28	1	3.332	4	1	2.039	5	0.783	0.783	15

We evaluate the 28 DMUs with random inputs and random outputs that are normally distributed as shown in Table 1 and Table 2, respectively. We use the average and variance of observed data in Table 1, 2 during 2015. The means and variances of all random input and output data are known. For instance, the first input for DMU3 has a normal distribution, its mean and variance is equal to 2.33, 0.42, respectively. Note that the outputs and inputs of different DMUs are independent. This independence assumption implies that all covariances become zero.

The stochastic efficiency and super-efficiency scores are obtained from the implementation of Models (**??**) and (3.4) for various amounts of α (i.e. α=0.5, α=0.6, α=0.7). The computational results which are obtained by GAMS software are presented in Table 3. Columns 1, 4, 7, Columns 2, 5, 8 and 3, 6, 9 of Table 3 present the stochastic efficiency scores, the super efficiency scores and the ranking of the 28 DMUs, respectively. Obviously, the stochastic efficiency scores in Model 3.4 are greater than 0 and less than or equal to 1. A super-efficiency score greater than 1 implies that the DMU is super-efficient, scores equal to 1 imply they are just efficient; scores less than 1 which are equal to their correspond efficiency scores imply that they are inefficient.

For example, as shown in Table 3, when α=0.5, number of efficient DMUs equal to 15, that some of them include: General Electric unit, Lathe, Installation, repair of pots and etc. The stochastic super-efficiency scores of these efficient DMUs are 1.199, 1.614, 1.122 and 13.251, respectively. Auto shop, DMU11, has score 0.718 which means the unit is inefficient, while repair of pots has super-efficiency score 13.251 which ranks it as the number one unit in terms of super-efficiency. The worst DMUs are Autoshop, DMU11, and power nailing, D27, with scores of 0.718 and 0.725, respectively when α=0.6, number of efficient DMUs equal to 11 and for α=0.7, number of efficient DMUs equal to 8, so, when α value increase, number of efficient units decrease and super-efficiency score don't increase.

It is mentioned, for different α level, the repair of pots unit is number one. This example provides the different ranking for the 28 DMUs for the α values reported in the columns of Table 3. In addition, as shown in Table 3, the stochastic efficiency scores and the stochastic super-efficiency scores are identical when the efficiency scores are less than 1. In Table 3, SES, SSES and R are stand for Stochastic efficiency score, Stochastic super-efficiency score and rank.

5 Conclusions and future research directions

In this study, we used the input-oriented efficiency and super – efficiency measures in stochastic data envelopment analysis. For the convenience of calculations, this paper calculated the deterministic equivalent for the stochastic version. This deterministic equivalent can be converted to a quadratic problem. We evaluated maintenance groups of IRALCO efficiency in the year 2015, by using these models.

Application of different α levels leads to various results. From the results it is clear that, when α value increase, the number of efficient units decreases and super-efficiency score don't increase. In this evaluation, repair of pots maintenance group was the best unit in different α level. This group compared to other units has the lowest manpower. In addition to, number of finished order type 1 is ok. Therefore, other units can choose the group as a template.

IRALCO planning unit can use these models to assess the maintenance groups, too. For further research, we plan to extend the proposed approach to other types of DEA models. Researchers can use this method to evaluate the performance of other factories maintenance groups. We suggest using data envelopment analysis with interval data to evaluate the performance of group maintenance or other decision-making units, too. Finally, developing the proposed super-efficiency measure in fuzzy DEA can be suggested for further research.

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