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Some notes on the existence of an inequality in Banach algebra

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Abstract

We shall prove an existence inequality for two maps on Banach algebra, with an example and in sequel we have some results on $\mathbb{R}$ and $\mathbb{R}^n$ spaces. This way can be applied for generalization of some subjects of mathematics in teaching which how we can extend a math problem to higher level.

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1 Introduction

We generalize a problem which was a simple existence inequality in [1, page 229] to Banach algebra ([2]) and in continue we obtain some results on $\mathbb{R}$ and $\mathbb{R}^n$ spaces.

2 Main results

Theorem 2.1 *Let E be a nonempty Banach algebra over $\mathbb{C}$, f and g two self-maps on E. If there exists $a \in E$ such that $\|a^2\| \geq 1$, then*

$$\|xy - f(x) - g(y)\| \geq \frac{1}{4},$$

for some $x, y \in E$.

Proof. Let $\|xy - f(x) - g(y)\| < \frac{1}{4}$ for all $x, y \in E$. So we have $\|f(0) + g(0)\| < \frac{1}{4}$, $\|f(a) + g(0)\| < \frac{1}{4}$, $\|f(0) + g(a)\| < \frac{1}{4}$. But

$$\begin{aligned}\|a^2 - f(a) - g(a)\| &\geq \|a^2\| - \|f(a) + g(a)\| \\ &> 1 - \|f(a) + g(0)\| - \|g(0) + f(0)\| \\ &- \|f(0) + g(a)\| \\ &> \frac{1}{4}.\end{aligned}$$

□

Corollary 1 *If E be a nonempty unital Banach algebra over $\mathbb{C}$, f and g two self-maps on E, then*

$$\|xy - f(x) - g(y)\| \geq \frac{1}{4},$$

for some $x, y \in E$.

Proof. If $\mathbf{1}$ denotes the unit in the Banach algebra E, then $\|\mathbf{1}\| = 1$. □

Example 2.2 *([1]) Let f and g be real valued functions of a real variable. Therefore there exist $x, y \in [0,1]$ such that $|xy - f(x) - g(y)| \geq \frac{1}{4}$.*

In sequel, fixed $n \in \mathbb{N}$.

Corollary 2 *If $f_1, \cdots, f_n : \mathbb{R} \to \mathbb{R}$ be maps, then*

$$\forall a_i \geq 0 \quad \exists x_i \in [0, a_i] \quad \left| x_1 \cdots x_n - \sum_{i=1}^{n} f_i(x_i) \right| \geq \frac{a_1 \cdots a_n}{2^n},$$

for all $i = 1, 2, \cdots, n$.

Lemma 2.3 *If $f, g : \mathbb{R}^n \to \mathbb{R}$, $I = [0, a_1] \times \cdots \times [0, a_n]$, $J = [0, b_1] \times \cdots \times [0, b_n]$, $a = (a_1, \cdots, a_n)$ and $b = (b_1, \cdots, b_n)$, then*

$$\exists x \in I, y \in J \quad |x.y - f(x) - g(y)| \geq \frac{a.b}{4},$$

where $x.y$ is inner product x and y.

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