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An Extension of Mixed Monotone Mapping to Tripled Fixed Point Theorem in Fuzzy Metric Spaces

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ABSTRACT

In this paper, we prove the concept of fuzzy metric spaces of tripled fixed point via mixed monotone mappings and prove the existence and uniqueness theorem for contractive type mapping. In order to do that, we consider a modification to results on tripled fixed point theorem in fuzzy metric spaces available in literature. Additionally, we prove some tripled fixed point theorems for metric spaces via mixed monotone mappings. These results extend and generalize some recent results in literature.

1 Introduction

The idea of coupled fixed point via mixed monotone operators of the form $T: X^2 \to X$, where X is a partially ordered metric space, was initiated by Bhaskar and Lakshmikantham in [1], thereby establishing some interesting coupled fixed point theorems. Also, in their work, they exemplified the significance of these results by showing the existence and uniqueness of the solution for a parabolic boundary value problem.

Fixed point theorems have been researched in several contexts, one of which is in the fuzzy settings. The concept of fuzzy set in metric space was firstly introduced by Zadeh [2] in 1965. in order to utilize this concept in mathematical analysis, many renowned researchers have extensively broadened the scope of the theory of fuzzy set and its applications. One of the most remarkable work in fuzzy topology is to determine an appropriate definition of fuzzy metric space

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for its possible applications in numerous fields of sciences. It has been well established that fuzzy metric space is a generalization of the metric space, hence many researchers have examined this scope of study and have explored it in many ways. For instance, George and Veeramani [3] modified the concept of a fuzzy metric space that was introduced by Kramosil and Michalek [4] and defined the Hausdorff topology of a fuzzy metric space. Hence, there exist a considerable number of literatures about fixed point properties defined on a complete metric and fuzzy metric spaces, which have earlier been studied by some authors (see [5-12]).

Also continuing in the vein of the generalization of metric spaces, the concept of tripled fixed point theorem in fuzzy metric spaces has been introduced by Roldan et al. [13], and in their submission, the existence and uniqueness theorem for contractive type mappings in fuzzy metric spaces was studied. In doing so, they consider a modification to the concept of fixed point theorems that were introduced by Berinde and Borcut [7], and generalize the work into fuzzy metric space.

In this paper, our aim is to obtain the existence and uniqueness theorems for contractive type mixed monotone mapping via a fuzzy metric space, which will consolidate and generalize some existing results already available in literature.

2 Preliminaries

As a way of simplification, let X denote a non-empty set and $\mathrm{X}^3 = \mathrm{X} \star \mathrm{X} \star \mathrm{X}$. subscripts will be utilized to signify the argument of a function. That is $F(\eta, \lambda, \mu)$ will be represented by $F_{\eta\,\lambda\,\mu}$, $M\ (\eta, \lambda, \tau)$ will be represented by $M_{\kappa\,\lambda}(t)$ and Fuzzy metric space by FMS. Furthermore, $g(\eta)$ will also be represented by g_η.

Definition 2. 1 [14] A metric on X is a mapping $\delta: \mathrm{X} \star \mathrm{X} \to \mathbb{R}$, for all $\eta, \lambda, \mu \in \mathrm{X}$, satisfying;

(i) $\delta_{\eta\,\lambda} \geq 0$;

(ii) $\delta_{\eta\,\lambda} = \delta_{\lambda\,\eta}$;

(iii) $\delta_{\eta\,\lambda} \leq \delta_{\eta\,\mu} + \delta_{\mu\,\lambda}$

Hence, if δ is metric on X, then (X, δ) is a metric space (concisely as written as MS).

Definition 2.2 [14] Let (X, δ) be a MS. A mapping $f: \mathrm{X} \rightarrow \mathrm{X}$ is Lipschitzian if these exists $\kappa \geq 0$ such that $\delta(f_\eta, f_\lambda) \leq \kappa 8_{\eta\, \lambda}$ for all$\eta, \lambda \in \mathrm{X}$. The least κ (written as κ_f) is said to be the lipschitz constant. The Lipschitz map is a contraction if $\kappa_f < 1$.

Definition 2.3 [14] A triangular norm (denoted as τ −norm) is a map $*: [0,1]^2 \rightarrow [0,1]$ that satisfies the properties of associativity, commutativity, which in both arguments if is nondecreasing and has its identity to be 1. For each $\alpha \in [0,1]$, the sequence $\{*^n\, \alpha\}_{n=1}^{\infty}$is inductively defined by $*^1\, \alpha$ and $*^n\, \alpha = (*^{n-1}\, \alpha) * \alpha$. A τ −norm * is of $\mathbb{H}$ −type (see [15]) if $\{*^n\, \alpha\}_{n=1}^{\infty}$ is equicontinuous at $\alpha = 1$, i. e., for al $\varepsilon \in (0,1)$, there exists $\theta \in (0,1)$ such that if $\alpha \in (1-\theta, 1]$, then $*^m\, \alpha > 1 - \varepsilon$ for all $m \in \mathbb{N}$.

The most significant and notable continuous τ −norm of $\mathbb{H}$ −type is $*= min$, that verifies $\min(\alpha, \beta) \geq \alpha\beta$ for $\alpha, \beta \in [0,1]$. The next result presents a broad range of τ −norms of $\mathbb{H}$ −type.

Lemma 2.1 [11] Let $\gamma \in (0,1]$ be real and let * be a τ −norm. Define $*_r$ as $\eta *_r = \eta * \lambda$, if $\max(\eta, \lambda) \leq 1 - \gamma$, and $\eta *_\gamma \lambda = \min(\eta, \lambda)$, if $\max(\eta, \lambda) > 1 - \gamma$, then $*_\gamma$ is a τ −norm of $\mathbb{H}$ −type.

Definition 2.4 [11] A triple $(\mathrm{X}, M, *)$ is called FMS if X is non-empty, * a continuous τ −norm and $M: \mathrm{X} \star \mathrm{X} \star [0, \infty) \rightarrow [0,1]$ is a Fuzzy set fulfilling the conditions, for each $\eta, \lambda, \mu \in \mathrm{X}$, and $\tau, \mu > 0$;

(i) $M_{\eta\, \lambda}(0) = 0$;

(ii) $M_{\eta\, \lambda}(\tau) = 1$ if and only if $\eta = \lambda$;

(iii) $M_{\eta\, \lambda}(\tau) = M_{\lambda\, \eta}(\tau)$;

(iv) $M_{\eta\, \lambda}(\cdot): [0, \infty) \rightarrow [0,1]$ is left continuous;

(v) $M_{\eta\, \lambda}(\tau) * M_{\lambda\, \eta}(\mu) \leq M_{\eta\, \mu}(\tau + \mu)$;

(vi) $\lim_{\tau \to \infty} M_{\eta\, \lambda}(t) = 1$ for all $\eta, \lambda \in \mathrm{X}$.

Hence, (X, M) is a FMS under $*$.

Lemma 2.2 [11] $M_{\eta\, \lambda}(*)$ is a non-decreasing function on $[0, \infty)$.

Definition 2.5 [14] Let (X, M) be a FMS under $*$ some contain τ −norm. A sequence $\{\eta_n\} \subset \eta$ is Cauchy if, for $\varepsilon > 0$ and $\tau > 0$, there exists $n_0 \in \mathbb{N}$ such that $M_{\eta_n \eta_m}(\tau) > 1 - \varepsilon$ for $n, m\ \geq n_0$. Then,

$\{\eta_n\} \subset \mathrm{X}$ is convergent to $\eta \in \mathrm{X}$, denoted by $\lim_{n\to\infty} \eta_n = \eta$ if, for $\varepsilon > 0$ and $\tau > 0$, there exists $n_0 \in \mathbb{N}$ such that $M_{\eta_n}(\tau) > 1 - \varepsilon$ for $n \geq n_0$. A FMS where every Cauchy sequence is convergent is a complete FMS.

Definition 2.6 [14] A function $g: \mathrm{X} \to \mathrm{X}$ on a FMS is continuous at a point $\eta_0 \in \mathrm{X}$ if for any $\{\eta_n\} \in \mathrm{X} \to \eta_0$, then $\{g\eta_n\} \to g\eta_0$. If g is continuous at every $\eta \in \mathrm{X}$, then g is continuous on X. Also, if $\eta_0 \in \mathrm{X}$, we will signify $g^{-1}(\eta_0) = \{\eta \in \mathrm{X}: g\eta = \eta_0\}$.

Remark 2.1 If $\eta \in [0,1]$ and $\alpha, \beta \in (0, \infty)$, then$\alpha \leq \beta$ implies that $\eta^\alpha \geq \eta^\beta$. Hence, this establishment will be applied as; $0 < \alpha \leq \beta \leq 1$ which implies that$M_{\eta\,\lambda}(\tau)^\alpha \geq M_{\eta\,\lambda}(\tau)^\beta \geq M_{\eta\,\lambda}(\tau)$.

3 Main results

Definition 3.1 Let $F: \mathrm{X}^3 \to \mathrm{X}$ and $g: \mathrm{X} \to \mathrm{X}$ be mappings.

(i) F and g are commuting if $gF_{\eta\,\lambda\,\mu} = F_{g_\eta g_\lambda g_\mu}$ for all $\eta, \lambda, \mu \in \mathrm{X}$,

(ii) Point $(\eta, \lambda, \mu) \in \mathrm{X}^3$ is a tripled coincidence point of the mappings F and g if $F_{\eta\,\lambda\,\mu} = g_\eta$, $F_{\lambda\,\eta\,\lambda} = g\lambda$ and $F_{\mu\,\lambda\,\eta} = g\mu$.

Theorem 3.1 Let $*$ be a τ −norm of $\mathbb{H}$ −type such that $\nu \star \tau \geq \nu\tau$ for all$\tau, \nu \in [0, 1]$. Let $\kappa \in (0, 1]$ and $\alpha, \beta, \gamma \in [0, 1]$ be real and $\alpha + \beta + \gamma \leq 1$, let $(\mathrm{X}, M, *)$ be a complete FMS and $F: \mathrm{X}^3 \to \mathrm{X}$ and $g: \mathrm{X} \to \mathrm{X}$ be mappings that$f(\mathrm{X}^3) \subseteq g(\mathrm{X})$ and g is continuous and commuting with F. If for all $\eta, \lambda, \mu, \phi, \psi \in \mathrm{X}$ and all $\tau > 0$.

$$M_{F_{\eta\,\lambda\,\mu} F_{\phi\,\psi\,\omega}}(\kappa\tau) \geq M_{g\eta g\phi}(\tau)^\alpha \star M_{g\lambda w\psi}(\tau)^\beta \star M_{g\mu g\omega}(\tau)^\gamma \tag{1}$$

Then, $\eta \in \mathrm{X}$ is unique such that $\eta = g\eta = F_{\eta\,\lambda\,\mu}$. Hence, F and g have at least a tripled coincidence point. Also, (η, λ, μ) is a unique tripled coincidence point of F and g if $g^{-1}(\eta_0) = \{\eta_0\}$ only if $F = \eta_0$ is constant on X^3.

Hence, we assume that $M_{g\eta g\phi}(\tau)^0 = 1$ for $\tau > 0$ and $\eta, \lambda \in \mathrm{X}$.

Proof. Suppose that F is constant in X^3, there exists $\eta_0 \in X$ such that $F_{\eta\,\lambda\,\mu} = \eta_0$ for $\eta, \lambda, \mu \in X$. If F and g are commuting maps, we obtain that $g\eta_0 = gF_{\eta\,\lambda\,\mu} = F_{g\eta\,g\lambda\,g\mu} = \eta_0$. Therefore, $\eta_0 = g\eta_0 = F_{\eta_0\eta_0\eta_0}$ and $(\eta_0\eta_0\eta_0)$ is in tripled coincidence point of F and g. At this point, suppose $g^{-1}(\eta_0) = \{\eta_0\}$ and $(\eta, \lambda, \mu) \in X^3$ is also a tripled coincidence point of F and g then, $g\eta = F_{\eta\,\lambda\,\mu} = \eta_0$, so that $\eta \in g^{-1}(\eta_0) = \{\eta_0\}$. Again, $\eta = \lambda = \mu = \eta_0$ and (η_0, η_0, η_0) is a coincidence point of maps F and g that is unique.

Now, considering that F is not a constant in X^3. In this circumstance, $(\alpha, \beta, \gamma) \neq (0, 0,0)$ and which leads to the proof subdivision into five steps. Through this proof, q and r denote a non-negative integer and $\tau \in (0, \infty)$.

Step 1

Let $\eta_0, \lambda_0, \mu_0 \in X$ be arbitrary points of X. Some $F(X^3) \subseteq g(X)$, then $\eta_1, \lambda_1, \mu_1 \in X$ such that $g\eta_1 = F_{\eta_0,\lambda_0,\mu_0}$, $g\lambda_1 F_{\lambda_0,\mu_0,\lambda_0}$and $g\mu_1 F_{\mu_0,\lambda_0,\mu_0}$ which is as a result of its mixed monotone property. Furthermore, we can construct $\{\eta_n\}, \{\lambda_n\}$ and $\{\mu_n\}$ such that, for $n \geq 0, g\eta_{n+1} = F_{\eta_n,\lambda_n,\mu_n}, g\lambda_{n+1} = F_{\lambda_n,\eta_n,\mu_n}$ and $g\mu_{n+1} = F_{\mu_n,\lambda_n,\eta_n}$.

Step 2

If $\{g\eta_n\}, \{g\lambda_n\}$ and $\{g\mu_n\}$ are Cauchy sequences. For $n \geq 0$ and all $\tau > 0$, $\sigma_n(\tau) = M_{g\eta_n,g\eta_{n+1}}(\tau) * M_{g\lambda_n,g\lambda_{n+1}}(\tau) * M_{g\mu_n,g\mu_{n+1}}(\tau)$. Since σ_n is a non-decreasing function and $\tau = \kappa\tau \leq \tau \leq {}^{\tau}/_{\kappa}$, we have that;

$$\sigma_n(\tau - \kappa\tau) \leq \sigma_n(\tau) \leq \sigma_n({}^{\tau}/_{\kappa}) \text{ for } \tau > 0 \text{ and } n \geq 0 \tag{2}$$

Now, we can deduce from inequality (1) that for all $n \in \mathbb{N}$ and $\tau \geq 0$, and also because of the mixed monotone property;

$$\begin{aligned} M_{g\eta_n g\eta_{n+1}}(\tau) &= M_{F_{\eta_{n-1}\mu_{n-1}\lambda_{n-1}}F_{\lambda_n\mu_n\lambda_n}}(\tau) \\ &\geq M_{g\eta_{n-1}g\eta_n}({}^{\tau}/_{\kappa})^{\alpha} * M_{g\lambda_{n-1}g\lambda_n}({}^{\tau}/_{\kappa})^{\beta} * M_{g\mu_{n-1}g\mu_n}({}^{\tau}/_{\kappa})^{\gamma}\,; \end{aligned} \tag{3}$$

$$\begin{aligned} M_{g\lambda_n g\lambda_{n+1}}(\tau) &= M_{F_{\lambda_{n-1}\lambda_{n-1}\mu_{n-1}}F_{\eta_n\lambda_n\mu_n}}(\tau) \\ &\leq M_{g\lambda_{n-1}g\lambda_n}({}^{\tau}/_{\kappa})^{\alpha} * M_{g\mu_{n-1}g\mu_n}({}^{\tau}/_{\kappa})^{\beta} * M_{g\lambda_{n-1}g\lambda_n}({}^{\tau}/_{\kappa})^{\gamma}\,; \end{aligned} \tag{4}$$

and

$$M_{g\mu_n g\mu_{n+1}}(\tau) = M_{F_{\mu_{n-1}\lambda_{n-1}\mu_{n-1}}F_{\mu_n\lambda_n\mu_n}}(\tau)$$
$$\geq M_{g\mu_{n-1}g\mu_n}(\tau/\kappa)^{\alpha} * M_{g\lambda_{n-1}g\lambda_n}(\tau/\kappa)^{\beta} * M_{g\eta_{n-1}g\eta_n}(\tau/\kappa)^{\gamma} \quad (5)$$

Going by (3), (4), (5)and Remark 9, we obtain;

$$M_{g\eta_n g\eta_{n+1}}(\tau) \geq M_{g\eta_{n-1}g\eta_n}(\tau/\kappa)^{\alpha} * M_{g\lambda_{n-1}g\lambda_n}(\tau/\kappa)^{\beta} * M_{g\mu_{n-1}g\mu_n}(\tau/\kappa)^{\gamma}$$
$$\geq M_{g\eta_{n-1}g\eta_n}(\tau/\kappa) * M_{g\lambda_{n-1}g\lambda_n}(\tau/\kappa) * M_{g\mu_{n-1}g\mu_n}(\tau/\kappa) = \sigma_{n-1}(\tau/\kappa)$$

$$M_{g\lambda_n g\lambda_{n+1}}(\tau) \leq M_{g\lambda_{n-1}g\lambda_n}(\tau/\kappa)^{\alpha} * M_{g\mu_{n-1}g\mu_n}(\tau/\kappa)^{\beta} * M_{g\lambda_{n-1}g\lambda_n}(\tau/\kappa)^{\gamma}$$
$$\leq M_{g\lambda_{n-1}g\lambda_n}(\tau/\kappa) * M_{g\mu_{n-1}g\mu_n}(\tau/\kappa) * M_{g\lambda_{n-1}g\lambda_n}(\tau/\kappa) = \sigma_{n-1}(\tau/\kappa)$$

and

$$M_{g\mu_n g\mu_{n+1}}(\tau) \geq M_{g\mu_{n-1}g\mu_n}(\tau/\kappa)^{\alpha} * M_{g\lambda_{n-1}g\lambda_n}(\tau/\kappa)^{\beta} * M_{g\eta_{n-1}g\eta_n}(\tau/\kappa)^{\gamma}$$
$$\geq M_{g\mu_{n-1}g\mu_n}(\tau/\kappa) * M_{g\lambda_{n-1}g\lambda_n}(\tau/\kappa) * M_{g\eta_{n-1}g\eta_n}(\tau/\kappa) = \sigma_{n-1}(\tau/\kappa)$$

This shows that for all $\tau > 0$ and $n \geq 0$;

$$M_{g\eta_n g\eta_{n+1}}(\tau) * M_{g\lambda_n g\lambda_{n+1}}(\tau) * M_{g\mu_n g\mu_{n+1}}(\tau) \geq \sigma_{n-1}(\tau/\kappa) \ \geq \sigma_{n-1}(\tau) \quad (6)$$

Substituting τ by$\tau - \kappa\tau$ in (6), we deduce for $\tau > 0$ and $n \geq 0$, that

$$M_{g\eta_n g\eta_{n+1}}(\tau) * M_{g\lambda_n g\lambda_{n+1}}(\tau) * M_{g\mu_n g\mu_{n+1}}(\tau) \geq \sigma_{n-1}(\tau - \kappa\tau) \quad (7)$$

Now, considering that $*$ is commutative, from (3), (4), (5), we see that;

$$\sigma_n(\tau) = M_{g\eta_n g\eta_{n+1}}(\tau) * M_{g\lambda_n g\lambda_{n+1}}(\tau) * M_{g\mu_n g\mu_{n+1}}(\tau)$$
$$\geq \left(M_{g\eta_{n-1}g\eta_n}(\tau/\kappa)^{\alpha} * M_{g\lambda_{n-1}g\lambda_n}(\tau/\kappa)^{\beta} * M_{g\mu_{n-1}g\mu_n}(\tau/\kappa)^{\gamma}\right)$$
$$* \left(M_{g\lambda_{n-1}g\lambda_n}(\tau/\kappa)^{\alpha} * M_{g\lambda_{n-1}\lambda\mu_n}(\tau/\kappa)^{\beta} * M_{g\mu_{n-1}g\mu_n}(\tau/\kappa)^{\gamma}\right)$$
$$* \left(M_{g\eta_{n-1}g\eta_n}(\tau/\kappa)^{\alpha} * M_{g\lambda_{n-1}g\lambda_n}(\tau/\kappa)^{\beta} * M_{g\mu_{n-1}g\mu_n}(\tau/\kappa)^{\gamma}\right)$$

$$= \left(M_{g\eta_{n-1}g\eta_n}(\tau/\kappa)^{\alpha} * M_{g\eta_{n-1}g\eta_n}(\tau/\kappa)^{\gamma}\right)$$
$$* \left(M_{g\lambda_{n-1}g\lambda_n}(\tau/\kappa)^{\beta} * M_{g\lambda_{n-1}g\lambda_n}(\tau/\kappa)^{\alpha} * M_{g\lambda_{n-1}g\lambda_n}(\tau/\kappa)^{\gamma} * M_{g\lambda_{n-1}g\lambda_n}(\tau/\kappa)^{\beta}\right.$$
$$\left. * M_{g\mu_{n-1}g\mu_n}(\tau/\kappa)^{\alpha}\right)$$

$$= M_{g\eta_{n-1}\eta}(\tau/\kappa)^{\alpha+\gamma} * M_{g\lambda_{n-1}g\lambda_n}(\tau/\kappa)^{\gamma+2\beta+\gamma} * M_{g\mu_{n-1}g\mu_n}(\tau/\kappa)^{\alpha+\beta+\gamma}$$

$$\geq M_{g\eta_{n-1}g\eta_n}(\tau/\kappa) * M_{g\lambda_{n-1}g\lambda_n}(\tau/\kappa) * M_{g\mu_{n-1}g\mu_n}(\tau/\kappa) = \sigma_{n-1}(\tau/\kappa).$$

On making use of (2);

$$\sigma_n(\tau) \geq \sigma_{n-1}(\tau/\kappa) \geq \sigma_{n-1}(\tau) \geq \sigma_{n-1}(\tau - \kappa\tau) \text{ for all } \tau > 0 \text{ and } n \geq 1 \tag{8}$$

Continuously using inequality (1), we deduce that $\sigma_n(\tau) \geq \sigma_{n-1}(\tau/\kappa) \geq \sigma_{n-2}\left(\tau/_{\kappa^2}\right) \geq \cdots \geq \sigma_0(\tau/_{\kappa^n})$ for all $\tau > 0$ and $n \geq 1$. This implies that;

$$\lim_{n\to\infty} \sigma_n(\tau) \geq \lim_{n\to\infty} \sigma_0\left(\tau/_{\kappa^n}\right) = 1 \Rightarrow \lim_{n\to\infty} \sigma_n(\tau) \geq = 1 \tag{9}$$

On using properties (6) and (8), we obtain;

$$M_{g\eta_n g\eta_{n+1}}(\tau), M_{g\lambda_n g\lambda_{n+1}}, M_{g\mu_n g\mu_{n+1}}(\tau) \geq \sigma_{n-1}(\tau - \kappa\tau) \tag{10}$$

We can now deduce by induction that;

$$M_{g\eta_n g\eta_{n+q}}(\tau) * M_{g\lambda_n g\lambda_{n+q}}(\tau) * M_{g\mu_n g\mu_{n+q}}(\tau) \geq *^q \ \sigma_{n-1}(\tau - \kappa\tau) \ \forall \ \tau > 0, n, q \geq 1. \tag{11}$$

Now, if $q = 1$, (11) is confined for $n \geq 1$ and $\tau > 0$ from (10). If (11) is true for some q, we now have o show for $q + 1$ to be true. On making use of (1), we have;

$$M_{g\eta_{n+1}g\eta_{n+q+1}}(\kappa\tau) = M_{F_{\eta_n\lambda_n\mu_n}F_{\eta_{n+q}\lambda_{n+q}\mu_{n+q}}}(\kappa\tau) \geq M_{g\lambda_n g\lambda_{n+q}}(\tau)^{\alpha} * M_{g\eta_n g\eta_{n+q}}(\tau)^{\beta} * M_{g\mu_n g\mu_{n+q}}(\tau)^{\gamma} *$$

$$\geq \left(*^q \ \sigma_{n-1}(\tau - \kappa\tau)\right)^{\alpha} * \left(*^q \ \sigma_{n-1}(\tau - \kappa\tau)\right)^{\beta} * \left(*^q \ \sigma_{n-1}(\tau - \kappa\tau)\right)^{\gamma}$$

$$\geq \left(*^q \ \sigma_{n-1}(\tau - \kappa\tau)\right)^{\alpha} * \left(*^q \ \sigma_{n-1}(\tau - \kappa\tau)\right)^{\beta} * \left(*^q \ \sigma_{n-1}(\tau - \kappa\tau)\right)^{\gamma}$$

$$= \left(*^q \ \sigma_{n-1}(\tau - \kappa\tau)\right)^{\alpha+\beta+\gamma} \geq *^q \ \sigma_{n-1}(\tau - \kappa\tau).$$

In the same vein, $M_{g\lambda_{n+1}g\lambda_{n+q+1}}(\kappa\tau), M_{g\mu_{n+1}g\eta_{\mu+q+1}}(\kappa\tau) \geq *^q \ \sigma_{n-1}(\tau - \kappa\tau)$.

Now, on using axiom (v) of FMS and (7);

$$M_{g\eta_n g\eta_{n+q+1}}(\tau) = M_{g\eta_n g\eta_{n+q+1}}(\tau - \kappa\tau + \kappa\tau) \geq M_{g\eta_n g\eta_{n+1}}(\tau - \kappa\tau) * M_{g\eta_{n+1}g\eta_{n+q+1}}(\kappa\tau)$$

$$\geq \sigma_{n-1}(\tau - \kappa\tau) * \left(*^q \ \sigma_{n-1}(\tau - \kappa\tau)\right) = *^{q+1} \ \sigma_{n-1}(\tau - \kappa\tau)$$

Similarly,$M_{g\lambda_n g\lambda_{n+q+1}}(\tau) = M_{g\mu_n g\mu_{n+q+1}}(\tau) =*^{q+1}\sigma_{n-1}(\tau-\kappa\tau)$,

Therefore, (11) is true. Hence, it can now be shown that $\{g\eta_n\}$ is Cauchy. Suppose $\tau > 0$ and $\varepsilon \in (0,1)$ as $*$ is a τ −norm of $\mathbb{H}$-type, there exists $0 < \Pi < 1$ such that $*^q\,\alpha > 1-\varepsilon$ for all $\alpha \in (1-\Pi, 1]$ and for $q \geq 1$. From (9), $\lim_{n\to\infty}\sigma_n(\tau) = 1$, there exists $n_0 \in \mathbb{N}$ such that $\sigma_n(\tau-\kappa\tau) > 1-\Pi$ for$n \geq n_0$.Therefore, from (11) we obtain $M_{g\eta_n g\eta_{n+q}}(\tau), M_{g\lambda_n g\lambda_{n+q}}(\tau), M_{g\mu_n g\mu_{n+q}}(\tau) > 1-\varepsilon$ for $n \geq n_0$ and $q \geq 1$. Hence, $\{g\eta_n\}$ is a Cauchy sequence. Also, $\{g\lambda_n\}$ and $\{g\mu_n\}$ are Cauchy.

Step 3

Since g and F have tripled coincidence point and X is complete, there exists $\eta, \lambda, \mu \in X$ such that $\lim_{n\to\infty} g\eta_n = \eta$, $\lim_{n\to\infty} g\lambda_n = \lambda$ and $\lim_{n\to\infty} g\mu_n = \mu$.

As a result of g being continuous, we obtain $\lim_{n\to\infty} gg\mu_n = g\mu$, $\lim_{n\to\infty} gg\lambda_n = g\lambda$ and $\lim_{n\to\infty} gg\eta_n = g\eta$. Then, for the commutative property of F with g, it implies $\lim_{n\to\infty} gg\mu_{n+1} = gF(\eta_n, \lambda_n, \mu_n) = F(g\eta_n, g\lambda_n, g\mu_n)$. By (1),

$$M_{gg\eta_{n+1}F_{\eta\lambda\mu}}(\kappa\tau) = M_{F_{g\eta_n g\lambda_n}F_{\eta\,\lambda\,\mu}}(\kappa\tau) \geq M_{gg\eta_n g_\eta}(\tau)^\alpha * M_{gg\lambda_n g_\lambda}(\tau)^\beta * M_{gg\mu_n g_\mu}(\tau)^\gamma$$

$$\geq M_{gg\eta_n g\eta}(\tau) * M_{gg\lambda_n g\lambda}(\tau) * M_{gg\mu_n g\mu}(\tau)$$

As $n \to \infty$, we obtain that $\lim_{n\to\infty} gg\eta_n = F_{\eta\,\lambda\,\mu}$. Hence, $F_{\eta\,\lambda\,\mu} = g\eta$. Also, it can be shown that $F_{\lambda\,\eta\,\mu} = g\lambda$ and $F_{\mu\,\lambda\,\eta} = g\mu$ and (η, λ, μ) is a tripled coincidence point of F and g.

$$F_{\eta\,\lambda\,\mu} = g\eta, \qquad F_{\lambda\,\eta\,\mu} = g\lambda \text{ and } F_{\mu\,\lambda\,\eta} = g\mu \tag{12}$$

Step 4

From the claims of (12) and applying condition (1) and as a result of its mixed monotone property, we obtain;

$$M_{g\eta g\lambda_{n+1}}(\kappa\tau) = M_{F_{\eta\,\lambda\,\mu}F_{\lambda_n\eta_n\lambda_n}}(\kappa\tau)$$
$$\geq M_{g\eta g\lambda_n}(\tau)^\alpha * M_{g\eta g\lambda_n}(\tau)^\alpha * M_{g\lambda g\mu_n}(\tau)^\beta * M_{g\mu g\lambda_n}(\tau)^\gamma; \tag{13}$$

$$M_{g\lambda g\mu_{n+1}}(\kappa\tau) = M_{F_{\lambda\,\eta\,\mu}F_{\mu_n\lambda_n\eta_n}}(\kappa\tau)$$
$$\leq M_{g\lambda g\mu_n}(\tau)^{\alpha} * M_{g\eta g\lambda_n}(\tau)^{\alpha} * M_{g\eta g\lambda_n}(\tau)^{\beta} * M_{g\lambda g\eta_n}(\tau)^{\gamma}, \qquad (14)$$

$$M_{g\mu g\eta_{n+1}}(\kappa\tau) = M_{F_{\mu\,\lambda\,\eta}F_{\eta_n\lambda_n\mu_n}}(\kappa\tau)$$
$$\geq M_{g\eta g\lambda_n}(\tau)^{\alpha} * M_{g\mu g\eta_n}(\tau)^{\alpha} * M_{g\lambda g\lambda_n}(\tau)^{\beta} * M_{g\eta g\mu_n}(\tau)^{\gamma} \qquad (15)$$

Let $\rho_n(\tau) = M_{g\eta g\lambda_n}(\tau) * M_{g\lambda g\mu_n}(\tau) * M_{g\mu g\eta_n}(\tau)$ for $\tau > 0$ and $n \geq 0$. From the use of (13), (14) and (15), we obtain;

$$\rho_{n+1}(\kappa\tau) = M_{g\eta g\lambda_{n+1}}(\kappa\tau) * M_{g\lambda g\mu_{n+1}}(\kappa\tau) * M_{g\mu g\eta_{n+1}}(\kappa\tau)$$

$$\geq \left(M_{g\eta g\lambda_n}(\tau)^{\alpha} * M_{g\lambda g\mu_n}(\tau)^{\beta} * M_{g\mu g\eta_n}(\tau)^{\gamma}\right) * \left(M_{g\lambda g\mu_n}(\tau)^{\alpha} * M_{g\eta g\lambda_n}(\tau)^{\beta} * M_{g\lambda g\eta_n}(\tau)^{\gamma}\right)$$
$$* \left(M_{g\mu g\eta_n}(\tau)^{\alpha} * M_{g\lambda g\lambda_n}(\tau)^{\beta} * M_{g\eta g\mu_n}(\tau)^{\gamma}\right)$$

$$= \left(M_{g\eta g\lambda_n}(\tau)^{\alpha} * M_{g\eta g\lambda_n}(\tau)^{\beta} * M_{g\eta g\mu_n}(\tau)^{\gamma}\right) * \left(M_{g\lambda g\mu_n}(\tau)^{\alpha} * M_{g\lambda g\lambda_n}(\tau)^{\beta} * M_{g\lambda g\eta_n}(\tau)^{\beta} * M_{g\lambda g\eta_n}(\tau)^{\gamma}\right)$$
$$* \left(M_{g\mu g\eta_n}(\tau)^{\alpha} * M_{g\mu g\lambda_n}(\tau)^{\gamma}\right)$$

Now, because of the mixed monotone property, take $\eta_n = \lambda_n = \mu_n$, and we obtain;

$$\rho_{n+1}(\kappa\tau) = \left(M_{g\eta g\lambda_n}(\tau)^{\alpha} * M_{g\eta g\lambda_n}(\tau)^{\beta} * M_{g\eta g\lambda_n}(\tau)^{\gamma}\right)$$
$$* \left(M_{g\lambda g\mu_n}(\tau)^{\alpha} * M_{g\lambda g\mu_n}(\tau)^{\beta} * M_{g\lambda g\mu_n}(\tau)^{\beta} * M_{g\lambda g\mu_n}(\tau)^{\gamma}\right) * \left(M_{g\mu g\eta_n}(\tau)^{\alpha} * M_{g\mu g\eta_n}(\tau)^{\gamma}\right)$$

$$= M_{g\eta g\lambda_n}(\tau)^{\alpha+\beta+\gamma} * M_{g\lambda g\mu_n}(\tau)^{\alpha+2\beta+\gamma} * M_{g\mu g\eta_n}(\tau)^{\alpha+\gamma}$$

$$\geq M_{g\eta g\lambda_n}(\tau) * M_{g\lambda g\mu_n}(\tau) * M_{g\mu g\eta_n}(\tau) = \rho_n(\tau)$$

At this point, we have proven that $\rho_{n+1}(\kappa\tau) \geq \rho_n(\tau)$ for $n \geq 1$ and $\tau > 0$.

Going through the processes over again, we have;

$$\rho_n(\tau) \geq \rho_{n-1}(\tau/\kappa) \geq \rho_{n-2}\left(\tau/\kappa^2\right) \ldots \geq \rho_0\left(\tau/\kappa^n\right) \text{ for } n \geq 1 \text{and } \tau > 0. \qquad (16)$$

On applying(13) – (16), we obtain;

$$M_{g\eta g\lambda_{n+1}}(\kappa\tau) \geq M_{g\eta g\lambda_n}(\tau)^{\alpha} * M_{g\lambda g\eta_n}(\tau)^{\beta} * M_{g\mu g\lambda_n}(\tau)^{\gamma} \geq \rho_n(\tau) \geq \rho_0\left(\tau/\kappa^n\right) \qquad (17)$$

$$M_{g\lambda g\mu_{n+1}}(\kappa\tau) \leq M_{gg\mu_n}(\tau)^{\alpha} * M_{g\eta g\lambda_n}(\tau)^{\beta} * M_{g\lambda g\eta_n}(\tau)^{\gamma} \leq \rho_n(\tau) \leq \rho_0\left(\tau/\kappa^n\right) \qquad (18)$$

$$M_{g\mu g\eta_{n+1}}(\kappa\tau) \geq M_{g\mu g\eta_n}(\tau)^{\alpha} * M_{g\lambda g\lambda_n}(\tau)^{\beta} * M_{g\eta g\mu_n}(\tau)^{\gamma} \geq \rho_n(\tau) \geq \rho_0\left(\tau/_{\kappa^n}\right) \tag{19}$$

Also, $M_{g\eta g\lambda_{n+1}}(\kappa\tau), M_{g\lambda g\mu_{n+1}}(\kappa\tau), M_{g\mu g\eta_{n+1}}(\kappa\tau) \geq \rho_0\left(\tau/_{\kappa^n}\right)$ for all $n \geq 1$ and $\tau > 0$.

Since $\lim_{n\to\infty} \rho_0\left(\tau/_{\kappa^n}\right) = 1$ for all $\tau > 0$, on applying limit on (17), (18) and (19) and considering the mixed monotone property, we have;

$\lim_{n\to\infty} g\eta_n = g\mu$, $\lim_{n\to\infty} g\lambda_n = g\eta$ and $\lim_{n\to\infty} g\mu_n = g\lambda$. With the use of (12), it shows that;

$F_{\eta\,\lambda\,\mu} = g\eta = \lim_{n\to\infty} g\lambda_n = \lambda,\ F_{\lambda\,\eta\,\mu} = g\lambda = \lim_{n\to\infty} g\mu_n = \mu, F_{\mu\,\lambda\,\eta} = g\mu = \lim_{n\to\infty} g\eta_n =$

$\eta.$ (20)

Step 5

Here, we will show that $\eta = \lambda = \mu$. Let $\theta(\tau) = M_{\eta\lambda}(\tau) * M_{\lambda\mu}(\tau) * M_{\mu\eta}(\tau)$ for all $\tau > 0$. Then, by conditions (1), (2) and by applying the mixed monotone condition of theorem;

$$\begin{aligned} M_{\eta\lambda}(\kappa\tau) = M_{F_{\eta\lambda\mu}F_{\lambda\eta\lambda}}(\kappa\tau) &\geq M_{g\eta g\lambda}(\tau)^{\alpha} * M_{g\lambda g\eta}(\tau)^{\beta} * M_{g\mu g\lambda}(\tau)^{\gamma} \\ &= M_{\lambda\mu}(\tau)^{\alpha} * M_{\mu\lambda}(\tau)^{\beta} * M_{\eta\mu}(\tau)^{\gamma}; \end{aligned} \tag{21}$$

$$\begin{aligned} M_{\lambda\mu}(\kappa\tau) = M_{F_{\lambda\eta\lambda}F_{\mu\lambda\eta}}(\kappa\tau) &\leq M_{g\lambda g\mu}(\tau)^{\alpha} * M_{g\eta g\lambda}(\tau)^{\beta} * M_{g\lambda g\eta}(\tau)^{\gamma} \\ &= M_{\mu\eta}(\tau)^{\alpha} * M_{\lambda\mu}(\tau)^{\beta} * M_{\mu\lambda}(\tau)^{\gamma}; \end{aligned} \tag{22}$$

$$\begin{aligned} M_{\mu\eta}(\kappa\tau) = M_{F_{\mu\lambda\eta}F_{\eta\lambda\mu}}(\kappa\tau) &\geq M_{g\mu g\eta}(\tau)^{\alpha} * M_{g\lambda g\lambda}(\tau)^{\beta} * M_{g\eta g\mu}(\tau)^{\gamma} \\ &= M_{\eta\lambda}(\tau)^{\alpha} * M_{\mu\mu}(\tau)^{\beta} * M_{\lambda\eta}(\tau)^{\gamma}; \end{aligned} \tag{23}$$

If we use inequalities $(21) - (23)$ together, then;

$$\begin{aligned} \theta(\kappa\tau) &= M_{\eta\lambda}(\kappa\tau) * M_{\lambda\mu}(\kappa\tau) * M_{\mu\eta}(\kappa\tau) \\ &\geq \left(M_{\lambda\mu}(\tau)^{\alpha} * M_{\mu\lambda}(\tau)^{\beta} * M_{\eta\mu}(\tau)^{\gamma}\right) * \left(M_{\mu\eta}(\tau)^{\alpha} * M_{\lambda\mu}(\tau)^{\beta} * M_{\mu\lambda}(\tau)^{\gamma}\right) \\ &\quad * \left(M_{\eta\lambda}(\tau)^{\alpha} * M_{\mu\mu}(\tau)^{\beta} * M_{\lambda\eta}(\tau)^{\gamma}\right) \end{aligned}$$

$$\begin{aligned} &= \left(M_{\eta\lambda}(\tau)^{\alpha} * M_{\eta\mu}(\tau)^{\gamma} * M_{\mu\mu}(\tau)^{\beta}\right) * \left(M_{\lambda\mu}(\tau)^{\alpha} * M_{\lambda\mu}(\tau)^{\beta} * M_{\lambda\eta}(\tau)^{\gamma}\right) \\ &\quad * \left(M_{\mu\lambda}(\tau)^{\beta} * M_{\mu\lambda}(\tau)^{\lambda} * M_{\mu\eta}(\tau)^{\alpha}\right) \end{aligned}$$

Now if we let $\eta = \lambda = \mu$, then;

$$\theta(\kappa\tau) \geq \left(M_{\eta\lambda}(\tau)^{\alpha} * M_{\eta\lambda}(\tau)^{\beta} * M_{\eta\lambda}(\tau)^{\gamma}\right) * \left(M_{\lambda\mu}(\tau)^{\alpha} * M_{\lambda\mu}(\tau)^{\beta} * M_{\lambda\mu}(\tau)^{\gamma}\right) * \left(M_{\mu\lambda}(\tau)^{\alpha} * M_{\mu\lambda}(\tau)^{\beta} * M_{\mu\lambda}(\tau)^{\gamma}\right)$$

$$= \left(M_{\eta\lambda}(\tau)^{\alpha+\beta+\gamma} * M_{\lambda\mu}(\tau)^{\alpha+\beta+\gamma} * M_{\mu\lambda}(\tau)^{\alpha+\beta+\gamma}\right) \geq M_{\eta\lambda}(\tau) * M_{\lambda\mu}(\tau) * M_{\mu\lambda}(\tau) = \theta(\tau) \tag{24}$$

We found that $\theta(\kappa\tau) \geq \theta(\tau)$, by implication $\theta(\tau) \geq \theta(\tau/_{\kappa}) \geq \theta\left(\tau/_{\kappa^2}\right) \geq \cdots \geq \theta(\tau/_{\kappa^n})$ for $\tau > 0$ and $n \geq 1$. By $(21) - (23)$, and for $\eta = \lambda = \mu$;

$$M_{\eta\lambda}(\kappa\tau) \geq M_{\lambda\mu}(\tau)^{\alpha} * M_{\mu\lambda}(\tau)^{\beta} * M_{\eta\mu}(\tau)^{\gamma} = M_{\lambda\mu}(\tau)^{\alpha} * M_{\mu\eta}(\tau)^{\beta} * M_{\eta\lambda}(\tau)^{\gamma} \geq M_{\lambda\mu}(\tau) * M_{\mu\eta}(\tau) * M_{\eta\lambda=}(\tau) = \theta(\tau) \geq \theta(\tau/_{\kappa^n})$$

$$M_{\lambda\mu}(\kappa\tau) \leq M_{\mu\eta}(\tau)^{\alpha} * M_{\lambda\mu}(\tau)^{\beta} * M_{\mu\lambda}(\tau)^{\gamma} = M_{\mu\eta}(\tau)^{\alpha} * M_{\eta\lambda}(\tau)^{\beta} * M_{\lambda\mu}(\tau)^{\gamma} \leq M_{\mu\eta}(\tau) \star M_{\eta\lambda}(\tau) \star M_{\lambda\mu}(\tau) = \theta(\tau) \leq \theta(\tau/_{\kappa^n}),$$

$$M_{\mu\eta}(\kappa\tau) \geq M_{\eta\lambda}(\tau)^{\alpha} * M_{\mu\mu}(\tau)^{\beta} * M_{\lambda\eta}(\tau)^{\gamma} = M_{\eta\lambda}(\tau)^{\alpha} * M_{\lambda\mu}(\tau)^{\beta} * M_{\mu\eta}(\tau)^{\gamma} \geq M_{\eta\lambda}(\tau)^{\alpha} * M_{\lambda\mu}(\tau)^{\beta} * M_{\mu\eta}(\tau)^{\gamma} = \theta(\tau) \leq \theta(\tau/_{\kappa^n}).$$

As $n \to \infty$ and $\tau > 0$, we obtain $\lim_{n\to\infty} \theta(\tau/_{\kappa^n}) = 1$, which implies;

$M_{\eta\lambda=}(\tau) = M_{\lambda\mu}(\tau) = M_{\mu\eta}(\tau) = 1$. That is $\eta = \lambda = \mu$.

Example 3.1 Consider $(\mathrm{X} = \mathbb{R}, \delta = M^{\phi})$ and let (X, δ) for $\tau > 0$ and $\eta \neq \lambda$ defines: be a metric space $M_{\eta\lambda}{}^{\gamma}(\tau) = \gamma^{-\frac{\delta_{\eta\lambda}}{\tau}}$.considering $(\mathrm{X} = \mathbb{R}, \delta = M^{\phi})$ and let $\eta, \lambda > 0$ and $\kappa \in (0,1)$ be such that $6a \leq b\kappa$. $F: \mathbb{R}^3 \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ and $F(\eta, \lambda, \mu) = a(\eta - \lambda)$ and $g\eta = b\eta$ for $\eta, \lambda, \mu \in \mathrm{X}$. Evidently, g is continuous, F and g are commuting and $F(\mathbb{R}^3) = \mathbb{R} = g(\mathbb{R})$. We now point out that M^{ϕ} verifies;

$$M^{\gamma}_{F_{\eta\lambda\mu}F_{\phi\psi\omega}}(\kappa\tau) = \left(\gamma^{|(\eta-\phi)+|\psi-\lambda||}\right)^{-\frac{a}{\kappa\tau}} \geq \left(\gamma^{-\left(\frac{2\max(|\eta-\phi|,|\psi-\lambda|)}{\tau}\right)}\right)^{a/\kappa} \geq \left(\gamma^{-\left(\frac{2\max(|\eta-\phi|,|\psi-\lambda|)}{\tau}\right)}\right)^{b/6}$$

$$= \left(\gamma^{-\frac{b}{3\tau}}\right)^{\max(|\eta-\phi|,|\psi-\lambda|)} = \min\left(\gamma^{-\frac{b|\eta-\phi|}{3\tau}}, \gamma^{-\frac{b|\omega-\lambda|}{3\tau}}\right)$$

$$\geq \min\left(\gamma^{-\frac{|b\eta-b\phi|}{3\tau}}, \gamma^{-\frac{|b\psi-b\lambda|}{3\tau}}, \gamma^{-\frac{|b\kappa-b\omega|}{3\tau}}\right)$$

$$= \min\left(\left[M^{\gamma}_{g\eta g\phi}(\tau)\right]^{1/3}, \left[M^{\gamma}_{g\psi g\lambda}(\tau)\right]^{1/3}, \left[M^{\gamma}_{g\mu g\omega}(\tau)\right]^{1/3}\right)$$

Consequently, we conclude that F and g have tripled coincidence point on the account of the application of Theorem 3.1.

4 Conclusions

This work has shown the existence and uniqueness of tripled fixed point in fuzzy metric spaces via mixed monotone operators of contractive-type condition. Consequently, it generalizes the ideology of tripled fixed point in metric spaces to tripled fixed point in a mixed monotone of fuzzy type mapping.

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