Fixed point of generalized contractive maps on $S^{JS}-$ metric spaces with two metrics

Kushal Roy[a], Mantu Saha[a], Ismat Beg[b,*]

[a] *Department of Mathematics, The University of Burdwan, Purba Bardhaman -713104, West Bengal, India*

[b] *Centre for Mathematics and Statistical Sciences, Lahore School of Economics, Lahore - 53200, Pakistan*

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Abstract

In this paper, we prove the existence of fixed point theorems for $\mathcal{Z}$-contractive map, Geraghty type contractive map and interpolative Hardy-Rogers type contractive mapping in the setting of $S^{JS}-$ metric spaces with two metrics, Examples are constructed to highlight the significance of newly obtained results.

1 Introduction

Fixed point theory in distance spaces and its applications has attracted many researchers in last five decades due to its wide applicability [9]. Maia [10] generalized the classical Banach Contraction Principle in the setting of a metric space with two metrics and proved that if T is a contraction mapping with respect to some non complete metric on a nonempty set X, while X is complete with respect to some metric, then T has a fixed point under certain conditions. In the past few years Maia's theorem and its applications in study of differential equations has been generalized in many directions by several researchers, see Ravi and O'Regan [1], Smet [3], Khan et al. [7], Rus et al. [12], Soni [14] and references therein..

Recently Beg et al. [2] in an attempt to generalize the notion of metric, introduced a new type of $S^{JS}-$metric and an $S^{JS}-$metric space, and studied its several topological properties. This newly introduced $S^{JS}-$metric space include the concepts of $S-$metric [13] and S_b-metric spaces [15] and has generalized those spaces in a unique way. Afterward Roy et al. [11] proved integral type fixed point and coupled fixed point theorems on $S^{JS}-$metric spaces.

In this paper, we consider a nonempty set X together with two $S^{JS}-$metrics and prove several fixed point results for $\mathcal{Z}$-contractive map, Geraghty type contractive map and interpolative Hardy-Rogers type contractive mapping. Examples are constructed to high light the significance of newly obtained fixed point theorems.

* Corresponding Author's E-mail: ibeg@lahoreschool.edu.pk(I. Beg)

2 Preliminaries

Let X be a nonempty set and $E : X^3 \to [0, \infty]$ be a function. Let us define the set

$$S(E, X, x) = \{\{x_n\} \subset X : \lim_{n\to\infty} E(x, x, x_n) = 0\}$$

for all $x \in X$.

Definition 2.1. *[2] Let X be a nonempty set and $E : X^3 \to [0, \infty]$ satisfies the following conditions:*
(E_1) $E(x, y, z) = 0$ implies $x = y = z$ for any $x, y, z \in X$;
(E_2) there exists some $b > 0$ such that for any $(x, y, z) \in X^3$ and $\{z_n\} \in S(E, X, z)$, we have

$$E(x, y, z) \leq b \limsup_{n\to\infty}(E(x, x, z_n) + E(y, y, z_n))$$

Then the pair (X, E) is called an $S^{JS}-$ metric space.

From [2, 11] we see that any $S-$ metric and S_b- metric spaces are $S^{JS}-$ metric spaces but the converse need not be true in general.

Definition 2.2. *[2] Let (X, E) be an $S^{JS}-$ metric space, then*
(i) a sequence $\{x_n\} \subset X$ is said to be convergent to an element $x \in X$ if $\{x_n\} \in S(E, X, x)$,
(ii) a sequence $\{x_n\} \subset X$ is said to be Cauchy if $\lim_{n,m\to\infty} E(x_n, x_n, x_m) = 0$,
(iii) (X, E) is said to be complete if every Cauchy sequence in X is convergent.

Definition 2.3. *[2] Let (X, E) be an $S^{JS}-$metric space and $T : X \to X$ be a self mapping. Then T is called continuous at $a \in X$ if for any $\epsilon > 0$ there exists $\delta > 0$ such that for any $x \in X$, $E(Ta, Ta, Tx) < \epsilon$ whenever $J(a, a, x) < \delta$.*

Proposition 2.1. *[2] In an $S^{JS}-$metric space (X, J) if $\{x_n\}$ converges to both x and y for $x, y \in X$, then $x = y$.*

Proposition 2.2. *[2] Let (X, E) be an $S^{JS}-$metric space and $\{x_n\} \subset X$ converges to some $x \in X$. Then $E(x, x, x) = 0$.*

Proposition 2.3. *[2] In an $S^{JS}-$metric space (X, E) if T is continuous at $a \in X$ then for any sequence $\{x_n\} \in S(E, X, a)$ implies $\{Tx_n\} \in S(E, X, Ta)$.*

Definition 2.4. *[4] A function $\zeta : [0, \infty)^2 \to \mathbb{R}$ is called a simulation function, if it satisfies the following conditions:*
(ζ_1) $\zeta(0, 0) = 0$,
(ζ_2) $\zeta(t, s) < s - t$ for all $s, t > 0$,
(ζ_3) If $\{t_n\}$ and $\{s_n\}$ are sequences in $(0, \infty)$ such that $\lim_{n\to\infty} t_n = \lim_{n\to\infty} s_n > 0$, then $\limsup_{n\to\infty} \zeta(t_n, s_n) < 0$.

3 Main Result

In an $S^{JS}-$metric space (X, F), $D_F : X^2 \to [0, \infty]$ stands for the function defined as $D_F(x, y) = F(x, x, y)$ for any $x, y \in X$. Set of all simulation functions is denoted by $\mathcal{Z}$. Also we denote by (X, F, J) a nonempty set X together with two $S^{JS}-$ metrics $F, J : X^3 \to [0, \infty]$. An example of (X, F, J) is;

Example 3.1. *Let $X=[-\infty,\infty]$ and $F,J:X^3\to\infty$ be defined by $F(x,y,z)=|x-\sqrt{2}|+|y-\sqrt{2}|+|z-\sqrt{2}|$ and $J(x,y,z)=|x|+|y|+|z|$ for all $x,y,z\in X$. Then (X,F,J) is an $S^{JS}-$metric space with two $S^{JS}-$metric, where both F and J are purely $S^{JS}-$metrics, neither $S-$metrics nor S_b-metrics.*

Before proving our main fixed point results we need to extend the notion of Z-contractive map [8], Geraghty contractive map [5] and Interpolative Hardy-Roger type contractive map [6] to the case of an $S^{JS}-$ metric space.

Definition 3.1. *Let $T:X\to X$ be a map defined on an $S^{JS}-$metric space (X,F), such that for any $x,y\in X$, $D_F(Tx,Ty)=\infty\Rightarrow D_F(x,y)=\infty$. Then T is said to be an $S^{JS}-\mathcal{Z}$-contractive if there exists $\zeta\in\mathcal{Z}$ such that for $x,y\in X$, $D_F(x,y)<\infty$ implies $\zeta(D_F(Tx,Ty),D_F(x,y))\geq 0$.*

Definition 3.2. *Let (X,F) be an $S^{JS}-$metric space. A map $T:X\to X$ is said to be an $S^{JS}-$Geraghty type contractive map if the map T satisfies the following contractive condition:*

$$D_F(Tx,Ty)\leq\beta(D_F(x,y))D_F(x,y)\text{ for all }x,y\in X\text{ with }D_F(x,y)>0 \tag{3.1}$$

Where $\beta:(0,\infty]\to[0,1)$ is a function, satisfying (a) $\beta(\infty)=0$ and (b) for any sequence $\{t_n\}\subset(0,\infty]$, $\beta(t_n)\to 1$ implies $t_n\to 0$.

Definition 3.3. *Let (X,F) be an $S^{JS}-$metric space and $T:X\to X$. The map T is said to be an $S^{JS}-$Interpolative Hardy-Rogers type contractive map if there exists $\mu\in[0,1)$, $\xi,\eta,\zeta\in(0,1)$ with $\xi+\eta+\zeta<1$ such that*

$$\begin{aligned} & D_F(Tx,Ty) \\ \leq\ & \mu D_F(x,y)^{\xi}D_F(x,Tx)^{\eta}D_F(y,Ty)^{\zeta}\left[\frac{1}{2}(D_F(x,Ty)+D_F(y,Tx))\right]^{1-\xi-\eta-\zeta} \end{aligned} \tag{3.2}$$

for all $x,y\in X\backslash Fix^*(T)$, where $Fix^*(T)=\{x\in X:D_F(x,Tx)=0\}\subset Fix(T)$, $Fix(T)$ is the set of all fixed points of T.

Definition 3.4. *Let (X,F) be an $S^{JS}-$metric space and T be a self mapping on X. Then X is called $T-$orbitally complete if for any $x_0\in X$ whenever $\{x_m\}\subset\mathcal{O}(T,x_0)=\{x_0,Tx_0,T^2x_0,...\}$, is a Cauchy sequence then there exists an element $x\in X$ such that $\{x_m\}\in S(F,X,x)$.*

Definition 3.5. *A self mapping T on an $S^{JS}-$metric space (X,F) is said to be orbitally continuous if for any $x_0\in X$, $\{T^{n_i}x_0\}_{i\geq 1}\in S(F,X,u)$, $u\in X$, implies $\{TT^{n_i}x_0\}_{i\geq 1}\in S(F,X,Tu)$.*

Now we state and prove our main results.

Theorem 3.1. *Let (X,F,J) be a $S^{JS}-$metric space with two metrics F and J. Assume that for $T:X\to X$ the following conditions are satisfied:*

(1) *$F(x,y,z)=\infty$ if and only if $J(x,y,z)=\infty$ otherwise $F(x,y,z)\leq J(x,y,z)<\infty$ for all $x,y,z\in X$;*
(2) *(X,F) is $T-$orbitally complete;*
(3) *T is orbitally continuous with respect to F;*
(4) *T is $S^{JS}-\mathcal{Z}$-contractive with respect to J;*
(5) *there exists $x_0\in X$ such that $\delta(J,T,x_0)=\sup\{d_J(T^ix_0,T^jx_0):i,j\geq 1\}<\infty$ and $D_J(T^px_0,T^qx_0)>0$ for all $p,q\geq 1$ $(p\neq q)$.*

Then T has a fixed point in X. Moreover if $z,z'\in X$ are two fixed points of T such that $D_J(z,z')<\infty$ then $z=z'$.

Proof. Let us consider the Picard iterating sequence $\{x_n\}$ by $x_n = T^n x_0$ for all $n \in \mathbb{N}$. Then for any $(i,j) \in \mathbb{N}^2$ with $i \neq j$ we have,

$$\begin{aligned} 0 &\leq \zeta(D_J(x_{n+i}, x_{n+j}), D_J(x_{n-1+i}, x_{n-1+j})) \\ &< D_J(x_{n-1+i}, x_{n-1+j}) - D_J(x_{n+i}, x_{n+j}) \end{aligned} \tag{3.3}$$

implies $D_J(x_{n+i}, x_{n+j}) < D_J(x_{n-1+i}, x_{n-1+j})$ for all $n \in \mathbb{N}$. So $\{D_J(x_{n+i}, x_{n+j})\}_{n\in\mathbb{N}}$ is a decreasing bounded sequence for any $i, j (i \neq j) \geq 1$. Thus there exists $\lambda \geq 0$ such that $\lim_{n\to\infty} d_J(x_{n+i}, x_{n+j}) = \lambda$ for all $i, j (i \neq j) \geq 1$.

If $\lambda > 0$ then for the sequences $t_n = D_J(x_{n+3}, x_{n+2})$ and $s_n = D_J(x_{n+2}, x_{n+1})$ for all $n \in \mathbb{N}$, we have $\lim_{n\to\infty} t_n = \lim_{n\to\infty} s_n = \lambda$ and thus

$$\begin{aligned} 0 &\leq \limsup_{n\to\infty} \zeta(D_J(x_{n+3}, x_{n+2}), D_J(x_{n+2}, x_{n+1})) \\ &= \limsup_{n\to\infty} \zeta(t_n, s_n) < 0, \end{aligned} \tag{3.4}$$

a contradiction. Therefore $\limsup_{n\to\infty} D_J(x_{n+i}, x_{n+j}) = 0$ for all $i, j \geq 1$ with $i \neq j$. Hence $\{x_n\}$ is Cauchy in (X, J). Now due to condition (1) it can be easily seen that $\{x_n\}$ is Cauchy in (X, F). Since (X, F) is $T-$orbitally complete, $\{x_n\}$ converges to some $\{x_n\} \in S(F, X, z)$. From condition (3) we get $\{T^{n+1}x_0\}$ converges to Tz. Hence we have $Tz = z$.

If possible let, z, z' be two fixed points of T such that $D_J(z, z') < \infty$. If $D_J(z, z') > 0$ then we get

$$\begin{aligned} 0 &\leq \zeta(D_J(Tz, Tz'), D_J(z, z')) \\ &< D_J(z, z') - D_J(Tz, Tz') \\ &= D_J(z, z') - D_J(z, z') = 0, \text{ a contradiction.} \end{aligned}$$

So we get $D_J(z, z') = 0$ implies $z = z'$. □

Theorem 3.2. *Let (X, F, J) be a $S^{JS}-$metric space with two metrics F and J. Assume that for $T : X \to X$ the following conditions are satisfied:*

(1) $F(x, y, z) = \infty$ *if and only if* $J(x, y, z) = \infty$ *otherwise* $F(x, y, z) \leq J(x, y, z) < \infty$ *for all* $x, y, z \in X$;;

(2) (X, F) *is* $T-$*orbitally complete;*

(3) T *is orbitally continuous with respect to* F;

(4) T *is* $S^{JS}-$*Geraghty type contractive mapping with respect to* J*;*

(5) *there exists* $x_0 \in X$ *such that* $\delta(J, T, x_0) = \sup\{d_J(T^i x_0, T^j x_0) : i, j \geq 1\} < \infty$ *and* $D_J(T^p x_0, T^q x_0) > 0$ *for all* $p, q \geq 1$ $(p \neq q)$.

Then T has a fixed point z in X. Moreover if $z' \in X$ is another fixed point of T such that $D_J(z, z') < \infty$ then $z = z'$.

Proof. Let us construct the Picard iterating sequence $\{x_n\}$ by $x_n = T^n x_0$ for all $n \in \mathbb{N}$. Then for any particular $(i, j) \in \mathbb{N}^2$ we have,

$$\begin{aligned} D_J(x_{n+i}, x_{n+j}) &= D_J(Tx_{n-1+i}, Tx_{n-1+j}) \\ &\leq \beta(D_J(Tx_{n-1+i}, Tx_{n-1+j}))D_J(Tx_{n-1+i}, Tx_{n-1+j}) \\ &< D_J(Tx_{n-1+i}, Tx_{n-1+j}) \leq \delta(D_J, T, x_0) < \infty \text{ for all } n \in \mathbb{N}. \end{aligned} \tag{3.5}$$

So $\{D_J(x_{n+i}, x_{n+j})\}_{n\geq 0}$ is a decreasing sequence of reals, which is bounded below. So $\{D_J(x_{n+i}, x_{n+j})\}_{n\geq 0}$ converges in $[0,\infty)$. We show that $D_J(x_{n+i}, x_{n+j}) \to 0$ as $n \to \infty$. If possible let $D_J(x_{n+i}, x_{n+j}) \to q$ as $n \to \infty$ for some $q > 0$.. Then we have

$$1 > \beta(D_J(x_{n-1+i}, x_{n-1+j})) \geq \frac{D_J(x_{n+i}, x_{n+j})}{D_J(x_{n-1+i}, x_{n-1+j})} \to 1 \text{ as } n \to \infty.$$

Thus $\beta(D_J(x_{n-1+i}, x_{n-1+j})) \to 1$ as $n \to \infty$, a contradiction. Therefore $D_J(x_{n+i}, x_{n+j}) \to 0$ as $n \to \infty$. Since $(i,j) \in \mathbb{N}^2$ is arbitrary we get $\{x_n\}$ is Cauchy in (X, J). Now from condition (1) it can be easily seen that $\{x_n\}$ is Cauchy in (X, F). Since (X, F) is $T-$orbitally complete, $\{x_n\}$ converges to some $z \in X$ in (X, F). From condition (3) we get $\{T^{n+1}x_0\} \in S(F, X, Tz)$. Hence we have $Tz = z$.

If possible let, there exists $z' \in X$ such that $Tz' = z'$ and $D_J(z, z') < \infty$. If $D_J(z, z') > 0$ then we get

$$\begin{aligned} D_J(z, z') &= D_J(Tz, Tz') \\ &\leq \beta(D_J(z, z'))D_J(z, z') < D_J(z, z'), \text{ a contradiction.} \end{aligned}$$

So we get $D_J(z, z') = 0$ implies $z = z'$. □

Theorem 3.3. *Let (X, F, J) be a $S^{JS}-$metric space with two metrics F and J. Assume that for $T : X \to X$ the following conditions are satisfied:*

(1) $F(x, y, z) = \infty$ *if and only if* $J(x, y, z) = \infty$ *otherwise* $F(x, y, z) \leq J(x, y, z) < \infty$ *for all* $x, y, z \in X$;

(2) (X, F) *is* $T-$*orbitally complete;*

(3) T *is orbitally continuous with respect to* F;

(4) T *is* $S^{JS}-$*Interpolative Hardy-Rogers type contractive mapping with respect to* J*;*

(5) *there exists* $x_0 \in X$ *such that* $\delta(J, T, x_0) = \sup\{d_J(T^i x_0, T^j x_0) : i, j \geq 1\} < \infty$.

Then T has a fixed point w in X.

Proof. Let us construct the Picard iterating sequence $\{x_n\}$ by $x_n = T^n x_0$ for all $n \in \mathbb{N}$. If for some $m \in \mathbb{N}$, $D_F(x_m, x_{m+1}) = 0$ then we have $Tx_m = x_m$ and T has a fixed point trivially. So without loss of generality we assume that $D_F(x_l, x_{l+1}) > 0$ for all $l \geq 1$. Let us take $\delta(J, T^{p+1}, x_0) = \sup\{d_J(T^{p+i}x_0, T^{p+j}x_0) : i, j \in \mathbb{N}\}$ for any non-negative integer p. Clearly $\delta(J, T^{p+1}, x_0) \leq \delta(J, T, x_0) < \infty$ for any $p \geq 1$. Then for all $i, j \geq 1$

$$\begin{aligned} &D_J(T^{n+i}x_0, T^{n+j}x_0) \\ &\leq \mu D_J(T^{n-1+i}x_0, T^{n-1+j}x_0)^{\xi} D_J(T^{n-1+i}x_0, T^{n+i}x_0)^{\eta} D_J(T^{n-1+j}x_0, T^{n+j}x_0)^{\zeta} \\ &\left[\frac{1}{2}(D_J(T^{n-1+i}x_0, T^{n+j}x_0) + D_J(T^{n-1+j}x_0, T^{n+i}x_0))\right]^{1-\xi-\eta-\zeta} \\ &\leq \mu\delta(J, T^n, x_0)^{\xi}\delta(J, T^n, x_0)^{\eta}\delta(J, T^n, x_0)^{\zeta} \left[\frac{1}{2}(\delta(J, T^n, x_0) + \delta(J, T^n, x_0))\right]^{1-\xi-\eta-\zeta} \\ &= \mu\delta(J, T^n, x_0) \text{ for all } n \geq 1. \end{aligned} \tag{3.6}$$

Therefore $\delta(J, T^{n+1}, x_0) \leq \mu\delta(J, T^n, x_0)$ for all $n \geq 1$. Thus $\delta(J, T^{n+1}, x_0) \leq \mu^n\delta(J, T, x_0)$ for all $n \in \mathbb{N}$. From which it follows that $\delta(J, T^{n+1}, x_0) \to 0$ as $n \to \infty$. So $\lim_{n\to\infty} D_J(T^n x_0, T^{n+k}x_0) \leq \lim_{n\to\infty} \delta(J, T^n, x_0) = 0$ that is $\lim_{n\to\infty} D_J(T^n x_0, T^{n+k}x_0) = 0$, $k \geq 1$. Hence by applying condition (1) we have $\lim_{n\to\infty} D_F(T^n x_0, T^{n+k}x_0) = 0$, $k \in \mathbb{N}$. So $\{x_n\}$ is Cauchy in

(X, F). Since (X, F) is $T-$orbitally complete it follows that $\{x_n\}$ is convergent in (X, F) that is there exists some $w \in X$ such that $\{T^n x_0\} \in S(F, X, w)$. From condition (3) we get $\{T^{n+1}x_0\} \in S(F, X, Tw)$. Thus $Tw = w$. □

Example 3.2. *Let $X = [0,1]$, $F(x,y,z) = |x-z| + |y-z|$ and $J(x,y,z) = |x| + |y| + 2|z|$ for all $x, y, z \in X$. Then both F and J are $S^{JS}-$metrics on X. Let $T : X \to X$ be defined as $Tx = \frac{x^2}{3(1+x)}$ for all $x \in X$ and $\zeta : [0,\infty)^2 \to \mathbb{R}$ be defined by*

$$\zeta(t,s) = \begin{cases} \frac{s}{2} - t, & \text{if } 0 \leq s < 1 \\ s - \frac{1}{3} - t, & \text{if } s \geq 1. \end{cases} \tag{3.7}$$

Then one can verify that T satisfies all the conditions of Theorem 3.7. Here 0 is the unique fixed point of T in X.

Example 3.3. *Let $X = [0,1]$ and F, J be the $S^{JS}-$metrics defined on X as in above example. Also let $T : X \to X$ be defined by $Tx = \frac{x}{e}$ for all $x \in X$ and $\beta : (0,\infty] \to [0,1)$ be defined by $\beta(t) = e^{-t}$ for all $t \in (0,\infty]$. Then one can easily verify that all the conditions of Theorem 3.8 are satisfied and T has a unique fixed point in X.*

Example 3.4. *Let $X = \{-1,0,1\}$, $F : X^3 \to [0,\infty]$ be defined by $F(x,y,z) = |x-z| + |y-z|$ for all $x,y,z \in X$ and $J : X^3 \to [0,\infty]$ be defined by $J(x,y,z) = |x| + |y| + 2|z|$ for all $x,y,z \in X$. Then both F and J are $S^{JS}-$metrics on X. Let $T : X \to X$ be defined by $T(-1) = 1$ and $T(0) = 0 = T(1)$. Then $Fix^*(T) = \{0\}$. Therefore $X \backslash Fix^*(T) = \{-1,1\}$. Now if we choose $\xi = \frac{1}{4} = \eta$, $\zeta = \frac{1}{3}$ and $\mu \in [\frac{1}{\sqrt{2}}, 1)$ be any fixed element then we see that T satisfies contractive condition (3.2) with respect to J. Also one can verify that T satisfies all the additional conditions of Theorem 3.9 and T has a fixed point in X.*

In this paper we considered two $S^{JS}-$metrics on a nonempty set X. So we can take several combinations of metric type structures on a nonempty set X.

Remark 3.1. *We can consider the following combinations:*
(1) F and J both are $S-$metrics;
(2) both F and J are S_b-metrics;
(3) one is $S-$metric another is S_b-metric;
(4) one is either $S-$metric or S_b-metric and another is purely $S^{JS}-$metric neither $S-$metric nor S_b-metric;
(5) both F and J are purely $S^{JS}-$metrics neither $S-$metric nor S_b-metric.

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