

# Thermomechanical Interactions Due to Hall Current in Transversely Isotropic Thermoelastic with and Without Energy Dissipation with Two Temperatures and Rotation

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## ABSTRACT

The present paper is concerned with the investigation of disturbances in a homogeneous transversely isotropic thermoelastic rotating medium with two temperatures, in the presence of the combined effects of Hall currents and magnetic field due to thermomechanical sources. The formulation is applied to the thermoelasticity theories developed by Green-Naghdi Theories of Type-II and Type-III. Laplace and Fourier transform technique is applied to solve the problem. As an application, the bounding surface is subjected to concentrated and distributed sources (mechanical and thermal sources). The analytical expressions of displacement, stress components, temperature change and current density components are obtained in the transformed domain. Numerical inversion technique has been applied to obtain the results in the physical domain. Numerical simulated results are depicted graphically to show a comparison of effect of Hall current on the two theories GN-II and GN-III on resulting quantities. Some special cases are also deduced from the present investigation.

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## 1 INTRODUCTION

**D**URING the past few decades, widespread attention has been given to thermoelasticity theories that admit a finite speed for the propagation of thermal signals. In contrast to the conventional theories based on parabolic-type heat equation, these theories are referred to as generalized theories. Because of the experimental evidence in support of the finiteness of the speed of propagation of a heat wave, generalized thermoelasticity theories are more realistic than conventional thermoelasticity theories in dealing with practical problems involving very short time intervals and high heat fluxes such as those occurring in laser units, energy channels, nuclear reactors, etc. The phenomenon of coupling between the thermomechanical behavior of materials and magnetic behavior of materials have been studied since the 19<sup>th</sup> century.

Chen and Gurtin [7], Chen et al. [8] and Chen et al. [9] have formulated a theory of heat conduction in deformable bodies which depends upon two distinct temperatures, the conductive temperature  $\psi$  and the thermo dynamical temperature  $T$ . For time independent situations, the difference between these two temperatures is proportional to the heat supply, and in absence of heat supply, the two temperatures are identical. For time

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dependent problems, the two temperatures are different, regardless of the presence of heat supply. The two temperatures  $T$ ,  $\psi$  and the strain are found to have representations in the form of a travelling wave plus a response, which occurs instantaneously throughout the body (Boley and Tolins[5]). The wave propagation in two temperatures theory of thermoelasticity was investigated by Warren and Chen[36].

Green and Naghdi [13] postulated a new concept in thermoelasticity theories and proposed three models which are subsequently referred to as GN-I, II, and III models. The linearised version of model-I corresponds to classical thermoelastic model (based on Fourier's law). The linearised version of model-II and III permit propagation of thermal waves at finite speed. Green-Naghdi's second model (GN-II), in particular exhibits a feature that is not present in other established thermoelastic models as it does not sustain dissipation of thermal energy [15]. In this model the constitutive equations are derived by starting with the reduced energy equation and by including the thermal displacement gradient among other constitutive variables. Green-Naghdi's third model (GN-III) admits dissipation of energy. In this model the constitutive equations are derived by starting with the reduced energy equation, where the thermal displacement gradient in addition to the temperature gradient, are among the constitutive variables. Green and Naghdi[14] included the derivation of a complete set of governing equations of a linearised version of the theory for homogeneous and isotropic materials in terms of the displacement and temperature fields and a proof of the uniqueness of the solution for the corresponding initial boundary value problem.

A comprehensive work has been done in thermoelasticity theory with and without energy dissipation and thermoelasticity with two temperatures. Youssef [40], constructed a new theory of generalized thermoelasticity by taking into account two-temperature generalized thermoelasticity theory for a homogeneous and isotropic body without energy dissipation. Youssef et al. [39] investigated State space approach of two temperatures generalized thermoelasticity of infinite body with a spherical cavity subjected to different types of thermal loading. Abbas[2] discussed two dimensional problem with energy dissipation. Quintanilla [26] investigated thermoelasticity without energy dissipation of materials with microstructure. Abbas, Kumar and Reen[1] discussed response of thermal source in transversely isotropic thermoelastic materials without energy dissipation and with two temperatures. Several researchers studied various problems involving two temperatures e.g. (Youssef and Al-Lehaibi [38]; Youssef [37]; Youssef [41]; Kumar, Sharma and Garg [23]; Kaushal et al[17]; Kaushal Sharma and Kumar[18]; Kumar and Mukhopdhyay [21]; Ezzat and Awad [12]; Sharma and Marin[30]; Sharma and Bhargav [29]; Sharma, Sharma and Bhargav [34]; Sharma and Kumar[31]; Sharma and Kumar[32]; Sharma, Kumar and Ram[33]).

In view of the fact that most of the large bodies like the earth, the moon and other planets have an angular velocity, as well as earth itself behaves like a huge magnet, it is important to study the propagation of thermoelastic waves in a rotating medium under the influence of magnetic field. So, the attempts are being made to study the propagation of finite thermoelastic waves in an infinite elastic medium rotating with angular velocity. Several authors (Das and Kanoria[10]; Kumar and Kansal[20]; Kumar and Rupender[22]; Kumar and Devi [19]; Atwa and Jahangir[4], Mahmoud[24]) have studied various problems in generalized thermoelasticity to study the effect of rotation.

When the magnetic field is very strong, the conductivity will be a tensor and the effect of Hall current cannot be neglected. The conductivity normal to the magnetic field is reduced due to the free spiraling of electrons and ions about the magnetic lines of force before suffering collisions and a current is induced in a direction normal to both the electric and magnetic fields. This phenomenon is called the Hall effect. Authors like (Zakaria[42,43]; Salem[27]; Attia[3], Sarkar and Lahiri[28]) have considered the effect of Hall current for two dimensional problems in micropolar thermoelasticity.

In spite of these, not much work has been done in thermoelastic solid with the combined effects of Hall current, rotation and two temperatures. Keeping these considerations in mind, we formulated a two dimensional problem in transversely isotropic thermoelastic solid with and without energy dissipation in the presence of magnetic field, two temperatures and rotation taking into consideration the effect of Hall current. The components of normal displacement, normal stress, tangential stress, conductive temperature and current density are obtained by using Laplace and Fourier transforms. Numerical computation is performed by using a numerical inversion technique and the resulting quantities are shown graphically.

## 2 BASIC EQUATIONS

The constitutive relations for a transversely isotropic thermoelastic medium are given by

$$t_{ij} = C_{ijkl}e_{kl} - \beta_{ij}T \quad (1)$$

Equation of motion for a transversely isotropic thermoelastic medium rotating uniformly with an angular velocity  $\Omega = \Omega n$ , where  $n$  is a unit vector representing the direction of axis of rotation and taking into account Lorentz force

$$t_{ij,j} + F_i = \rho \{ \ddot{u}_i + (\Omega \times (\Omega \times u))_i + (2\Omega \times \dot{u})_i \} \quad (2)$$

Following Chandrasekharaiah [6] and Youssef [38], The heat conduction equation with two temperatures and with and without energy dissipation is given by

$$K_{ij}\varphi_{,ij} + K_{Lj}^*\dot{\varphi}_{,ij} = \beta_{ij}T_0\ddot{e}_{Lj} + \rho C_E \ddot{T} \quad (3)$$

The above equations are supplemented by generalized Ohm's law for media with finite conductivity and including the Hall current effect

$$J = \frac{\sigma_0}{1+m^2} \left( E + \mu_0 \left( \dot{u} \times H - \frac{1}{en_e} J \times H_0 \right) \right) \quad (4)$$

and the strain displacement relations are

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad i, j = 1, 2, 3 \quad (5)$$

Here  $F_i = \mu_0(J \times H_0)_i$ , are the components of Lorentz force.  $\beta_{ij} = C_{ijkl}\alpha_{ij}$  and  $T = \varphi - a_{ij}\varphi_{,ij}$ ,  $\beta_{ij} = \beta_i\delta_{ij}$ ,  $K_{ij} = K_i\delta_{ij}$ ,  $K_{ij}^* = K_i^*\delta_{ij}$ ,  $i$  is not summed.  $C_{ijkl}$  ( $C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk}$ ) are elastic parameters,  $\beta_{ij}$  is the thermal tensor,  $T$  is the temperature,  $T_0$  is the reference temperature,  $t_{ij}$  are the components of stress tensor,  $e_{kl}$  are the components of strain tensor,  $u_i$  are the displacement components,  $\rho$  is the density,  $C_E$  is the specific heat,  $K_{ij}$  is the thermal conductivity,  $K_{Lj}^*$  is the materialistic constant,  $a_{ij}$  are the two temperature parameters,  $\alpha_{ij}$  is the coefficient of linear thermal expansion,  $\Omega$  is the angular velocity of the solid,  $H$  is the magnetic strength,  $\dot{u}$  is the velocity vector,  $E$  is the intensity vector of the electric field,  $J$  is the current density vector,  $m \left( = \omega_e t_e = \frac{\sigma_0 \mu_0 H_0}{en_e} \right)$

is the Hall parameter,  $t_e$  is the electron collision time,  $\omega_e = \frac{e\mu_0 H_0}{m_e}$  is the electronic frequency,  $e$  is the charge of an

electron,  $m_e$  is the mass of the electron,  $\sigma_0 = \frac{e^2 t_e n_e}{m_e}$ , is the electrical conductivity and  $n_e$  is the number of density of electrons.

### 3 FORMULATION AND SOLUTION OF THE PROBLEM

We consider a homogeneous perfectly conducting transversely isotropic thermoelastic medium which is rotating uniformly with an angular velocity  $\Omega$  initially at uniform temperature  $T_0$ . The rectangular Cartesian co-ordinate system  $(u_1, u_2, u_3)$  having origin on the surface ( $x_3 = 0$ ) with  $x_3$ -axis pointing vertically downwards into the medium is introduced. The surface of the half-space is subjected to thermomechanical sources. For two dimensional problem in  $xz$ -plane, we take

$$(u_1, 0, u_3) \tag{6}$$

We also assume that

$$E = 0, \quad \Omega = (0, \Omega, 0) \tag{7}$$

The generalized Ohm's law

$$J_2 = 0 \tag{8}$$

The current density components  $J_1$  and  $J_3$  using (4) are given as:

$$J_1 = \frac{\sigma_0 \mu_0 H_0}{1+m^2} \left( m \frac{\partial u_1}{\partial t} - \frac{\partial u_3}{\partial t} \right) \tag{9}$$

$$J_3 = \frac{\sigma_0 \mu_0 H_0}{1+m^2} \left( \frac{\partial u_1}{\partial t} + m \frac{\partial u_3}{\partial t} \right) \tag{10}$$

Following Slaughter [35], using appropriate transformations, on the set of Eqs.(2) and (3) and with the aid of (6)-(10), we obtain the equations for transversely isotropic thermoelastic solid as:

$$C_{11} \frac{\partial^2 u_1}{\partial x^2} + C_{13} \frac{\partial^2 u_3}{\partial x_1 \partial x_3} + C_{44} \left( \frac{\partial^2 u_1}{\partial x_3^2} + \frac{\partial^2 u_3}{\partial x_1 \partial x_3} \right) - \beta \frac{\partial}{\partial x_1} \left\{ \varphi - \left( a_1 \frac{\partial^2 \varphi}{\partial x_1^2} + a_3 \frac{\partial^2 \varphi}{\partial x_3^2} \right) \right\} - \mu_0 J_3 H_0 = \rho \left( \frac{\partial^2 u_1}{\partial t^2} - \Omega^2 u_1 + 2\Omega \frac{\partial u_3}{\partial t} \right) \tag{11}$$

$$(C_{13} + C_{44}) \frac{\partial^2 u_1}{\partial x_1 \partial x_3} + C_{44} \frac{\partial^2 u_3}{\partial x_1^2} + C_{33} \frac{\partial^2 u_3}{\partial x_3^2} - \beta_3 \frac{\partial}{\partial x_3} \left\{ \varphi - \left( a_1 \frac{\partial^2 \varphi}{\partial x_1^2} + a_3 \frac{\partial^2 \varphi}{\partial x_3^2} \right) \right\} + \mu_0 J_1 H_0 = \rho \left( \frac{\partial^2 u_3}{\partial t^2} - \Omega^2 u_3 - 2\Omega \frac{\partial u_1}{\partial t} \right) \tag{12}$$

$$\left( k_1 + k_1^* \frac{\partial}{\partial t} \right) \frac{\partial^2 \varphi}{\partial x_1^2} + \left( k_3 + k_3^* \frac{\partial}{\partial t} \right) \frac{\partial^2 \varphi}{\partial x_3^2} = T_0 \frac{\partial^2}{\partial t^2} \left\{ \beta_1 \frac{\partial u_1}{\partial x_1} + \beta_3 \frac{\partial u_3}{\partial x_3} \right\} + \rho C_e \ddot{T} \tag{13}$$

And

$$t_{11} = c_{11} e_{11} + c_{13} e_{33} - \beta_1 T \tag{14}$$

$$t_{33} = c_{13} e_{11} + c_{33} e_{33} - \beta_3 T \tag{15}$$

$$t_{13} = 2c_{44} e_{13} \tag{16}$$

where

$$T = \varphi - \left( a_1 \frac{\partial^2 \varphi}{\partial x_1^2} + a_3 \frac{\partial^2 \varphi}{\partial x_3^2} \right)$$

$$\beta_1 = (c_{11} + c_{12}) \alpha_1 + c_{13} \alpha_3, \quad \beta_3 = 2c_{13} \alpha_1 + c_{33} \alpha_3$$

In the above equations we use the contracting subscript notations (1 → 11, 2 → 22, 3 → 33, 4 → 23, 5 → 31, 6 → 12) to relate  $c_{ijkl}$  to  $c_{mn}$ .

We assume that medium is initially at rest. The undisturbed state is maintained at reference temperature. Then we have the initial and regularity conditions are given by

$$\begin{aligned} u_1(x_1, x_3, 0) &= 0 = \dot{u}_1(x_1, x_3, 0) \\ u_3(x_1, x_3, 0) &= 0 = \dot{u}_3(x_1, x_3, 0) \\ 0 &= \dot{\varphi}(x_1, x_3, 0) \quad \text{for } x_3 \geq 0, \quad -\infty < x_1 < \infty \\ u_1(x_1, x_3, t) &= u_3(x_1, x_3, t) = \varphi(x_1, x_3, t) = 0 \quad \text{for } t > 0 \quad \text{when } x_3 \rightarrow \infty \end{aligned} \quad (17)$$

To facilitate the solution, following dimensionless quantities are introduced:

$$\begin{aligned} x'_1 &= \frac{x_1}{L}, \quad x'_3 = \frac{x_3}{L}, \quad u'_1 = \frac{\rho c_1^2}{L \beta_1 T_0} u_1, \quad u'_3 = \frac{\rho c_1^2}{L \beta_1 T_0} u_3, \quad T' = \frac{T}{T_0}, \quad t' = \frac{c_1}{L} t, \quad t'_{11} = \frac{t_{11}}{\beta_1 T_0}, \quad J' = \frac{\rho c_1^2}{\beta_1 T_0} J, \quad t'_{33} = \frac{t_{33}}{\beta_1 T_0}, \\ t'_{31} &= \frac{t_{31}}{\beta_1 T_0}, \quad \varphi' = \frac{\varphi}{T_0}, \quad a'_1 = \frac{a_1}{L}, \quad a'_3 = \frac{a_3}{L}, \quad H' = \frac{h}{H_0}, \quad M = \frac{\sigma_0 \mu_0 H_0}{\rho c_1 L}, \quad \Omega' = \frac{L}{c_1} \Omega \end{aligned} \quad (18)$$

Making use of (18) in Eqs. (11)-(13), after suppressing the primes, yield

$$\frac{\partial^2 u_1}{\partial x_1^2} + \delta_4 \frac{\partial^2 u_3}{\partial x_1 \partial x_3} + \delta_2 \left( \frac{\partial^2 u_1}{\partial x_3^2} + \frac{\partial^2 u_3}{\partial x_1 \partial x_3} \right) - \frac{\partial}{\partial x_1} \left\{ \varphi - \left( \frac{a_1}{L} \frac{\partial^2 \varphi}{\partial x_1^2} + \frac{a_3}{L} \frac{\partial^2 \varphi}{\partial x_3^2} \right) \right\} - \frac{M}{1+m^2} \mu_0 H_0 \left( \frac{\partial u_1}{\partial t} + m \frac{\partial u_3}{\partial t} \right) = \frac{\partial^2 u_1}{\partial t^2} - \Omega^2 u_1 + 2\Omega \frac{\partial u_3}{\partial t} \quad (19)$$

$$\delta_1 \frac{\partial^2 u_1}{\partial x_1 \partial x_3} + \delta_2 \frac{\partial^2 u_3}{\partial x_1^2} + \delta_3 \frac{\partial^2 u_3}{\partial x_3^2} - \frac{\beta_3}{\beta_1} \frac{\partial}{\partial x_3} \left\{ \varphi - \left( \frac{a_1}{L} \frac{\partial^2 \varphi}{\partial x_1^2} + \frac{a_3}{L} \frac{\partial^2 \varphi}{\partial x_3^2} \right) \right\} + \frac{M}{1+m^2} \mu_0 H_0 \left( m \frac{\partial u_1}{\partial t} - \frac{\partial u_3}{\partial t} \right) = \frac{\partial^2 u_3}{\partial t^2} - \Omega^2 u_3 - 2\Omega \frac{\partial u_1}{\partial t} \quad (20)$$

$$\varepsilon_1 \left( 1 + \frac{\varepsilon_3}{\varepsilon_1} \frac{\partial}{\partial t} \right) \frac{\partial^2 \varphi}{\partial x_1^2} + \varepsilon_2 \left( 1 + \frac{\varepsilon_4}{\varepsilon_2} \frac{\partial}{\partial t} \right) \frac{\partial^2 \varphi}{\partial x_3^2} = \varepsilon'_5 \beta_1^2 \frac{\partial^2}{\partial t^2} \left( \frac{\partial u_1}{\partial x_1} + \frac{\beta_3}{\beta_1} \frac{\partial u_3}{\partial x_3} \right) + \frac{\partial^2}{\partial t^2} \left\{ \varphi - \frac{a_1}{L} \frac{\partial^2 \varphi}{\partial x_1^2} + \frac{a_3}{L} \frac{\partial^2 \varphi}{\partial x_3^2} \right\} \quad (21)$$

$$\begin{aligned} \delta_1 &= \frac{(c_{13} + c_{44})}{c_{11}}, \quad \delta_2 = \frac{c_{44}}{c_{11}}, \quad \delta_3 = \frac{c_{33}}{c_{11}}, \quad \delta_4 = \frac{c_{13}}{c_{11}}, \\ \varepsilon_1 &= \frac{k_1}{\rho C_E c_1^2}, \quad \varepsilon_2 = \frac{k_3}{\rho C_E c_1^2}, \quad \varepsilon_3 = \frac{k_1^*}{L \rho C_E c_1}, \quad \varepsilon_4 = \frac{k_3^*}{L \rho C_E c_1}, \quad \varepsilon'_5 = \frac{T_0}{\rho^2 C_E c_1^2} \end{aligned}$$

Apply Laplace and Fourier transforms defined by

$$\bar{f}(x_1, x_3, s) = \int_0^\infty f(x_1, x_3, t) e^{-st} dt \quad (22)$$

$$\hat{f}(\xi, x_3, s) = \int_{-\infty}^\infty \bar{f}(x_1, x_3, s) e^{i\xi x_1} dx_1 \quad (23)$$

On Eqs. (19)-(21), we obtain a system of homogeneous equations in terms of  $\tilde{u}_1, \tilde{u}_3$  and  $\tilde{\varphi}$  which yield a non trivial solution if determinant of coefficient  $\{\tilde{u}_1, \tilde{u}_3, \tilde{\varphi}\}^T$  vanishes and we obtain the following characteristic equation

$$PD^6 + QD^4 + RD^2 + S(\tilde{u}_1, \tilde{u}_3, \tilde{\varphi}) = 0 \quad (24)$$

where  $P, Q, R$  and  $S$  are given in appendix A.

The solution of the Eq. (24) satisfying the radiation condition that  $\tilde{u}_1, \tilde{u}_3, \tilde{\varphi} \rightarrow 0$  as  $x_3 \rightarrow \infty$ , can be written as:

$$\tilde{u}_1 = A_1 e^{-\lambda_1 x_3} + A_2 e^{-\lambda_2 x_3} + A_3 e^{-\lambda_3 x_3} \tag{25}$$

$$\tilde{u}_3 = d_1 A_1 e^{-\lambda_1 x_3} + d_2 A_2 e^{-\lambda_2 x_3} + d_3 A_3 e^{-\lambda_3 x_3} \tag{26}$$

$$\tilde{\varphi} = l_1 A_1 e^{-\lambda_1 x_3} + l_2 A_2 e^{-\lambda_2 x_3} + l_3 A_3 e^{-\lambda_3 x_3} \tag{27}$$

where  $\pm\lambda_i, (i = 1, 2, 3)$ , are the roots of (24) and  $d_i$  and  $l_i$  are given in appendix B

#### 4 BOUNDARY CONDITIONS

On the half-space surface ( $x_3 = 0$ ) normal point force and thermal point source are applied. The appropriate boundary conditions are

$$t_{33} = -F_1 \psi_1(x) \delta(t) \tag{28}$$

$$t_{31} = 0 \tag{29}$$

$$\frac{\partial \varphi}{\partial x_3} = F_2 \psi_2(x) \delta(t) \quad \text{at } x_3 = 0 \tag{30}$$

where  $F_1$  is the magnitude of the force applied,  $F_2$  is the constant temperature applied on the boundary,  $\psi_1(x)$  specifies the source distribution function along  $x$  axis. Applying the Laplace and Fourier transform defined by (22)-(23) on the boundary conditions (28)-(30) and with the help of Eqs. (14)-(16), (18), (25)-(27), we obtain the components of displacement, normal stress, tangential stress, conductive temperature and current density components as given in appendix C (C.1-C.7)

##### 4.1 Mechanical force on the surface of half-space

Taking  $F_2 = 0$  in Eqs. (C.1)-(C.7), we obtain the components of displacement, normal stress, tangential stress, conductive temperature and current density components due to mechanical force.

##### 4.2 Thermal source on the surface of half-space

Taking  $F_1 = 0$  in Eqs. (C.1)-(C.7), we obtain the components of displacement, normal stress, tangential stress, conductive temperature and current density components due to thermal source.

##### 4.3 Green's function

To synthesize the Green's function, i.e. the solution due to concentrated normal force and thermal point source on the half-space is obtained by setting

$$\psi_1(x) = \delta(x), \quad \psi_2(x) = \delta(x) \tag{31}$$

In Eqs. (28) and (30). Applying the Laplace and Fourier transform defined by (22)-(23) on the Eq. (31) gives

$$\hat{\psi}_1(\xi) = 1, \quad \hat{\psi}_2(\xi) = 1 \quad (32)$$

Using (32) in (C.1)-(C.7), we obtain the components of displacement, stress and conductive temperature and current density components.

#### 4.4 Influence function

The method to obtain the half-space influence function, i.e. the solution due to distributed force/source applied on the half space is obtained by setting

$$\psi_1(x), \psi_2(x) = \begin{cases} 1 & \text{if } |x| \leq m \\ 0 & \text{if } |x| > m \end{cases} \quad (33)$$

In Eqs. (28) and (30). The Laplace and Fourier transforms of  $\psi_1(x)$  and  $\psi_2(x)$  with respect to the pair  $(x, \xi)$  for the case of a uniform strip load of non dimensional width  $2m$  applied at origin of co-ordinate system  $x_1 = x_3 = 0$  in the dimensionless form after suppressing the primes becomes

$$\hat{\psi}_1(\xi), \hat{\psi}_2(\xi) = [2 \sin(\xi m) / \xi], \xi \neq 0 \quad (34)$$

The expressions for displacement, stresses and conductive temperature and current density components can be obtained for uniformly distributed normal force and thermal source by replacing  $\hat{\psi}_1(\xi)$  and  $\hat{\psi}_2(\xi)$  from (34) respectively in Eqs. (C.1)-(C.7)

## 5 PARTICULAR CASES

If  $k_1^* = k_3^* = 0$ , then from (C.1)-(C.7), we obtain the corresponding expressions for displacements, and stresses, conductive temperature and components of current density for transversely isotropic magnetoelastostatic solid without energy dissipation and with two temperature with Hall current effect and rotation.

If  $a_1 = a_3 = 0$ , then from (C.1)-(C.7), we obtain the corresponding expressions for displacements, stresses, conductive temperature and components of current density for transversely isotropic magnetoelastostatic solid with and without energy dissipation alongwith with Hall current effect and rotation.

If we take  $c_{11} = \lambda + 2\mu = c_{33}$ ,  $c_{12} = c_{13} = \lambda$ ,  $c_{44} = \mu$ ,  $\beta_1 = \beta_3 = \beta$ ,  $\alpha_1 = \alpha_3 = \alpha$ ,  $K_1 = K_3 = K$  in Eqs. (C.1)-(C.7), we obtain the corresponding expressions for displacements, stresses, conductive temperature components of current density in isotropic magnetoelastostatic solid with two temperatures and with and without energy dissipation alongwith combined effects of Hall current and rotation.

If  $m = 0$ , in Eqs. (C.1)-(C.7), we obtain the components of displacements, stresses, conductive temperature and components of current density for transversely isotropic magnetoelastostatic solid and with and without energy dissipation and with two temperatures alongwith rotation.

## 6 INVERSION OF THE TRANSFORMATION

To obtain the solution of the problem in physical domain, we must invert the transforms in Eqs. (C.1)-(C.7). Here the displacement components, normal and tangential stresses and conductive temperature are functions of  $x_3$ , the parameters of Laplace and Fourier transforms  $s$  and  $\xi$  respectively and hence are of the form  $f(\xi, x_3, s)$ . To obtain the function  $f(x_1, x_3, t)$  in the physical domain, we first invert the Fourier transform using

$$\bar{f}(x_1, x_3, s) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\xi x_1} \hat{f}(\xi, x_3, s) d\xi = \frac{1}{2\pi} \int_{-\infty}^{\infty} |\cos(\xi x) f_e - i \sin(\xi x) f_0| d\xi \tag{35}$$

where  $f_e$  and  $f_0$  are respectively the odd and even parts of  $\hat{f}(\xi, x_3, s)$ . Thus the expression (35) gives the Laplace transform  $\bar{f}(x_1, x_3, s)$  of the function  $f(x_1, x_3, t)$ . Following Honig and Hirdes [16], the Laplace transform function  $\bar{f}(x_1, x_3, s)$  can be inverted to  $f(x_1, x_3, t)$ .

The last step is to calculate the integral in Eq. (35). The method for evaluating this integral is described in Press et al. [25]. It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

### 7 NUMERICAL RESULTS AND DISCUSSION

For the purpose of numerical evaluation, cobalt material has been chosen following Dhaliwal and Singh[11], as:

$$\begin{aligned} c_{11} &= 3.071 \times 10^{11} \text{ Nm}^{-2}, c_{33} = 3.581 \times 10^{11} \text{ Nm}^{-2}, c_{13} = 1.027 \times 10^{11} \text{ Nm}^{-2}, c_{44} = 1.510 \times 10^{11} \text{ Nm}^{-2}, \rho = 8.836 \times 10^3 \text{ Kg m}^{-3}, \\ T_0 &= 298^\circ \text{ K}, C_E = 4.27 \times 10^2 \text{ JK g}^{-1} \text{ deg}^{-1}, K_1 = .690 \times 10^2 \text{ wm}^{-1} \text{ deg}^{-1}, K_3 = .690 \times 10^2 \text{ wm}^{-1} \text{ deg}^{-1}, \beta_1 = 7.04 \times 10^6 \text{ Nm}^{-2} \text{ deg}^{-1}, \\ \beta_3 &= 6.90 \times 10^6 \text{ Nm}^{-2} \text{ deg}^{-1}, K_1^* = 0.02 \times 10^2 \text{ N sec}^{-2} \text{ deg}^{-1}, K_3^* = 0.04 \times 10^2 \text{ N sec}^{-2} \text{ deg}^{-1}, \mu_0 = 1.2571 \times 10^{-6} \text{ Hm}^{-1}, \\ H_0 &= 1 \text{ Jm}^{-1} \text{ nb}^{-1}, \varepsilon_0 = 8.838 \times 10^{-12} \text{ Fm}^{-1} \end{aligned}$$

with non-dimensional parameter  $L = 1$  and  $\sigma_0 = 9.36 \times 10^5 \text{ col}^2 / \text{ Cal.cm.sec}, \Omega = 3, t_0 = 0.02, M = 3$  and two temperature parameters is taken as  $a_1 = 0.03$  and  $a_3 = 0.06$ . Using the above values the graphical representation of values of normal stress  $t_{33}$ , tangential stress  $t_{31}$ , conductive temperature  $\varphi$ , transverse conduction current density  $J_x$  and normal conduction current density  $J_z$  for a transversely isotropic magneto-thermoelastic have been investigated for normal force/thermal source and uniformly distributed force/source to show a comparison of the effect of Hall current on the two theories of Green and Naghdi i.e. GN-II and GN-III and is presented in the Figs. 1-20. The computations are carried out in the range  $0 \leq x \leq 10$ .

The solid line, small dashed line corresponds to GN-III with  $m=0.6$  and  $m=0$  respectively.

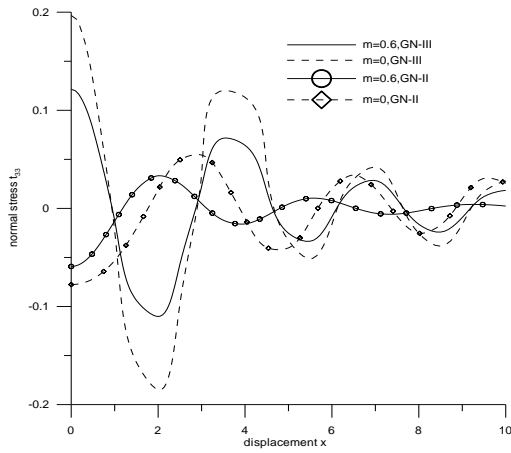
The solid line with centre symbol circle, the small dashed line with centre symbol diamond corresponds GN-II with  $m=0.6$  and  $m=0$  respectively.

### 8 MECHANICAL FORCE ON THE SURFACE OF HALF-SPACE

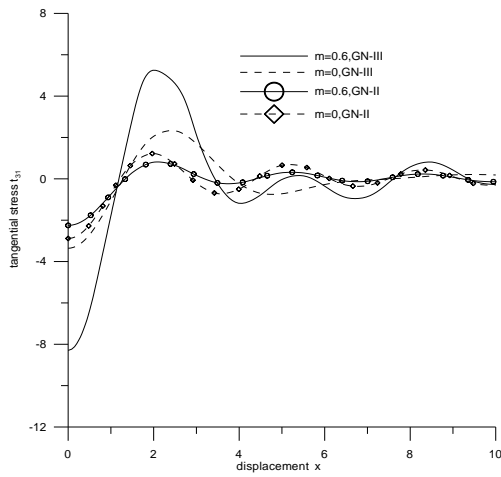
#### 8.1 Concentrated normal force

Fig.1 depicts variations of normal stress  $t_{33}$  with distance  $x$ . Here in all the cases, behaviour is oscillatory with amplitude decreasing as  $x$  increases. For  $m = 0$ , variations in GN-III are very high. For  $m = 0.6$ , variations in GN-II are comparatively small. There are more variations in GN-III than in GN-II. Fig. 2 presents the variations of tangential stress  $t_{31}$  with distance  $x$ . Here, an oscillatory pattern is observed with variations decreasing as  $x$  increases. Maximum variations are observed in the range  $0 \leq x \leq 4$ . Fig.3 exhibits variations in conductive temperature  $\varphi$  with distance  $x$ . Here, we observe that values corresponding to GN-III with  $m = 0$  are greater than for  $m = 0.6$  and are varying most in the range  $0 \leq x \leq 4$ . Behaviour is similar oscillatory in the rest for all the cases. Also an opposite oscillatory behaviour is observed corresponding to the two theories with difference in magnitude. Figs. 4 and 5 describe the variations of current density components  $J_x$  and  $J_z$  respectively. In  $J_x$  and  $J_z$ , for  $m = 0$ , variations are similar corresponding to GN-III but are purely opposite in case of GN-II. For  $m = 0.6$ , totally opposite pattern is observed in both the components for GN-III. whereas for  $m = 0.6$ , in case of GN-II, variations are small near the point of application of the source but behaviour is opposite in the rest.

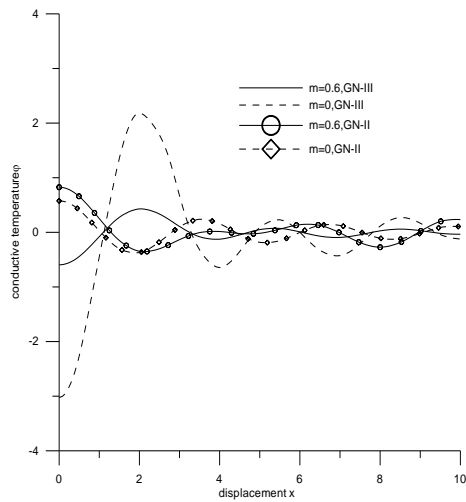




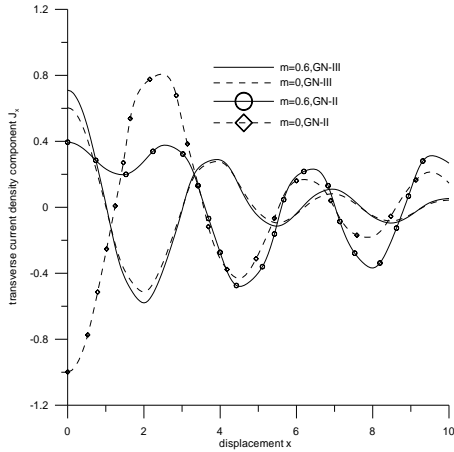
**Fig.1**  
Variations of normal stress  $t_{33}$  with distance  $x$  (concentrated normal force).



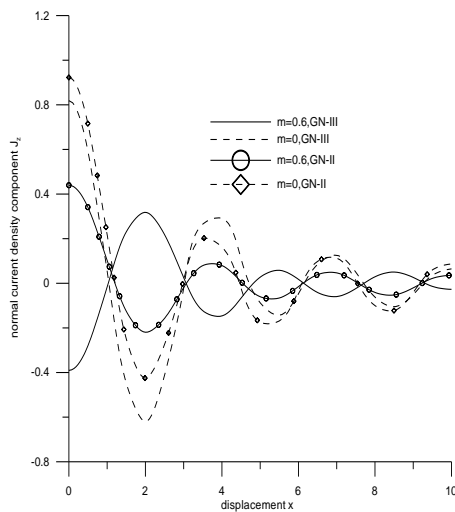
**Fig.2**  
Variations of tangential stress  $t_{31}$  with distance  $x$  (concentrated normal force).



**Fig.3**  
Variations of conductive temperature  $\phi$  with distance  $x$  (concentrated normal force).



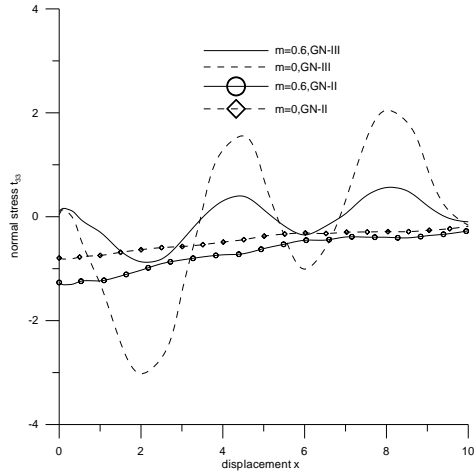
**Fig.4** Variations of transverse current density  $J_x$  with distance  $x$  (concentrated normal force).



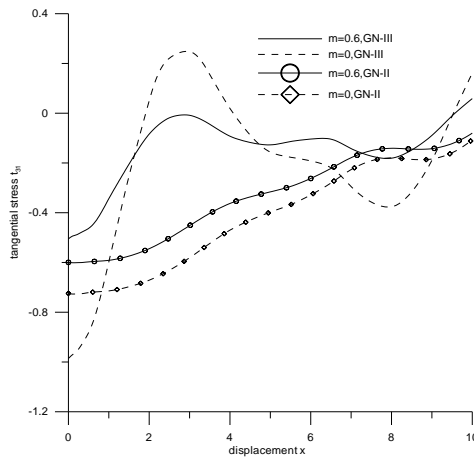
**Fig.5** Variations in normal current density component  $J_z$  with distance  $x$ (concentrated normal force).

8.2 Uniformly distributed force

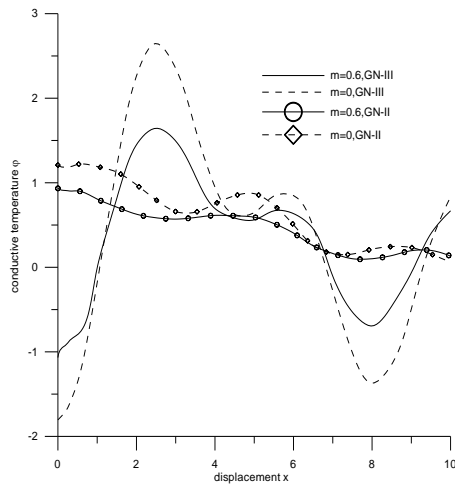
Fig.6 shows variations of normal stress  $t_{33}$  with distance  $x$ . Here small variations are observed in GN-II for both the values of Hall parameter whereas behaviour is ascending oscillatory in GN-III corresponding to both the cases i.e. presence and absence of Hall current. Fig.7 presents the variations of tangential stress  $t_{31}$  with distance  $x$ . For GN-III, there is a sharp increase in the range  $0 \leq x \leq 2$  and variations are oscillatory afterwards. Here, an ascending pattern is observed with less variations in case of GN-II. Fig.8 exhibits variations in conductive temperature  $\varphi$  with distance  $x$ . Here, we observe that values in GN-III with  $m = 0$  are greater than for  $m = 0.6$  in the range  $1 \leq x \leq 7$  and trend is opposite in the rest. Variations in GN-II are descending with less variations and values in GN-II for  $m = 0$  are greater than for  $m = 0.6$  in the whole range. Figs. 9 and 10 describe the variations of current density components  $J_x$  and  $J_z$  respectively. In Fig. 9, we observe a similar pattern in GN-III, corresponding to  $m = 0$  and  $m = 0.6$  whereas in GN-II, small variations are observed corresponding to both the cases. In Fig. 10, a similar oscillatory pattern is observed in all the cases with difference in magnitude except for the case  $m = 0.6$  in GN-III.



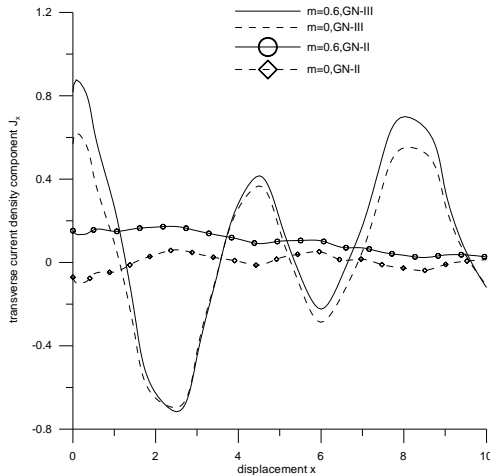
**Fig.6**  
Variations of normal stress  $t_{33}$  with distance  $x$  (Uniformly distributed mechanical force).



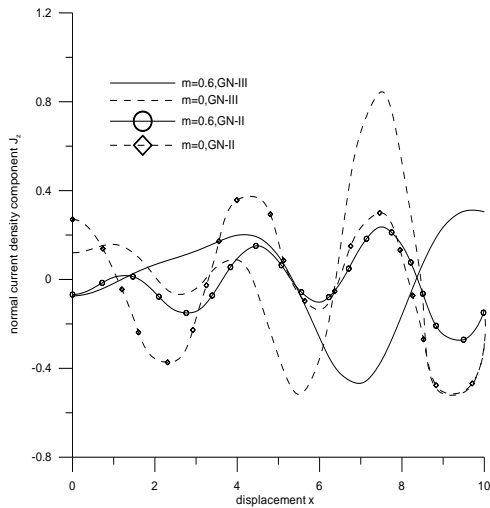
**Fig.7**  
Variations of tangential stress  $t_{31}$  with distance  $x$  (Uniformly distributed mechanical force).



**Fig.8**  
Variations of conductive temperature  $\varphi$  with distance  $x$  (Uniformly distributed mechanical force).



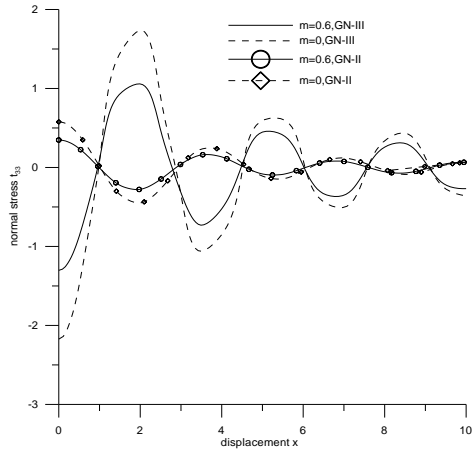
**Fig.9**  
Variations of transverse current density  $J_x$  with distance  $x$  (Uniformly distributed mechanical force).



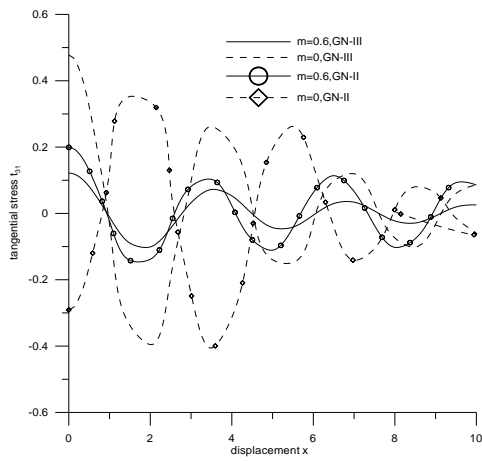
**Fig.10**  
Variations in normal current density component  $J_z$  with distance  $x$  (Uniformly distributed mechanical force).

8.3 Thermal source on the Surface of Half-Space  
8.3.1 Concentrated thermal source

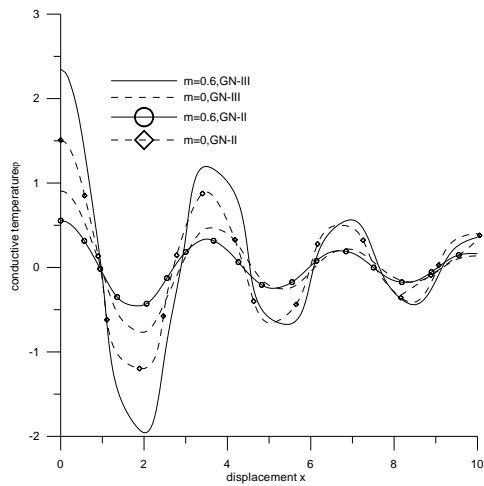
Fig.11 depicts variations of normal stress  $t_{33}$  with distance  $x$ . Here an opposite oscillatory pattern with difference in magnitude is observed in both the theories GN-II and GN-III. Variations in GN-III are observed more as compared with GN-II. Fig. 12 presents the variations of tangential stress  $t_{31}$  with distance  $x$ . Here, a similar oscillatory pattern is observed for  $m = 0.6$  in GN-III and GN-II whereas for  $m = 0$ , trend is opposite oscillatory. Fig.13 exhibits variations in conductive temperature  $\varphi$  with distance  $x$ . Here behaviour is similar oscillatory for all the cases. Figs. 14 and 15 describe the variations of current density components  $J_x$  and  $J_z$  respectively. In Fig.14, variations are similar oscillatory corresponding to all the cases with difference in magnitude. But in Fig. 15 variations are opposite oscillatory for  $m = 0$  in GN-II and GN-III and are similar for  $m = 0.6$ .



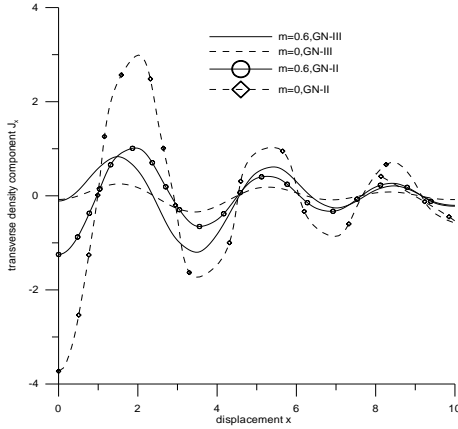
**Fig.11**  
Variations of normal stress  $t_{33}$  with distance  $x$  (concentrated normal source).



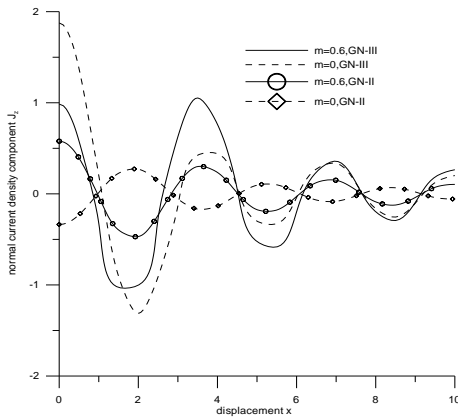
**Fig.12**  
Variations of tangential stress  $t_{31}$  with distance  $x$  (concentrated normal source).



**Fig.13**  
Variations of conductive temperature  $\varphi$  with distance  $x$  (concentrated normal source).



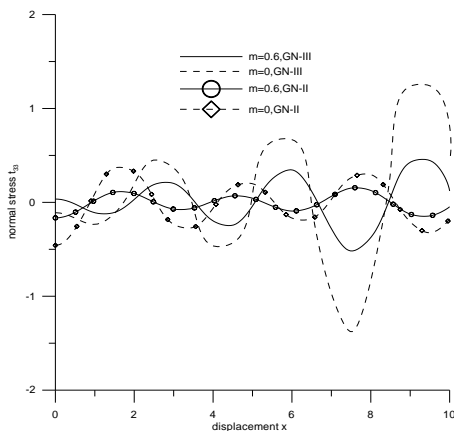
**Fig.14**  
Variations of transverse current density  $J_x$  with distance  $x$  (concentrated normal source).



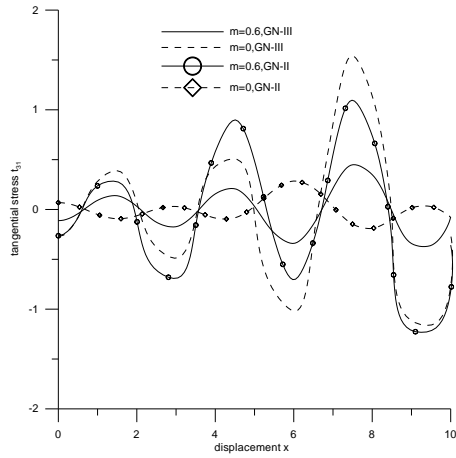
**Fig.15**  
Variations in normal current density component  $J_z$  with distance  $x$  (concentrated normal source).

8.3.2 Uniformly distributed source

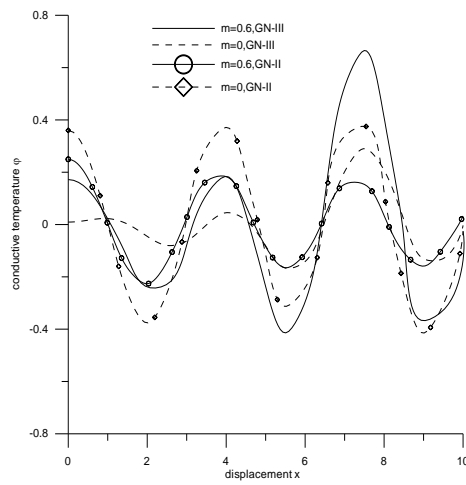
Fig.16 shows variations of normal stress  $t_{33}$  with distance  $x$ . Here we observe that variations increase as  $x$  increases and are oscillatory in all the cases. Fig.17 presents the variations of tangential stress  $t_{31}$  with distance  $x$ . Here for  $m = 0.6$ , trend is similar oscillatory whereas for  $m = 0$ , trend is opposite oscillatory with difference in magnitude in GN-II and GN-III. Variations also increase as  $x$  increases. Fig.18 exhibits variations in conductive temperature  $\varphi$  with distance  $x$ . Here a similar oscillatory pattern is observed in all the cases with difference in magnitude. Figs. 19 and 20 describe the variations of current density components  $J_x$  and  $J_z$  respectively. In Fig. 19, we observe that variations increase as  $x$  increases in all the cases. In Fig.20, we find that variations decrease as  $x$  increases.



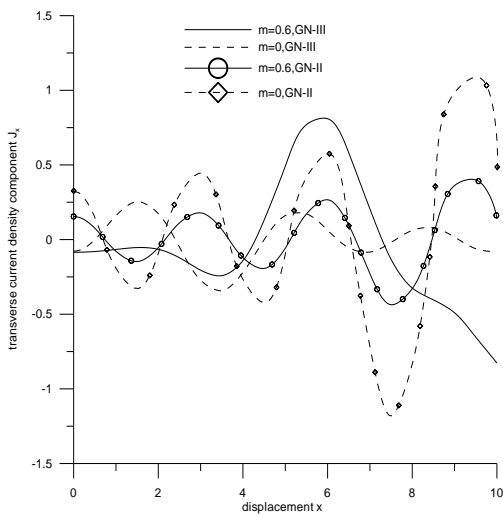
**Fig.16**  
Variations of normal stress  $t_{33}$  with distance  $x$  (Uniformly distributed thermal source).



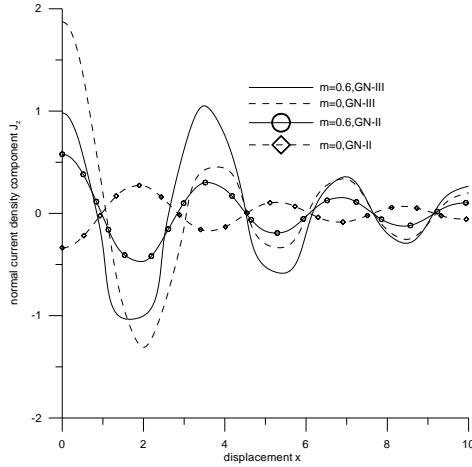
**Fig.17**  
Variations of tangential stress  $t_{31}$  with distance  $x$  (Uniformly distributed thermal source).



**Fig.18**  
Variations of conductive temperature  $\phi$  with distance  $x$  (Uniformly distributed thermal source).



**Fig.19**  
Variations of transverse current density  $J_x$  with distance  $x$  (Uniformly distributed thermal source).



**Fig.20**  
Variations in normal current density component  $J_z$  with distance  $x$  (Uniformly distributed thermal source).

**9 CONCLUSIONS**

From the graphs, it is clear that Hall parameter has different impacts on the two theories of GN-II and GN-III. In case of concentrated normal force/ thermal point source, variations decrease as  $x$  increases whereas while applying uniformly distributed thermal source, opposite trend is observed i.e. variations increase as  $x$  increases. Non uniform pattern of graphs is observed while applying uniformly distributed mechanical force. Opposite trends are observed in both the current density components. Presence of hall current leads to uniform oscillatory trends in GN-II and GN-III. In presence of hall current, both the curves corresponding to GN-II and GN-III move in opposite oscillatory trend except in case of tangential stress as behaviour is similar oscillatory in that case.

**APPENDIX A**

$$\begin{aligned}
 P &= \delta_2 \delta_3 \zeta_5 + \varepsilon_5 \zeta_5 \delta_2 \\
 Q &= \zeta_{10} \zeta_7 \delta_3 + \delta_2 \zeta_7 \zeta_{11} - \delta_3 \delta_2 \zeta_6 - \varepsilon_5 \beta_3^2 \zeta_4 \delta_2 - \zeta_3^2 \zeta_7 - i \xi \zeta_3 p_3 \zeta_5 \varepsilon_5 \beta_1^2 + \xi^2 \varepsilon_5 \delta_3 \zeta_5 \beta_1^2 - i \xi \varepsilon_5 \zeta_5 \beta_1 \beta_3 \zeta_3 - \xi^2 \varepsilon_5 \beta_1^2 \delta_3 \zeta_5 \\
 R &= \zeta_{10} \zeta_{11} \zeta_7 - \zeta_{10} \delta_3 \zeta_6 - \varepsilon_5 \beta_3^2 \zeta_{10} \zeta_4 - \delta_2 \zeta_6 \zeta_{11} + \zeta_3^2 \zeta_6 + \zeta_2^2 \zeta_7 + p_3 \zeta_3 \zeta_4 \varepsilon_5 \beta_1^2 i \xi + i \xi \zeta_3 \zeta_4 \beta_1 \beta_3 \varepsilon_3 - \xi^2 \zeta_5 \zeta_{11} \beta_1^2 \varepsilon_5 \\
 S &= -\zeta_{10} \zeta_{11} \zeta_6 - \zeta_2^2 \zeta_6 + \xi^2 \zeta_{11} \beta_1^2 \zeta_4 \varepsilon_5 \\
 \zeta_6 &= (\varepsilon_1 + \varepsilon_3 s) \xi^2 + s^2 \left(1 + \frac{a_1}{L} \xi^2\right), \quad \zeta_7 = \varepsilon_2 + \varepsilon_4 s - \frac{a_3}{L} s^2, \quad \zeta_{10} = \zeta_1 - \xi^2, \quad \zeta_{11} = \zeta_1 - \delta_2 \xi^2, \quad \varepsilon_5 = \frac{T_0 s^2}{\rho^2 C_E c_1^2}
 \end{aligned}$$

**APPENDIX B**

$$\begin{aligned}
 d_i &= \frac{\lambda_i^2 (\varepsilon_3 i \xi \delta_3 - \zeta_3 \beta_3 \beta_1 \xi) - \lambda_i (\xi \beta_1 \beta_3 \zeta_2) + \varepsilon_5 i \xi \zeta_{11}}{\lambda_i^4 (\beta_3 \zeta_7 + \varepsilon_5 \zeta_5 \beta_3^2) + \lambda_i^2 (\zeta_7 \zeta_{11} - \delta_3 \zeta_6 - \beta_3^2 \varepsilon_5 \zeta_4) - \zeta_6 \zeta_{11}} & i = 1, 2, 3 \\
 l_i &= \frac{-\lambda_i^3 (\zeta_3 \zeta_7 + \varepsilon_5 i \xi \zeta_5 p_3) - \zeta_2 \zeta_7 \lambda_i^2 + (\zeta_3 \zeta_6 + \varepsilon_5 i \xi p_3 \zeta_4) \lambda_i + \zeta_6 \zeta_2}{\lambda_i^4 (\delta_3 \zeta_7 + \varepsilon_5 \zeta_5 \beta_3^2) + \lambda_i^2 (\zeta_7 \zeta_{11} - \delta_3 \zeta_6 - \beta_3^2 \varepsilon_5 \zeta_4) - \zeta_6 \zeta_{11}} & i = 1, 2, 3
 \end{aligned}$$



## APPENDIX C

$$\tilde{u}_1 = \frac{-F_1\tilde{\psi}_1(\xi)}{\Delta}(\Delta_1 e^{-\lambda_1 x_3} + \Delta_2 e^{-\lambda_2 x_3} + \Delta_3 e^{-\lambda_3 x_3}) + \frac{F_2\tilde{\psi}_2(\xi)}{\Delta}(\Delta_1^* e^{-\lambda_1 x_3} + \Delta_2^* e^{-\lambda_2 x_3} + \Delta_3^* e^{-\lambda_3 x_3}) \quad (\text{C.1})$$

$$\tilde{u}_3 = \frac{-F_1\tilde{\psi}_1(\xi)}{\Delta}(d_1\Delta_1 e^{-\lambda_1 x_3} + d_2\Delta_2 e^{-\lambda_2 x_3} + d_3\Delta_3 e^{-\lambda_3 x_3}) + \frac{F_2\tilde{\psi}_2(\xi)}{\Delta}(d_1\Delta_1^* e^{-\lambda_1 x_3} + d_2\Delta_2^* e^{-\lambda_2 x_3} + d_3\Delta_3^* e^{-\lambda_3 x_3}) \quad (\text{C.2})$$

$$\tilde{t}_{33} = \frac{-F_1\tilde{\psi}_1(\xi)}{\Delta}(\Delta_{11}\Delta_1 e^{-\lambda_1 x_3} + \Delta_{12}\Delta_2 e^{-\lambda_2 x_3} + \Delta_{13}\Delta_3 e^{-\lambda_3 x_3}) + \frac{F_2\tilde{\psi}_2(\xi)}{\Delta}(\Delta_{11}\Delta_1^* e^{-\lambda_1 x_3} + \Delta_{12}\Delta_2^* e^{-\lambda_2 x_3} + \Delta_{13}\Delta_3^* e^{-\lambda_3 x_3}) \quad (\text{C.3})$$

$$\tilde{t}_{31} = \frac{-F_1\tilde{\psi}_1(\xi)}{\Delta}(\Delta_{21}\Delta_1 e^{-\lambda_1 x_3} + \Delta_{22}\Delta_2 e^{-\lambda_2 x_3} + \Delta_{23}\Delta_3 e^{-\lambda_3 x_3}) + \frac{F_2\tilde{\psi}_2(\xi)}{\Delta}(\Delta_{21}\Delta_1^* e^{-\lambda_1 x_3} + \Delta_{22}\Delta_2^* e^{-\lambda_2 x_3} + \Delta_{23}\Delta_3^* e^{-\lambda_3 x_3}) \quad (\text{C.4})$$

$$\tilde{\varphi} = \frac{-F_1\tilde{\psi}_1(\xi)}{\Delta}(\Delta_{31}\Delta_1 e^{-\lambda_1 x_3} + \Delta_{32}\Delta_2 e^{-\lambda_2 x_3} + \Delta_{33}\Delta_3 e^{-\lambda_3 x_3}) + \frac{F_2\tilde{\psi}_2(\xi)}{\Delta}(\Delta_{31}\Delta_1^* e^{-\lambda_1 x_3} + \Delta_{32}\Delta_2^* e^{-\lambda_2 x_3} + \Delta_{33}\Delta_3^* e^{-\lambda_3 x_3}) \quad (\text{C.5})$$

$$\tilde{J}_x = \frac{-F_1\tilde{\psi}_1(\xi)}{\Delta} \frac{c_1\sigma_0 H_0 \mu_0}{1+m^2} \left( (m-d_1)\Delta_1 e^{-\lambda_1 x_3} + (m-d_2)\Delta_2 e^{-\lambda_2 x_3} + (m-d_3)\Delta_3 e^{-\lambda_3 x_3} \right) + \frac{F_2\tilde{\psi}_2(\xi)}{\Delta} \frac{c_1\sigma_0 H_0 \mu_0}{1+m^2} \left( (m-d_1)\Delta_1^* e^{-\lambda_1 x_3} + (m-d_2)\Delta_2^* e^{-\lambda_2 x_3} + (m-d_3)\Delta_3^* e^{-\lambda_3 x_3} \right) \quad (\text{C.6})$$

$$\tilde{J}_z = \frac{-F_1\tilde{\psi}_1(\xi)}{\Delta} \frac{c_1\sigma_0 H_0 \mu_0}{1+m^2} \left( (1+md_1)\Delta_1 e^{-\lambda_1 x_3} + (1+md_2)\Delta_2 e^{-\lambda_2 x_3} + (1+md_3)\Delta_3 e^{-\lambda_3 x_3} \right) + \frac{F_2\tilde{\psi}_2(\xi)}{\Delta} \frac{c_1\sigma_0 H_0 \mu_0}{1+m^2} \left( (1+md_1)\Delta_1^* e^{-\lambda_1 x_3} + (1+md_2)\Delta_2^* e^{-\lambda_2 x_3} + (1+md_3)\Delta_3^* e^{-\lambda_3 x_3} \right) \quad (\text{C.7})$$

where

$$\begin{aligned} (\Delta_{22}\Delta_{33} - \Delta_{23}\Delta_{32}) &= \Delta_1, (\Delta_{23}\Delta_{31} - \Delta_{21}\Delta_{33}) = \Delta_2, (\Delta_{21}\Delta_{32} - \Delta_{22}\Delta_{31}) = \Delta_3, \sqrt{\xi^2 + \frac{c_1^2}{c^2}s^2} = \lambda_4 \\ (\Delta_{12}\Delta_{23} - \Delta_{13}\Delta_{22}) &= \Delta_1^*, (\Delta_{11}\Delta_{23} - \Delta_{13}\Delta_{21}) = \Delta_2^*, (\Delta_{11}\Delta_{22} - \Delta_{12}\Delta_{21}) = \Delta_3^* \\ \Delta_{1j} &= \frac{c_{13}}{\rho c_1^2} i\xi - \frac{c_{33}}{\rho c_1^2} d_j \lambda_j - \frac{\beta_3}{\beta_1} l_j + \frac{\beta_3}{\beta_1} a_3 l_j \lambda_j^2 - \frac{\beta_3}{\beta_1} l_j a_1 \xi^2 \quad j = 1, 2, 3 \\ \Delta_{2j} &= -\lambda_j + i\xi d_j \quad j = 1, 2, 3 \\ \Delta_{3j} &= -\lambda_j l_j, \quad j = 1, 2, 3 \end{aligned}$$

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