Transversely Isotropic Magneto-Visco Thermoelastic Medium with Vacuum and without Energy Dissipation

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ABSTRACT

In the present investigation the disturbances in a homogeneous transversely isotropic magneto-Visco thermoelastic rotating medium with two temperature due to thermomechanical sources has been addressed. The thermoelasticity theories developed by Green-Naghdi (Type II and Type III) both with and without energy dissipation has been applied to the thermomechanical sources. The Laplace and Fourier transform techniques have been applied to solve the present problem. As an application, the bounding surface is subjected to concentrated and distributed sources (mechanical and thermal sources). The analytical expressions of displacement, stress components, temperature change and induced magnetic field are obtained in the transformed domain. Numerical inversion techniques have been applied to obtain the results in the physical domain. Numerical simulated results are depicted graphically to show the effect of viscosity on the resulting quantities. Some special cases of interest are also deduced from the present investigation.

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1 INTRODUCTION

Duration the past few decades, widespread attention has been given to thermoelasticity theories that admit a finite speed for the propagation of thermal signals. In contrast to the conventional theories based on parabolic-type heat equation, these theories are referred to as generalized theories. Because of the experimental evidence in support of the finiteness of the speed of propagation of a heat wave, generalized thermoelasticity theories are more realistic than conventional thermoelasticity theories in dealing with practical problems involving very short time intervals and high heat fluxes such as those occurring in laser units, energy channels, nuclear reactors, etc. The phenomenon of coupling between the thermomechanical behavior of materials and magnetic behavior of materials has been studied since the 19th century. Chen and Gurtin [7], Chen et al. [8] and Chen et al. [9] have formulated a theory of heat conduction in deformable bodies which depends upon two distinct temperatures, the conductive temperature φ and the thermo dynamical temperature T. In case of time independent situations, the difference between these two temperatures is proportional to the heat supply, and in absence of heat supply, the two

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temperatures are identical. For time dependent problems, the two temperatures are different, regardless of the presence of heat supply. The two temperatures T, φ and the strain are found to have representations in the form of a travelling wave plus a response, which occurs instantaneously throughout the body (Boley and Tolins [4]).The wave propagation in two temperature theory of thermoelasticity was investigated by Warren and Chen [46]. Arani, Salari, Khademizadeh and Arefmanesh [1] have discussed magneto thermoelastic transient response of a functionally graded thick hollow sphere subjected to magnetic and thermoelastic fields. Khademizadeh, Arani and Salari [25] have studied stress analysis of magneto thermoelastic and induction magnetic field in FGM hallow sphere. Singh and Bala [43] have discussed propagation of waves in a two- temperature rotating thermoelastic solid half- space without energy dissipation. Green and Naghdi [16] postulated a new concept in thermoelasticity theories and proposed three models which are subsequently referred to as GN-I, II, and III models. The linearized version of model-I corresponds to classical thermoelastic model (based on Fourier's law). The linearized version of model-II and III permit propagation of thermal waves at finite speed. Green-Naghdi's second model (GN-II), in particular exhibits a feature that is not present in other established thermoelastic models as it does not sustain dissipation of thermal energy [17]. In this model the constitutive equations are derived by starting with the reduced energy equation and by including the thermal displacement gradient among other constitutive variables. Green-Naghdi's third model (GN-III) admits dissipation of energy. In this model the constitutive equations are derived by starting with the reduced energy equation where the thermal displacement gradient in addition to the temperature gradient is among the constitutive variables. Green and Naghdi [18] included the derivation of a complete set of governing equations of a linearized version of the theory for homogeneous and isotropic materials in terms of the displacement and temperature fields and a proof of the uniqueness of the solution for the corresponding initial boundary value problem.

A comprehensive work has been done in thermoelasticity theory with and without energy dissipation and thermoelasticity with two temperatures. Youssef [49] constructed a new theory of generalized thermoelasticity by taking into account two-temperature generalized thermoelasticity theory for a homogeneous and isotropic body without energy dissipation. Quintanilla [37] investigated thermoelasticity without energy dissipation of materials with microstructure. Kumar and Devi [27] discussed magneto thermoelastic with and without energy dissipation Half-Space in contact with Vacuum. Several researchers studied various problems involving two temperature e.g. (Kumar, Sharma and Garg [31]; Kaushal et al [23]; Kaushal Sharma and Kumar [24]; Kumar and Mukhopdhyay [29]; Ezzat and Awad [13]; Ezzat [14]; Sharma and Marin [41]; Sharma and Bhargav [41]; Sharma, Sharma and Bhargav [42]). Different authors have discussed different types of problems in viscoelasticity. Freudenthal [15] pointed out that most solids when subjected to dynamic loading exhibit viscous effects. The Kelvin -Voigt model [45] is one of the macroscopic mechanical models often used to describe the viscoelastic behavior of a material. This model represents the delayed elastic response subjected to stress where the deformation is time dependent. Iesan and Scalia [21] studied some theorems in the theory of thermo-viscoelasticity. Borrelli and Patria [5] investigated the discontinuity of waves through a linear thermoviscoelastic solid of integral type. Corr et al. [10] investigated the nonlinear generalized Maxwell fluid model for viscoelastic materials. Pal [35] studied the problem of torsional body forces in viscoelastic half-space. Effect of viscosity on wave propagation in anisotropic thermoelastic medium with three-phase-lag model was discussed by Kumar, Chawla and Abbas [26]. Effect of rotation, magnetic field and a periodic loading on radial vibrations thermo-viscoelastic non-homogeneous media was investigated by Basyouni, Mahmoud and Alzahrani [3]. Hilton [19] analyzed coupled longitudinal 1–d thermal and viscoelastic waves in media with temperature dependent material properties. Yadav, Kalkal and Deswal [47] investigated a state space problem of Two-Temperature generalized thermo-viscoelasticity with fractional order strain subjected to moving heat source. Sharma, Kumar and Lata [39] have studied the problem of disturbance due to inclined load in transversely isotropic thermoelastic medium with two temperatures and without energy dissipation.

In view of the fact that most of the large bodies like the earth, the moon and other planets have an angular velocity, as well as earth itself behaves like a huge magnet. It is important to study the propagation of thermoelastic waves in a rotating medium under the influence of magnetic field. So, the attempts are being made to study the propagation of finite thermoelastic waves in an infinite elastic medium rotating with angular velocity. Several authors (Das and Kanoria [11]; Kumar and Kansal [28]; Kumar and Rupender [30]; Atwa and Jahangir [2]; Mahmoud [33]; Sarkar and Lahiri [38]; Othman [34]; Lofty and Hassan [32]) have studied two-dimensional problem of generalized thermoelasticity to study the effect of rotation. In spite of all these investigations, no attempt has been made yet to study the response of thermomechanical sources in transversely magneto-Visco thermoelastic solid with two temperature and magnetic effect and in contact with vacuum in the context of Green Naghdi theories of type-II and type-III. The components of normal displacement, normal stress, tangential stress and conductive temperature subjected to concentrated normal force, uniformly distributed force and linearly distributed source are obtained by

using Laplace and Fourier transforms. Numerical computation has been performed by using a numerical inversion technique and the resulting quantities are shown graphically. Some particular cases are also discussed.

2 BASIC EQUATIONS

Following Ezzat [14], the simplified Maxwell's linear equation of electrodynamics for a slowly moving and perfectly conducting elastic solid are

$$curl\ \boldsymbol{h} = j + \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} \tag{1}$$

$$curl\ \boldsymbol{E} = -\mu_0 \frac{\partial \boldsymbol{h}}{\partial t} \tag{2}$$

$$\boldsymbol{E} = -\mu_0 \left(\frac{\partial u}{\partial t} \times \boldsymbol{H}_0\right) \tag{3}$$

$$div\ \boldsymbol{h} = 0 \tag{4}$$

Maxwell stress components are given by

$$T_{ij} = \mu_0 \left(H_i\ h_j + H_j\ h_i - H_k\ h_k\ \delta_{ij}\right) \tag{5}$$

where $\mathbf{H_0}$ − the external applied magnetic field intensity vector, $\mathbf{h}$ − the induced magnetic field vector, $\mathbf{E}$ − the induced electric field vector, $\mathbf{j}$ − the current density vector, $\mathbf{u}$ − is the displacement vector, the magnetic and electric permeabilities respectively, μ_0 and ε_0 are magnetic and electric permeability respectively , T_{ij} the component of Maxwell stress tensor and δ_{ij} the Kronecker delta.

The constitutive relations for a transversely isotropic thermoelastic medium are given by

$$t_{ij} = C_{ijkl}\ e_{kl} - \beta_{ij}\ T \tag{6}$$

Equation of motion for a transversely isotropic thermoelastic medium rotating uniformly with an angular velocity $\boldsymbol{\Omega} = \Omega n$, where *n* is a unit vector representing the direction of axis of rotation and taking into account Lorentz force

$$t_{ij,j} + F_i = \rho\{\ddot{u}_i + (\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times u))_i + (2\boldsymbol{\Omega} \times \dot{u})_i\} \tag{7}$$

The heat conduction equation, following Chandrasekharaiah [6] and Youssef [48] is

$$K_{ij}\ \varphi_{,ij} + K_{ij}^*\ \dot{\varphi}_{,ij} = \beta_{ij}\ T_0 \ddot{e}_{ij} + \rho C_E \ddot{T} \tag{8}$$

The strain displacement relations are

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \qquad i,j = 1,2,3 \tag{9}$$

where $F_i = \mu_0(\boldsymbol{j} \times \boldsymbol{H}_0)_i$ are the components of Lorentz force. $\beta_{ij} = C_{ijkl}\alpha_{ij}$ and $T = \varphi - a_{ij}\varphi_{,ij}$

$\beta_{ij} = \beta_i \delta_{ij}$, $K_{ij} = K_i \delta_{ij}$, $K_{ij}^* = K_i^* \delta_{ij}$, i is not summed C_{ijkl} ($C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk}$) are elastic parameters, β_{ij} is the thermal elastic coupling tensor, T is the temperature, T_0 is the reference temperature, t_{ij} are the components of stress tensor, e_{kl} are the components of strain tensor, u_i are the displacement components, ρ is the density, C_E is the specific heat, K_{ij} is the materialistic constant, K_{ij}^* is the thermal conductivity, a_{ij} are the two temperature parameters, α_{ij} is the coefficient of linear thermal expansion, Ω is the angular velocity of the solid.

3   FORMULATION AND SOLUTION OF THE PROBLEM

We consider a homogeneous perfectly conducting transversely isotropic magneto-Visco thermoelastic medium in contact with vacuum permeated by an initial magnetic field $\mathbf{H_0}$ acting along y-axis. The rectangular Cartesian co-ordinate system (x, y, z) having origin on the surface (z=0) with z-axis pointing vertically downwards into the medium is introduced. The surface of the half-space is subjected to thermomechanical load.

H = (0, H$_0$, 0)   **0**

$E_1(x, 0, t) = E_1^0(x, 0, t)$, $h(x, 0, t) = h^0(x, 0, t)$

Co-ordinate system and geometry of the problem
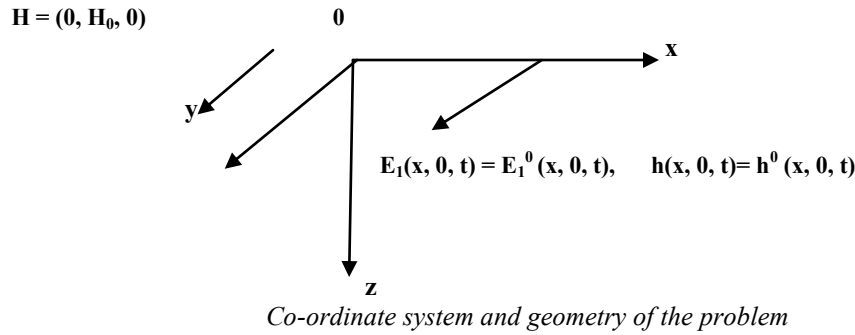
Following Kumar [31], we also assume that

$$\mathbf{\Omega} = (0, \Omega, 0) \tag{10a}$$

From the generalized Ohm's law

$$j_2 = 0 \tag{10b}$$

The current density components j_1 and j_3 are given as:

$$j_1 = -\varepsilon_0 \mu_0 H_0 \frac{\partial^2 w}{\partial t^2} \tag{10c}$$

$$j_3 = \varepsilon_0 \mu_0 H_0 \frac{\partial^2 u}{\partial t^2} \tag{10d}$$

In the vacuum, contacting the transversely isotropic thermoelastic half-space, the system of equations of electrodynamics is

$$curl\,\mathbf{h}^0 = \varepsilon_0 \frac{\partial \mathbf{E}^0}{\partial t} \tag{11}$$

$$curl\,\mathbf{E}^0 = -\mu_0 \frac{\partial \mathbf{h}^0}{\partial t} \tag{12}$$

$$div \ \boldsymbol{h}^0 = 0 \tag{13}$$

where $\boldsymbol{h}^0, \boldsymbol{E}^0$ are the induced magnetic and electric field vectors respectively in vacuum and μ_0, ε_0 are magnetic and electric permeability respectively. The above equations reduce to

$$(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}) \boldsymbol{h}^0 = 0 \tag{14}$$

where c is velocity of light given by $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$, and ∇^2 is the Laplacian operator. In this case, Maxwell stress becomes

$$T_{ij}^{\ 0} = \mu_0 (H_i \ h_j^{\ 0} + H_j h_i^{\ 0} - H_k \ h_k^{\ 0} \delta_{ij}) \tag{15}$$

$T_{ij}^{\ 0}$ are the components of Maxwell stress in vacuum. Following Slaughter [44], using appropriate transformations, on the set of Eqs. (6)- (7), we derive the basic equations for transversely isotropic thermoelastic solid. The components of displacement vector *u, v, w* and conductive temperature φ for the two dimensional problem have the form

$$\vec{u} = u(x,z,t), \ \vec{w} = w(x,z,t) \quad \text{and} \quad \varphi = \varphi(x,z,t) \tag{16}$$

Eqs. (7) and (8) with the aid of (16) ,yield

$$c_{11} \frac{\partial^2 u}{\partial x^2} + c_{13} \frac{\partial^2 w}{\partial x \partial z} + c_{44} (\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x \partial z}) - \beta_1 \frac{\partial}{\partial x} \{\varphi - (a_1 \frac{\partial^2 \varphi}{\partial x^2} + a_3 \frac{\partial^2 \varphi}{\partial z^2})\} - \mu_0 j_3 H_0 = \rho(\frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega \frac{\partial w}{\partial t}) \tag{17}$$

$$(c_{13} + c_{44}) \frac{\partial^2 u}{\partial x \partial z} + c_{44} \frac{\partial^2 w}{\partial x^2} + c_{33} \frac{\partial^2 w}{\partial z^2} - \beta_3 \frac{\partial}{\partial z} \{\varphi - (a_1 \frac{\partial^2 \varphi}{\partial x^2} + a_3 \frac{\partial^2 \varphi}{\partial z^2})\} + \mu_0 j_1 H_0 = \rho(\frac{\partial^2 w}{\partial t^2} - \Omega^2 w - 2\Omega \frac{\partial u}{\partial t}) \tag{18}$$

$$\left(k_1 + k_1^* \frac{\partial}{\partial t} \right) \frac{\partial^2 \varphi}{\partial x^2} + \left(k_3 + k_3^* \frac{\partial}{\partial t} \right) \frac{\partial^2 \varphi}{\partial z^2} = T_0 \frac{\partial^2}{\partial t^2} \left\{ \beta_1 \frac{\partial u}{\partial x} + \beta_3 \frac{\partial w}{\partial z} \right\} + \rho C_E \ddot{T} \tag{19}$$

$$(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}) \boldsymbol{h}^0 = 0 \tag{20}$$

and

$$t_{11} = c_{11} e_{11} + c_{13} e_{33} - \beta_1 T \tag{21}$$

$$t_{33} = c_{13} e_{11} + c_{33} e_{33} - \beta_3 T \tag{22}$$

$$t_{13} = 2 c_{44} e_{13} \tag{23}$$

where $T = \varphi - (a_1 \frac{\partial^2 \varphi}{\partial x^2} + a_3 \frac{\partial^2 \varphi}{\partial z^2})$

$$\beta_1 = (c_{11} + c_{12}) \alpha_1 + c_{13} \alpha_3, \qquad \beta_3 = 2 c_{13} \alpha_1 + c_{33} \alpha_3 \tag{24}$$

Assuming that the viscoelastic nature of the material is described by the Voight [45] model of linear viscoelasticity (Kaliski [22]), we replace the elastic constants C_{11}, C_{12}, C_{13}, C_{33}, C_{44} by $\overline{C_{11}}, \overline{C_{12}}, \overline{C_{13}}, \overline{C_{33}}, \overline{C_{44}}$.

Following Kumar [31], $\overline{\psi} = \psi \left(1 + Q_1 \dfrac{\partial}{\partial t}\right)$ where

$$\psi = C_{11},\ C_{12,} C_{13}, C_{33},\ C_{44} \tag{25}$$

We assume that medium is initially at rest. The undisturbed state is maintained at reference temperature. Then we have the initial and regularity conditions are given by

$$u(x,z,0) = 0 = u(x,z,0)$$
$$w(x,z,0) = 0 = w(x,z,0)$$
$$\varphi(x,z,0) = 0 = \varphi(x,z,0) \qquad For \quad z \geq 0,\ -\infty < x < \infty$$
$$(x,z,t) = w(x,z,t) = \varphi(x,z,t) = 0 \quad For \quad t > 0 \ when\ z \rightarrow \infty$$

To facilitate the solution, following dimensionless quantities are introduced:

$$x' = \frac{x}{L},\ z' = \frac{z}{L},\ u' = \frac{\rho c_1^2}{L \beta_1 T_0} u,\ w' = \frac{\rho c_1^2}{L \beta_1 T_0} w,\ T' = \frac{T}{T_0},\ t' = \frac{c_1}{L} t,\ t'_{11} = \frac{t_{11}}{\beta_1 T_0},\ t'_{33} = \frac{t_{33}}{\beta_1 T_0},$$

$$t'_{31} = \frac{t_{31}}{\beta_1 T_0}, \varphi' = \frac{\varphi}{T_0},\ a_1' = \frac{a_1}{L},\ a_3' = \frac{a_3}{L},\ h' = \frac{h}{H_0},\ \Omega' = \frac{L}{C_1}\Omega \tag{26}$$

Making use of (26) in Eqs. (17)- (20), with the aid of Eq. (25) after suppressing the primes, yield

$$\frac{\partial^2 u}{\partial x^2} + \delta_4 \frac{\partial^2 w}{\partial x \partial z} + \delta_2 \left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x \partial z}\right) - \frac{\partial}{\partial x}\left\{\varphi - \left(\frac{a_1}{L}\frac{\partial^2 \varphi}{\partial x^2} + \frac{a_3}{L}\frac{\partial^2 \varphi}{\partial z^2}\right)\right\} = \left(\frac{\varepsilon_0 \mu_0^2 H_0^2}{\rho} + 1\right)\frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega \frac{\partial w}{\partial t} \tag{27}$$

$$\delta_1 \frac{\partial^2 u}{\partial x \partial z} + \delta_2 \frac{\partial^2 w}{\partial x^2} + \delta_3 \frac{\partial^2 w}{\partial z^2} - \frac{\beta_3}{\beta_1}\frac{\partial}{\partial z}\left\{\varphi - \left(\frac{a_1}{L}\frac{\partial^2 \varphi}{\partial x^2} + \frac{a_3}{L}\frac{\partial^2 \varphi}{\partial z^2}\right)\right\} = \left(\frac{\varepsilon_0 \mu_0^2 H_0^2}{\rho} + 1\right)\frac{\partial^2 w}{\partial t^2} - \Omega^2 w - 2\Omega \frac{\partial u}{\partial t} \tag{28}$$

$$\varepsilon_1 \left(1 + \frac{\varepsilon_3}{\varepsilon_1}\frac{\partial}{\partial t}\right)\frac{\partial^2 \varphi}{\partial x^2} + \varepsilon_2 \left(1 + \frac{\varepsilon_4}{\varepsilon_2}\frac{\partial}{\partial t}\right)\frac{\partial^2 \varphi}{\partial z^2} = \varepsilon_5 \beta_1^2 \frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial x} + \frac{\beta_3}{\beta_1}\frac{\partial w}{\partial z}\right) + \frac{\partial^2}{\partial t^2}\left(\left\{\varphi - \frac{a_1}{L}\frac{\partial^2 \varphi}{\partial x^2} + \frac{a_3}{L}\frac{\partial^2 \varphi}{\partial z^2}\right\}\right) \tag{29}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} - \frac{c_1^2}{c^2}\frac{\partial^2}{\partial t^2}\right)h_0 = 0 \tag{30}$$

$$\delta_1 = \frac{\left(\overline{c_{13}} + \overline{c_{44}}\right)}{\overline{c_{11}}},\ \delta_2 = \frac{\overline{c_{44}}}{\overline{c_{11}}},\ \delta_3 = \frac{\overline{c_{33}}}{\overline{c_{11}}},\ \delta_4 = \frac{\overline{c_{13}}}{\overline{c_{11}}},\ \varepsilon_1 = \frac{k_1}{\rho C_E c_1^2},\ \varepsilon_2 = \frac{k_3}{\rho C_E c_1^2},$$

$$\varepsilon_3 = \frac{k_1^*}{L \rho C_E c_1},\ \varepsilon_4 = \frac{k_3^*}{L \rho C_E c_1},\ \varepsilon_5' = \frac{T_0}{\rho^2 C_E c_1^2},\ c_1^2 = \frac{\overline{c_{11}}}{\rho}$$

Apply Laplace and Fourier transforms defined by

$$\overline{f}(x,z,s) = \int_0^\infty f(x,z,t)e^{-st}dt \tag{31}$$

$$\hat{f}(\xi,z,s) = \int_{-\infty}^{\infty} \overline{f}(x,z,s)e^{i\xi x}dx_1 \tag{32}$$

On Eqs. (27) - (30), we obtain a system of homogeneous equations in terms of u, w, φ and h_0 which yield a non-trivial solution if determinant of coefficient $\{\tilde{u}, \tilde{w}, \tilde{\varphi}, \tilde{h}\}^T$ vanishes and we obtain the following characteristic equation

$$(PD^6 + QD^4 + RD^2 + S)(u, w, \varphi) = 0 \tag{33}$$

$$(D^2 - \varsigma_8)h = 0 \tag{34}$$

where

$$P = \delta_2\delta_3\varsigma_7 + \varepsilon_5\varsigma_5\delta_2$$

$$Q = \varsigma_{10}\varsigma_7\delta_3 + \delta_2\varsigma_7\varsigma_{11} - \delta_3\delta_2\varsigma_6 + \varepsilon_5\beta_3^2\varsigma_4\delta_2 - \varsigma_3^2\varsigma_7 - i\xi\varsigma_3 p_3\varsigma_5\varepsilon_5\beta_1^2 + \xi^2\varepsilon_5\delta_3\varsigma_5\beta_1^2 + i\xi\varepsilon_5\varsigma_5\beta_1\beta_3\varsigma_3 - \xi^2\varepsilon_5\beta_1^2\delta_3\varsigma_5$$

$$R = \varsigma_{10}\varsigma_{11}\varsigma_7 - \varsigma_{10}\delta_3\varsigma_6 - \varepsilon_5\beta_3^2\varsigma_{10}\varsigma_4 - \delta_2\varsigma_6\varsigma_{11} + \varsigma_3^2\varsigma_6 + \varsigma_2^2\varsigma_7 + p_3\varsigma_3\varsigma_4\varepsilon_5\beta_1^2 i\xi + i\xi\varsigma_3\varsigma_4\beta_1\beta_3\varepsilon_5 - \xi^2\varsigma_5\varsigma_{11}\beta_1^2\varepsilon_5$$

$$S = \varsigma_{10}\varsigma_{11}\varsigma_6 - \varsigma_2^2\varsigma_6 + \xi^2\varsigma_{11}\beta_1^2\varsigma_4\varepsilon_5$$

$$\zeta_1 = -\left(\frac{\varepsilon_0\mu_0^2 H_0^2}{\rho} + 1\right)s^2 + \Omega^2 \ , \ \varsigma_2 = -2\Omega s \ , \ \zeta_3 = (\delta_4 + \delta_2)i\xi \ , \ \zeta_4 = 1 + \frac{a_1}{L}\xi^2 \ , \ \varsigma_5 = \frac{a_3}{L}$$

$$\varsigma_6 = (\varepsilon_1 + \varepsilon_3 s)\xi^2 + s^2(1 + \frac{a_1}{L}\xi^2) \ , \ \varsigma_7 = \varepsilon_2 + \varepsilon_4 s + \frac{a_3}{L}s^2 \ , \ \varsigma_8 = \xi^2 + \frac{c_1^2}{c^2}s^2 \ , \ \varsigma_{10} = \varsigma_1 - \xi^2 \ , \ \varsigma_{11} = \varsigma_1 - \delta_2\xi^2,$$

$$\varepsilon_5 = \frac{T_0}{\rho^2 C_E c_1^2}s^2 \ , \ p_3 = \frac{\beta_3}{\beta_1}$$

The roots of the Eq. (31) and (32) are $\pm\lambda_i$, $(i = 1, 2, 3)$, the solution of the Eqs. (33) and (34) satisfying the radiation condition that u, w, φ and $h_0 \rightarrow 0$ *as* $z \rightarrow \infty$, can be written as:

$$\tilde{u} = A_1 e^{-\lambda_1 z} + A_2 e^{-\lambda_2 z} + A_3 e^{-\lambda_3 z} \tag{35}$$

$$\tilde{w} = d_1 A_1 e^{-\lambda_1 z} + d_2 A_2 e^{-\lambda_2 z} + d_3 A_3 e^{-\lambda_3 z} \tag{36}$$

$$\tilde{\varphi} = l_1 A_1 e^{-\lambda_1 z} + l_2 A_2 e^{-\lambda_2 z} + l_3 A_3 e^{-\lambda_3 z} \tag{37}$$

$$\tilde{h} = A_4 e^{-\lambda_4 z} \tag{38}$$

where d_i and l_i are coupling constants and given by

$$d_i = \frac{(\lambda_i^2(\varepsilon_5 i \xi\delta_3 - \varsigma_3\beta_3\beta_1\xi) - \lambda_i(\xi\beta_1\beta_3\varsigma_2) + \varepsilon_5 i\xi\varsigma_{11})}{(\lambda_i^4(\delta_3\varsigma_7 + \varepsilon_5\varsigma_5\beta_3^2) + \lambda_i^2(\varsigma_7\varsigma_{11} - \delta_3\varsigma_6 - \beta_3^2\varepsilon_5\varsigma_4) - \varsigma_6\varsigma_{11})} \quad i = 1, 2, 3$$

$$l_i = \frac{(\lambda_i^3(\varepsilon_5\varsigma_3 + \varepsilon_3 i\xi\varsigma_5 p_3) - \varsigma_2\varsigma_7\lambda_i^2(\varsigma_3\varsigma_6 + \varepsilon_3 i\xi p_3\varsigma_4)\lambda_i + \varsigma_6\varsigma_2)}{(\lambda_i^4(\delta_3\varsigma_7 + \varepsilon_5\varsigma_5\beta_3^2) + \lambda_i^2(\varsigma_7\varsigma_{11} - \delta_3\varsigma_6 - \beta_3^2\varepsilon_5\varsigma_4) - \varsigma_6\varsigma_{11})} \quad i = 1, 2, 3$$

4   BOUNDARY CONDITIONS

On the half-space surface $(z = 0)$ normal point force and thermal point source are applied. The appropriate boundary conditions are

$$t_{33} + T_{33} - T_{33}^0 = -F_1\psi_1(x)\delta(t) \tag{39}$$

$$t_{31} = 0 \tag{40}$$

$$\frac{\partial\varphi}{\partial z} = F_2\psi_1(x)\delta(t) \qquad at \quad z = 0 \tag{41}$$

where F_1 is the magnitude of the force applied, F_2 is the constant temperature applied on the boundary, $\psi_1(x)$ specifies the source distribution function along x axis.

The transverse components of the magnetic field intensity are continuous across the surface of the half- space

$$h(x,0,t) = h^0(x,0,t) \tag{42}$$

The transverse components of the electric field intensity are continuous across the surface of the half-space.

$$E_1(x,0,t) = E_1^0(x,0,t) \tag{43}$$

Since the relative permeabilities are very nearly unity, it follows from Eqs. (5), (15) and (40) that

$$T_{33} = T_{33}^0 \tag{44}$$

and the condition (37) reduces to

$$t_{33} = -F_1\psi_1(x)\delta(t) \tag{45}$$

Applying the Laplace and Fourier transform defined by (31)-(32) on the boundary conditions (39)-(43) and with the help of Eqs. (5), (23)-(25), (26), (35)-(38), we obtain the components of displacement, normal stress, tangential stress, conductive temperature and induced magnetic field (in vacuum) as:

$$u = \frac{-F_1\psi_1(\xi)}{\Delta}(\Delta_1 e^{-\lambda_1 z} + \Delta_2 e^{-\lambda_2 z} + \Delta_3 e^{-\lambda_3 z}) + \frac{F_2\psi_2(\xi)}{\Delta}(\Delta_1^* e^{-\lambda_1 z} + \Delta_2^* e^{-\lambda_2 z} + \Delta_3^* e^{-\lambda_3 z}) \tag{46}$$

$$w = \frac{-F_1\psi_1(\xi)}{\Delta}(d_1\Delta_1 e^{-\lambda_1 z} + d_2\Delta_2 e^{-\lambda_2 z} + d_3\Delta_3 e^{-\lambda_3 z}) + \frac{F_2\psi_2(\xi)}{\Delta}(d_1\Delta_1^* e^{-\lambda_1 z} + d_2\Delta_2^* e^{-\lambda_2 z} + d_3\Delta_3^* e^{-\lambda_3 z}) \tag{47}$$

$$t_{33} = \frac{-F_1\psi_1(\xi)}{\Delta}(\Delta_{11}\Delta_1 e^{-\lambda_1 z} + \Delta_{12}\Delta_2 e^{-\lambda_2 z} + \Delta_{13}\Delta_3 e^{-\lambda_3 z}) + \frac{F_2\psi_2(\xi)}{\Delta}(\Delta_{11}\Delta_1^* e^{-\lambda_1 z} + \Delta_{12}\Delta_2^* e^{-\lambda_2 z} + \Delta_{13}\Delta_3^* e^{-\lambda_3 z}) \tag{48}$$

$$t_{31} = \frac{-F_1\psi_1(\xi)}{\Delta}(\Delta_{21}\Delta_1 e^{-\lambda_1 z} + \Delta_{22}\Delta_2 e^{-\lambda_2 z} + \Delta_{23}\Delta_3 e^{-\lambda_3 z}) + \frac{F_2\psi_2(\xi)}{\Delta}(\Delta_{21}\Delta_1^* e^{-\lambda_1 z} + \Delta_{22}\Delta_2^* e^{-\lambda_2 z} + \Delta_{23}\Delta_3^* e^{-\lambda_3 z}) \tag{49}$$

$$\varphi = \frac{-F_1\psi_1(\xi)}{\Delta}(\Delta_{31}\Delta_1 e^{-\lambda_1 z} + \Delta_{32}\Delta_2 e^{-\lambda_2 z} + \Delta_{33}\Delta_3 e^{-\lambda_3 z}) + \frac{F_2\psi_2(\xi)}{\Delta}(\Delta_{31}\Delta_1^* e^{-\lambda_1 z} + \Delta_{32}\Delta_2^* e^{-\lambda_2 z} + \Delta_{33}\Delta_3^* e^{-\lambda_3 z}) \tag{50}$$

$$h = \frac{-F_1\varepsilon_0\mu_0 H_0 s^2}{\lambda_4\Delta}(d_1\Delta_1 e^{-\lambda_1 z} + d_2\Delta_2 e^{-\lambda_2 z} + d_3\Delta_3 e^{-\lambda_3 z}) + \frac{F_2\varepsilon_0\mu_0 H_0 s^2}{\lambda_4\Delta}(d_1\Delta_1^* e^{-\lambda_1 z} + d_2\Delta_2^* e^{-\lambda_2 z} + d_3\Delta_3^* e^{-\lambda_3 z}) \tag{51}$$

where

$$(\Delta_{22}\Delta_{33}-\Delta_{23}\Delta_{32})=\Delta_1,(\Delta_{23}\Delta_{31}-\Delta_{21}\Delta_{33})=\Delta_2,(\Delta_{21}\Delta_{32}-\Delta_{22}\Delta_{31})=\Delta_3,\ \sqrt{\xi^2+\frac{c_1^2}{c^2}s^2}=\lambda_4$$

$$(\Delta_{12}\Delta_{23}-\Delta_{13}\Delta_{22})=\Delta^*_1,\ \ (\Delta_{11}\Delta_{23}-\Delta_{13}\Delta_{21})=\Delta^*_2,\ \ (\Delta_{11}\Delta_{22}-\Delta_{12}\Delta_{21})=\Delta^*_3$$

$$\Delta_{1j}=\frac{c_{13}}{\rho c_1^2}i\,\xi-\frac{c_{33}}{\rho c_1^2}d_i\,\lambda_j-\frac{\beta_3}{\beta_1}l_j+\frac{\beta_3}{\beta_1}a_3l_j\,\lambda_j^{\ 2}-\frac{\beta_3}{\beta_1}a_1l_j\,\xi^2\qquad j=1,2,3$$

$$\Delta_{2j}=-\lambda_j+i\,\xi d_j\qquad j=1,2,3$$

$$\Delta_{3j}=-\lambda_j l_j\qquad\qquad j=1,2,3$$

where d_i, l_i are coupling constants and λ_i are the roots of the Eqs. (31) and (32).

4.1 Mechanical force on the surface of half-space

Taking $(F_2=0)$ in Eqs. (46)- (51), we obtain the components of displacement , normal stress, tangential stress , conductive temperature and induced magnetic field (in vacuum) due to mechanical force.

4.2 Thermal source on the surface of half-space

Taking $(F_1=0)$ in Eqs. (46)- (51), we obtain the components of displacement, normal stress, tangential stress, conductive temperature and induced magnetic field (in vacuum) due to thermal source.

4.3 Green's function

Following Kumar [31], to synthesize the Green's function, i.e. the solution due to concentrated normal force and thermal point source on the half-space is obtained by setting

$$\psi_1(x)=\delta(x)\tag{52}$$

In Eqs. (39) and (41), applying the Laplace and Fourier transforms defined by (31)-(32) on the Eq. (52) gives

$$\hat{\psi}_1(\xi)=1\tag{53}$$

Using (53) in (46)-(51), we obtain the components of displacement, stress and conductive temperature and induced magnetic effect.

4.4 Influence function

The method to obtain the half-space influence function, i.e. the solution due to distributed force/source applied on the half space is obtained by setting

$$\psi_1(x)=\begin{cases}1\ if\ |x|\le m\\0\,if\ |x|>m\end{cases}\tag{54}$$

In Eqs. (39) and (41), the Laplace and Fourier transforms of $\psi_1(x)$ with respect to the pair (x,ξ) for the case of a uniform strip load of non-dimensional width $2m$ applied at origin of co-ordinate system $x=z=0$ in the dimensionless form after suppressing the primes becomes

$$\hat{\psi}_1(\xi) = \left[\frac{2\sin(\xi m)}{\xi}\right] , \quad \xi \neq 0 \tag{55}$$

The expressions for displacement, stresses and conductive temperature can be obtained for uniformly distributed normal force and thermal source by replacing $\psi_1(x)$ from (53) respectively in Eqs. (46)-(51).

5   PARTICULAR CASES

i.   If $k_1^* = k_3^* = 0$, $\Omega = 0$ and $Q_1 = Q_2 = Q_3 = Q_4 = Q_5 = 0$ in Eqs. (46)- (51), we obtain the resulting expressions for transversely isotropic thermoelastic solid without energy dissipation and with two temperature.

ii.   If $k_1 = k_3 = 0$, $Q_1 = Q_2 = Q_3 = Q_4 = Q_5 = 0$ in Eqs. (46)- (51), we obtain the resulting expressions for transversely isotropic thermoelastic solid with and without energy dissipation and with two temperature without rotation.

iii. If $a_1 = a_3 = 0$, $Q_1 = Q_2 = Q_3 = Q_4 = Q_5 = 0$ in Eqs. (46)- (51), we obtain the corresponding expressions for displacements, and stresses and conductive temperature for transversely isotropic thermoelastic solid with rotation and with and without energy dissipation.

iv.   If we take $\overline{c_{11}} = \lambda + 2\mu = \overline{c_{33}}$, $\overline{c_{44}} = \mu$, $\beta_1 = \beta_3 = \beta$, $\alpha_1 = \alpha_3 = \alpha$, $k_1 = k_3 = k$, $k_1^* = k_3^* = k^*$ in Eqs. (46)-(51), we obtain the corresponding expressions for displacements, and stresses and conductive temperature for isotropic Visco thermoelastic solid with combined effects of rotation, two temperature and with and without energy dissipation.

6   INVERSION OF THE TRANSFORMATION

To obtain the solution of the problem in physical domain, we must invert the transforms in Eqs. (46)- (51). Here the displacement components, normal and tangential stresses and conductive temperature are functions of z, the parameters of Laplace and Fourier transforms s and ξ respectively and hence are of the form $f(\xi, z, s)$. To obtain the function $f(x, z, t)$ in the physical domain, we first invert the Fourier transform using

$$\bar{f}(x, z, s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\xi x_1}\hat{f}(\xi, z, s)d\xi = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\cos(\xi x)f_e - i\sin(\xi x)f_o\right|d\xi \tag{56}$$

where f_e and f_o are respectively the odd and even parts of $\hat{f}(\xi, z, s)$ Thus the expression (56) gives the Laplace transform $\bar{f}(x, z, s)$ of the function $f(x, z, t)$. Following Honig and Hirdes [20], the Laplace transform function $\bar{f}(x, z, s)$ can be inverted to $f(x, z, t)$. The last step is to calculate the integral in Eq. (56). The method for evaluating this integral is described in Press et al. [36]. It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

7   NUMERICAL RESULTS AND DISCUSSION

Following Dhaliwal and Singh [12], cobalt material has been taken for thermoelastic material as:

$$c_{11} = 3.071 \times 10^{11} Nm^{-2}, c_{33} = 3.581 \times 10^{11} Nm^{-2}, c_{13} = 1.027 \times 10^{11} Nm^{-2},$$

$$c_{44} = 1.510 \times 10^{11} Nm^{-2}, \rho = 8.836 \times 10^3 Kgm^{-3}, T_0 = 298^0 K,$$

$$C_E = 4.27 \times 10^2 JKg^{-1} deg^{-1}, K_1 = 0.690 \times 10^2 wm^{-1} deg^{-1},$$

$$K_3 = 0.690 \times 10^2 wm^{-1} deg^{-1}, \beta_1 = 7.04 \times 10^6 Nm^{-2} deg^{-1},$$

$$\beta_3 = 6.90 \times 10^6 Nm^{-2} deg^{-1}, K_1^* = 0.02 \times 10^2 Nsec^{-2} deg^{-1},$$

$$K_3^* = 0.04 \times 10^2 Nsec^{-2} deg^{-1}, \mu_0 = 1.2571 \times 10^{-6} Hm^{-1},$$

$$H_0 = iJm^{-1}nb^{-1}, \varepsilon_0 = 8.838 \times 10^{-12} Fm^{-1}, F_1 = 1 and F_2 = 1$$

In case of non-zero value with non-dimensional parameter $L = 1$. Using the above values, the graphical representations of normal displacement, induced magnetic effect, normal stress and conductive temperature for transversely isotropic magneto thermoelastic have been investigated for normal force/ thermal source and uniformly distributed force/source. Effect of viscosity on the various quantities with distance x has been shown.

For a particular model of heat conducting transversely isotropic magneto – Visco thermoelastic solid half space, we take the values $Q_1 = 0.5, Q_2 = 0.75, Q_3 = 1.0, Q_4 = 1.5, Q_5 = 2.0$ and for without viscous effect, we take $Q_i = 0 (i = 1, 2, 3, 4, 5)$.

Solid line represents the transversely isotropic magneto thermoelastic with viscosity (VS).

Solid line with centre symbol circle represents transversely isotropic magneto thermoelastic without viscosity (W VS).

8   MECHANICAL FORCES ON THE SURFACE OF HALF-SPACE
8.1 Concentrated force

Fig.1 shows the variation of normal displacement with distance x. We notice that the values of u_3 (VS) increase smoothly for the whole range whereas u_3 (WVS) first faces an increase for the range $0 \le x \le 2$ followed by a decrease for $2 \le x \le 4$ and increases monotonically for the rest. Fig.2 exhibits the variations of normal stress t_{33} with distance x. We notice that the values of t_{33} (VS) decrease monotonically for the whole range whereas the trends are oscillatory with descending amplitudes corresponding to WVS. Variations of conductive temperature φ with distance x are examined in the Fig.3 We find that variations of φ (VS) increase monotonically for the whole range whereas corresponding to WVS, first we find a decrease for the range $0 \le x \le 3$ and then the variations increase smoothly for the range $3 \le x \le 7$ followed by oscillatory trends. Fig.4 exhibits the variations of induced magnetic effect h with distance x. Here, we notice that the trends of variations corresponding to VS are decreasing for the whole range whereas corresponding to W VS the trends are also decreasing for the range $4 \le x \le 10$ and are oscillatory for the range $0 \le x \le 4$.
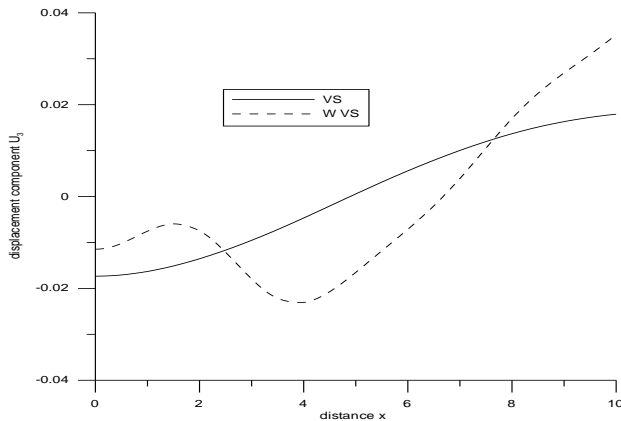

Fig.1
Variation of normal displacement w with distance x (concentrated normal force).
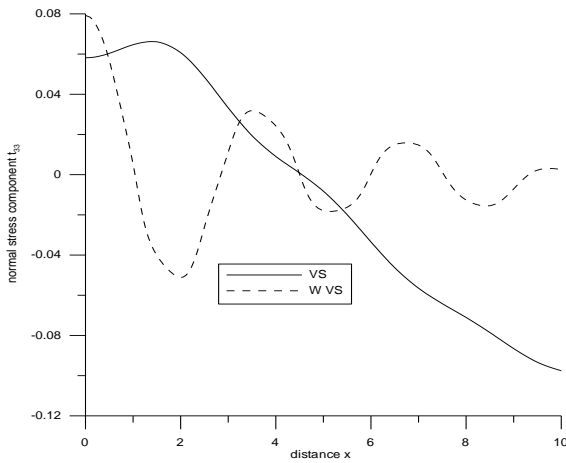
Fig.2
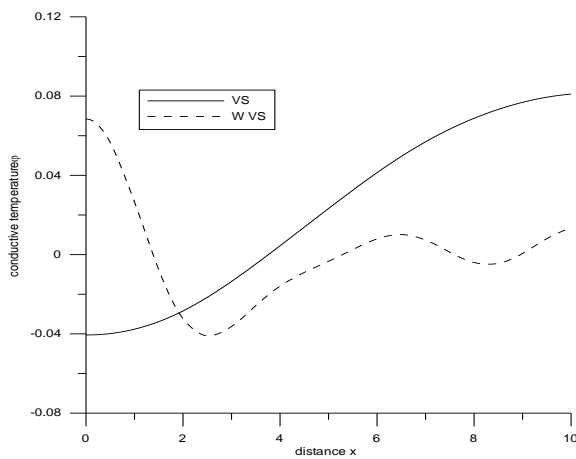Variation of normal stress t_{33} with distance x (concentrated normal force).

Fig.3
Variation of conductive temperature φ with distance x (concentrated normal force).
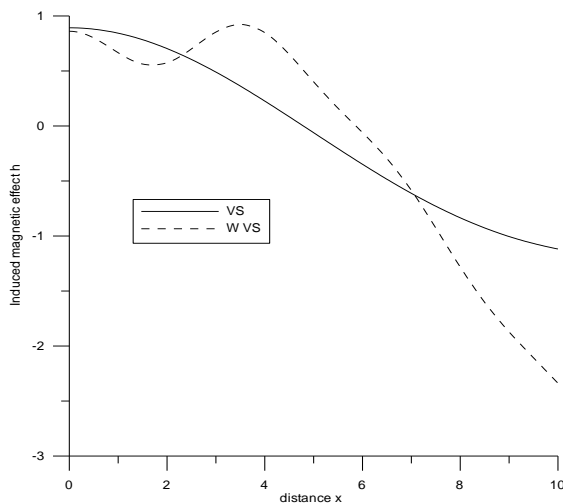

Fig.4
Variation of induced magnetic effect h with distance x (concentrated normal force).

8.2 Uniformly distributed force

Fig.5 exhibits normal displacement u_3 with distance x. Here, we notice that corresponding to WVS, the variations decrease sharply for the range the $0 \leq x \leq 4$ and increase slowly with vibrations for the rest. Corresponding to VS, the trends are oscillatory with varying amplitudes for the whole range. Fig.6 displays the variations of normal stress

t_{33} with distance x. It is seen that with viscosity, trends are oscillatory with small sharp amplitudes whereas without viscosity the variations of t_{33} increase sharply for the range $0 \le x \le 4$ and follow oscillatory trends afterwards.

Fig. 7 shows the variations in conductive temperature φ with distance x. Here, we find that the trends are oscillatory corresponding to both the cases with different amplitudes. Fig.8 gives variations of induced magnetic effect h with distance x. Due to viscosity, the trends of variations are oscillatory near the boundary surface whereas without viscosity, initially, there is a sharp increase for the range $0 \le x \le 4$ and the trends are oscillatory for the rest.
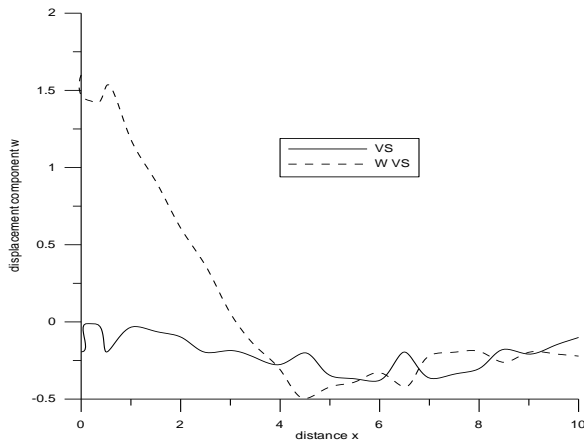
Fig.5
Variation of normal displacement w with distance x (uniformly distributed force).
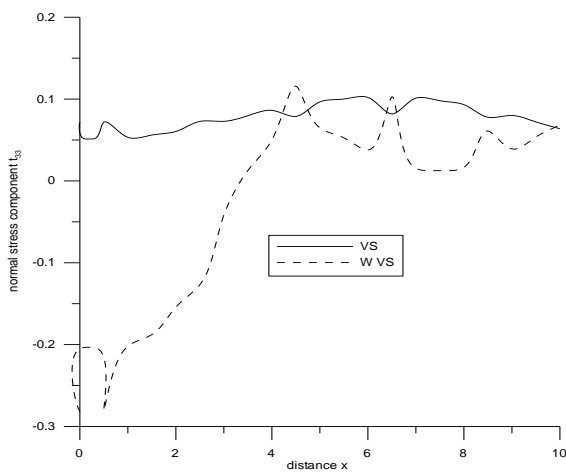
Fig.6
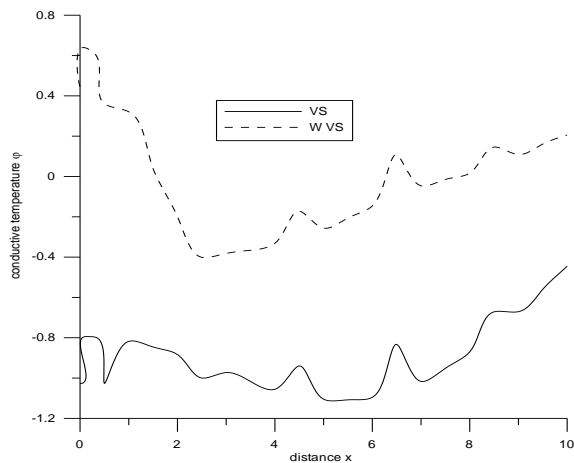Variation of normal stress t_{33} with distance x (uniformly distributed force).

Fig.7
Variation of conductive temperature φ with distance x (uniformly distributed force).
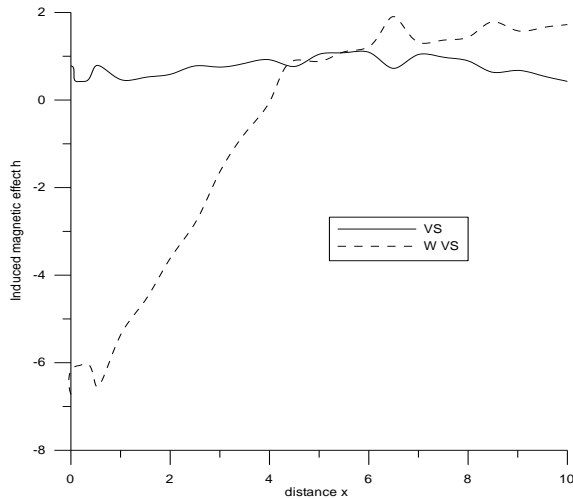
Fig.8
Variation of induced magnetic effect h with distance x (uniformly distributed force).

8.3 Thermoelastic interaction due to thermal sources
8.3.1 Thermal point source

Fig. 9 exhibits the behaviour of normal displacement u_3 with distance x. Here we find that the variations corresponding to VS increase for the range $0 \leq x \leq 5$ and decrease for the rest whereas corresponding to WVS, the variations follow oscillatory pattern with descending amplitudes. Fig. 10 displays the variations for normal stress t_{33}. Here we notice that corresponding to VS, the trends are increasing with oscillations in between. Corresponding to WVS, the trends are oscillatory. Fig. 11 displays the variations of conductive temperature φ with distance x. Here, we find that, corresponding to VS, there is a small increase for the range $0 \leq x \leq 5$ which is followed by a sharp decrease for the rest. Corresponding to WVS, the trends are oscillatory with decreasing amplitudes. Fig.12 shows the variations of induced magnetic effect with distance x. Here, we notice that corresponding to VS, the variations are decreasing whereas corresponding to WVS the trends are oscillatory.
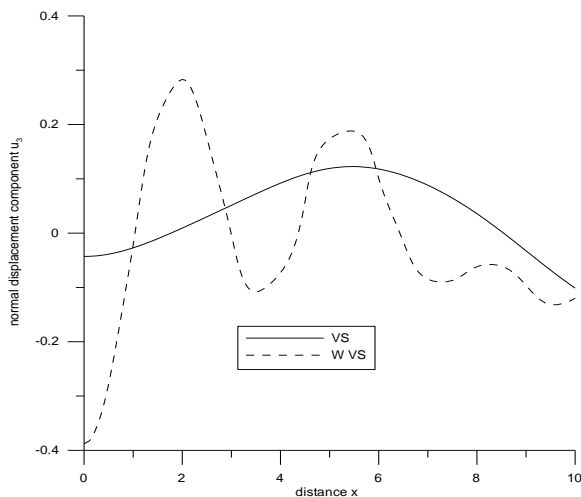

Fig.9
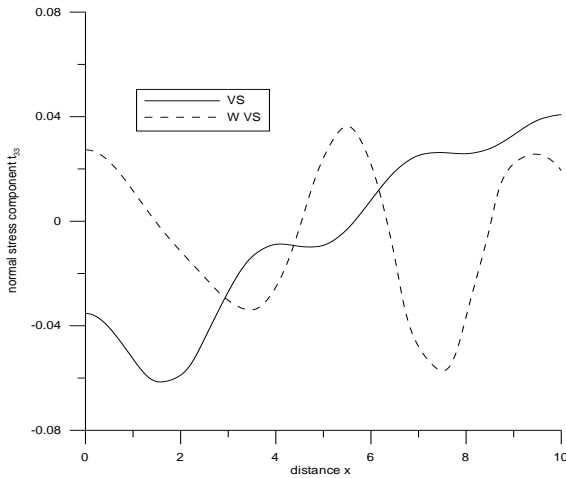Variation of normal displacement w with distance x (thermal point source).

Fig.10
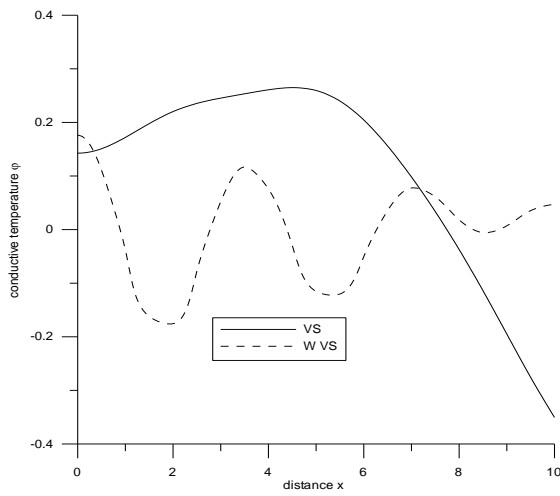Variation of normal stress t_{33} with distance x (thermal point source).

Fig.11
Variation of conductive temperature φ with distance x (thermal point source).
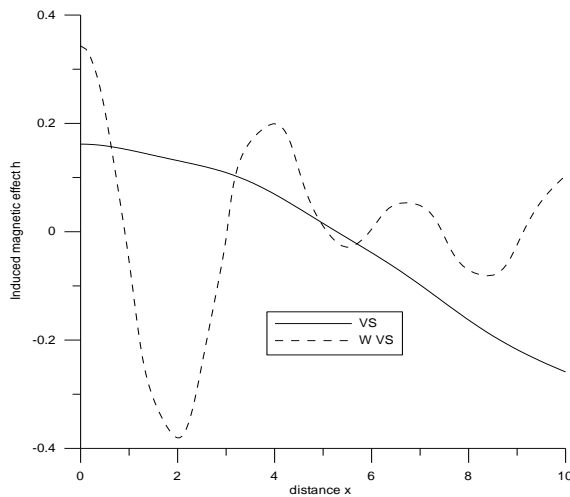

Fig.12
Variation of induced magnetic effect h with distance x (thermal point source).

8.3.2 Uniformly distributed thermal source

Fig.13 exhibits the trends of normal displacement u_3 with distance x. Here we notice that corresponding to VS, the variations are near the boundary surface and are in form of vibrations whereas corresponding to WVS the trends are

oscillatory. Fig.14 displays the variations of normal stress t_{33} .We find that the trends are in form of vibrations which corresponding to VS, increase for the range $0 \leq x \leq 6$ and decrease for the rest whereas corresponding to WVS ,the trends are opposite. Fig.15 shows the variations of conductive temperature φ with distance x. Here the variations are opposite as discussed in Fig.14. Fig.16 shows variations of induced magnetic effect h with distance x. Here, corresponding to both the cases, the variations are similar with change of amplitude and are in form of vibrations.
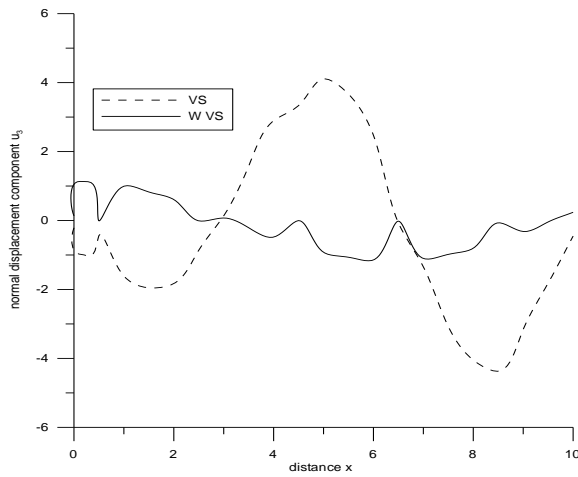
Fig.13
Variation of normal displacement w with distance x (uniformly distributed thermal source).
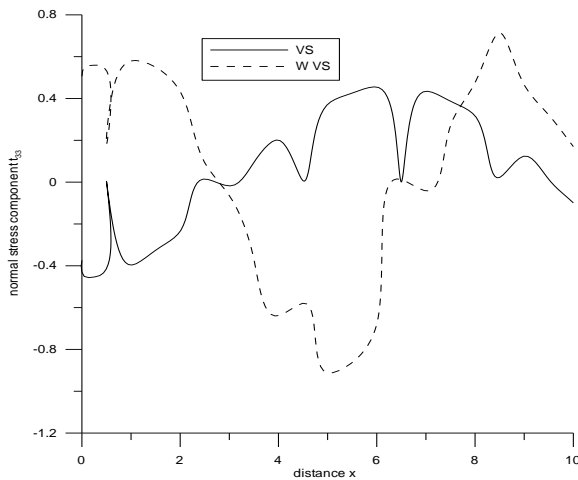
Fig.14
Variation of normal stress t_{33} with distance x (uniformly distributed thermal source).
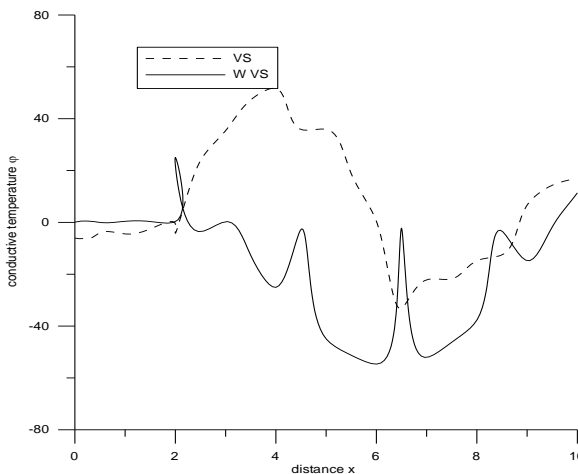
Fig.15
Variation of conductive temperature φ with distance x (uniformly distributed thermal source).
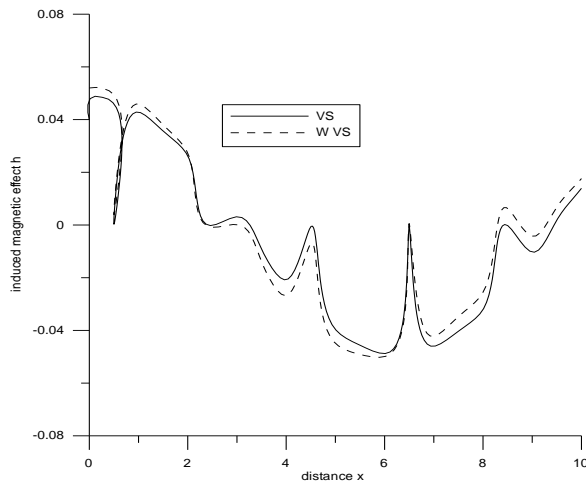
Fig.16
Variation of induced magnetic effect h with distance x (uniformly distributed thermal source).

9 CONCLUSIONS

It is observed from the graphs that viscosity has a sound impact on the deformation of transversely isotropic magneto-Visco thermoelastic solid. With viscosity, the trends of variations are either increasing or decreasing and somewhere in the form of vibrations whereas without viscosity the trends are oscillatory. From the figures, it is observed that the viscosity decreases the values of normal displacement, normal stress t_{33}, conductive temperature φ and induced magnetic effect h due to CNF (Concentrated Normal Force) near the application of the source. For UDF (Uniformly Distributed Force), viscosity decreases the value of normal displacement, φ and increases the value of normal stress t_{33}, magnetic effect h. Due to thermal source, viscosity increases the value of normal displacement w whereas it decreases the value of normal stress t_{33}, φ and h. For UDTS (Uniformly Distributed Thermal Source), viscosity increases the value of normal displacement u_3 and φ whereas it decreases the value of normal stress t_{33}, h near the application of source. The results provide a motivation to investigate conducting thermoelastic materials as a new class of applicable thermoelastic solids. The results presented in this paper will be useful for researchers in material science, physicists as well as for those are working on the development of magneto- Visco thermoelasticity and in particular situations as in geophysics, optics, acoustics, geomagnetic and oil prospecting etc. The used methods in the present article are applicable to a wide range of problems in thermodynamics and Visco thermoelasticity.

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