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Wave Propagation at the Boundary Surface of Elastic Layer Overlaying a Thermoelastic Without Energy Dissipation Half-space

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ABSTRACT

The present investigation is to study the surface wave propagation at imperfect boundary between an isotropic thermoelastic without energy dissipation half-space and an isotropic elastic layer of finite thickness. The penetration depth of longitudinal, transverse, and thermal waves has been obtained. The secular equation for surface waves in compact form is derived after developing the mathematical model. The components of temperature distribution, normal and tangential stress are computed at the interface and presented graphically. The effect of stiffness is shown on the resulting amplitudes and the effect of thermal is shown on the penetration depth of various waves. A particular case of interest is also deduced. Some special cases of interest are also deduced from the present investigation.

Keywords: Thermoelasticity without energy dissipation; Stiffness; Amplitudes.

1 INTRODUCTION

THERE are two possible boundary conditions at an interface of a solid state interface. One boundary condition is for perfectly bonded interface and other is slip boundary condition. A generalization of this concept is that of imperfectly bonded interface for which the displacement across the surface need not be continuous. Imperfect bonding mean that the traction is continuous across the interface but the small displacement is not. The small vector difference in the displacement is assumed to depend linearly on the traction vector. Precisely jumps in the displacement components are assumed to be proportional (in terms of spring factor type interface parameters) to their respective interface components. The infinite values of interface parameters mean vanishing of displacement jumps and therefore correspond to perfect bond interface and zero values of interface parameter correspond to free semi space. Recently, various authors have used the imperfect conditions at an interface to study various types of problems. Interface modeling has been subject of numerous studies in material science and composite structure. The importance of researches in this topic cannot be overemphasized as it is directly related to the prediction of the overall materials properties, delamination, transmission of force, etc (Refs [1-9] have been introduced the three phase and like spring models).

The classical theory of heat conduction based on the Fourier's law (i.e. the heat flux is proportional to temperature gradient) leads to the diffusion equation, which is not well accepted from a physical point of view. Theoretical consideration of this problem has been the subject of many investigations, both in cases of stationary rigid solid and deformable elastic solid. A comprehensive review is given by Joseph and Preziosi [10] who cite a large number of papers on this subject. The articles of Dreyer and Struchtrup [11] and Caviglia et al. [12] provide an

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extensive survey of work on experiments involving the propagation of heat as thermal wave. Extensive reviews on the second sound theories can be found in the work of Chandrasekharaiah [13] and Muller and Ruggeri [14].

In the recent survey of Chandrasekharaiah [15] and Hetnarski and Ignazack [16], the theory proposed by Green and Naghdi [17-21] is considered as an alternative way of formulation of the propagation of heat. This theory is developed in a rational way to produce a fully consistent theory which is capable of incorporating thermal pulse transmission in a logical manner. Green and Naghdi use an entropy equality rather than entropy inequality. The characterization of material response for the thermal phenomena is based on three types of constitutive functions, labeled as type I, II and III. The nature of these types of constitutive equations are such as, when the theory of type I is linearized, the parabolic equation of heat conduction arises(i.e. theory is based on Fourier's Law), whereas linearized version of type II and III permit propagation of thermal wave finite speed, which is more suitable from physical point of view. In the present paper, we are dealing with theory of type II. This type of theory does not involve the dissipation of energy. So it is generally known as thermoelasticity without energy dissipation. It is pertinent to recall that Green and Naghdi [18-20] wrote: "this type of theory …type II, since it involves no dissipation of energy is perhaps a more natural candidate for its identification as thermoelasticity than the usual theory". In this paper, linear model is adopted to represent the imperfectly bonded interface conditions. The linear model is simplified and idealized situation of imperfectly bonded interface, where the discontinuities in displacements at interfaces have a linear relationship with the interface stresses.

Scott [22] studied the energy and dissipation of Inhomogeneous plane waves in thermoelasticity. Ciarletta [23] has extended thermoelasticity without energy dissipation to take into account the micropolar effects. Kalpakides and Maugin [25] derived the conservation laws for the Green-Nagadi theory of thermoelasticity without energy dissipation. Recently, Othman and Song [25] discussed the reflection of plane waves from an elastic solid half-space under hydrostatic initial stress without energy dissipation. Chirita and Ciarletta [26] investigated the spatial behavior for non-standard problem in linear thermoelasticity without energy dissipation. Jiangong Zhang and Xue [27] studied the generalized thermoelastic waves in functionally graded plates without energy dissipation, Jiangong, et al. [28] discussed circumferential thermoelastic waves in orthotropic cylindrical curved plates without energy dissipation. Youssef [29] investigated the theory of two temperature thermoelasticity without energy dissipation. Jiangong et al. [30] studied the guided thermoelastic wave propagation in layered plates without energy dissipation.

The aim of the present paper is to study the wave propagation at imperfect boundary between an isotropic elastic layer and isotropic thermoelastic without energy dissipation half space. The exact nature of the layers beneath the earth's surface is not known. One has, therefore, to consider various appropriate models for the purpose of theoretical investigations. These problems not only provide better information about the internal composition of the earth but also helpful in exploration of valuable materials beneath the earth surface. The penetration depth of different waves has been plotted graphically. The expression for amplitude ratio of normal and tangential stress are also obtained and depicted graphically. Some special cases of interest are also deduced from the present investigation.

2 BASIC EQUATIONS

Following Green and Naghdi [20], the basic governing equations for thermoelastic without energy dissipation solid in the absence of body forces and heat sources are

(i) Constitutive relations:

$$\sigma_{ij} = \lambda u_{r,r}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) - \beta\delta_{ij}T \tag{1}$$

(ii) Equation of motion

$$\mu u_{i,jj} + (\lambda + \mu)u_{j,ij} - \beta T_{,i} = \rho\ddot{u}_i \tag{2}$$

(iii) Heat conduction equation

$$\rho C^* \frac{\partial^2 T}{\partial t^2} + \beta T_0 \frac{\partial^2(\nabla\cdot\vec{u})}{\partial t^2} = KT_{,ii}, \qquad i,j = 1,2,3 \tag{3}$$

Here, $\beta = (3\lambda + 2\mu)\alpha_t$ where λ and μ are Lame's constants, α_t is the coefficient of linear thermal expansion ρ and C^* are, respectively. The density and specific heat at constant strain $\sigma_{ij}(=\sigma_{ji})$, are respectively, the components of stress and strain tensor, u is the component of displacement vector and K is the thermal conductivity, T_0 is the reference temperature assumed to be such that and t is the time. The symbols "," and "." correspond to partial derivative and time derivative w.r.t. spatial variable, respectively, δ_{ij} is the Kronecker delta and $\nabla = \hat{i}(\partial/\partial x_1) + \hat{j}(\partial/\partial x_2) + \hat{k}(\partial/\partial x_2)$. Following Bullen [21], the equations of motion and constitutive relations in isotropic elastic medium are given by

$$(\lambda^e + \mu^e)\nabla(\nabla \cdot u^e) + \mu^e \nabla^2 u^e = \rho^e \ddot{u}^e \tag{4}$$

$$\sigma_{ij}^e = \lambda^e \theta^e \delta_{ij} + 2\mu^e \varepsilon_{ij} \tag{5}$$

where

$$\theta^e = u_{k,k}^e \tag{6}$$

$$\varepsilon_{ij}^e = (u_{i,j}^e + u_{j,i}^e)/2, \qquad i,j = 1,2,3 \tag{7}$$

Here, $\vec{u}^e = (u_1^e, u_2^e, u_3^e)$ is the displacement vector, ρ^e is the density of the isotropic medium and λ^e and μ^e are the Lame's constants and $\nabla^2 = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2 + \partial^2/\partial x_3^2$.

3 FORMULATION AND SOLUTION OF THE PROBLEM

We consider an isotropic elastic layer (Medium M_1) of thickness H overlaying an isotropic, thermoelastic without energy dissipation half-space (Medium M_2). The origin of the co-ordinate system $(x_1, x_2\ x_3)$ is taken at any point on the horizontal surface and x_1-axis in the direction of wave propagation and x_3- axis pointing vertically downward into the half space so that all particles on a line parallel to x_2- axis are equally displaced. Therefore, all the field quantities will be independent of x_2- axis co-ordinate. The interface between isotropic elastic layer and thermoelastic without energy dissipation half space has been taken at an imperfect boundary see Fig. 1. The displacement vector $\vec{u}$, and temperature T for medium M_2 are taken as

$$\vec{u} = (u_1, 0, u_3), \mathrm{T}(x_1, x_2, t) \tag{8}$$

and displacement vector $\vec{u}^e$ for the layer (Medium M_1) is taken as

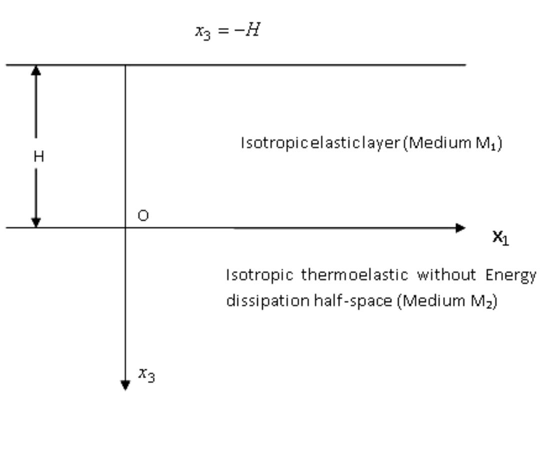

Fig.1
Geometry of the problem.

$$\vec{u}^e = (u_1^e, 0, u_3^e) \tag{9}$$

The dimensionless quantities are defined as:

$$x_i' = \frac{\omega_1^* x_i}{v_1}, \quad t' = \omega_1^* t, \quad u_i' = \frac{\omega_1^* u_i}{v_1}, \quad \mathrm{T}' = \frac{a_1 \mathrm{T}}{\rho v_1^2}, \quad \sigma_{ij}' = \frac{\sigma_{ij}}{a_1 \mathrm{T}_0}, \quad k_n' = \frac{v_1}{a_1 \mathrm{T}_0 \omega_1^*} k_n, \quad k_t' = \frac{v_1}{a_1 \mathrm{T}_0 \omega_1^*} k_t$$

where

$$v_1^2 = \frac{c_{11}}{\rho}, \quad \omega_1^* = \frac{\rho C_E v_1^2}{K_1} \tag{10}$$

and ω_1^* is the characteristic frequency of the medium, v_1 is the longitudinal wave velocity in the medium. For half space, we introduce potential function Φ and Ψ through the relations following [31].

$$u_1 = \frac{\partial \Phi}{\partial x_1} - \frac{\partial \Psi}{\partial x_3}, \qquad u_1 = \frac{\partial \Phi}{\partial x_1} - \frac{\partial \Psi}{\partial x_3} \tag{11}$$

and for an isotropic elastic layer, we introduce potential function Φ^e and Ψ^e through the relations

$$u_1^e = \frac{\partial \Phi^e}{\partial x_1} - \frac{\partial \Psi^e}{\partial x_3}, \qquad u_3^e = \frac{\partial \Phi^e}{\partial x_3} + \frac{\partial \Psi^e}{\partial x_1} \tag{12}$$

Making use of Eqs. (8)-(9) in Eqs. (2)-(4) and applying the dimensionless quantities defined by Eq. (10) on resulting equations, after suppressing the primes, with the aid of Eqs. (11)-(12) and assuming the solution of the resulting equations as

$$(\Phi, \Psi, \mathrm{T}, \Phi^e, \Psi^e) = (1, W, S, P_1, P_2) U \exp\left[i\xi(x_1 + m x_3 - ct)\right] \tag{13}$$

We obtain, after some algebraic calculation the components of displacement in layer, displacement components and temperature distribution in the half-space (satisfying the radiation condition Re $(m_p) \geq 0$) are given as

$$u_1^e = \left[i\xi(B_1 c_4 + B_2 s_4) + \xi m_5 (D_1 s_5 - D_2 c_5)\right] \exp\left\{i\xi(x_1 - ct)\right\} \tag{14}$$

$$u_3^e = \left[\xi m_4 (B_2 c_4 - B_1 s_4) + i\xi(D_1 c_5 + D_2 s_5)\right] \exp\left\{i\xi(x_1 - ct)\right\} \tag{15}$$

and

$$u_1 = \left[\sum_{p=1}^{2} i\xi e^{-\xi m_p x_3} A_p - \xi m_3 A_3 e^{-\xi m_3 x_3}\right] \exp\left\{i\xi(x_1 - ct)\right\} \tag{16}$$

$$u_3 = \left[\sum_{p=1}^{2} -\xi m_p e^{-\xi m_p x_3} A_p + i\xi m_3 A_3 e^{-\xi m_3 x_3}\right] \exp\left\{i\xi(x_1 - ct)\right\} \tag{17}$$

$$T = \sum_{P=1}^{2} A_p n_{1p} \exp\left[i\xi(x_1 + i m_p x_3 - ct)\right] \tag{18}$$

where $c = \omega / \xi$ is the non- dimensional phase velocity, ω is the frequency; m is unknown parameter, l, W, S are respectively, the amplitude ratio of u_1, u_3, T w.r.t. u_1

and

$$m_3 = \sqrt{1-\frac{c^2}{\delta_1^2}}, \quad m_4 = \sqrt{c^2-1}, \quad m_5 = \sqrt{c^2\delta_2^2-1}, \quad c_4 = \cos(\xi m_4 x_3), \quad c_5 = \cos(\xi m_5 x_3),$$

$$s_4 = \sin(\xi m_4 x_3), \quad s_5 = \sin(\xi m_5 x_3), \quad \delta_1^2 = \frac{\mu}{\lambda+2\mu}, \quad \delta_2^2 = \frac{\mu^e}{\lambda^e+\mu^e}, \quad \delta_3^2 = \frac{\rho^e v_1^2}{\mu^e}$$

where $A_p(p=1,2), B_1, B_2, D_1, D_2$ are arbitrary constants. The couplings constants $n_{1p}(p=1,2,3,4)$ are given in Appendix B and $m_p^2(p=1,2)$ are the roots of quadratic equation given in Appendix A.

4 BOUNDARY CONDITIONS

In this paper, linear model is adopted to represent the imperfectly bonded interface conditions. The boundary conditions are that the normal force of magnitude P_1 tangential force of magnitude P_2 are acting at the free surface. The discontinuities in displacements have linear relations with stresses, continuity of normal and tangential stress, vanishing of the gradient of temperature at the interface between the isotropic elastic layer and thermoelastic without energy dissipation half. Mathematically, these can be written as

(i) Mechanical conditions

$$\left.\begin{aligned}(\sigma_{33}^e)_{M_1} &= -P_1 e^{i\xi(x_1-ct)}\\(\sigma_{31}^e)_{M_1} &= -P_2 e^{i\xi(x_1-ct)}\end{aligned}\right\} \quad x_3=-H \tag{19}$$

$$\left.\begin{aligned}(\sigma_{33})_{M_1} &= k_n\left[(u_3)_{M_2}-(u_3^e)_{M_1}\right]\\(\sigma_{31})_{M_1} &= k_t\left[(u_1)_{M_2}-(u_1^e)_{M_1}\right]\\(\sigma_{33}^e)_{M_1} &= (\sigma_{33})_{M_2}\\(\sigma_{31}^e)_{M_1} &= (\sigma_{31})_{M_2}\end{aligned}\right\} \quad x_3=0 \tag{20}$$

(ii) Thermal condition

$$\frac{\partial T}{\partial x_3}=0 \quad \text{at} \quad x_3=0 \tag{21}$$

where k_n and k_t normal and transverse stiffness of layer having dimension N/m^3 and H is the thickness of the layer.

5 BEHAVIOUR OF THE COMPONENTS OF STRESSES AND TEMPERATURE DISTRIBUTION

Substituting the values of $u_1, u_3, T, u_1^e, u_3^e$ from Eqs. (14)-(18) in Eqs. (19)-(21) and with the aid of Eqs. (1), (5), (8)-(10), after simplification, we obtain the value of arbitrary constant $A_i(=1,2,3), B_p(p=1,2),$ and $D_p(p=1,2)$ .Using these values, we obtain the component of stresses and temperature distribution as

$$\sigma_{33}=\left[U\frac{\Delta_1}{D}e^{-\xi m_1 x_3}-U_2A_2\frac{\Delta_2}{D}e^{-\xi m_2 x_3}+U_3\frac{\Delta_3}{D}e^{-\xi m_3 x_3}\right]\mathrm{e}^{i\xi(x_1-ct)}$$
$$\sigma_{31}=\left[V_1\frac{\Delta_1}{D}e^{-\xi m_1 x_3}-V_2\frac{\Delta_2}{D}e^{-\xi m_2 x_3}+V_3\frac{\Delta_3}{D}e^{-\xi m_3 x_3}\right]\mathrm{e}^{i\xi(x_1-ct)}$$
$$\sigma_{33}^e=\left[R_1c_4\frac{\Delta_4}{D}-R_2s_4\frac{\Delta_5}{D}+iR_3s_5\frac{\Delta_6}{D}+ic_5R_4\frac{\Delta_7}{D}\right]\mathrm{e}^{i\xi(x_1-ct)} \tag{22}$$
$$\sigma_{31}^e=\left[2im_4s_4\frac{\Delta_4}{D}+2im_4c_4\frac{\Delta_5}{D}+(m_5^2-1)c_5\frac{\Delta_6}{D}+(1-m_5^2)s_5\frac{\Delta_7}{D}\right]\mathrm{e}^{i\xi(x_1-ct)}$$
$$T=\left[n_{11}\frac{\Delta_1}{D}e^{-\xi m_1 x_3}-n_{12}\frac{\Delta_2}{D}e^{-\xi m_2 x_3}+n_{13}\frac{\Delta_3}{D}e^{-\xi m_3 x_3}\right]\mathrm{e}^{i\xi(x_1-ct)}$$

where

$$A_1=\frac{\Delta_1}{D},\qquad A_2=\frac{\Delta_2}{D},\qquad A_3=\frac{\Delta_3}{D},\qquad B_1=\frac{\Delta_4}{D},\qquad B_2=\frac{\Delta_5}{D},\qquad D_1=\frac{\Delta_6}{D},\qquad D_2=\frac{\Delta_7}{D}$$

where $\Delta_i(i=1,2....7)$ and D are determinant of 7×7 given in Appendix B.

6 PARTICULAR CASE: DERIVATION OF SECULAR EQUATION

Substituting the values of u_1,u_3,T,u_1^e,u_3^e from Eqs. (15)-(18) in Eqs. (19)-(21) without normal and tangential forces and with the aid of Eqs. (1), (5), (8)-(10), after simplification, we obtain

$$r_3\tan(\xi m_5 H)\Delta^*{}_1-r_4\Delta^*{}_2-r_2\Delta^*{}_3-r_1\Delta^*{}_4=0 \tag{23}$$

where $\Delta^*{}_p(p=1,2,3,4)=\left|R_{ij}\right|_{6\times 6}$, the entries R_{ij} of the determinant are given in Appendix C and $\Delta^*{}_2$ obtained by replacing the first column of Δ_1 by $\left[R_{11}^*\,R_{21}^*\ \ R_{31}^*\,0\ R_{51}^*\ 0\right]^{\mathrm{T}}$, $\Delta^*{}_3$ obtained by replacing the second column of Δ_2 by $\left[R_{12}^*\ R_{11}^*\ 0\ R_{42}^*\ R_{52}^*\ R_{62}^*\ \ R_{72}^*\ \right]^{\mathrm{T}}$, $\Delta^*{}_4$ obtained by replacing the third column of $\Delta^*{}_3$ by $\left[R_{13}^*\ 0\ \ 0\ R_{43}^*\ R_{53}^*\ R_{63}^*\ \ R_{73}^*\ \right]^{\mathrm{T}}$. The entries of $\Delta_4^*(p=1,2,3,4)$ are given in Appendix C.

7 SPACIAL CASES

(i) Normal stiffness
In this case, $K_n\neq 0$, $K_t\to\infty$ and the secular Eq. (23) remain the same. But the following will be replaced in the values of $\Delta_p(p=1,2,3,4)$

$$R_{41}=\xi m_4,\qquad R_{42}=0,\qquad R_{43}=-\xi,\qquad R_{44}=-\xi m_3,$$
$$R_{45}=\xi,\qquad R_{46}=\xi m_3,\qquad R_{41}^*=0,\qquad R_{42}^*=0,\qquad R_{73}^*=0 \tag{24}$$

(ii) Tangential stiffness

In this case, $K_t \neq 0$, $K_n \rightarrow \infty$ and the secular Eq. (23) remain the same with the change values of $\Delta_p (p=1,2,3,4)$ by taking

$$\begin{aligned} &R_{61} = 0, \quad R_{62} = \xi, \quad R_{63} = 0, \quad R_{64} = \xi, \\ &R_{65} = -\xi, \quad R_{66} = -\xi, \quad R^*_{61} = \xi, \quad R^*_{62} = 0, \quad R^*_{63} = \xi \end{aligned} \tag{25}$$

(iii) Welded contact
In this case $K_n \rightarrow \infty$, $K_t \neq 0$ and the secular Eq. (23) remain the same. But the value of $\Delta_p (p=1,2,3,4)$ are given by replacing

$$\begin{aligned} &R_{41} = \xi m_4, \quad R_{42} = 0, \quad R_{43} = -\xi, \quad R_{44} = -\xi m_3, \quad R_{45} = \xi, \\ &R_{46} = \xi m_3, \quad R^*_{41} = 0, \quad R^*_{42} = 0, \quad R^*_{43} = 0, \quad R_{61} = 0, \\ &R_{62} = \xi, \quad R_{63} = 0, \quad R_{64} = \xi, \quad R_{65} = -\xi, \quad R_{66} = -\xi, \\ &R^*_{61} = \xi, \quad R^*_{62} = 0, \quad R^*_{63} = \xi \end{aligned} \tag{26}$$

8 NUMERICAL RESULTS AND DISCUSSION

The material chosen for this purpose of numerical calculation is magnesium which is isotropic material. The physical data for a single crystal of magnesium material is given as [32]:

$$\begin{aligned} &\lambda = 2.696\times10^{10}\,\text{Kgm}^{-1}\text{s}^{-2},\ \mu = 1.639\times10^{10}\,\text{Kgm}^{-1}\text{s}^{-2},\ T_0 = 0.298\times10^{3}\,\text{K}, \\ &C^* = .104\times10^{4}\,\text{JKg}^{-1}\text{K}^{-1},\ \alpha_t = 2.33\times10^{-5}\,\text{K}^{-1},\ \rho = 1.74\times10^{3}\,\text{Kgm}^{-3}, K = 0.170\times10^{3}\,\text{Wm}^{-1}\text{K}^{-1} \end{aligned}$$

The elastic parameters for Granite are given by Bullen [21].

$$\lambda^e = 2.238\times10^{3}\,\text{J Kg}^{-1}\text{K}^{-1}, \mu^e = 2.238\times10^{3}\,\text{J Kg}^{-1}\text{K}^{-1}, \rho^e = 2.65\times10^{3}\,\text{J Kgm}^{-3}$$

8.1 Penetration depth

The variations of penetration depth of the waves namely V_i (i=1, 2, 3) changes with respect to frequency see Fig. 2-4. In this figure, the solid curve represent of case of with thermal (WT) and star symbol on these lines correspond to without thermal (WDT). The value of penetration depth V_1 for the case WDT decrease monotonically for the range $0 \leq \omega \leq 2.8$ and for $\omega > 2.8$ it increases its oscillation and finally become constant, whereas for the case of WT, the value of penetration depth V_1 increases for the range $0 \leq \omega \leq 2.2$ and for $\omega > 2.2$, it decreases and finally becomes constant

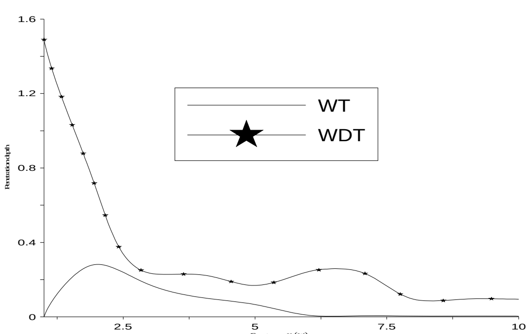

Fig. 2
Variation of penetration depth (V_1) w.r.t. frequency.

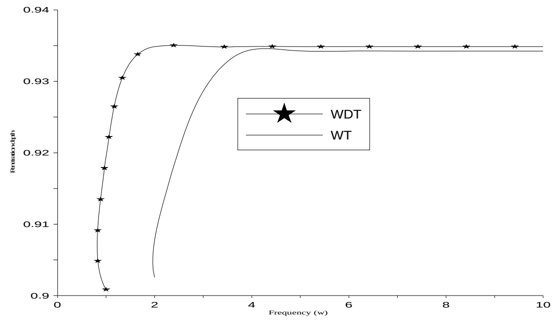

Fig. 3
Variation of penetration depth (V_2) w.r.t. frequency.

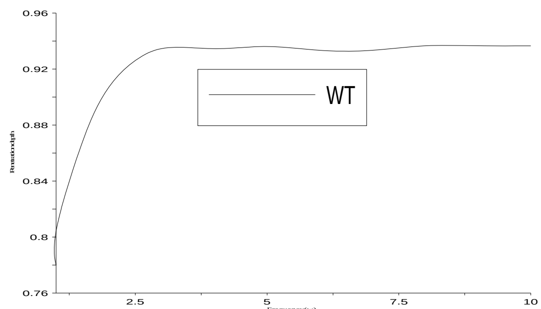

Fig. 4
Variation of penetration depth (V_3) w.r.t. frequency.

It is noticed that the value of penetration depth V_1 in case of WDT remain more (in comparison with WT) see Fig. 2. The value of penetration depth V_2, in both cases WT, WDT increases the small value of frequency, as frequency increases it becomes dispersionless for both cases see Fig. 3. It is evident that the value penetration depth V_1 remain higher in case of WDT (in comparison with WT).The value of penetration depth of V_3 increases monotonically $0 \le \omega \le 2.5$ and $\omega > 2.5$, it increases with oscillation and finally become constant see Fig. 4.

8.2 Behaviour of components of stresses and temperature distribution

The solid line, small dashes line, big dashes line and big dashes line with dotted correspond to with stiffness (WS), normal stiffness (NS), tangential stiffness (TS) and welded contact (WC) respectively. The values of t_{33}^e for all cases WS, NS, TS, WC remain oscillatory for the range $0 \le \omega \le 4.5$ and as $\omega > 4.5$, the values of t_{33}^e increase monotonically for NS, TS and decreases for WS, WC. It is noticed that the values of t_{33}^e in case of WC remain more (in comparison with WS, NS, TS for $0 \le \omega < 2$) and for $\omega > 4.5$ the values of NS remain more(in comparison with WS, TS, WC) see Fig. 5. The values of t_{31}^e initially oscillate for all the cases WS, NS, TS, WC for the range $1 \le \omega \le 4.5$ and for $\omega > 4.5$ the value of t_{31}^e for WS, NS, WC decrease monotonically .It is noticed that the values of t_{31}^e for the case of WS is greater than the value of NS, TS,WC for $1 \le \omega \le 2.5$, and for the range $2.7 \le \omega \le 4.5$, the value of WC remain more (in comparison with NS, TS, WS) and again for $\omega > 4.5$ the value of t_{31}^e for WS remain more (in comparison with NS, TS, WC) see Fig. 6.

The values of t_{33} have small oscillation for all cases WS, NS, TS, WC near the application of the source as $0 \le \omega \le 4.5$ and for $\omega > 4.5$, the values of t_{33} decrease monotonically (for WS , WC) and increases (for NS, TS). It is noticed that the value of t_{33} remain more as ω varies (i) $0 \le \omega \le 2$ for WS, (ii) $2.7 \le \omega \le 4.5$ for NS, (iii) $\omega > 4.5$ for WS. To compare other cases of stiffness see Fig. 7. The value of t_{31} oscillate for all cases WS, NS, TS, WC when $0 \le \omega \le 4.2$, as ω increases, the values of t_{31} increases for WS, WC and decreases for NS, TS. It is noticed that the value of t_{31} (i) remain smaller for WS when $0 \le \omega \le 2$, (ii) remain higher for NS when

$4 \le \omega \le 4.6$ (iii) remain higher for WC when $\omega > 4.6$, to compare with other corresponding stiffness see Fig. 8. The value of temperature (T) increase with oscillation for all cases WS, NS, TS, WC when $1 \le \omega \le 4$, as $\omega > 4.6$ the value of T decrease monotonically for WS and slightly increase for NS, TS and WC. It is evident that the value of T remains more as ω varies (i) $1.7 \le \omega \le 2.9$ for WS, (ii) $3 \le \omega \le 4.3$ for WC, (iii) $\omega > 4.3$ for WS, to compare with itself with each other cases of stiffness see Fig. 9.

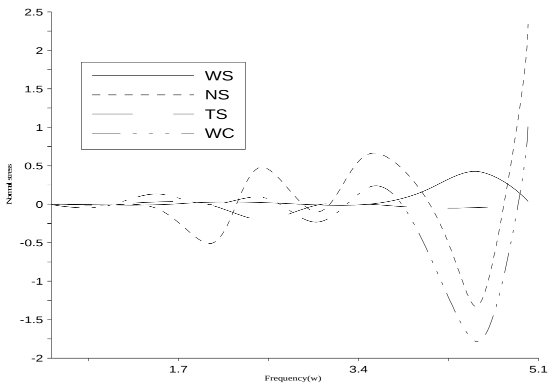

Fig. 5
Variation of normal stress t_{33}^{e} w.r.t. frequency.

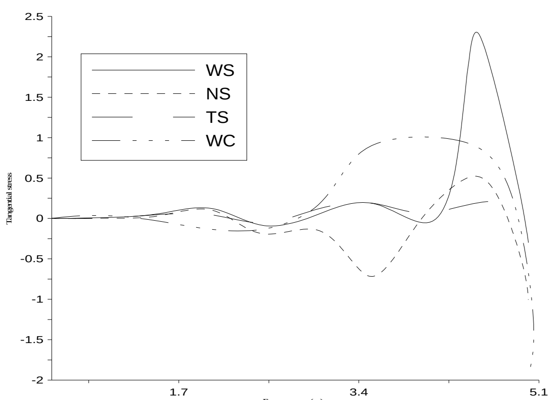

Fig. 6
Variation of tangential stress t_{31}^{e} w.r.t. frequency.

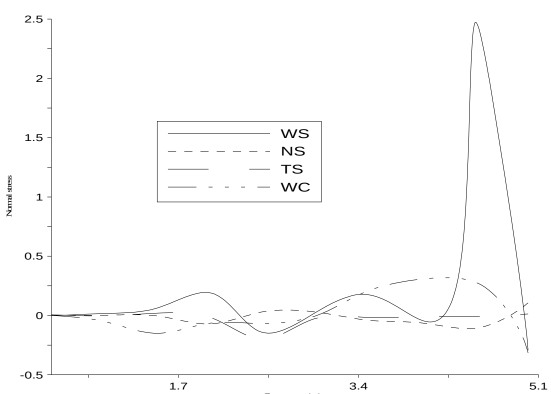

Fig. 7
Variation of normal stress t_{33} w.r.t. frequency.

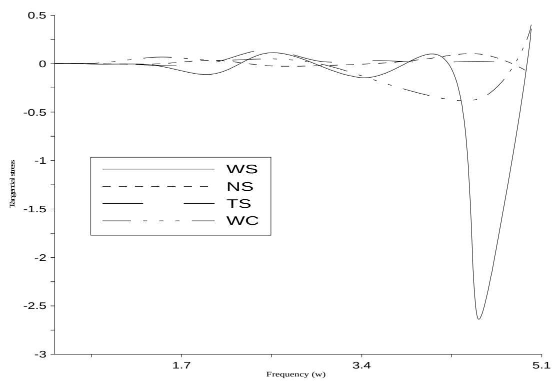

Fig. 8
Variation of tangential stress t_{31} w.r.t. frequency.

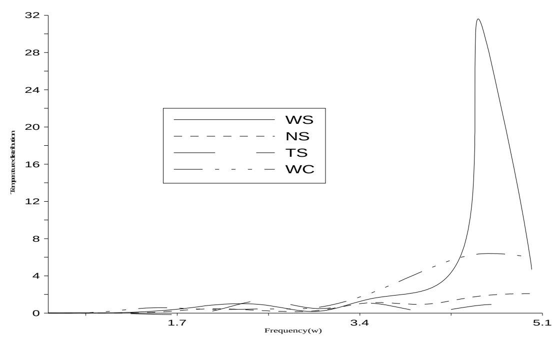

Fig. 9
Variation of temperature distribution (T) w.r.t. frequency.

9 CONCLUSIONS

The penetration depth of longitudinal, transverse, and thermal wave at the imperfect boundary between an isotropic thermoelastic without energy dissipation half-space and an isotropic elastic layer of finite thickness has been discussed. The secular equation in compact form has been derived. The components of temperature distribution, normal and tangential stress are computed at the interface and shown graphically. The effect of stiffness is shown on temperature distribution, normal and tangential stress and the effect of thermal is shown on the penetration depth of various waves. It is noticed that the values of penetration depth of longitudinal and transverse waves in case of without thermal remain (WTD) more in comparison with thermal (WT). The numerical results are found to be significantly in agreement with the corresponding analytic results. The effects of stiffness are observed on normal and tangential stress, and the effect of thermal are noticed on penetration depth of longitudinal and transverse waves.

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APPENDIX A

$$m^4 + A^* m^2 + B^* = 0 \tag{A.1}$$

where

$$A^* = \frac{f_2 + \xi^2 (f_1 + f_3)}{\xi^2}, \qquad B^* = \frac{f_2 + f_1 f_3}{\xi^2}$$

$$f_1 = \xi^2 (c^2 - 1), \quad f_2 = \chi_1 \omega^2 \xi^2, \quad f_3 = \omega_1^* + \xi^2 \quad q_1 = 2P_2 m_4 \xi^2, \quad q_2 = \xi^2 (m_5^2 - 1),$$

$$t_p = \xi^2 (2\Gamma_2 + \Gamma_1) m_p - \Gamma_p - \Gamma_3 n_{1p} \quad t_3 = 2\xi^2 (\Gamma_1 + \Gamma_2) m_3, \quad d_p = 2\xi^2 \Gamma_2 m_p, \quad d_3 = \xi^2 (m_3^2 - 1), \quad J_p = -\xi m_p + h \tag{A.2}$$

APPENDIX B

$$D = \begin{vmatrix} 0 & 0 & 0 & r_1 & -r_2 & ir_3 & r_4 \\ 0 & 0 & 0 & in_1 & in_1 & n_3 & n_4 \\ t_1 & -t_2 & -i & -l_1 & -t_4 & i & il_2 \\ ik_t & ik_t & -d_2 & -i & -id_1 & d_3 & -m_5 \\ h_1 & h_2 & ih_3 & -h_4 & 0 & 0 & ih_5 \\ ik_1 & ik_2 & k_3 & 0 & ik_4 & k_5 & 0 \\ q_1 & q_2 & 0 & 0 & 0 & 0 & 0 \end{vmatrix} \tag{B.1}$$

$$\Delta_1 = \begin{vmatrix} P_1 & 0 & 0 & r_1 & -r_2 & ir_3 & r_4 \\ P_2 & 0 & 0 & in_1 & in_1 & n_3 & n_4 \\ 0 & -t_2 & -i & -l_1 & -t_4 & i & il_2 \\ 0 & ik_t & -d_2 & -i & -id_1 & d_3 & -m_5 \\ 0 & h_2 & ih_3 & -h_4 & 0 & 0 & ih_5 \\ 0 & ik_2 & k_3 & 0 & ik_4 & k_5 & 0 \\ 0 & q_2 & 0 & 0 & 0 & 0 & 0 \end{vmatrix} \tag{B.2}$$

where Δ_2 is obtained by replacing the second column of D by $[P_1 \;\; P_2 \;\;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0]$, Δ_3 is obtained by replacing the second column of D by $[P_1 \;\; P_2 \;\;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0]$ and so on Δ_7 is obtained by replacing the seventh column of D by $[P_1 \;\; P_2 \;\;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0]$.

APPENDIX C

$$R_{14} = R_{15} = R_{17} = R_{42} = R_{51} = R_{52} = R_{53} = R_{61} = R_{62} = R_{63} = R_{64} = 0$$
$$R_{11} = n_4, \quad R_{12} = n_2, \quad R_{13} = n_1, \quad R_{21} = l_2, \quad R_{22} = t_2, \quad R_{23} = l_1, \quad R_{24} = \xi, \quad R_{25} = -t_1,$$
$$R_{26} = -t_2, \quad R_{31} = -w_3, \quad R_{32} = -q_1, \quad R_{33} = -w_1, \quad R_{34} = -w_2, \quad R_{35} = -w_1, \quad R_{36} = w_2,$$
$$R_{41} = -l_2, \quad R_{43} = l_1, \quad R_{44} = -t_3, \quad R_{45} = -t_1, \quad R_{46} = -t_2, \quad R_{54} = -d_3, \quad R_{55} = d_1,$$
$$R_{54} = d_2, \quad R_{65} = J_1, \quad R_{66} = J_2, \quad R_{41}^* = R_{61}^* = R_{52}^* = R_{63}^* = R_{62}^* = R_{43}^* = 0,$$
$$R_{11}^* = n_3, \quad R_{21}^* = t_1, \quad R_{31}^* = -q_2, \quad R_{51}^* = q_2, \quad R_{12}^* = n_4, \quad R_{22}^* = l_2, \quad R_{32}^* = -w_3,$$
$$R_{42}^* = -l_2, \quad R_{13}^* = n_2, \quad R_{23}^* = l_2, \quad R_{33}^* = -q_1, \quad R_{53}^* = q_1 \tag{C.1}$$

$$P_1^* = \frac{\lambda^e}{\beta T_0}, \quad P_2^* = \frac{\mu^e}{\beta T_0}, \quad \Gamma_1 = \frac{\lambda}{\beta T_0}, \quad \Gamma_2 = \frac{\mu}{\beta T_0},$$

$$r_1 = (P_1^* + 2P_2^*)c_4 m_4^2 + P_1^*, \quad r_2 = (P_1^* + 2P_2^*)s_4 m_4^2 + P_1^*, \quad r_3 = 2m_5 P_2^*,$$

$$n_1 = 2m_4 s_4, \quad n_2 = 2m_4 c_4, \quad n_3 = (m_5^2 - 1)c_5, \quad n_4 = (1 - m_5^2)s_5$$

$$d_1 = 2\xi m_4 P_2^*, \quad d_2 = k_t m_3, \quad d_3 = \xi(1 - m_5^2), \quad t_1^* = k_n m_1, \quad t_2^* = k_n m_2, \quad t_4^* = k_n m_4,$$

$$h_1 = \Gamma_1(1 - m_1) - 2\Gamma_2 - n_{11}\Gamma_3, \quad h_2 = \Gamma_1(1 - m_2) - 2\Gamma_2 - n_{12}\Gamma_3, \quad h_3 = (\Gamma_1 + 2\Gamma_2)m_3,$$

$$h_4 = P_1^* + (P_1^* + 2P_2^*)m_4^2, \quad h_5 = 2m_5 P_2, \quad k_1 = 1 + m_1, \quad k_1 = 1 + m_2, \quad k_1 = 1 - m_3^2,$$

$$k_4 = 2m_4, \quad k_5 = m_5^2 - 1, \quad l_1 = \left[P_1(1 + m_4^2) + 2P_2^*\right]\xi^2, \quad l_2 = 2\xi^2 m_5 P_2^*,$$

$$w_1 = \xi k_t, \quad w_2 = \xi k_t m_3, \quad w_3 = \xi k_t m_4. \tag{C.2}$$

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