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Application of Piezoelectric and Functionally Graded Materials in Designing Electrostatically Actuated Micro Switches

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ABSTRACT

In this research, a functionally graded microbeam bonded with piezoelectric layers is analyzed under electric force. Static and dynamic instability due to the electric actuation is studied because of its importance in micro electro mechanical systems, especially in micro switches. In order to prevent pull-in instability, two piezoelectric layers are used as sensor and actuator. A current amplifier is used to supply input voltage of the actuator from the output of the sensor layer. Using Hamilton's principle and Euler-Bernoulli theory, equation of motion of the system is obtained. It is shown that the load type (distributed or concentrated) applied to the micro beam from the piezoelectric layer, depends on the shape of the actuator layer (E.g. rectangle, triangular). Finite element method is implemented for evaluation of displacement field in the micro beam and dynamic response of the micro beam under electric force is calculated using finite difference method. Effect of squeeze film damping on pull-in voltage and time-response of the system is considered using nonlinear Reynolds equation. Effect of several parameters such as gain value between piezoelectric sensor and actuator layer, profile of functionally material, and geometry of the system is considered on dynamic behavior of the micro beam especially on pull-in instability. Results are verified for simple cases with previous related studies in the literature and good agreements were achieved. Results indicate that increasing gain value between sensor and actuator enhances stiffness of the system and will raise pull-in voltage. Also, dependency of dynamic properties of the system such as amplitude and frequency of vibration on functionally graded material profile is shown. The material distribution of the functionally graded material is designed in such a way that results in a specific pull-in voltage.

Keywords: Piezoelectric, Functionally graded material, Microbeam, Static and dynamic instability

1 INTRODUCTION

MICRO electro mechanical devices are widely used in the past two decades. Based on the vast applications of micro beams and plates in many devices such as sensors and actuators, many researchers have been interested in studying these systems. Electrostatic force is a common way of actuation in MEMS because of its advantages such as simplicity and high speed of actuation. Capacitive accelerometers, capacitive pressure sensors, comb drivers and electrostatically actuated MEMS power switches are examples of sensing and actuating devices utilize electrostatic deformation [1]. Vibration and dynamic analysis of these systems is a necessity for designing accurate and efficient sensors and actuators. Pull-in instability is widespread phenomena in micro electro mechanical systems. It results due to insufficiency of restoring force of micro beam or plate in contrasting with electric force.

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Although pull-in is undesired in some cases of MEMS, there are examples in which pull-in is deliberately used such as micro switches. Dynamic response of micro structures along with pull-in phenomenon has been studied by many researchers. Osterberg [2] has studied pull-in instability of microstructures with rectangular and circular shapes. He has also compared his closed form models with experiments. Ahmadian et al. [3] developed a nonlinear co-rotational finite element model for coupled-domain MEMS devices with electrostatic actuation and squeeze film effect. They considered micro beam as an Euler-Bernoulli beam. They also used an incremental-iterative method based on the Newmark direct integration and the Newton-Raphson algorithm to solve nonlinear equations. Another important effect in MEMS is squeeze film damping. Nayfeh and Younis [4] in an analytical work presented a new approach to the modeling of flexible microstructures under squeeze film damping using perturbation method and utilizing compressible Reynolds equation coupled with the equation governing the plate deflection. Moghimi Zand et al. [5] used finite element and finite difference method to model vibrational behavior of electrostatically actuated micro plates subjected to nonlinear squeeze film damping effect. Vibration control of structures by means of piezoelectric laminates has been researched recently. Lin and Huang [6] studied vibration control of beam-plates with bonded piezoelectric sensors and actuators using a control algorithm based on Lyapunov energy function. They used Hamilton's principle to derive governing equation of motion and finite element method for analysis. M. Collet et al. [7] analyzed damping on a piezoelectric laminated beam. They considered the effect of piezoelectric actuator as a pure moment at the end of the beam. Using experimental tests, they studied dynamic response and effect of damping on frequency response of the system.

Although a lot of researches have been performed in the field of MEMS, application of functionally graded materials in micro beams and plates has not been yet studied. Also, the control effect of piezoelectric layers on dynamic behavior of micro beams and their pull-in voltage is a novel subject which has not scrutinized perfectly. In present research, the micro beam is considered to be made of functionally graded material. One of the main objects in this research is to present applicability of functionally graded materials in micro beams. This capability helps us to design micro switches with desired pull-in voltage. Two piezoelectric layers are also used as sensor and actuator to control vibration of the system, especially near pull-in instability.

2 MODELING AND FORMULATION

For deriving the equations of motion, we first use Euler- Bernoulli beam theory which assumes displacement fields as below [8]:

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) - z\frac{\partial w_0}{\partial x} \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{1}$$

In the above equation, u_0 and w_0 are displacements of an arbitrary point of the mid-plane in the direction of x, y and z. So, longitudinal strain of the micro beam can be obtained in terms of displacement field as follows

$$\varepsilon_{xx} = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w_0}{\partial x^2} \tag{2}$$

Hamilton's principle is used to derive governing equations of the system. So, kinetic and potential energies of the system must be obtained. Kinetic energy of the system can be calculated as

$$T^* = \int_0^l \left(\int_A \rho \dot{w}^2 dA \right) \mathrm{d}x \tag{3}$$

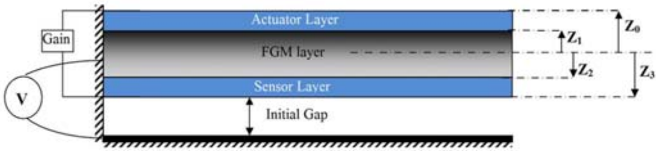

Fig .1
A cantilever FGM micro beam boned with two piezoelectric laminates.

In which A is cross-sectional area of the micro beam including (Functionally Graded Material) FGM layer and two piezoelectric layers. Above integral can be expanded as

$$T^{*}=\int_{0}^{l}\rho_{a}.A_{a}.\dot{w}^{2}\mathrm{d}x+\int_{0}^{l}\rho_{s}.A_{s}.\dot{w}^{2}\ \mathrm{d}x+\int_{0}^{l}\left(\int_{-\frac{h_b}{2}}^{-\frac{h_b}{2}}\rho(z).\dot{w}^{2}.b.dz\right)\mathrm{d}x \tag{4}$$

From Eq. (1) it can be assumed that

$$w_{s}(x)=w_{a}(x)=w_{FGM}(x) \tag{5}$$

Total potential energy of the system can be expressed as

$$U=U_{s}+U_{a}+U_{FGM} \tag{6}$$

In the above equation, U_s, U_a and U_{FGM} are potential energies of piezoelectric sensor, actuator, and FGM layer, respectively. In order to obtain potential energy density of piezoelectric layers, we first use constitutive equations of a piezoelectric material [9]

$$\begin{aligned}\{\sigma\}&=[\mathrm{C}]\{\varepsilon\}-[\mathrm{e}]^{\mathrm{T}}\{\mathrm{E}\}\\ \{\mathrm{D}\}&=[\mathrm{e}]\{\varepsilon\}+[\in]\{\mathrm{E}\}\end{aligned} \tag{7}$$

Considering that polarization of piezoelectric layers is in the z-direction, the above equation becomes

$$\begin{aligned}\sigma_{xx}&=c_{11}^{piezo}\varepsilon_{xx}-e_{13}E_{z}\\ D_{z}&=e_{31}\varepsilon_{xx}+\in_{33}E_{z}\end{aligned} \tag{8}$$

where σ and ε are mechanical stress and strain, respectively. E_z and D_z are electric field and electric displacement. $\in$ and e are electric permittivity and piezoelectric coefficient factor. So, the density potential energy is as follows [10]

$$\hat{U}_{p}=c_{11}^{piezo}\varepsilon_{xx}^{2}-\in_{33}E_{z}^{2}-2e_{31}\varepsilon_{xx}E_{z} \tag{9}$$

In Eq. (9) the first term is potential energy of an elastic material. The second and third terms are electric energy and piezoelectric energy.

2.1 Functionally graded materials

FGM are typically mixture of metal and ceramic with varying volume fraction in specific direction such as thickness. So, properties vary from fully metallic surface to fully ceramic one [11]

$$\begin{aligned}E(z)&=(E_{c}-E_{m})\left(\frac{z}{h}+\frac{1}{2}\right)^{k}+E_{m}\\ \rho(z)&=(\rho_{c}-\rho_{m})\left(\frac{z}{h}+\frac{1}{2}\right)^{k}+\rho_{m}\\ v_{FGM}&=\text{const.}\end{aligned} \tag{10}$$

Here, h is thickness of the micro beam and z is the thickness coordinate, m and c subscripts are referred to metal and ceramic, respectively. Superscript k indicates the variation profile of volume fraction. It is obvious that k=0 represents a fully ceramic micro beam and for the case of k=1 properties of the micro beam change linearly from the metallic surface to ceramic surface. The potential energy of the FGM layer due to its elastic deformation is

$$\hat{U}_{FGM} = \frac{1}{2} c_{11}^{FGM} \varepsilon_{xx}^{2} \tag{11}$$

In which c_{11}^{FGM} is modulus of elasticity of FGM material. Also, from Hamilton's principle we have

$$\int_{t_1}^{t_2} (\delta T - \delta U + \delta W_{nc})\,\mathrm{d}t = 0 \tag{12}$$

Substituting Eqs. (4), (9) and (10) into (12) and considering displacement field and electric field in the piezoelectric layers as independent variables , the governing equations of electric filed in piezoelectric layers are obtained

$$\begin{aligned} &\delta E_{3s} : \ -\epsilon_{33}\ E_{3s} + z w'' e_{31} = D_z^s \\ &\delta E_{3a} : \ -\epsilon_{33}\ E_{3s} + z w'' e_{31} = -\epsilon_{33}\ \frac{V_a}{t_a} \end{aligned} \tag{13}$$

We use a charge amplifier in the feedback as shown in Fig. 2. It is enforced by the amplifier that $E_z^s \approx 0$. So, Eq. (13) becomes

$$D_{3s} = -z_m^s w'' e_{31} \tag{14}$$

$$E_{3a} = \frac{z w'' e_{31}}{\epsilon_{33}} + \frac{V_a}{t_a} \tag{15}$$

The output electric charge of the sensor layer by integrating over its surface is

$$Q_s = \int_A D_z^s\ \mathrm{d}A = -z_m^s b e_{31} \int_0^l w'' \mathrm{d}x \tag{16}$$

Also, for the charge amplifier shown in Fig. 2 we have

$$\left.\begin{aligned} e_0 &= -A e_- \\ e_- &= e_0 + i/(sC) \end{aligned}\right\} e_0 = \frac{-Ai}{(1+A)sC} \simeq -i/sC = -Q/C \tag{17}$$

Thus, applied voltage on actuator layer, V_a is obtained:

$$V_a = \frac{Q}{C} = \frac{1}{C}\left(z_m^s b e_{31} \int_0^l w'' \mathrm{d}x \right) \tag{18}$$

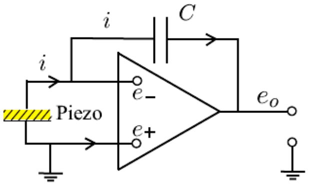

Fig. 2
Charge amplifier [10].

Equation of motion of the micro beam and boundary conditions can be expressed as

$$-I\ddot{w}-(k_{2a})''-(Kw'')''+p=0 \tag{19}$$

$$\left[\left\{(k_{2a})'+(Kw'')'\right\}\delta w-(k_{2a}+Kw'')\delta w'\right]_0^l=0 \tag{20}$$

in which

$$\Lambda=\rho_a A_a+\rho_s A_s+b\int_{-\frac{h_b}{2}}^{\frac{h_b}{2}}\rho(z)z\ \mathrm{d}z \tag{21}$$

$$K=k_a+k_s+k_{FGM}+k_{3a} \tag{22}$$

$$k_s=\int_{A_s}c_{11}^{s}z^2\mathrm{d}A\ ,\ k_a=\int_{A_a}c_{11}^{a}z^2\mathrm{d}A,$$

$$k_{FGM}=\int_{A_{FGM}}c_{11}^{FGM}z^2\mathrm{d}A,$$

$$k_{2a}=\int_{A_a}ze_{31}\frac{V_a}{t_a}\mathrm{d}A=be_{31}V_a z_a^m \tag{23}$$

Also, f_e and f_p are electric and pressure forces applied on the micro beam. Electric force between charged plates can be calculated as

$$f=\frac{\epsilon_0 V^2}{2H^2} \tag{24}$$

To calculate pressure force, it is needed to obtain pressure distribution under the micro beam. Boundary conditions obtained from (20) are presented in Table 1. In order to obtain these boundary conditions it is assumed that the micro beam width is constant with respect to *x*. So, the terms including derivatives of the width (such as $(k_{2a})^{''}$ and $(k_{2a})^{'}$) are to be zero. It can be seen from boundary conditions of Eq. (20) that if the width of actuator is constant with respect to x, the equivalent load of actuator (in a cantilever system) can be considered as a pure moment at the end of the micro beam. But if it (width of the micro beam) varies in longitudinal direction, boundary conditions and the load type applied from actuator layer will change. In this research, it is considered that cross sectional-area of the system is constant with respect to *x*.

2.2 Squeeze film damping

Nonlinear Reynolds equation is used for modeling squeeze film damping as follows [12]

$$\frac{\partial}{\partial x}\left(H^3P\frac{\partial P}{\partial x}\right)=12\eta_{eff}\frac{\partial}{\partial t}\left(HP\right) \tag{25}$$

Table 1
Boundary conditions of Eq. (20)

$k_{2a}+Kw''=0,\qquad w'''=0$	$x=l$	Cantilever
$w'=w=0$	$x=0$	
$w'=w=0$	$x=0,l$	Fixed- fixed

in which H=g+w is the actual gap and g is initial air gap between the micro beam and electrode surface and w is the deflection of the micro beam that is positive in upward direction. P is the pressure in the squeeze film layer which is applied on the micro beam as a resistive force against deformation in downward direction. So, coupled domain equations of the system are:

$$\begin{aligned} &B\frac{\partial^4 w}{\partial x^4}+\bar{\rho}\frac{\partial^2 w}{\partial t^2}=f_e-P \\ &\frac{\partial}{\partial x}\left(H^3 P\frac{\partial P}{\partial x}\right)=12\ \eta_{eff}\frac{\partial}{\partial t}(HP) \end{aligned} \tag{26}$$

in which η_{eff} is the effective viscosity for squeeze film layer and is calculated as [12]

$$\eta_{eff}=\frac{\mu}{1+9.658K_n^{1.159}} \tag{27}$$

where μ is viscosity of the film and K_n is Knudsen number which is defined as[13]

$$K_n=\frac{\lambda}{H} \tag{28}$$

where λ is the mean free path of the gas molecules and H is the gap between micro beam and electrode surface.

3 NUMERICAL SOLUTION

In this section, we use finite element method to solve equation of motion for the micro beam while finite difference method is used simultaneously for solving Reynold's equation. Using FEM, Eq. (12) can be written as:

$$[K]\{W\}+[M]\{\ddot{W}\}=\{F\} \tag{29}$$

The mass matrix and stiffness matrix can be obtained as follows

$$\begin{aligned} &M_{ij}^e=\int_0^l N_i N_j\ \mathrm{d}x \\ &K_{ij}^e=\int_0^l N_i'' N_j''\ \mathrm{d}x \end{aligned} \tag{30}$$

where N_i's are shape functions for the linear beam element

$$\begin{aligned} &N_1=\frac{1}{l^3}(2x^3-3x^2l+l^3) \\ &N_2=\frac{1}{l^3}(x^3l-2x^2l^2+xl^3) \\ &N_3=\frac{1}{l^3}(-2x^3+3x^2l) \\ &N_4=\frac{1}{l^3}(x^3l-x^2l^2) \end{aligned} \tag{31}$$

To solve Eqs. (26), time-domain finite difference method is used. Therefore, explicit form for obtaining deflection and pressure in each time step is

$$\{W\}^{t+1} = \Delta t^2[M]^{-1}(\{F_{elec}\} - [K]\{W^t\}) + 2\{W\}^t - \{W\}^{t-1} \tag{32}$$

$$\begin{aligned} P_i^{t+1} = P_i^t + [\frac{1}{24\eta_{eff}}\{ & \left(3(g+w_i^t)^2 \frac{w_{i+1}^t - w_{i-1}^t}{2\Delta x} \times \frac{(P_{i+1}^t)^2 - (P_{i-1}^t)^2}{2\Delta x}\right) + \left(\frac{(P_{i+1}^t)^2 - 2(P_i^t)^2 + (P_{i+1}^t)^2}{\Delta x^2}\right) \\ & \times (g+w_i^t)^3\} - \frac{w_i^{t+1} - w_i^t}{\Delta t} P_i^t] \frac{1}{g+w_i^t} \end{aligned} \tag{33}$$

Boundary and initial conditions for Reynolds equation are

$$\begin{aligned} & P(x,t)\big|_{free\ edge} = P_0 \\ & \frac{\partial P(x,t)}{\partial x}\Big|_{clamped\ edge} = 0 \\ & P(x,t)\big|_{t=0} = P_0 \end{aligned} \tag{34}$$

Initial conditions for deflection in (19) are

$$w(x,t)\big|_{t=0} = \frac{\partial w(x,t)}{\partial t}\Big|_{t=0} = 0 \tag{35}$$

4 RESULTS

One of the most important phenomena related to micro beams is pull-in instability as mentioned in the introduction because it can be used for analyzing sensitivity of sensors. Basically, pull-in instability occurs when electric actuation applies on the micro beam. It is caused because of insufficiency of elastic restoring force of the micro beam in contrast with electric force. Due to the nature of this type of actuation, electric force increases progressively as the micro beam is deformed.

Pull-in instability occurs in two types: static pull-in and dynamic pull-in. Static pull-in occurs when actuation voltage is incremented slowly until electric force conquest the stiffening force of the micro beam and instability occurs. In the dynamic pull-in, however, electric voltage is applied instantaneously and the micro beam contacts with the electrode. Due to this importance, pull-in instability is frequently studied by researchers and therefore can be used for validation of results. To validate our results, FGM profile is considered in simple case being made of only one material (n=0 or n=∞). To calculate static pull-in voltage, we solve Eq. (29) without considering inertia term. Therefore, for voltages smaller than V_{pi} we have

$$[K]\{W\} = \{F\} \tag{36}$$

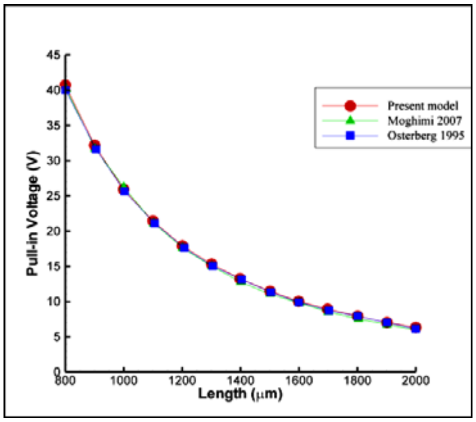

Fig. 3
Static pull-in voltage vs. length of a fixed- fixed micro beam.

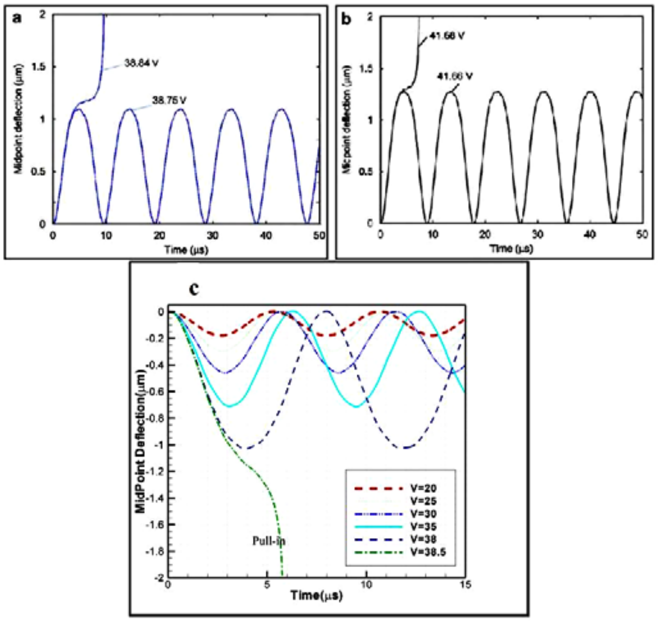

Fig. 4
Midpoint deflection of a fixed-fixed micro beam vs. Time. (*E*=169 Gpa, gap=2 µm, *h*=2µm, *w*=20µm, *L*=300µm); (a) model presented by M. Moghimi Zand [5], (b) Model presented by Krylov [15] and (c) present model.

Because the force vector is a function of displacement, it is updated in each step and new values for micro beam displacement are calculated. This procedure is repeated until deformation of the micro beam converges. It is obvious that for voltages above the pull-in voltage convergence doesnot occur. In Fig. 3, static pull-in voltage is presented in different lengths of micro beam. Comparison between results obtained by Osterberg [1] and Moghimi [14] shows good agreement between them. Transient behavior of the system can also be used to validate this model. A computer code was used to obtain dynamic behavior of the system by solving (26) numerically. For voltages smaller than dynamic pull-in voltage as it can be seen in Fig. 4, the motion of each point of the micro beam was harmonic with respect to time. Applying voltages bigger than dynamic pull-in, voltage of the system results in contact of micro beam with electrode. We compared our results with those obtained by Moghimi Zand [5] and Krylov [15] in Fig. 4. Also, a comparison between damped and undamped response is performed to indicate the effect of squeeze film. As mentioned in the introduction, utilizing piezoelectric sensors and actuators simultaneously in a micro beam can be an operative tool to control its vibration. Deflection of the micro beam is sensed by the sensor and the resultant voltage is applied on the actuator, gained in the feedback. In Fig. 5, time history of tip deflection of a cantilever micro beam is presented for several gain values between piezoelectric layers. It can be seen that amplitude and time period of harmonic motion varies by changing gain value between sensor and actuator.

Also, in the condition that pull-in occurs- decreasing gain value- the micro beam contacts with electrode in less time. It can be realized that gain value can be chosen so that prevents pull-in instability (comparing gain=1 and gain=10). To illustrate the effect of gain variation, squeeze film damping has not been considered in this case. Material properties of piezoelectric material are presented in Table 2. Dependency of pull-in voltage on gain value between piezoelectric layers is shown Fig. 6. It is obvious that by increasing gain value pull-in voltage of the system increases, too. In the next step, we are going to peruse the effect of using functionally graded materials on pull-in voltage of the micro beam. Relation between pull-in voltage and volume fraction in a FGM micro beam with polynomial profile is presented in Fig. 7. It can be seen that by increasing *K*, pull-in voltage of the micro beam decreases. It is worthy to mention that "*K*", the horizontal axis in the diagram, is FGM profile index and

$$E(z) = (E_c - E_m)\left(\frac{z}{h} + \frac{1}{2}\right)^K + E_m \tag{37}$$

Dynamic behavior of a FGM micro beam for several FG profiles under squeeze film damping is presented in Fig. 8. It can be easily seen that decreasing k in the FGM profile (which is equivalent to increase the volumetric ratio of ceramic part) stiffens the micro beam against pull-in instability. So, material distribution in FGM layer can be chosen in a way that results in an arbitrary pull-in voltage or a desired time response which can be useful in designing micro switches working with electrostatic actuation. Also, preventing pull-in instability is possible by changing the FGM profile. Material properties of metal and ceramic components used in FGM layer is presented in Table 3.

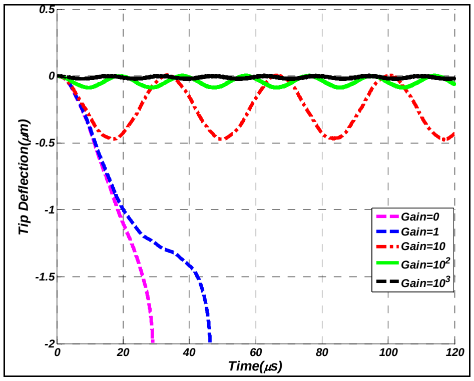

Fig. 5
Time response of cantilever laminate micro beam for different gain values.

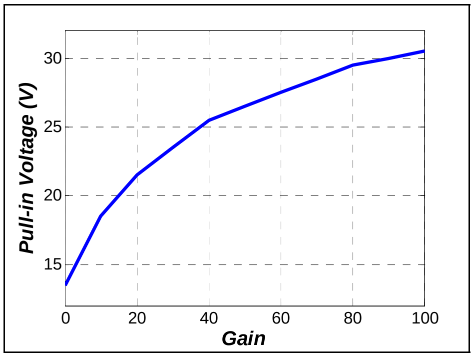

Fig. 6
Pull-in Voltage vs. Gain value between piezoelectric layers.

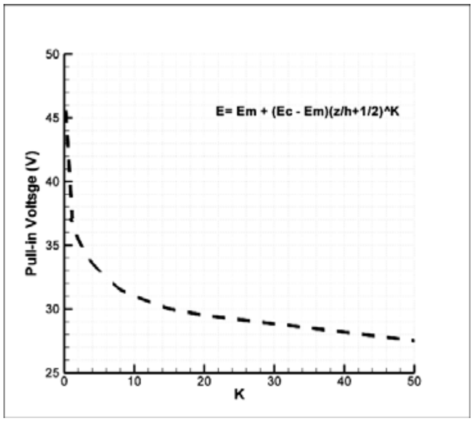

Fig 7
Dynamic pull-in voltage of fixed-fixed FGM micro beam vs. profile of FGM layer.

Table 2
Mechanical properties of metal and ceramic components of FGM layer used in the micro beam

ρ_c (Kgm^{-3})	ρ_m (Kgm^{-3})	ν (PZT)	E_c (GPa)	E_m (GPa)
3750	4429	0.3	320.2	105

Table 3
Material properties of the piezoelectric patches (PZT)

E (Nm^{-2})	h (thickness) (μm)	ν (PZT)	e_{31} (C m^{-2})	$\in_{33}/\in_0$	ρ (Kg /m^3)
12.6×10^{10}	1	0.3	-6.5	1475	7500

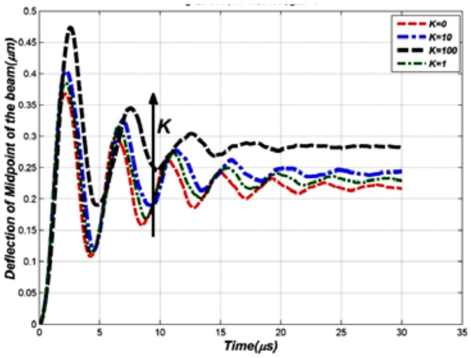

Fig 8
Dynamic behavior of fixed- fixed FGM micro beam under squeeze film damping for different material distributions (gap=2μm, h=2μm, w=40μm, L=500μm).

Increasing *K* from zero to infinity is equal to changing the micro beam material distribution from fully ceramic to fully metallic. Because the modulus of elasticity of ceramic materials is usually more than metals, it is reasonable that by increasing *K* total stiffness of the FGM micro beam decreases and pull-in instability occurs with smaller excitation voltages.

5 CONCLUSION

Dynamic behavior of a FGM micro beam bonded with two piezoelectric layers was studied. Using Hamilton's principle, governing equation of motion is derived. Finite element and finite difference method were used to solve equation of vibration of the micro beam, considering squeeze film damping effect. Both static pull-in voltage and transient response of the micro beam were calculated and relationship between the behavior of micro beam and system parameters was shown. Dependency of pull-in instability on properties of FGM profile and gain between piezoelectric layers make it possible to design micro switches with desired excitation voltage. In the cases that pull-in effect is unfavorable, moreover, using piezoelectric sensor and actuator simultaneously can be an effective way to control undesirable motions of the micro beam near the instability region.

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