Journal of Solid Mechanics Vol. 1, No. 3 (2009) pp. 159-170

Vibration Analysis of Functionally Graded Spinning Cylindrical Shells Using Higher Order Shear Deformation Theory

M. Mehrparvar*

Department of Mechanical Engineering, Islamic Azad University, Khomeinishahr Branch, Khomeinishahr, Iran

Received 19 July 2009; received in revised form 2 October 2009; accepted 6 October 2009

ABSTRACT

In this paper the vibration of a spinning cylindrical shell made of functional graded material is investigated. After a brief introduction of FG materials, by employing higher order theory for shell deformation, constitutive relationships are derived. Next, governing differential equation of spinning cylindrical shell is obtained through utilizing energy method and Hamilton's principle. Making use of the principle of minimum potential energy, the characteristic equation of natural frequencies is derived. After verification of the results, the effect of changing different parameters such as material grade, geometry of shell and spinning velocity on the natural frequency are examined.

Keywords: Vibration; Functionally graded material; Spinning cylindrical shell; Higher order shear deformation theory

1 INTRODUCTION

THE concept of functionally gradient materials (FGMs) was first introduced in 1984 by a group of materials scientists in Japan when they were in a process of preparing thermal barrier materials [1, 2]. Since then, FGMs have attracted much of the interests as a heat-shielding material. FGMs are made by combining different materials using powder metallurgy methods [3]. They possess property variations in the constituent volume fractions that lead to continuous change in the composition, microstructure, porosity, etc. as well as results in gradients in the mechanical and thermal properties [2, 4-7]. Studies on FGMs have been extensive but are largely confined to analysis of thermal stress and deformation [8-12]. Related to the vibration of cylindrical shells numerous researches were conducted. Many of these studies are for isotropic and composite shells. Arnold and Warburton [10], Ludwig and Krieg [11], Chung [12], Soedel [13], Forsberg [14], Bhimaraddi [15], Soldatos and Hajigeoriou [16], Soldatos [17], Lam and Loy [18], Loy and Lam [19], and Loy, Lam and Shu [20] are among those who have carried out studies on the vibration of cylindrical shells. Nevertheless, associated with the vibration of cylindrical shells made of FG material only one work is reported [21] in which the bases of vibration analysis were on the usage of classical theory. To move one step further on this analysis, in this work the higher order shear deformation theory (HSDT) is employed on the spinning shells which have various applications in the industry.

In this paper, the vibration of a spinning cylindrical shell made of functional graded material (FGM) is investigated. The considered functionally graded material is composed of stainless steel and nickel where the volume fractions follow a power-law distribution. The objective is to study the frequency characteristics, the influence of the constituent volume fractions, and the effects of the configurations of the constituent materials on the natural frequencies. A functionally graded cylindrical shell is essentially an inhomogeneous shell consisting of a mixture of isotropic materials. The analysis of the functionally graded cylindrical shell is carried out using higher order theory for shell deformation and solved using Hamilton's principle method.

* *E-mail address:* m007me@yahoo.com.

1 FUNCTIONALLY GRADED MATERIALS

One of the most advantages of the FGM over other engineering materials is its superior resistant against harsh temperature condition. The combined material properties of such a material can be expressed as [21]

$$P_F(T,z) = \sum_{i=1} P_{mi}(T) V_{mi}(z) \tag{1}$$

in which, P_F can be any effective FGM property, P_{mi} is a parameter depending on temperature and V_{mi} is a volume fraction for ith material. Moreover, T represents the temperature (°K) and z is the coordinate along the body thickness. The parameter for each material usually is expressed as [21]

$$P_{mi}(T) = P_\circ\left(\frac{P-1}{T} + 1 + P_1 T + P_2 T^2 + P_3 T^3\right) \tag{2}$$

in which P_0, P_1, P_2, P_3, and P are some temperature dependent parameters. In order to model the property distribution, according to Voigt model which assumes that the strain is the same in the ceramic and in the metal, the FGM mechanical property is expressed as

$$\begin{aligned} P &= \sum_{i=1}^{k} v_{fi}\, P_i \\ P(z) &= P_c V_c + P_m V_m \end{aligned} \tag{3}$$

in which P is the effective FGM property, indices c and m, stand for ceramic and metal respectively, and V represents the FGM volume fraction subjected to the law

$$V_c + V_m = 1 \tag{4}$$

For a cylindrical shell with thickness h, the variation of the volume fraction of phases can be expressed according to the following power law in terms of thickness z

$$V_{mi} = \left(\frac{2z+h}{h}\right)^N \tag{5}$$

For the mechanical properties of the cylindrical shell either of following power law models can be used

$$\begin{aligned} P(z) &= (P_c - P_m)\left(\frac{2z+h}{h}\right)^N + P_m \\ P(z) &= (P_m - P_c)\left(\frac{2z+h}{h}\right)^N + P_c \end{aligned} \tag{6}$$

For example, the Young's modulus for functionally graded (FG) cylindrical shell with N=0.5 can be expressed as

$$E(z) = (E_c - E_m)\left(\frac{2z+h}{h}\right)^{0.5} + E_m \tag{7}$$

This means that the cylindrical shell at z=h/2 is completely metal and in the z=-h/2 it is completely ceramic. In other words, shell starts from inside surface by ceramic material and becomes completely metal at the outside wall.

2 GOVERNING EQUATIONS

Consider a cylindrical spinning shell made of FGM, with length *L*, radius *R*, and thickness of *h*, spinning around its symmetric axis with constant angular velocity of Ω (see Fig. 1). In vibration analysis of shell in this work, higher order shear deformation theory is considered. Based on the assumptions prevailing on this theory, differential equations are derived. For a point located somewhere on the spinning shell and referred to the coordinate system shown in Fig. 1, the position and velocity of such point are [22]

$$\begin{aligned} &\vec{r} = u\hat{i} + v\hat{j} + w\hat{k} \\ &\vec{V} = \dot{u}\hat{i} + \dot{v}\hat{j} + \dot{w}\hat{k} + (\Omega\hat{i} \times w\hat{k}) + (\Omega\hat{i} \times v\hat{j}) \,, \end{aligned} \tag{8}$$

in which *i*, *j* and *k* are unit vectors in the cylindrical coordinates of x,θ and *z*, respectively. Moreover, Ω is shell rotational speed, *u*, *v* and *w* are displacement components in x,θ and *z* directions, respectively and $\dot{u}, \dot{v}$, and $\dot{w}$ are time derivative of displacement components. As it was mentioned, in this study we use the Reddy higher order theory [23] assumptions in which the general form of displacement components are defined as

$$\begin{aligned} &u(x,\theta,z) = u_0(x,\theta) + z\beta_x(x,\theta) + z^2\mu_x(x,\theta) + z^3\psi_x(x,\theta) \\ &u(x,\theta,z) = v_0(x,\theta) + z\beta_\theta(x,\theta) + z^2\mu_\theta(x,\theta) + z^3\psi_\theta(x,\theta) \\ &w(x,\theta,z) = w_0(x,\theta) \end{aligned} \tag{9}$$

in which, u_0, v_0, w_0, β_x, β_θ, μ_x, μ_θ, ψ_x, and ψ_θ are defined as

$$\begin{aligned} &u_0 = u(x,\theta,0,t), \qquad v_0 = v(x,\theta,0,t), \qquad w_0 = w(x,\theta,0,t) \\ &\beta_x = \left(\frac{\partial u}{\partial z}\right)_{z=0}, \qquad \mu_x = \left(\frac{\partial^2 u}{\partial z^2}\right)_{z=0}, \qquad \psi_x = \left(\frac{\partial^3 u}{\partial z^3}\right)_{z=0} \\ &\beta_\theta = \left(\frac{\partial v}{\partial z}\right)_{z=0}, \qquad \mu_\theta = \left(\frac{\partial^2 v}{\partial z^2}\right)_{z=0}, \qquad \psi_\theta = \left(\frac{\partial^3 v}{\partial z^3}\right)_{z=0} \end{aligned} \tag{10}$$

Moreover, based on this theory, we further have

$$\begin{aligned} &\varepsilon_z = \circ \\ &\sigma_{xz}\left(x,\theta,\pm\frac{h}{2},t\right) = \sigma_{\theta z}\left(x,\theta,\pm\frac{h}{2},t\right) = 0 \end{aligned} \tag{11}$$

Since the cylindrical shell has a thin wall, the strain components can be defined as

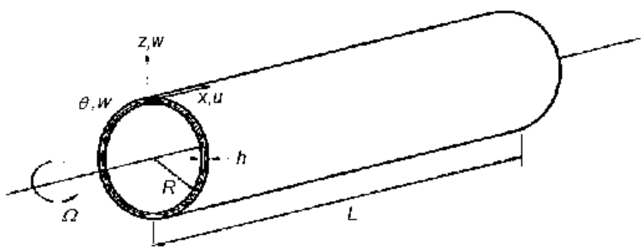

Fig. 1
Spinning cylindrical shell, geometry and coordinates.

$$\begin{pmatrix} e_x \\ e_\theta \\ e_{x\theta} \end{pmatrix} = \begin{pmatrix} \overset{\circ}{\varepsilon}_x \\ \overset{\circ}{\varepsilon}_\theta \\ \overset{\circ}{\varepsilon}_{x\theta} \end{pmatrix} + z \begin{pmatrix} \overset{\circ}{k}_x \\ \overset{\circ}{k}_\theta \\ \overset{\circ}{k}_{x\theta} \end{pmatrix} + z^3 \begin{pmatrix} k_x^2 \\ k_\theta^2 \\ k_{x\theta}^2 \end{pmatrix}$$
$$\begin{pmatrix} e_{\theta z} \\ e_{xz} \end{pmatrix} = \begin{pmatrix} \overset{\circ}{\varepsilon}_{\theta z} \\ \overset{\circ}{\varepsilon}_{xz} \end{pmatrix} + z^2 \begin{pmatrix} k_{\theta z}^1 \\ k_{xz}^1 \end{pmatrix} \tag{12}$$

where

$$\begin{pmatrix} \overset{\circ}{k}_x \\ \overset{\circ}{k}_\theta \\ \overset{\circ}{k}_{x\theta} \end{pmatrix} = \begin{pmatrix} \dfrac{\partial \beta_x}{\partial x} \\ \dfrac{\partial \beta_\theta}{R\partial \theta} \\ \dfrac{\partial \beta_\theta}{\partial x} + \dfrac{\partial \beta_x x}{R\partial x} \end{pmatrix}, \quad \begin{pmatrix} k_{\theta z}^1 \\ k_{x\theta}^1 \end{pmatrix} = -\frac{4}{h^2} \begin{pmatrix} \beta_\theta + \dfrac{\partial w_\circ}{R\partial \theta} \\ \beta_x + \dfrac{\partial w_\circ}{\partial x} \end{pmatrix}, \quad \begin{pmatrix} k_x^2 \\ k_\theta^2 \\ k_{x\theta}^2 \end{pmatrix} = -\frac{4}{3h^2} \begin{pmatrix} \dfrac{\partial \beta_x}{\partial x} + \dfrac{\partial^2 w_\circ}{\partial x^2} \\ \dfrac{\partial \beta_\theta}{R\partial \theta} + \dfrac{\partial^2 w_\circ}{R^2 \partial \theta^2} \\ \dfrac{\partial \beta_\theta}{\partial x} + \dfrac{\partial \beta_x}{R\partial \theta} + 2\dfrac{\partial^2 w_\circ}{R\partial x \partial \theta} \end{pmatrix}$$
$$\begin{pmatrix} \overset{\circ}{\varepsilon}_{\theta z} \\ \overset{\circ}{\varepsilon}_{xz} \end{pmatrix} = \begin{pmatrix} \beta_\theta + \dfrac{\partial w_\circ}{R\partial \theta} \\ \beta_x + \dfrac{\partial w_\circ}{\partial x} \end{pmatrix}, \quad \begin{pmatrix} \overset{\circ}{\varepsilon}_x \\ \overset{\circ}{\varepsilon}_\theta \\ \overset{\circ}{\varepsilon}_{x\theta} \end{pmatrix} = \begin{pmatrix} \dfrac{\partial u_\circ}{\partial x} \\ \dfrac{\partial v_\circ}{R\partial \theta} + \dfrac{w_\circ}{R} \\ \dfrac{\partial v_\circ}{\partial x} + \dfrac{\partial u_\circ}{R\partial \theta} \end{pmatrix} \tag{13}$$

Furthermore, by employing Hook's law one can easily calculate the following forces (normal and shear) and moments (bending and torsional) all per unit of length as

$$(N_x, N_\theta, N_{x\theta}) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_\theta, \sigma_{x\theta})\, \mathrm{d}z$$
$$(M_x, M_\theta, M_{x\theta}) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_\theta, \sigma_{x\theta})\, z\mathrm{d}z$$
$$(P_x, P_\theta, P_{x\theta}) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_\theta, \sigma_{x\theta})\, z^2\mathrm{d}z$$
$$(Q_x, S_x) = \int_{-h/2}^{h/2} \sigma_{xz} (1, z^2)\, \mathrm{d}z$$
$$(Q_\theta, S_\theta) = \int_{-h/2}^{h/2} \sigma_{\theta z} (1, z^2)\, \mathrm{d}z \tag{14}$$

in which N_i, M_i, P_i, Q_i and S_i are system of forces or moments and can be represented in matrix form as

$$\{N\} = [S] * \{\varepsilon\} \tag{15a}$$

in which

$$[N] = [N_x, N_\theta, N_{x\theta}, M_x, M_\theta, M_{x\theta}, P_x, P_\theta, P_{x\theta}]$$
$$[\varepsilon] = [\overset{\circ}{\varepsilon}_x, \overset{\circ}{\varepsilon}_\theta, \overset{\circ}{\varepsilon}_{x\theta}, \overset{\circ}{k}_x, \overset{\circ}{k}_\theta, \overset{\circ}{k}_{x\theta}, k_x^2, k_\theta^2, k_{x\theta}^2] \tag{15b}$$

$$[S]=\begin{bmatrix} A_{11} & A_{12} & \circ & B_{11} & B_{12} & \circ & E_{11} & E_{12} & \circ \\ A_{12} & A_{22} & \circ & B_{12} & B_{22} & \circ & E_{12} & E_{22} & \circ \\ \circ & \circ & A_{66} & \circ & \circ & B_{66} & \circ & \circ & E_{66} \\ B_{11} & B_{12} & \circ & D_{11} & D_{12} & \circ & F_{11} & F_{12} & \circ \\ B_{12} & B_{22} & \circ & D_{12} & D_{22} & \circ & F_{12} & F_{22} & \circ \\ \circ & \circ & B_{66} & \circ & \circ & D_{66} & \circ & \circ & F_{66} \\ E_{11} & E_{12} & \circ & F_{11} & F_{12} & \circ & H_{11} & H_{12} & \circ \\ E_{12} & E_{22} & \circ & F_{12} & F_{22} & \circ & H_{12} & H_{22} & \circ \\ \circ & \circ & E_{66} & \circ & \circ & F_{66} & \circ & \circ & H_{66} \end{bmatrix}$$

Furthermore, related to shear terms in the Hook's law, one can obtain the following shear forces and bending moments due to shear forces (all per unit of length) as

$$\{T\}=\left[X\right]*\{\varepsilon_s\} \tag{16a}$$

in which

$$\begin{aligned} \{T\}^T &= [Q_x, Q_\theta, S_x, S_\theta] \\ \{\varepsilon_S\}^T &= [\overset{\circ}{\varepsilon}_{\theta z}, \overset{\circ}{\varepsilon}_{x\theta}, k^1_{\theta z}, k^1_{xz}] \\ \left[X\right] &= \begin{pmatrix} A_{55} & A_{54} & D_{44} & D_{54} \\ A_{54} & A_{44} & D_{54} & D_{55} \\ D_{55} & D_{54} & F_{44} & F_{54} \\ D_{54} & D_{55} & F_{54} & F_{55} \end{pmatrix} \end{aligned} \tag{16b}$$

in which the elements of [S] and [X] matrices i.e., A_{ij}, D_{ij}, E_{ij}, F_{ij} and H_{ij} are defined as

$$(A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}) = \int_{-h/2}^{h/2} Q_{ij}\,(1, z, z^2, z^3, z^4, z^6)\,\mathrm{d}z \tag{17}$$

It should be mentioned that for the isotropic materials the elements of Q_{ij} (i,j = 1,2,6) matrix turned to be as

$$\begin{aligned} Q_{11} &= Q_{22} = \frac{E}{1-\nu^2} \\ Q_{12} &= \frac{\nu E}{1-\nu^2} \\ Q_{44} &= Q_{55} = Q_{66} = \frac{E}{2+2\nu} \end{aligned} \tag{18}$$

2.1 Boundary conditions

In this study the boundary condition of both ends are taken to be simply supported. Based on this type of boundary condition, following admissible displacement field for any point on the mid-plane surface is given by

$$\begin{aligned} u_0(x,\theta,t) &= A \;\; \sin\frac{m\pi x}{L}\;\cos(n\theta+\omega t) \\ v_0(x,\theta,t) &= B \;\; \cos\frac{m\pi x}{L}\;\sin(n\theta+\omega t) \\ w_0(x,\theta,t) &= C \;\; \cos\frac{m\pi x}{L}\;\cos(n\theta+\omega t) \end{aligned} \tag{19}$$

$$\beta_x(x,\theta,t) = D \ \sin\frac{m\pi x}{L}\ \cos(n\theta + \omega t)$$

$$\beta_\theta(x,\theta,t) = E \ \cos\frac{m\pi x}{L}\ \sin(n\theta + \omega t)$$

In Eq. (19), the coefficients A, B, C, D, and E represent the magnitude of the vibration amplitude in the aforementioned directions. By employing energy method i.e., defining the kinetic energy, internal energy (due to existing stresses) and work done by external forces of the system and imposing the Hamilton principle, differential equation of the motion can be obtained. The variational form of the functional (potential energy) is expressed as

$$\delta\int_{t_1}^{t_2}(T - U + W)\,\mathrm{d}t = 0 \tag{20}$$

in which T, U and W represent the kinetic energy, internal energy and work done by external forces, respectively. For the geometry under consideration, i.e. cylindrical shell, the kinetic energy is defined by

$$T = \frac{1}{2}\rho_t h\int_0^l\int_0^{2\pi} \vec{v}\bullet\vec{v}\, R\,\mathrm{d}\theta\,\mathrm{d}x \tag{21}$$

in which $\rho^{th}=\rho$, i.e., ρ_t is the density per unit length. Substituting Eq. (8) into Eq. (21) satisfy the following equation

$$T = \frac{1}{2}\rho_t h\int_0^l\int_0^{2\pi}\left[\overset{\circ}{u}^2 + \overset{\circ}{v}^2 + \overset{\circ}{w}^2 + 2\Omega(v\overset{\circ}{w} - w\overset{\circ}{v}) + \Omega^2(v^2 + w^2)\right] R\,\mathrm{d}\theta\,\mathrm{d}x \tag{22}$$

Related to the internal energy, we break it into two parts; i.e. strain energy due to centrifugal force ($\hat{N}_\theta = \rho h\Omega^2 R^2$) which is shown by U_h and defined as [22]

$$U_h = \frac{1}{2}\int_0^l\int_0^{2\pi} N_\theta\left\{\left(\frac{1}{R}\frac{\partial u}{\partial x}\right)^2 + \left[\frac{1}{R}\left(\frac{\partial v}{\partial\theta} + w\right)\right]^2 + \left[\frac{1}{R}\left(\frac{\partial w}{\partial\theta} - v\right)\right]^2\right\} R\,\mathrm{d}\theta\,\mathrm{d}x \tag{23}$$

and shell strain energy due to internal stresses shown by U_ε is defined by

$$U_\varepsilon = \frac{1}{2}\int_0^l\int_0^{2\pi}\{\varepsilon\}^T[s]\{\varepsilon\}\, R\,\mathrm{d}\theta\,\mathrm{d}x \tag{24}$$

After substituting Eqs. (22-24) into Eq. (20), the equations of motion is obtained

$$\begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} \end{pmatrix}\begin{pmatrix} A \\ B \\ C \\ D \\ E \end{pmatrix} = 0 \tag{25}$$

Now by setting the determinant of above linear algebraic equation to zero, the characteristic equation of natural frequencies which is an equation of order tenth in ω is obtained

$$\lambda_\circ\omega^{10} + \lambda_1\omega^9 + \lambda_2\omega^8 + \lambda_3\omega^7 + \lambda_4\omega^6 + \lambda_5\omega^5 + \lambda_6\omega^4 + \lambda_7\omega^3 + \lambda_8\omega^2 + \lambda_9\omega + \lambda_{10} = 0 \tag{26}$$

where λ_i (i=0,1,…10) in Eq. (26) are all functions of elements of [C] matrix.

3 RESULTS AND DISCUSSIONS

By solving Eq. (26), natural frequencies of spinning cylindrical shell can be determined. Table 1 lists the value of different parameters such as E, ν and ρ of stainless steel and nickel part of the FG material. The result of solving Eq. (26) i.e. natural frequency vs. wave number n, for different values of volume fraction power N, L/R=20, h/R=0.05, Ω=50 (rps) and m=1 is listed in Table 2. It should be mentioned that the values under N^{SS} and N^{N} columns on this table represent the natural frequency of the cylindrical shell made of only stainless steel or only nickel, respectively. If we just change the value of h/R from 0.05 to 0.002 which means for reducing the thickness 25 times less and keeping the other parameters unchanged, the results of natural frequencies vs. wave number are listed in Table 3. The pictorial variations of the natural frequencies given in Tables 2-3 are shown in Figs. 2-3. As it is seen from Tables 2-3, the variation of natural frequency for different values of volume fraction from 0.5 to 20, has only a discrepancy of about 2.3% and the highest value for the natural frequency in this range belongs to the case of isotropic state made of stainless steel (N^{N}=0). The lowest value for the natural frequency in this range belongs to the case of isotropic state made of ceramic (N^{SS}=0). It can be verified that for the FG material of first kind, by increasing the value of N, the natural frequency approaches to the case of N^{N} i.e. an isotropic media made of nickel which is an indication of reduction of natural frequency. Referred to Fig. 3, it can be noticed that for a thin cylindrical shell the natural frequency has its minimum value for n=2 and 3, a condition which compared to Fig. 2 is created due to thinness (h/R<<1) of the shell.

Now, let us calculate the values of the natural frequency while h/R is changing for different values of volume fraction under condition of L/R=20, Ω=50 (rps) and m=1. The result of this variation is given in Table 4. Moreover, the given results in Table 4 are converted into different curves depicted in Fig. 4.

Table 1
Mechanical properties of FGM at T=300 K [21]

Coefficients	Stainless Steel			Nickel		
	E (N m^{-2})	ν	ρ (kg m^{-3})	E (N m^{-2})	ν	ρ (kg m^{-3})
P_0	201.04×10^{9}	0.3262	8166	223.95×10^{9}	0.3100	8900
P_{-1}	0	0	0	0	0	0
P_1	3.079×10^{-4}	-2.002×10^{-4}	0	-2.794×10^{-4}	0	0
P_2	-6.534×10^{-7}	3.797×10^{-7}	0	-3.998×10^{-9}	0	0
P_3	0	0	0	0	0	0
	2.07788×10^{11}	0.317756	8166	2.05098×10^{11}	0.3100	8900

Table 2
Natural frequency vs. wave number n, for different values of volume fraction power N, (L/R=20, h/R=0.05, Ω=50 (rps) and m=1)

n	N^{SS}=0	N^{N}=0	N=0.5	N=0.7	N=1	N=5	N=10	N=20
1	13.461	12.806	13.234	13.182	13.124	12.91	12.903	12.887
2	33.385	31.592	32.691	32.461	32.42	31.799	31.86	31.79
3	92.89	88.156	91.208	90.842	90.442	88.998	88.822	88.692
4	177.949	168.879	174.719	174.019	173.249	170.479	169.798	169.72
5	287.679	273.029	282.459	281.319	280.089	275.619	273.959	273.799
6	422.04	400.45	414.281	412.619	410.809	404.249	403.32	402.719
7	580.669	551.109	570.139	567.849	565.349	556.339	554.47	552.959
8	763.868	724.97	750.008	746.988	743.708	731.865	728.52	727.418
9	971.737	922.287	954.127	950.277	946.117	931.037	927.189	926.027
10	1203.86	1142.56	1182.06	1177.26	1172.06	1153.46	1149.46	1146.36

Table 3
Natural frequency vs. wave number n, for different values of volume fraction, N (L/R=20, h/R=0.002, Ω=50 (rps) and m=1)

n	N^{SS}=0	N^{N}=0	N=0.5	N=0.7	N=1	N=5	N=10	N=20
1	13.437	12.783	13.21	13.158	13.1	12.887	12.829	12.815
2	4.565	4.342	4.4888	4.4324	4.453	4.3798	4.3564	4.3495
3	4.2313	4.0169	4.1591	4.1429	4.1249	4.0571	4.0333	4.0256
4	7.121	6.7537	6.9932	6.9651	6.9344	6.8211	6.7816	6.7686
5	11.431	10.844	11.225	11.179	11.13	10.95	10.888	10.867
6	16.806	15.946	16.503	16.436	16.364	16.101	16.01	15.98
7	23.173	21.99	22.755	22.664	22.564	22.202	22.077	22.037
8	30.548	28.992	29.998	29.878	29.746	29.271	29.107	29.053
9	39.5185	37.5395	38.8185	38.6655	38.4995	37.8945	37.6855	37.6185
10	48.29	45.838	47.423	47.233	47.027	46.277	46.019	45.935

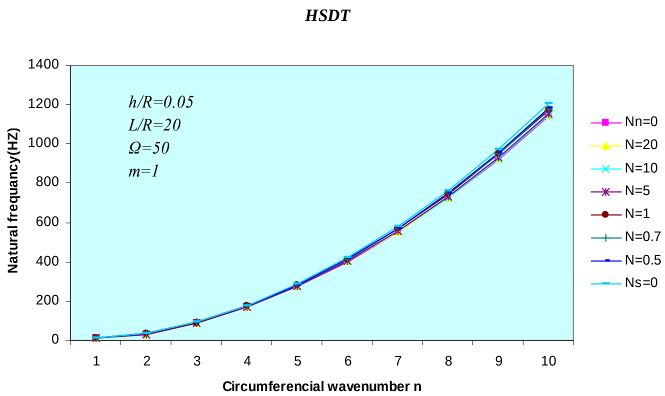

Fig. 2
Variation of the natural frequency vs. n, wave number for different values of volume fraction L/R=20, h/R=0.05, Ω=50 (rps) and m=1.

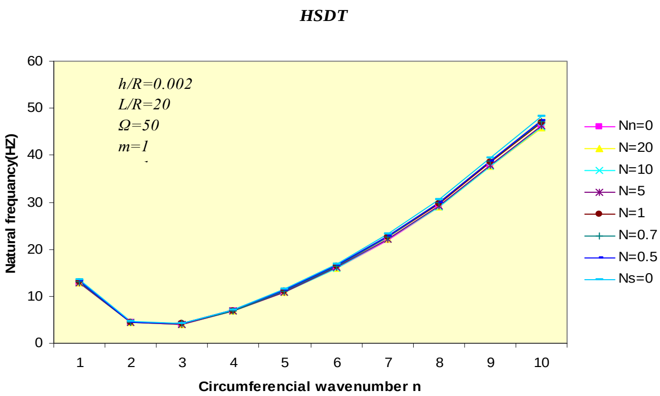

Fig. 3
Variation of the natural frequency vs. n, wave number for different values of volume fraction L/R=20, h/R=0.002, Ω=50 (rps) and m=1.

As can be seen from Fig. 4, the natural frequency increases more or less in a linear fashion up to h/R=0.024 and then it decreases a bit and remains unchanged afterwards. This trend is observed for all sort of different FG materials. In a parallel analysis, the values of the natural frequency while L/R is changing for different values of volume fraction and wave number under condition of h/R=0.002, Ω=50 (rps) and m=1. The result of such analysis is given in Table 5 and also depicted in Fig. 5. Related to Figs. 4 and 5, one can realize that the natural frequency of a short cylindrical shell of FG material is higher than the longer one. In other words, by reducing the value of L/R, the natural frequency of a cylindrical shell increases. By setting the value of L/R=20, h/R=0.05, n=1, m=1, the values of natural frequency of cylindrical shell for different FG materials are obtained while the spinning speed of the shell changes. The result of such calculation is listed in Table 6. By close examination of numbers in any specific column in this table, one can deduce that little variations exist. In other words, any variation in the spinning speed has insignificant effect on the natural frequency of the cylindrical shell made of FG material. This finding is consistent with the fact that the natural frequency is not a function of excitation frequency but rather function of intrinsic properties such as structural mass and stiffness of the body. To validate the accuracy of our analysis, the results of first ten natural frequencies for simply supported cylindrical shells based on higher order shear deformation theory (HSDT) and classical shell theory (CST) are compared with Loy et al. [21], when L/R=20, h/R=0.01, ν= 0.3, (see Table 7). Another comparison is made with work of Arnold and Warburton [10] for two specific natural frequencies but different wave numbers (see Table 8).

Table 4
Natural frequency vs. h/R for different values of volume fraction N, L/R=20, Ω=50 (rps) and m=1

h/R	n	N^{SS}=0	N^{N}=0	N=0.5	N=0.7	N=1	N=5	N=10	N=20
0.001	3	2.75990	2.62170	2.71410	2.70360	2.69190	2.64720	2.63820	2.63190
0.005	2	5.47220	5.20130	5.38240	5.36170	5.33860	5.25060	5.23960	5.22080
0.007	2	6.3530	6.03610	6.24760	6.22360	6.19690	6.09490	6.08730	6.05970
0.01	2	7.90630	7.50880	7.77310	7.7430	7.70970	7.58340	7.56740	7.53910
0.02	1	13.6680	13.0130	13.4410	13.3890	13.3310	13.1170	13.110	13.0940
0.03	1	13.6540	12.9990	13.4270	13.3750	13.3170	13.1030	13.0960	13.080
0.05	1	13.4610	12.8060	13.2340	13.1820	13.1240	12.910	12.9030	12.8870

Table 5
Natural frequency vs. L/R for different values of volume fraction N, h/R=0.002, Ω=50 (rps) and m=1

L/R	n	N^{SS}=0	N^{N}=0	N=0.5	N=0.7	N=1	N=5	N=10	N=20
0.2	20	438.46	416.64	431.22	429.56	427.72	420.7	419.93	418.27
0.5	15	174.81	166.08	171.91	171.25	170.51	167.7	167.34	166.73
1	11	86.801	82.463	85.36	85.031	84.665	83.268	83.082	82.786
2	8	42.992	40.836	42.275	42.112	41.93	41.237	41.051	40.997
5	5	16.786	15.948	16.508	16.445	16.374	16.103	16.07	16.01
10	4	8.5525	8.1213	8.4081	8.3755	8.3394	8.2023	8.1763	8.1542
20	3	4.0576	4.0653	4.0891	4.1569	4.1749	4.1911	4.0489	4.2633
50	2	1.4708	1.3957	1.4455	1.4398	1.4335	1.4098	1.4088	1.4015
100	1	0.5515	0.5245	0.5422	0.54	0.53761	0.5288	0.5272	0.5261

Table 6
Natural frequency vs. spinning speed Ω, for different values of volume fraction N, L/R=20, h/R=0.05, m=1, n=1

Ω	N^{SS}=0	N^{N}=0	N=0.5	N=0.7	N=1	N=5	N=10	N=20
5	33.190	31.596	32.694	32.465	32.322	31.902	31.886	31.876
10	33.190	31.595	32.694	32.465	32.321	31.901	31.886	31.875
15	33.189	31.594	32.693	32.464	32.321	31.901	31.885	31.875
20	33.188	31.593	32.692	32.463	32.320	31.900	31.884	31.873
30	33.188	31.592	32.692	32.462	32.320	31.900	31.884	31.872
40	33.187	31.592	32.691	32.462	32.319	31.899	31.883	31.871
50	33.187	31.591	32.691	32.461	32.319	31.899	31.883	31.871
100	33.186	31.591	32.690	32.460	32.318	31.898	31.882	31.870
150	33.186	31.590	32.689	32.460	32.318	31.897	31.881	31.869
200	33.176	31.589	32.689	32.459	32.317	31.897	31.880	31.868

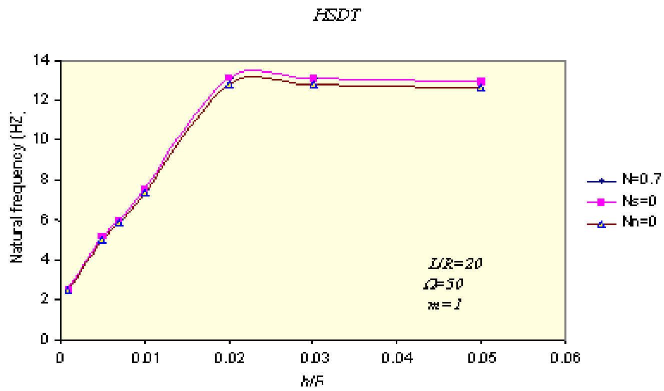

Fig. 4
Variation of the natural frequency vs. h/R, for different values of volume fraction L/R=20, Ω=50 (rps) and m=1.

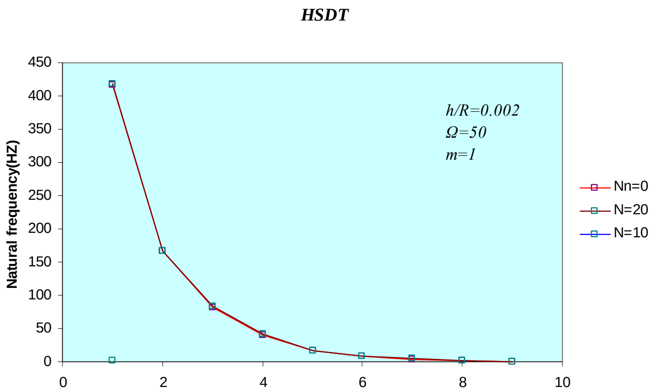

Fig. 5
Variation of the natural frequency vs. L/R for different values of volume fraction h/R=0.002, Ω=50 (rps) and m=1.

In both of these cases $\xi = \omega R\sqrt{(1-\nu^2)\rho / E}$. While vibrating analysis of simply supported FG cylindrical shells composed of stainless steel and nickel with its properties changing in the thickness-wise direction according to a volume fraction power-law, let us consider the effect of the changing the constituent volume fractions V_f and see how does it affect the FGM configuration. This can be done by varying-simultaneously-the volume fractions of the stainless steel and nickel, which is taking care of by changing the value of the power-law exponent N. The effects on the FGM configuration are studied by studying the frequencies of two FG cylindrical shells i.e. Type-I FG cylindrical shell and Type-II FG cylindrical shell. Type-I FG cylindrical shell has nickel on its inner surface and stainless steel on its outer surface and Type-II FG cylindrical shell has stainless steel on its inner surface and nickel on its outer surface. Figs. 6 and 7 show the variations of the volume fractions V_f of nickel and stainless steel, in the thickness-wise direction z for Type-I and Type-II FG cylindrical shells, respectively.

Table 7
Comparison of obtained first ten natural frequencies with different methods (L/R=20, h/R=0.01, ν = 0.3)

n	Loy et al. [21]	HSDT	CST
1	0.016101	0.016106	0.016104
2	0.009382	0.009387	0.009389
3	0.022105	0.022108	0.022110
4	0.042095	0.042097	0.042099
5	0.068008	0.068010	0.068011
6	0.099730	0.099732	0.099734
7	0.137239	0.137243	0.137244
8	0.180527	0.18053	0.180532
9	0.229594	0.229598	0.2296
10	0.284435	0.284439	0.284443

Table 8
Comparison of obtained results with different methods (L= 8 in, R=2 in, h=0.1 in, E= 30×10^6 lbf-in^{-2}, $\rho = 7.35\times10^{-4}$ lbf-s^{2}in^{-4})

m	Arnold and Warburton [10]	HSDT	CST
1 (n=2)	2046.8	20549.5	2050.7
2	5637.6	5640.7	5643.3
3	8935.3	8939.3	8941.3
4	11405	11410.9	11416.9
5	13245	13258.9	13262.9
6	14775	14789.3	14799.6
1 (n=3)	2199.3	2201.9	2204.0
2	4041.9	4049.3	4052.0
3	6620	6627.1	6633.3
4	9124	9137.5	9140.6
5	11357	11370.2	11378.8
6	13384	13401.9	13411.9

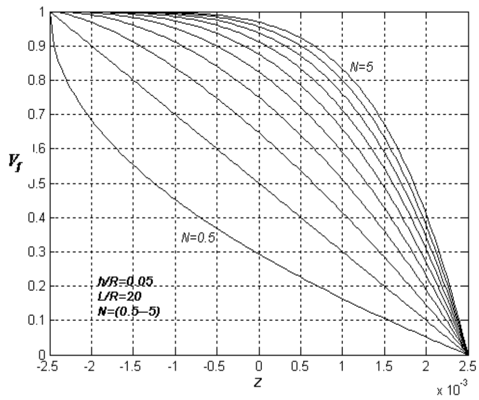

Fig. 6
Variation of volume fraction of nickel in the thickness-wise direction z, Type I.

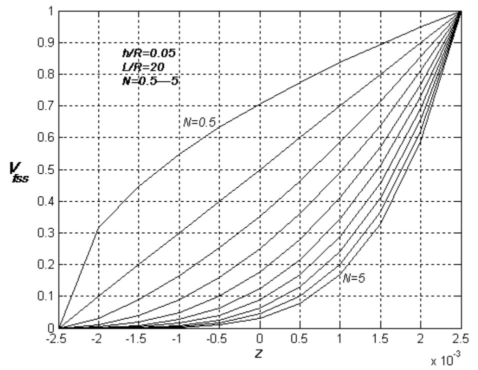

Fig. 7
Variation of volume fraction of Stainless Steel in the thickness-wise direction *z*, Type II.

The influence of the value of N, which affects the constituent volume fraction, can be seen from the tables. As N increased, the natural frequencies decreased. The decreased in the natural frequencies from N=1 to N=15 is about 2.3% at n=1 and about 2.4% at n=10. The natural frequencies approach those of N^{SS} when N is small and they approach those of N^N when N is large.

4 CONCLUSIONS

Based on the higher order shear deformation theory as well as calculated and depicted results of a spinning cylindrical shell made of different types of FG materials it can be concluded that:

(i). Variation on the shell rotational speed has no effect on the value of the natural frequency.
(ii). The natural frequency of short cylindrical shell is higher than the long one, having all other parameters the same.
(iii). The natural frequency of thick cylindrical shell is higher than the thin one, having all other parameters the same.
(iv). For the FG material of the first kind, by increasing the value of volume fraction N, the natural frequency approaches to the case of N^N, i.e. an isotropic media made of nickel. In other words, natural frequency decreases by increasing N.
(v). The least value for the natural frequency occurs at n=2 and 3 and $h/R \le 0.01$.

RFERENCES

[1] Yamanouchi M., Koizumi M., Hirai T., Shiota I., 1990, in: *Proceedings of the First International Symposium on Functionally Gradient Materials*, Japan.
[2] Koizumi M., 1993, The concept of FGM, *Ceramic Transactions*, *Functionally Gradient Materials* **34**: 3-10.
[3] Anon, 1996, FGM components: PM meets the challenge, *Metal Powder Report* **51**: 28-32.
[4] Sata N., 1993, Characteristic of SiC-TiB composites as the surface layer of SiC-TiB-Cu functionally gradient material produced by self-propagating high-temperature synthesis, *Ceramic Transactions*, *Functionally Gradient Materials* **34**: 109-116.
[5] Yamaoka H., Yuki M., Tahara K., Irisawa T., Watanabe R., Kawasaki A., 1993, Fabrication of functionally gradient material by slurry stacking and sintering process, *Ceramic Transactions*, *Functionally Gradient Materials* **34**: 165-172.
[6] Rabin B.H., Heaps R.J., 1993, Powder processing of NI- Al2O3 FGM, *Ceramic Transactions*, *Functionally Gradient Materials* **34**: 173-180.
[7] Fukui Y., 1991, Fundamental investigation of functionally gradient material manufacturing system using centrifugal force, *International Journal of Japan Society of Mechanical Engineers Series III* **34**: 144-148.
[8] Wetherhold R.C., Seelman S., Wang J.Z., 1996, Use of functionally graded materials to eliminate or control thermal deformation, *Composites Science and Technology* **56**: 1099-1104.
[9] Obata Y., Noda N., 1994, Steady thermal stresses in a hollow circular cylinder and a hollow sphere of a functionally gradient material, *Journal of Thermal Stresses* **17**: 471-87.
[10] Arnold R.N., Warburton G.B., 1949, Flexural vibrations of the walls of thin cylindrical shells having freely supported ends, in: *Proceedings of the Royal Society London A* **197**: 238-56.
[11] Ludwig A., Krieg R., 1981, An analytical quasi-exact method for calculating eigenvibrations of thin circular cylindrical shells, *Journal of Sound and Vibration* **74**: 155-74.
[12] Chung H., 1981, Free vibration analysis of circular cylindrical shells, *Journal of Sound and Vibration* **74**: 331-350.
[13] Soedel W., 1980, A new frequency formula for closed circular cylindrical shells for a large variety of boundary conditions, *Journal of Sound and Vibration* **70**: 309-317.
[14] Forsberg K., 1964, Influence of boundary conditions on modal characteristics of cylindrical shells, *AIAA Journal* **2**: 182-189.
[15] Bhimaraddi A., 1984, A higher order theory for free vibration analysis of circular cylindrical shells, *International Journal of Solids and Structures* **20**: 623-630.
[16] Soldatos K.P., Hajigeoriou V.P., 1990, Three-dimensional solution of the free vibration problem of homogeneous isotropic cylindrical shells and panels, *Journal of Sound and Vibration* **137**: 369-384.
[17] Soldatos K.P., 1984, A comparison of some shell theories used for the dynamic analysis of cross-ply laminated circular cylindrical panels, *Journal of Sound and Vibration* **97**: 305-319.
[18] Lam K.Y., Loy C.T., 1995, Effects of boundary conditions on frequencies characteristics for a multi-layered cylindrical shell, *Journal of Sound and Vibration* **188**: 363-384.
[19] Loy C.T., Lam K.Y., 1997, Vibration of cylindrical shells with ring support, *International Journal of Mechanical Sciences* **39**: 455-471.
[20] Loy C.T., Lam K.Y., Shu C., 1997, Analysis of cylindrical shells using generalized differential quadrature, *Shock and Vibration* **4**: 193-98.
[21] Loy C.T., Lam K.Y., Reddy J.N., 1999, Vibration of functionally graded cylindrical shells, *International Journal of Mechanical Sciences* **41**: 316-323.
[22] Zhao X, Liew K.M., Ng T.Y., 2002, Vibration of rotating cross ply laminated circular cylindrical shells with stringer and ring stiffeners, *International Journal of Solids and Structures* **39**(2): 529-545.
[23] Reddy J.N., 2004, *Mechanics of Laminated Composite Plates and Shells*, USA.