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Study of Wave Motion in an Anisotropic Fiber-Reinforced Thermoelastic Solid

R. Kumar[1,*], R.R. Gupta[2]

[1]*Department of Mathematics, Kurukshetra University Kurukshetra, Haryana, India*
[2]*Department of Mathematics, Maharishi Markandeshwar University, Mullana, Haryana, India*

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ABSTRACT

The present investigation deals with the propagation of waves in the layer of an anisotropic fibre reinforced thermoelastic solid. Secular equations for symmetric and skew-symmetric modes of wave propagation in completely separate terms are derived. The amplitude of displacements and temperature distribution were also obtained. Finally, the numerical solution was carried out for Cobalt material and the dispersion curves, amplitude of displacements and temperature distribution for symmetric and skew-symmetric wave modes to examine the effect of anisotropy. Some particular cases are also deduced.

Keywords: Wave propagation; Fiber-reinforced; Transversely isotropic; Amplitudes

1 INTRODUCTION

FIBER-REINFORCED are widely used in engineering structures, due to their superiority over the structural materials in applications requiring high strength and stiffness in lightweight components. Consequently, characterization of their mechanical behavior is of particular importance for structural design using these materials. Fibers are assumed an inherent material property, rather than some form of inclusion in models. In the case of an elastic solid reinforced by a series of parallel fibers it is usual to assume transverse isotropy.

The idea of continuous self-reinforcement at every point of an elastic solid was introduced by Belfield et al [1]. The characteristic property of reinforced concrete member is that its components, namely concrete and steel act together as a single anisotropic unit as main long as they remain in the elastic condition, i.e. the two components are bound together so that there can be no relative displacement between them. The dynamical interaction between the thermal and mechanical fields in solids has great practical applications in modern aeronautics, astronautics, nuclear reactors and high energy particle accelerators. The generalized theory of thermoelasticity has drawn widespread attention because it removes the physically unacceptable situation of the classical theory of thermoelasticity, that is, that the thermal disturbance propagates with infinite velocity. Lord- Shulman theory [2] is important generalized theories of thermoelasticity that become center of interest of recent research in this area. They incorporated a flux rate term into the Fourier's law of heat conduction (with one relaxation time) and formulated a generalized theory admitting finite speed for thermal signals. The temperature of a deformable body can vary both with time and from point to point. This variation can be caused both by heat exchange with external medium and by the process of deformation itself during which a part of the mechanical energy is transformed into heat. Acharya and Roy [3] discussed the propagation of plane waves and their reflection at the free/rigid boundary of a fiber-reinforced magnetoelstic semi space. Sengupta and Nath [4] investigated the problem of surface waves in fiber-reinforced anisotropic elastic media. Singh [5] showed that, for wave propagation in fiber-reinforced anisotropic media, this decoupling cannot be achieved by the introduction of the displacement potentials. However, no attempt has been made to discuss the propagation of waves in the layer of fiber-reinforced anisotropic generalized thermoelastic solid.

* Corresponding author.
E-mail address: rajneesh_kuk@rediffmail.com (R. Kumar).

The aim of the present study is to enhance our knowledge about the propagation of waves in a layer of fiber-reinforced transversely isotopic thermoelastic solid. This study has many applications in various fields of science and technology, namely, atomic physics, industrial engineering, thermal power plants, submarine structures, pressure vessel, aerospace, chemical pipes and metallurgy. After developing the solution, frequency equations connecting the phase velocity with wave number, for symmetric and skew-symmetric wave modes are derived. The amplitude ratios of displacements and temperature distribution are also obtained. The dispersion curves, attenuation coefficients, amplitude ratio of displacements and temperature distribution for symmetric and skew-symmetric waves are presented and illustrated graphically to evince the effect of anisotropy.

2 BASIC EQUATIONS

The linear equations governing thermoelastic interactions in homogeneous transversely isotropic fiber-reinforced solid are:

2.1 Constitutive relations

$$\tau_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km}\delta_{ij} + e_{kk}a_i a_j) \\ + 2(\mu_L - \mu_T)(a_i a_k e_{kj} + a_j a_k e_{ki}) + \beta(a_k a_m e_{km} a_i a_j) - \beta_{ij}T \tag{1}$$

The deformation tensor is defined by

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \qquad i, j = 1,2,3 \tag{2}$$

Balance law: The balance laws for fiber-reinforced linearly elastic medium whose preferred direction is that of **a** are

$$\tau_{ij,j} + \rho F_i = \rho \ddot{u}_i \tag{3}$$

2.2 Equation of heat conduction

$$K_{ij}T_{,ij} = (\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2})(T_0\beta_{ij}\dot{u}_{i,j} + \rho C_e \dot{T}), \qquad i, j = 1,2,3 \tag{4}$$

where ρ is the mass density, τ_{ij} are components of stress, u_i the mechanical displacement, e_{ij} are components of infinitesimal strain, *T* the temperature change of a material particle, T_0 the reference uniform temperature of the body, $K_{ij} = k_i\delta_{ij}$ (*i* not summed) the heat conduction tensor, $\beta_{ij} = \beta_i\delta_{ij}$ (*i* not summed) the thermal elastic coupling tensor, c_e the specific heat at constant strain, a_j are components of **a**, all referred to Cartesian coordinates. The vector **a** may be a function of position. The coefficients $\lambda, \mu_L, \mu_T, \alpha$ and β are elastic constants with the dimension of stress. We choose **a** [6] so that its components are (1, 0, 0). The comma notation is used for spatial derivatives and superimposed dot represents time differentiation.

3 PROBLEM FORMULATION

In the present paper, we consider an infinite layer with traction free surfaces at $x_2 = \pm H$ (layer of thickness 2*H*), which consists of homogeneous, transversely isotropic thermoelastic material. We take the origin of the coordinate system (x_1, x_2, x_3) on the middle surface of the layer. The $x_2 - x_3$ plane is chosen to coincide with the middle surface and x_2 axis normal to it along the thickness. For the two-dimensional problem, we assume the components of the displacement vector of the form

$$\vec{u} = (u_1, u_2, 0), \tag{5}$$

and assume that the solutions are independent of x_3 i.e. $\partial/\partial x_3 \equiv 0$. Thus the field equations and constitutive relations for such a medium reduces to:

$$c_{11}\frac{\partial^2 u_1}{\partial x_1{}^2} + (c_{13}+c_0)\frac{\partial^2 u_2}{\partial x_1 \partial x_2} + c_0\frac{\partial^2 u_1}{\partial x_2{}^2} - \beta_1\frac{\partial T}{\partial x_1} = \rho\frac{\partial^2 u_1}{\partial t^2} \tag{6}$$

$$c_{33}\frac{\partial^2 u_2}{\partial x_2{}^2} + (c_{13}+c_0)\frac{\partial^2 u_1}{\partial x_1 \partial x_1} + c_0\frac{\partial^2 u_2}{\partial x_1{}^2} - \beta_2\frac{\partial T}{\partial x_2} = \rho\frac{\partial^2 u_2}{\partial t^2} \tag{7}$$

$$k_1\frac{\partial^2 T}{\partial x_1{}^2} + k_2\frac{\partial^2 T}{\partial x_2{}^2} - \rho c_e\left(\frac{\partial T}{\partial t} + \tau_0\frac{\partial^2 T}{\partial t^2}\right) = T_0\left[\beta_1\left(\frac{\partial^2 u_1}{\partial x_1 \partial t} + \tau_0\frac{\partial^3 u_1}{\partial x_1 \partial t^2}\right) + \beta_2\left(\frac{\partial^2 u_2}{\partial x_2 \partial t} + \tau_0\frac{\partial^3 u_2}{\partial x_2 \partial t^2}\right)\right] \tag{8}$$

where

$$\beta_1 = (c_{11}+c_{13})\alpha_1 + c_{13}\alpha_2, \qquad \beta_2 = (c_{13}+c_{33}-c_{55})\alpha_1 + c_{33}\alpha_2, \qquad c_{11} = \lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta,$$
$$c_{13} = \lambda + \alpha, \qquad c_{33} = c_{22} = \lambda + 2\mu_T, \qquad c_0 = c_{44}/2,\ c_{44} = c_{66} = 2\mu_L, \qquad c_{55} = 2\mu_T,\ c_{23} = c_{33} - c_{55}$$

and $\lambda, \alpha, \beta, \mu_L, \mu_T$ are material constants, k_1, k_2 are coefficients of thermal conductivity, α_1, α_2 are coefficients of linear thermal expansion, τ_0 is thermal relaxation time, u_1, u_2 are the components of displacement vector. For further considerations, it is convenient to introduce the non-dimensional variables defined by

$$(x_1', x_2') = \frac{\omega_1^*}{v_1}(x_1, x_2), \qquad \tau'_{ij} = \frac{\tau_{ij}}{\beta_1 T_0}, \qquad T' = \frac{T}{T_0}, \qquad t' = \omega_1^* t \tag{9}$$

where $\omega_1^* = \frac{c_e c_{11}}{k_1}$, $v_1^2 = \frac{c_{11}}{\rho}$

4 BOUNDARY CONDITION

The boundary conditions for the thermally insulated fibre-reinforced layer are the vanishing of normal stress, tangential stress and temperature distribution. Therefore, we consider the following non-dimensional boundary conditions at $x_2 = \pm H$:

$$\tau_{22} = 0, \qquad \tau_{21} = 0, \qquad \frac{\partial T}{\partial x_2} = 0 \tag{10}$$

5 NORMAL MODE ANALYSIS AND SOLUTION OF THE PROBLEM

We assume the solution for (u_1, u_2, T) representing propagating waves in the $x_1 - x_2$ plane of the form

$$(u_1, u_2, T) = (1, \bar{u}_2, \bar{T})u_1 e^{i\xi(x_1 + mx_3 - ct)} \tag{11}$$

where ξ is the wave number, $\omega = \xi c$ is the angular frequency and c is the phase velocity of the wave, m is the unknown parameter which signifies the penetration depth of the wave, $\bar{u}_2, \bar{T}$ are respectively, the amplitude ratios of the displacement u_2 and temperature distribution T to that of the displacement u_1. With the help of Eqs. (9) and (11), field Eqs. (6)-(8) reduced to (after suppressing primes)

$$
\begin{aligned}
&[m^2 a_2 + a_1 + m a_3 \bar{u}_2 + a_4 \bar{T}]\, u_1 e^{i\xi(x_1 + m x_3 - ct)} = 0, \\
&[m a_3 + (a_5 m^2 + a_2)\bar{u}_2 + a_6 m \bar{T}]\, u_1 e^{i\xi(x_1 + m x_3 - ct)} = 0, \\
&[a_{10} + m a_8 \bar{u}_2 + (a_{12} + a_9 m^2)\bar{T}]\, u_1 e^{i\xi(x_1 + m x_3 - ct)} = 0,
\end{aligned} \tag{12}
$$

where

$$
a_1 = \frac{\rho c^2 \xi^2}{c_{11}} - \xi^2, \quad a_2 = \frac{c_0 \xi^2}{c_{11}}, \quad a_3 = -\frac{(c_0 + c_{13})\xi^2}{c_{11}}, \quad a_4 = -i\xi^2, a_5 = (1 - \frac{c_{33}}{c_{11}})\xi^2, \quad a_6 = a_4 \bar{\beta},
$$
$$
a_7 = i\omega(1 - i\omega\tau_0), \quad a_8 = i\xi\varepsilon\bar{\beta} a_7, \quad a_9 = -\bar{k}\xi^2, \quad a_{10} = i\xi\varepsilon a_7, \quad a_{11} = -\xi^2, \quad a_{12} = a_7 + a_{11}
$$

The condition for the non-trivial solution of system of Eqs. (12), yields a cubic equation in $q = m^2$ as

$$
Aq^3 + Bq^2 + Cq + D = 0 \tag{13}
$$

where

$$
A = a_2 a_5 a_9, \quad B = a_5(a_1 a_9 + a_2 a_{12}) + a_9(a_2 - a_3),
$$
$$
C = a_1 a_6 (a_{12} - a_8) + a_2(a_1 a_9 - a_6 a_8) + (a_2^2 - a_3^2) a_{12} + a_3 a_6 a_{10} + a_4(a_8 a_3 - a_{10} a_5), \quad D = a_{12}(a_1 a_2 - a_4 a_{10})
$$

The roots of this equation give three values of q, and hence of c. Three positive values of c will be the velocities of propagation of three possible waves viz., quasi-longitudinal displacement (qLD) wave, quasi-transverse displacement (qTD) wave and quasi-thermal (qT) wave. So Eq. (13) leads to the following solution for displacements and temperature distribution:

$$
(u_1, u_2, T) = \sum_{i=1}^{3} [A_i \cos(m_i x_2 \xi) + B_i \sin(m_i x_2 \xi)](1, r_i, t_i)\; e^{i\xi(x_1 - ct)} \tag{14}
$$

where

$$
r_i = \frac{m_i [a_3 a_9 m_i^2 + (a_3 a_{12} - a_6 a_{10})]}{a_5 a_9 m_i^4 + m_i^2 (a_5 a_{12} - a_6 a_8 + a_2 a_9) + a_2 a_{12}}, \quad t_i = \frac{(a_8 a_3 - a_{10} a_5) m_i^2 - a_2 a_{10}}{a_5 a_9 m_i^4 + m_i^2 (a_5 a_{12} - a_6 a_8 + a_2 a_9) + a_2 a_{12}}
$$

6 DERIVATION OF SECULAR EQUATION

Substituting the values of u_1, u_2 and T in the boundary conditions (10) at the surfaces $\pm H$ of the layer

$$
\begin{aligned}
&\sum_{i=1}^{3} [((g_1 + g_{2i}) c_i + g_{3i} s_i) A_i + (-(g_1 + g_{2i}) s_i + g_{3i} c_i) B_i] = 0 \\
&\sum_{i=1}^{3} [((g_1 + g_{2i}) c_i - g_{3i} s_i) A_i + (-(g_1 + g_{2i}) s_i + g_{3i} c_i) B_i] = 0
\end{aligned} \tag{15}
$$

$$\sum_{i=1}^{3}[(g_{4i}s_i+g_{5i}c_i)A_i+(g_{4i}c_i-g_{5i}s_i)B_i]=0$$
$$\sum_{i=1}^{3}[(-g_{4i}s_i+g_{5i}c_i)A_i+(g_{4i}c_i+g_{5i}s_i)B_i]=0$$
$$\sum_{i=1}^{3}[-g_{6i}s_iA_i+g_{6i}c_iB_i]=0$$
$$\sum_{i=1}^{3}[g_{6i}s_iA_i+g_{6i}c_iB_i]=0$$

where

$$s_i=\sin\ (m_i\xi x_2),\qquad c_i=\cos\ (m_i\xi x_2),\qquad g_1=c_{13}\mathrm{i}\xi,\qquad g_{2i}=-\beta_2 t_i,\qquad g_{3i}=\xi m_i r_i c_{33},\qquad g_{4i}=\xi m_i$$
$$g_{4i}=\xi m_i,\qquad g_{5i}=r_i\xi i,\qquad g_{6i}=t_i m_i\xi,\qquad i=1,2,3$$

In order that the six boundary conditions given by Eq. (10) be satisfied simultaneously, the determinant of the coefficients of A_i and B_i (i=1, 2, 3) in Eqs. (15) vanishes. This gives an equation for the frequency of the layer oscillations. The frequency equation for the waves in the present case, after applying lengthy algebraic reductions and manipulations of the determinant leads to the following secular equations

$$\begin{aligned}&[T_1]^{\pm}[g_{61}g_{42}(g_1+g_{23})-g_{61}g_{43}(g_1+g_{22})]+[T_2]^{\pm}[g_{62}g_{43}(g_1+g_{21})-g_{62}g_{41}(g_1+g_{23})]\\&\quad+[T_3]^{\pm}[g_{63}g_{41}(g_1+g_{22})-g_{63}g_{42}(g_1+g_{21})]=0\end{aligned}\tag{16}$$

These are the frequency equations which correspond to the symmetric and skew symmetric mode with respect to the medial plane $x_3=0.$ Here, the superscript '+' corresponds to skew symmetric and '-' refers to symmetric modes and $T_i=\tan\ (m_i\xi x_2),$ i=1,2,3.

6.1 Amplitudes of displacements and temperature distribution

In this section the amplitudes of displacement components and temperature distribution for symmetric and skew symmetric modes of plane waves can be obtained as

$$\begin{aligned}&[(u_1)_{sym},(u_1)_{asym}]=\sum_{i=1}^{3}[A_i\cos(m_ix_2\xi),B_i\sin(m_ix_2\xi)]\ \mathrm{e}^{\mathrm{i}\xi(x_1-ct)}\\&[(u_2)_{sym},(u_2)_{asym}]=\sum_{i=1}^{3}r_i[A_i\sin(m_ix_2\xi),B_i\cos(m_ix_2\xi)]\ \mathrm{e}^{\mathrm{i}\xi(x_1-ct)}\\&[(T)_{sym},(T)_{asym}]=\sum_{i=1}^{3}t_i[A_i\sin(m_ix_2\xi),B_i\cos(m_ix_2\xi)]\ \mathrm{e}^{\mathrm{i}\xi(x_1-ct)}\end{aligned}\tag{17}$$

6.2 Specific loss

The specific loss is the ratio of energy (ΔW) dissipated in taking a specimen through a stress cycle, to the elastic energy (W) stored in the specimen when the strain maximum. Kolsky [7] shows that specific loss $(\Delta W/W)$ is 4π time the absolute value of the ratio of the imaginary part of wave number to the real part of wave number i.e.

$$\frac{\Delta W}{W}=4\pi\left|\frac{\mathrm{Im}(k)}{\mathrm{Re}(k)}\right|\tag{18}$$

He noted that specific loss is the most direct method of defining internal friction for a material.

6.3 Particular case:

Isotropic elastic case: Taking $\mu_L = \mu_T = \mu$ and $\alpha = \beta = 0$ the Eq. (13), we obtain the corresponding expression for isotropic fiber-reinforced elastic solid.

7 NUMERICAL RESULTS AND DISCUSSION

In order to illustrate theoretical results obtained in the preceding sections, we now present some numerical results. For the purpose of numerical computations, we have used MATLAB's Program. The following relevant physical constants are taken for a fiber-reinforced transversely isotopic material

$$\rho = 2.66\times10^3\ \text{Kg/m}^3,\ \ \lambda = 5.65\times10^{10}\ \text{N/m}^2,\ \ \mu_T = 2.46\times10^{10}\ \text{N/m}^2,\ \ \mu_L = 5.66\times10^{10}\ \text{N/m}^2$$

$$\alpha = -1.28\times10^{10}\ \text{N/m}^2,\quad \beta = 220.90\times10^{10}\ \text{N/m}^2,\quad K_1 = .0921\times10^3\ \text{Jm}^{-1}\text{deg}^{-1}\text{s}^{-1},$$

$$K_2 = .0963\times10^3\ \text{Jm}^{-1}\text{deg}^{-1}\text{s}^{-1},\ \ \alpha_1 = .017\times10^4\ \text{deg}^{-1},\ \ \alpha_2 = .015\times10^4\ \text{deg}^{-1},$$

$$C_e = 0.787\times10^3\ \text{J Kg}^{-1}\ \text{deg}^{-1},\ \ T_0 = 293\text{K},\ \ \tau_0 = .05\text{s},\ \ \omega = 2\text{s}^{-1}.$$

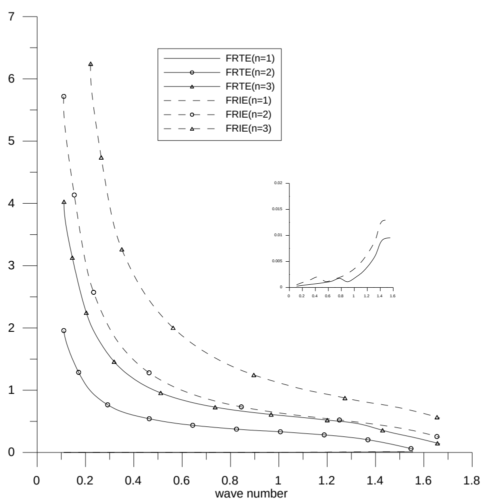

Fig. 1
Variation of phase velocity with wave number for symmetric mode.

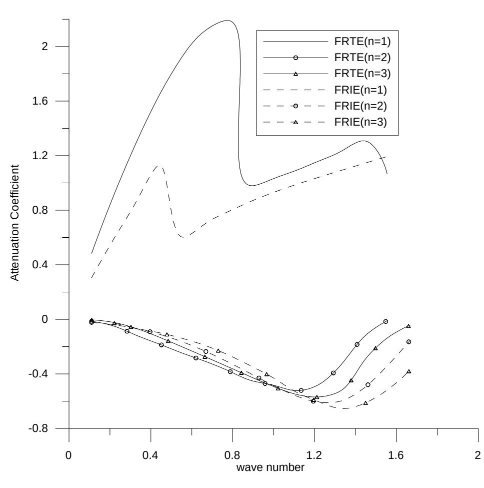

Fig. 2
Variation of attenuation coefficient with wave number for symmetric mode.

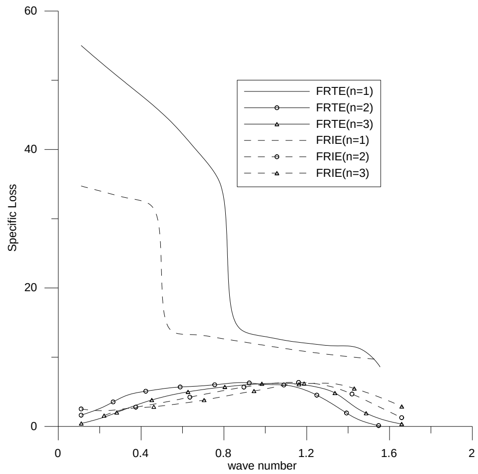

Fig. 3
Variation of specific loss with wave number for symmetric mode.

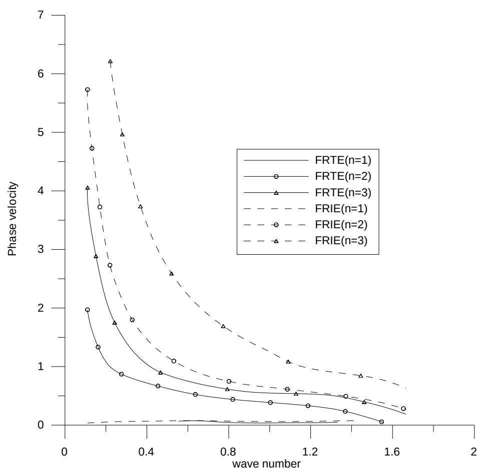

Fig. 4
Variation of phase velocity with wave number for skew symmetric mode.

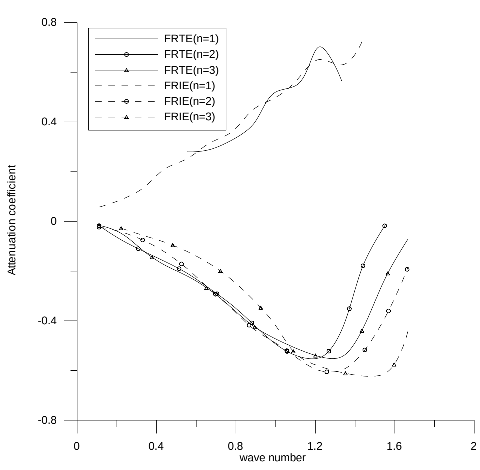

Fig. 5
Variation of attenuation coefficient with wave number for skew symmetric mode.

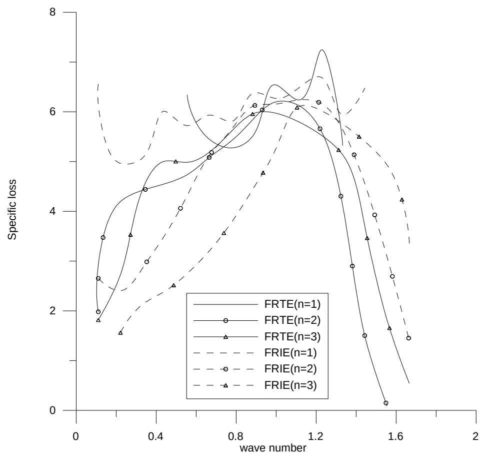

Fig. 6
Variation of specific loss with wave number for skew symmetric mode.

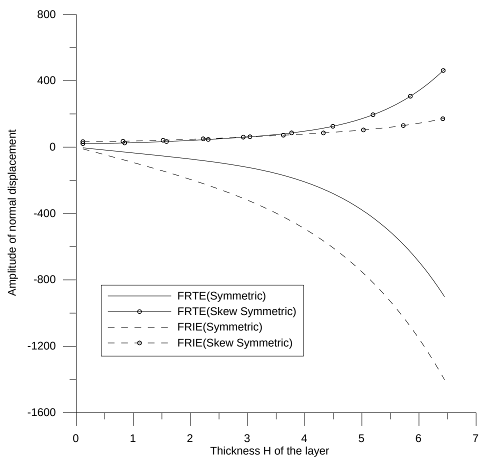

Fig. 7
Variation of amplitude of normal displacement.

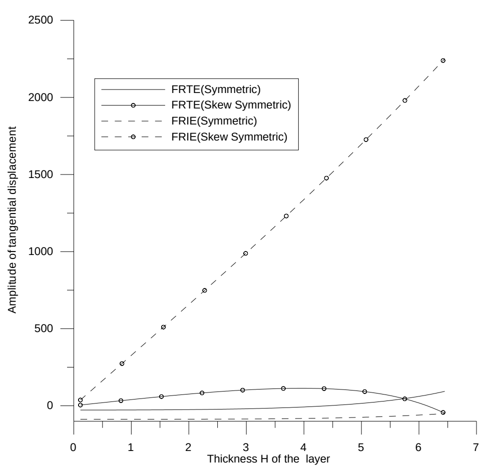

Fig. 8
Variation of amplitude of tangatial displacement.

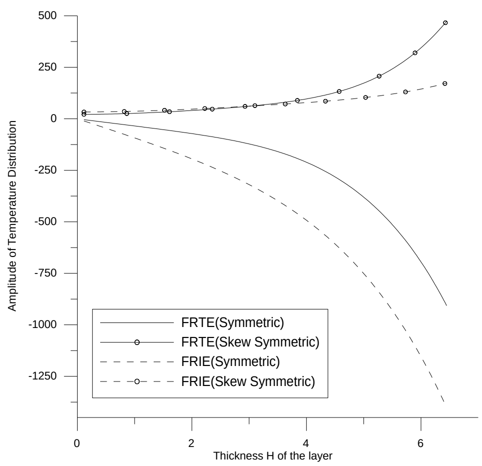

Fig. 9
Variation of amplitude of temperature displacement.

The plots of non-dimensional phase velocity, attenuation coefficient and specific loss with non-dimensional wave number restricted to thickness H=1 for symmetric and skew symmetric modes are shown in Figs. 1-6. Here, solid lines with and without center symbol represents the variation corresponding to transversely isotropic fiber-reinforced thermoelastic solid (FRTE) while, broken lines with and without center symbol represents the variation corresponding to fiber-reinforced isotropic elastic solid (FRIE). The lines shown in the figures without center symbol represent the variations corresponding to initial mode (n=1) of wave propagation, lines with center symbol $(-\circ-\circ-)$ represent the variations corresponding to second mode (n=2) and lines with center symbol $(-\times-\times-)$ represent the variations corresponding to final mode (n=3) of wave propagation.

Figs. 1 and 4 show the variation of phase velocity with respect to wave number for symmetric and skew symmetric modes, respectively. It is depicted from these figures that for all the modes of wave propagation, the value of phase velocity decreases sharply to attain a constant value at the end. The variation of attenuation coefficient with respect to wave number for symmetric and skew symmetric modes can be depicted from Figs. 2 and 5, respectively. It is seen from Fig. 2 that for initial mode, the value of attenuation coefficient sharply increases over the interval (0, 0.6), and then sharply decreases for FRTE. The variation pattern for FRIE is similar to that of FRTE with difference in their amplitude. As we move to higher modes, its value sharply decreases up to the range 1.2, and then sharply increases for both FTRE and FRIE. However, for the skew symmetric mode, the variation pattern is similar to those of symmetric mode, except for the initial mode, where its value constantly increases with increase in wave number. Figs. 3 and 6 illustrate the variations of specific loss with wave number for symmetric and skew symmetric modes. It is depicted from these figures that for symmetric mode, at the initial mode, its value decreases to attain a constant value. But for the higher modes, its value oscillates with small amplitude about the origin. However, for the skew symmetric mode, and for all the modes of wave propagation, its value oscillates arbitrarily with different amplitudes.

Figs. 7-9 indicate the trend of variations of amplitude of normal displacement, tangential displacement and temperature distribution with respect to thickness H of the layer. It is depicted from Figs. 7 and 9 that the amplitude of normal displacement and temperature distribution sharply decreases for symmetric mode and increases with increase in wave number for skew symmetric mode. The variation pattern for both FRTE and FRIE remain same. Fig. 8 depicts the variation of tangential displacement with thickness of the layer. For all the values, its value oscillates near the origin except for the skew symmetric case and for FRIE solid where its value sharply increases with increase in wave number.

8 CONCLUSIONS

The propagation of waves in an infinite layer of fiber reinforced thermoelastic transversely isotropic medium after deriving the secular equation is investigated. The phase velocities of highest mode of wave propagation attain quite large values at vanishing wave number, which sharply flattens out to become steady with increasing wave number for both symmetric and antisymmetric modes. The value of attenuation coefficient initially increases and then tends to zero at higher values of wave number. An appreciable anisotropy is evinced from all the curves. The values of phase velocity get decreased with increase in anisotropy, while that of attenuation coefficient and specific loss oscillates arbitrarily.

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