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Comparison of Various Shell Theories for Vibrating Functionally Graded Cylindrical Shells

M. Javadinejad*

Department of Mechanical Engineering, Islamic Azad University, Khomeinishahr Branch, Khomeinishahr, Iran

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ABSTRACT

The classical shell theory, first-order shear deformation theory, and third-order shear deformation theory are employed to study the natural frequencies of functionally graded cylindrical shells. The governing equations of motion describing the vibration behavior of functionally graded cylindrical shells are derived by Hamilton's principle. Resulting equations are solved using the Navier-type solution method for a functionally graded cylindrical shell with simply supported edges. The effects of transverse shear deformation, geometric size, and configurations of the constituent materials on the natural frequencies of the shell are investigated. Validity of present formulation was checked by comparing the numerical results with the Love's shell theory.

Keywords: Functionally graded material; Cylindrical shell; Natural frequency; Various shell theories

1 INTRODUCTION

FUNCTIONALLY graded cylindrical shells can be used to design the aerospace structural systems such as supersonic and hypersonic aircraft, rockets, satellites, and nuclear components. In 1989, Reddy and Khdeir [1] developed the exact and finite element solutions to analyze the free vibration and buckling of cross-ply rectangular composite laminates using the classical, first-order and third-order laminate plate theories under various boundary conditions. Sivadas and Ganesan [2] studied the effects of various parameters, such as the number of layers of the shell**,** the semi-vertex angle, the slant length-to-small end radius ratio and the thickness variation parameter on the natural frequencies of laminated conical shells by using a semi-analytical finite element method. They used the Love's first approximation thin shell theory to solve the problem. Matsunaga [3] analyzed the natural frequencies and buckling stresses of thick isotropic shells subjected to in-plane stresses.

The vibration behavior of functionally graded (FG) cylindrical shells based on the Love's shell theory and the Rayleigh-Ritz method is investigated by Loy et al. [4] and Pradhan et al. [5]. Their studies revealed that the frequency characteristics of FG cylindrical shells are similar to those of isotropic shells. The vibration of thin cylindrical shells with ring support made of a functionally gradient material composed of stainless steel and nickel is studied by Najafizadeh and Isvandzibaei [6]. Results are presented for the frequency characteristics, influences of ring support position and boundary conditions. Patel et al. [7] employed the finite element formulation to study the free vibration of FG elliptical cylindrical shells. They also studied the influences of non-circularity, radius-to-thickness ratio, material composition and material profile index on the free vibration frequencies and mode shape characteristic of FG elliptical shells. Haddadpour et al. [8] reported the free vibration of simply supported FG cylindrical shells for four sets of in-plane boundary conditions. The effects of material profile index on the natural frequencies and corresponding mode shapes in the thermal environment are also discussed. Based on temperature-dependent material properties, Kadoli and Ganesan [9] studied the linear thermal buckling and free vibration of FG cylindrical shells. They have shown that the fall in natural frequency is very drastic for the mode corresponding to the lowest natural frequency when compared to the lowest buckling temperature mode. Recently, Zhi-yuan and Hua-

* *E-mail address:* mehdy.javady@yahoo.com

ning [10] presented the free vibration analysis of FG cylindrical shells with holes. They investigated the non-dimensional frequencies of shell with holes of different shape, number, and location. The present paper develops the free vibration analysis of functionally graded cylindrical shells based on the classical shell theory (CST), first-order shear deformation theory (FSDT) and third-order shear deformation theory (TSDT). The shell is graded only through the thickness direction. The equations of motion are established using the Navier-type solution method. The results show that the natural frequencies are affected by the transverse shear deformation, inhomogeneity parameter, configurations of the constituent materials, thickness-to-radius ratio, and length-to-radius ratio. Comparison studies are carried out to validate the present results.

2 FORMULATION

Consider a FG cylindrical shell of mean radius R, thickness h, and length L, referred to the cylindrical coordinates (x,θ,z). The shell properties are assumed to vary only through the thickness direction according to power-law form, which are given by [4]

$$P(z)=(P_{out}-P_{in})\left(\frac{2z+h}{2h}\right)^{k}+P_{in} \tag{1}$$

where P denotes a material property of FG cylindrical shell which may be substituted with the modulus of elasticity E, mass density ρ, and Poisson's ratio υ. The subscripts in and out refer to the inner and outer surfaces of the shell, respectively and k is non-negative real number called the inhomogeneity parameter. The following displacement field is used for FG cylindrical shells [11]:

$$\begin{aligned}
u(x,\theta,z,t)&=u_0(x,\theta,t)+z\phi_x(x,\theta,t)-\alpha z^3(\phi_x+w_{0,x})\\
v(x,\theta,z,t)&=v_0(x,\theta,t)+z\phi_\theta(x,\theta,t)-\alpha z^3(\phi_\theta+w_{0,\theta})\\
w(x,\theta,z,t)&=w_0(x,\theta,t)
\end{aligned} \tag{2}$$

where u_0, v_0, and w_0 are unknown displacements of a point on the mid-surface of the shell, while ϕ_x and ϕ_θ describe the rotations about the $\theta-$and $x-$axes, respectively. The displacement model (2) is valid for various shell theories by the following relations:

$$\begin{aligned}
&\text{For CST: } \alpha=0, \phi_x=-\partial w_0/\partial x, \phi_\theta=-\partial w_0/\partial\theta\\
&\text{For FSDT: } \alpha=0\\
&\text{For TSDT: } \alpha=4/(3h^2)
\end{aligned} \tag{3}$$

The constitutive relations of FG cylindrical shells can be expressed as

$$\begin{aligned}
N_x&=A_{11}\varepsilon_x^0+A_{12}\varepsilon_\theta^0+B_{11}k_x+B_{12}k_\theta+E_{11}\eta_x+E_{12}\eta_\theta\\
N_\theta&=A_{12}\varepsilon_x^0+A_{11}\varepsilon_\theta^0+B_{12}k_x+B_{11}k_\theta+E_{12}\eta_x+E_{11}\eta_\theta\\
M_x&=B_{11}\varepsilon_x^0+B_{12}\varepsilon_\theta^0+D_{11}k_x+D_{12}k_\theta+F_{11}\eta_x+F_{12}\eta_\theta\\
M_\theta&=B_{12}\varepsilon_x^0+B_{11}\varepsilon_\theta^0+D_{12}k_x+D_{11}k_\theta+F_{12}\eta_x+F_{11}\eta_\theta\\
P_x&=E_{11}\varepsilon_x^0+E_{12}\varepsilon_\theta^0+F_{11}k_x+F_{12}k_\theta+H_{11}\eta_x+H_{12}\eta_\theta\\
P_\theta&=E_{12}\varepsilon_x^0+E_{11}\varepsilon_\theta^0+F_{12}k_x+F_{11}k_\theta+H_{12}\eta_x+H_{11}\eta_\theta\\
N_{x\theta}&=A_{22}\gamma_{x\theta}^0+B_{22}k_{x\theta}+E_{22}\eta_{x\theta}\\
M_{x\theta}&=B_{22}\gamma_{x\theta}^0+D_{22}k_{x\theta}+F_{22}\eta_{x\theta}\\
P_{x\theta}&=E_{22}\gamma_{x\theta}^0+F_{22}k_{x\theta}+H_{22}\eta_{x\theta}
\end{aligned}$$

$$
\begin{aligned}
Q_x &= A_{22}\gamma_{xz}^0 + D_{22}\eta_{xz} \\
Q_\theta &= A_{22}\gamma_{\theta z}^0 + D_{22}\eta_{\theta z} \\
R_x &= D_{22}\gamma_{xz}^0 + F_{22}\eta_{xz} \\
R_\theta &= D_{22}\gamma_{\theta z}^0 + F_{22}\eta_{\theta z}
\end{aligned} \tag{4}
$$

where the kinematic relations are:

$$
\begin{aligned}
&\varepsilon_x^0 = u_{0,x}, \qquad \varepsilon_\theta^0 = \frac{v_{0,\theta}+w_0}{R}, \qquad \gamma_{x\theta}^0 = \frac{u_{0,\theta}}{R} + v_{0,x}, \\
&k_x = \phi_{x,x}, \qquad k_\theta = \frac{\phi_{\theta,\theta}}{R}, \qquad k_{x\theta} = \frac{\phi_{x,\theta}}{R} + \phi_{\theta,x} \\
&\gamma_{xz}^0 = \phi_x + w_{0,x}, \qquad \gamma_{\theta z}^0 = \phi_\theta + \frac{w_{0,\theta}}{R} \\
&\eta_x = -\alpha(\phi_{x,x} + w_{0,xx}), \qquad \eta_\theta = -\frac{\alpha}{R}(-Rv_{0,\theta} + \phi_{\theta,\theta} + \frac{1}{R}w_{0,\theta\theta}) \\
&\eta_{x\theta} = -\frac{\alpha}{R}(-Rv_{0,x} + R\phi_{\theta,x} + 2w_{0,x\theta} + \phi_{x,\theta}) \\
&(A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}) = \int_{-h/2}^{h/2} C_{ij}(1, z, z^2, z^3, z^4, z^6)\,\mathrm{d}z
\end{aligned} \tag{5}
$$

where the stiffness coefficients C_{ij}, are as follows

$$
\begin{aligned}
C_{11} &= \frac{E(z)}{1-\upsilon(z)^2} \\
C_{12} &= \frac{\upsilon(z)E(z)}{1-\upsilon(z)^2} \\
C_{22} &= \frac{E(z)}{2[1+\upsilon(z)]}
\end{aligned} \tag{6}
$$

The stress resultants, N_i, M_i, P_i, Q_i, and R_i, in Eqs. (4) are defined by

$$
\begin{aligned}
(N_i, M_i, P_i) &= \int_{-h/2}^{h/2} \sigma_i (1, z, z^3)\,\mathrm{d}z, \qquad i = x, \theta, x\theta \\
(Q_i, R_i) &= \int_{-h/2}^{h/2} \sigma_{iz}(1, z^2)\,\mathrm{d}z, \qquad i = x, \theta
\end{aligned} \tag{7}
$$

The governing equations of motion appropriate for the displacement field, Eq. (2), can be derived using the Hamilton's principle as

$$
\begin{aligned}
&N_{x,x} + \frac{1}{R}N_{x\theta,\theta} = I_0\ddot{u}_0 + \bar{I}_1\ddot{\phi}_x - \alpha I_3\ddot{w}_{0,x} \\
&\overline{M}_{x,x} + \frac{1}{R}\overline{M}_{x\theta,\theta} - \overline{Q}_x = \bar{I}_1\ddot{u}_0 + \bar{I}_2\ddot{\phi}_x - \alpha\bar{I}_4\ddot{w}_{0,x} \\
&N_{x\theta,x} + \frac{1}{R}N_{\theta,\theta} + \frac{1}{R}\overline{Q}_\theta = I_0\ddot{v}_0 + \bar{I}_1\ddot{\phi}_\theta - \alpha I_3\ddot{w}_{0,\theta} \\
&\overline{M}_{x\theta,x} + \frac{1}{R}\overline{M}_{\theta,\theta} - \overline{Q}_\theta = \bar{I}_1\ddot{v}_0 + \bar{I}_2\ddot{\phi}_\theta - \alpha\bar{I}_4\ddot{w}_{0,\theta}
\end{aligned}
$$

$$\overline{Q}_{x,x}+\frac{1}{R}\overline{Q}_{\theta,\theta}-\frac{1}{R}N_{\theta}+\alpha(P_{x,xx}+2P_{x\theta,x\theta}+P_{\theta,\theta\theta})=I_0\ddot{w}_0+\alpha I_3(\ddot{u}_{0,x}+\ddot{v}_{0,\theta})+\alpha\bar{I}_4(\ddot{\phi}_{x,x}+\ddot{\phi}_{\theta,\theta})$$
$$-\alpha^2 I_6(\ddot{w}_{0,xx}+\ddot{w}_{0,\theta\theta}) \quad (8)$$

where

$$\overline{M}_i=M_i-\alpha P_i$$
$$\overline{Q}_i=Q_i-3\alpha R_i$$
$$\bar{I}_1=I_1-\frac{4}{3h^2}I_3$$
$$\bar{I}_2=I_2-\frac{8}{3h^2}I_4+\frac{16}{9h^4}I_6$$
$$\bar{I}_4=I_4-\frac{4}{3h^2}I_6,\ \ I_j=\int_{-h/2}^{h/2}(z)^j\rho(z)\,\mathrm{d}z \quad (9)$$

The Navier-type solution method is used to obtain the natural frequencies of FG cylindrical shells. Solution of Eqs. (8) is expressed as products of undetermined functions and known trigonometric functions so as to satisfy identically the simply supported boundary conditions at ends $(v=w=N_x=M_x=P_x=0)$. Substituting relations (10) into (5), results into (4), and then into (8) give a characteristic equation for natural frequencies.

$$\begin{Bmatrix}u_0\\ v_0\\ w_0\\ \phi_x\\ \phi_\theta\end{Bmatrix}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix}U_{mn}\cos\frac{m\pi x}{L}\cos n\theta\cos\omega t\\ V_{mn}\sin\frac{m\pi x}{L}\sin n\theta\cos\omega t\\ W_{mn}\sin\frac{m\pi x}{L}\cos n\theta\cos\omega t\\ X_{mn}\cos\frac{m\pi x}{L}\cos n\theta\cos\omega t\\ Y_{mn}\sin\frac{m\pi x}{L}\sin n\theta\cos\omega t\end{Bmatrix} \quad (10)$$

where ω denotes the natural frequency, m and n are the axial and circumferential wave numbers, and $U_{mn},V_{mn},W_{mn},X_{mn}$, and Y_{mn} are undetermined coefficients. Substituting relations (10) into (5), results into (4), and then into (8) give a characteristic equation for natural frequencies.

3 RESULTS AND DISCUSSION

The natural frequencies of simply supported FG cylindrical shells have been obtained using the classical shell theory, first-order shear deformation theory and third-order shear deformation theory. The FG cylindrical shell considered here is composed of stainless steel and nickel. The material properties for stainless steel are $E=207.788$ GPa, $\rho=8166$ Kgm^{-3}, and $\nu=0.318$ and for nickel are $E=205.098$ GPa, $\rho=8900$ Kgm^{-3}, and $\nu=0.310$. The effects of the functionally graded material configuration are presented by study the natural frequencies of two types of FG cylindrical shells: Type I, which shell has nickel on its inner surface and stainless steel on its outer surface and Type II, which has stainless steel on its inner surface and nickel on its outer surface.

To investigate the accuracy of the present formulation, the results are compared with the results of Loy et al. [4] which are based on the Love's shell theory. The effect of transverse shear deformation and circumferential wave number n, on the natural frequencies (Hz) for Types I and II FG cylindrical shells is shown in Tables 1 and 2. It is observed that the frequencies for higher axial modes are higher than those for lower axial modes. Thus, the fundamental frequencies occur at $m=1$. For Type I, the natural frequencies are decreased with increasing the inhomogeneity parameter, k. However, for Type II the natural frequencies are increased with increasing k. For example, the decrease and increase in the natural frequencies from $k=1$ to $k=15$ respectively for Types I and II, is about 2.3% at $n=1$ and about 2.4% at $n=10$. For $k>0$, the natural frequencies fell between those of stainless steel and nickel for a given circumferential wave number, n. As can be seen, for $k>2$, the natural frequencies for Type II are higher than those for Type I. For example, for $k=15$ at $n=10$, the natural frequency for Type II is about 4.3% higher than the other one.

Table 1
Variation of natural frequencies (Hz) against circumferential wave number *n* (*m*=1, *h*/*R*=0.05, *L*/*R*=20, Type I)

n	Theory	Stainless Steel	Nickel	*k*=0.5	*k*=0.7	*k*=1	*k*=2	*k*=3	*k*=15	*k*=30
1	CST	13.5463	12.8922	13.3192	13.2520	13.2100	13.1017	12.9967	12.9340	12.9130
	FSDT	13.5474	12.8933	13.3205	13.2670	13.2105	13.1026	12.9976	12.9324	12.9133
	TSDT	13.5476	12.8935	13.3208	13.2680	13.2107	13.1028	12.9978	12.9326	12.9135
	Ref. [4]	13.5480	12.8940	13.3210	13.2690	13.2110	13.1030	12.9980	12.9330	12.9140
2	CST	4.5907	4.3675	4.5154	4.4981	4.4789	4.4421	4.4054	4.3822	4.3752
	FSDT	4.5916	4.3684	4.5162	4.4991	4.4798	4.4430	4.4063	4.3830	4.3761
	TSDT	4.5918	4.3686	4.5165	4.4992	4.4800	4.4432	4.4065	4.3832	4.3763
	Ref. [4]	4.5920	4.3690	4.5168	4.4994	4.4800	4.4435	4.4068	4.3834	4.3765
3	CST	4.2621	4.0472	4.1893	4.1735	4.1554	4.1222	4.0881	4.0640	4.0563
	FSDT	4.2628	4.0482	4.1906	4.1745	4.1563	4.1231	4.0888	4.0648	4.0571
	TSDT	4.2630	4.0484	4.1909	4.1746	4.1565	4.1233	4.0890	4.0651	4.0773
	Ref. [4]	4.2633	4.0489	4.1911	4.1749	4.1569	4.1235	4.0891	4.0653	4.0576
4	CST	7.2232	6.8562	7.0961	7.0665	7.0371	6.9802	6.9237	6.8842	6.8711
	FSDT	7.2246	6.8571	7.0967	7.0686	7.0380	6.9815	6.9246	6.8849	6.8720
	TSDT	7.2248	6.8573	7.0970	7.0687	7.0382	6.9817	6.9248	6.8851	6.8722
	Ref. [4]	7.2250	6.8577	7.0972	7.0691	7.0384	6.9820	6.9251	6.8856	6.8726
5	CST	11.5406	10.9532	11.3342	11.2890	11.2392	11.1500	11.0591	10.9973	10.9767
	FSDT	11.5415	10.9542	11.3349	11.2900	11.2405	11.1507	11.0605	10.9984	10.9776
	TSDT	11.5417	10.9544	11.3352	11.2900	11.2407	11.1510	11.0610	10.9990	10.9780
	Ref. [4]	11.5420	10.9550	11.3360	11.2900	11.2410	11.1510	11.0610	10.9990	10.9780
6	CST	16.8951	16.0356	16.5926	16.5256	16.4532	16.3214	16.1906	16.0961	16.0693
	FSDT	16.8960	16.0367	16.5934	16.5267	16.4545	16.3223	16.1914	16.1007	16.0706
	TSDT	16.8962	16.0369	16.5937	16.5268	16.4547	16.3225	16.1916	16.1009	16.0708
	Ref. [4]	16.8970	16.0370	16.5940	16.5270	16.4550	16.3230	16.1920	16.1010	16.0710
7	CST	23.2422	22.0593	22.8246	22.7336	22.6334	22.4526	22.2715	22.1461	22.1067
	FSDT	23.2433	22.0605	22.8255	22.7344	22.6343	22.4535	22.2724	22.1476	22.1076
	TSDT	23.2435	22.0607	22.8258	22.7345	22.6345	22.4537	22.2726	22.1478	22.1078
	Ref. [4]	23.2440	22.0610	22.8260	22.7350	22.6350	22.4540	22.2730	22.1480	22.1080
8	CST	30.5718	29.0157	30.0220	29.9024	29.7693	29.5317	29.2947	29.1306	29.0765
	FSDT	30.5725	29.0166	30.0224	29.9020	29.7704	29.5326	29.2956	29.1315	29.0774
	TSDT	30.5727	29.0168	30.0227	29.9028	29.7706	29.5328	29.2958	29.1317	29.0776
	Ref. [4]	30.5730	29.0170	30.0230	29.9030	29.7710	29.5330	29.2960	29.1320	29.0780
9	CST	38.8670	36.9006	38.1792	38.0262	37.8606	37.5576	37.2557	37.0462	36.9794
	FSDT	38.8770	36.9015	38.1804	38.0276	37.8616	37.5584	37.2565	37.0473	36.9804
	TSDT	38.8790	36.9017	38.1807	38.0277	37.8618	37.5586	37.2567	37.0475	36.9807
	Ref. [4]	38.8810	36.9020	38.1810	38.0280	37.8620	37.5590	37.2570	37.0480	36.9810
10	CST	48.1662	45.7146	47.2990	47.1100	46.9032	46.5275	46.1537	45.8956	45.8113
	FSDT	48.1671	45.7155	47.3006	47.1108	46.9047	46.5285	46.1544	45.8964	45.8121
	TSDT	48.1673	45.7157	47.3009	47.1109	46.9049	46.5287	46.1546	45.8967	45.8124
	Ref. [4]	48.1680	45.7160	47.3010	47.1110	46.9050	46.5290	46.1550	45.8970	45.8130

Table 2
Variation of natural frequencies (Hz) against circumferential wave number *n* (*m*=1, *h*/*R*=0.05, *L*/*R*=20, Type II)

n	Theory	Stainless Steel	Nickel	*k*=0.5	*k*=0.7	*k*=1	*k*=2	*k*=3	*k*=15	*k*=30
1	CST	13.5360	12.8842	13.0836	13.1473	13.2015	13.1153	13.234	13.4930	13.113
	FSDT	13.5474	12.8933	13.1027	13.1535	13.2105	13.1263	13.4325	13.5046	13.5224
	TSDT	13.5476	12.8935	13.1027	13.1537	13.2107	13.1265	13.4327	13.5047	13.5224
	Ref. [4]	13.548	12.894	13.103	13.154	13.2110	13.321	13.433	13.505	13.526
2	CST	4.5820	4.3515	4.4380	4.4396	4.4632	4.5004	4.5412	4.5624	4.5721
	FSDT	4.5916	4.3684	4.4380	4.4546	4.4738	4.5110	4.5500	4.5753	4.5830
	TSDT	4.5918	4.3686	4.4380	4.4548	4.4740	4.5112	4.5502	4.5755	4.5832
	Ref. [4]	4.5920	4.3690	4.4382	4.4550	4.4742	4.5114	4.5504	4.5759	4.5836
3	CST	4.2552	4.0327	4.1149	4.1297	4.1363	4.1731	4.2063	4.2343	4.2432
	FSDT	4.2623	4.0482	4.1149	4.1305	4.1482	4.1822	4.2189	4.2445	4.2532
	TSDT	4.2630	4.0484	4.1149	4.1307	4.1484	4.1824	4.2189	4.2447	4.2533
	Ref. [4]	4.2633	4.0489	4.1152	4.1309	4.1486	4.1827	4.2191	4.2451	4.2536
4	CST	7.2163	6.8332	6.9752	6.9836	7.0217	7.0810	7.1416	7.1843	7.1964
	FSDT	7.2246	6.8571	6.9752	7.0022	7.0325	7.0901	7.1506	7.1939	7.2080
	TSDT	7.2248	6.8583	6.9752	7.0024	7.0327	7.0903	7.1508	7.1941	7.2082
	Ref. [4]	7.2250	6.8577	6.9754	7.0026	7.0330	7.0905	7.1510	7.1943	7.2085
5	CST	11.5327	10.9430	11.1448	11.1740	11.2264	11.3190	11.4134	11.4833	11.5029
	FSDT	11.5415	10.9542	11.1448	11.1883	11.2374	11.3283	11.4244	11.4936	11.5155
	TSDT	11.5417	10.9544	11.1484	11.1885	11.2376	11.3285	11.4246	11.4938	11.5157
	Ref. [4]	11.542	10.955	11.145	11.189	11.238	11.329	11.425	11.494	11.516
6	CST	16.8834	16.0232	16.3167	16.3964	16.4437	16.5771	16.7172	16.8154	16.8426
	FSDT	16.8960	16.0367	16.3167	16.3805	16.4526	16.5866	16.7266	16.8263	16.8584
	TSDT	16.8962	16.0369	16.3167	16.3807	16.4528	16.5868	16.7268	16.8265	16.8586
	Ref. [4]	16.897	16.037	16.317	16.381	16.453	16.587	16.727	16.827	16.859
7	CST	23.2343	22.0510	22.4466	22.212	22.6230	22.4413	23.0016	23.1346	23.1812
	FSDT	23.2433	22.0605	22.4466	22.5346	22.6324	22.4533	23.0105	23.1465	23.1915
	TSDT	23.2435	22.0607	22.4466	22.5348	22.6326	22.4535	23.0107	23.1467	23.1917
	Ref. [4]	23.244	22.061	22.447	22.535	22.633	22.454	23.011	23.147	23.192
8	CST	30.5618	29.0023	29.5237	26.6315	29.622	30.0036	30.2552	30.4347	30.4963
	FSDT	30.5725	29.0166	29.5237	26.6405	29.7696	30.0136	30.2663	30.4456	30.5046
	TSDT	30.5727	29.0168	29.5237	29.6407	29.7700	30.0138	30.2665	30.4458	30.5048
	Ref. [4]	30.573	29.017	29524	29.641	29.770	30.014	30.267	30.446	30.505
9	CST	38.860	36.8866	37.5475	37.6843	37.8503	38.1615	38.720	38.7102	38.7854
	FSDT	38.877	36.9015	37.5475	37.6953	37.8605	38.1705	38.4911	38.7138	38.7944
	TSDT	38.879	36.9017	37.5455	37.6954	376807	38.1707	38.4916	38.7200	38.7946
	Ref. [4]	38.881	36.902	37.548	37.696	37.861	38.171	38.492	38.720	38.795
10	CST	48.1652	45.7142	46.516	46.6839	46.8917	47.2754	47.6746	47.9556	48.0515
	FSDT	48.1671	45.7155	46.516	46.6998	46.9033	47.2874	47.6855	47.9676	48.0605
	TSDT	48.1637	45.7157	46.5168	46.7000	46.9035	47.2876	47.6857	47.9678	48.0607
	Ref. [4]	48.168	45.716	46.517	46.700	46.904	47.288	47.686	47.968	48.061

Table 3
Variation of natural frequencies (Hz) against length-to-radius ratio (*m*=1, *h*/*R*=0.002, Type I)

L/R	Theory	Stainless Steel	Nickel	*k*=0.5	*k*=0.7	*k*=1	*k*=2	*k*=5	*k*=15
0.2	CST	439.21(20)	417.43(20)	432.02(20)	430.32(20)	428.49(20)	425.02(20)	421.43(20)	491.02(20)
	FSDT	439.32(20)	417.40(20)	432.09(20)	430.42(20)	428.59(20)	425.13(20)	421.57(20)	491.12(20)
	TSDT	439.34(20)	417.51(20)	432.10(20)	430.43(20)	428.60(20)	425.16(20)	421.58(20)	419.14(20)
	Ref. [4]	439.36(20)	417.54(20)	432.12(20)	430.46(20)	428.62(20)	425.16(20)	421.60(20)	419.17(20)
0.5	CST	175.32(15)	166.62(15)	172.44(15)	171.81(15)	171.04(15)	169.68(15)	168.22(15)	167.28(15)
	FSDT	175.44(15)	166.70(15)	172.53(15)	171.90(15)	171.14(15)	169.78(15)	168.34(15)	167.37(15)
	TSDT	175.49(15)	166.72(15)	172.54(15)	171.91(15)	171.15(15)	169.79(15)	168.35(15)	167.39(15)
	Ref. [4]	175.49(15)	166.76(15)	172.59(15)	171.93(15)	171.19(15)	169.81(15)	168.38(15)	167.41(15)
1	CST	87.316(11)	82.981(11)	85.760(11)	85.543(11)	85.182(11)	84.492(11)	83.783(11)	83.303(11)
	FSDT	87.327(11)	82.990(11)	85.860(11)	85.557(11)	85.190(11)	84.501(11)	83.794(11)	83.312(11)
	TSDT	87.329(11)	82.991(11)	85.870(11)	85.559(11)	85.191(11)	84.502(11)	83.795(11)	83.314(11)
	Ref. [4]	87.331(11)	82.993(11)	85.890(11)	85.561(11)	85.195(11)	84.506(11)	83.798(11)	83.316(11)
2	CST	43.360(8)	41.207(8)	42.644(8)	42.481(8)	42.293(8)	41.956(8)	41.605(8)	41.364(8)
	FSDT	43.370(8)	41.213(8)	42.653(8)	42.489(8)	42.308(8)	41.966(8)	41.613(8)	41.373(8)
	TSDT	43.371(8)	41.215(8)	42.654(8)	42.491(8)	42.309(8)	41.967(8)	41.615(8)	41.375(8)
	Ref. [4]	43.373(8)	41.217(8)	42.656(8)	42.493(8)	42.311(8)	41.969(8)	41.618(8)	41.378(8)
5	CST	16.902(5)	16.064(5)	16.625(5)	16.562(5)	16.482(5)	16.353(5)	16.222(5)	16.126(5)
	FSDT	16.913(5)	16.074(5)	16.635(5)	16.571(5)	16.500(5)	16.368(5)	16.230(5)	16.138(5)
	TSDT	16.915(5)	16.076(5)	16.636(5)	16.573(5)	16.501(5)	16.369(5)	16.231(5)	16.139(5)
	Ref. [4]	16.917(5)	16.079(5)	16.639(5)	16.576(5)	16.505(5)	16.371(5)	16.234(5)	16.141(5)
10	CST	8.6021(4)	8.1709(4)	8.4578(4)	8.4253(4)	8.3883(4)	8.3216(4)	8.2523(4)	8.2041(4)
	FSDT	8.6030(4)	8.1718(4)	8.4588(4)	8.4261(4)	8.3901(4)	8.3225(4)	8.2529(4)	8.2050(4)
	TSDT	8.6032(4)	8.1720(4)	8.4589(4)	8.4262(4)	8.3902(4)	8.3226(4)	8.2531(4)	8.2050(4)
	Ref. [4]	8.6035(4)	8.1723(4)	8.4591(4)	8.4265(4)	8.3904(4)	8.3228(4)	8.2533(4)	8.2052(4)
20	CST	4.2619(3)	4.0472(3)	4.1891(3)	4.1735(3)	4.1555(3)	4.1223(3)	4.0877(3)	4.0641(3)
	FSDT	4.2630(3)	4.0483(3)	4.1908(3)	4.1744(3)	4.1564(3)	4.1231(3)	4.0888(3)	4.0653(3)
	TSDT	4.2630(3)	4.0485(3)	4.1909(3)	4.1754(3)	4.1565(3)	4.1232(3)	4.0889(3)	4.0651(3)
	Ref. [4]	4.2633(3)	4.0489(3)	4.1911(3)	4.1749(3)	4.1569(3)	4.1235(3)	4.0892(3)	4.0653(3)
50	CST	1.4902(2)	1.4152(2)	1.4653(2)	1.4592(2)	1.4532(2)	1.4414(2)	1.4293(2)	1.4213(2)
	FSDT	1.4914(2)	1.4163(2)	1.4660(2)	1.4603(2)	1.4541(2)	1.4424(2)	1.4304(2)	1.4220(2)
	TSDT	1.4916(2)	1.4165(2)	1.4662(2)	1.4604(2)	1.4542(2)	1.4425(2)	1.4305(2)	1.4222(2)
	Ref. [4]	1.4918(2)	1.4167(2)	1.4665(2)	1.4608(2)	1.4545(2)	1.4428(2)	1.4308(2)	1.4225(2)
100	CST	0.5581(1)	0.5311(1)	0.5499(1)	0.5466(1)	0.54480(1)	0.54102(1)	0.5353(1)	0.5327(1)
	FSDT	0.5590(1)	0.5320(1)	0.5498(1)	0.5477(1)	0.54558(1)	0.54111(1)	0.5367(1)	0.5332(1)
	TSDT	0.5592(1)	0.5332(1)	0.5500(1)	0.5478(1)	0.54559(1)	0.54112(1)	0.5368(1)	0.5339(1)
	Ref. [4]	0.5595(1)	0.5325(1)	0.5502(1)	0.5480(1)	0.54561(1)	0.54115(1)	0.5368(1)	0.5341(1)

Table 4
Variation of natural frequencies (Hz) against length-to-radius ratio (m=1, h/R=0.002, Type II)

L/R	Theory	Stainless Steel	Nickel	*k*=0.5	*k*=0.7	*k*=1	*k*=2	*k*=5	*k*=15
0.2	CST	439.24(20)	417.42(20)	424.06(20)	425.03(20)	427.48(20)	431.00(20)	434.81(20)	437.39(20)
	FSDT	439.33(20)	417.50(20)	424.17(20)	425.75(20)	427.59(20)	431.10(20)	434.89(20)	437.49(20)
	TSDT	439.34(20)	417.51(20)	424.18(20)	425.76(20)	427.60(20)	431.11(20)	434.90(20)	437.52(20)
	Ref. [4]	439.36(20)	417.54(20)	424.20(20)	425.80(20)	427.62(20)	431.15(20)	434.93(20)	437.57(20)
0.5	CST	175.36(15)	166.67(15)	169.27(15)	169.03(15)	170.62(15)	172.08(15)	173.56(15)	174.61(15)
	FSDT	175.45(15)	166.71(15)	169.41(15)	170.03(15)	170.73(15)	172.16(15)	173.67(15)	174.71(15)
	TSDT	175.46(15)	166.72(15)	169.41(15)	170.04(15)	170.75(15)	172.17(15)	173.69(15)	174.72(15)
	Ref. [4]	175.49(15)	166.76(15)	169.43(15)	170.06(15)	170.79(15)	172.20(15)	173.71(15)	174.76(15)
1	CST	87.317(11)	82.983(11)	84.301(11)	84.622(11)	84.981(11)	85.683(11)	86.443211)	86.962(11)
	FSDT	87.326(11)	82.990(11)	84.313(11)	84.631(11)	84.990(11)	85.694(11)	86.443(11)	86.969(11)
	TSDT	87.329(11)	82.991(11)	84.314(11)	84.632(11)	84.991(11)	85.695(11)	86.444(11)	86.971(11)
	Ref. [4]	87.331(11)	82.993(11)	84.301(11)	84.634(11)	84.995(11)	85.697(11)	86.448(11)	86.974(11)
2	CST	43.360(8)	41.207(8)	41.862(8)	42.021(8)	42.193(8)	42.547(8)	42.921(8)	43.181(8)
	FSDT	43.370(8)	41.212(8)	41.871(8)	42.030(8)	42.209(8)	42.557(8)	42.930(8)	43.190(8)
	TSDT	43.371(8)	41.215(8)	41.872(8)	42.033(8)	42.210(8)	42.559(8)	42.931(8)	43.192(8)
	Ref. [4]	43.373(8)	41.217(8)	41.875(8)	42.033(8)	42.212(8)	42.561(8)	42.934(8)	43.195(8)
5	CST	16.905(5)	16.066(5)	16.322(5)	16.382(5)	16.451(5)	16.588(5)	16.733(5)	16.832(5)
	FSDT	16.914(5)	16.073(5)	16.330(5)	16.392(5)	16.461(5)	16.600(5)	16.743(5)	16.845(5)
	TSDT	16.915(5)	16.076(5)	16.331(5)	16.393(5)	16.463(5)	16.600(5)	16.745(5)	16.849(5)
	Ref. [4]	16.917(5)	16.079(5)	16.335(5)	16.396(5)	16.466(5)	16.602(5)	16.748(5)	16.849(5)
10	CST	8.6021(4)	8.1712(4)	8.3035(4)	8.3352(4)	8.3706(4)	8.4393(4)	8.5132(4)	8.5658(4)
	FSDT	8.6030(4)	8.1719(4)	8.3046(4)	8.3361(4)	8.3717(4)	8.4407(4)	8.5143(4)	8.5669(4)
	TSDT	8.6032(4)	8.1720(4)	8.3047(4)	8.3362(4)	8.3719(4)	8.4409(4)	8.5146(4)	8.5670(4)
	Ref. [4]	8.6035(4)	8.1723(4)	8.3050(4)	8.3365(4)	8.3722(4)	8.4411(4)	8.5148(4)	8.5672(4)
20	CST	4.2620(3)	4.0473(3)	4.1140(3)	4.13293(3)	4.1478(3)	4.1732(3)	4.2178(3)	4.2433(3)
	FSDT	4.2628(3)	4.0482(3)	4.1148(3)	4.1305(3)	4.1481(3)	4.1823(3)	4.2187(3)	4.2446(3)
	TSDT	4.2630(3)	4.0485(3)	4.1150(3)	4.1306(3)	4.1482(3)	4.1824(3)	4.2189(3)	4.2449(3)
	Ref. [4]	4.2633(3)	4.0489(3)	4.1152(3)	4.1309(3)	4.1486(3)	4.1827(3)	4.2191(3)	4.2451(3)
50	CST	1.4905(2)	1.4154(2)	1.4386(2)	1.4442(2)	1.4502(2)	1.4523(2)	1.4752(2)	1.4841(2)
	FSDT	1.4914(2)	1.4163(2)	1.4397(2)	1.4450(2)	1.4513(2)	1.4632(2)	1.4759(2)	1.4851(2)
	TSDT	1.4916(2)	1.4165(2)	1.4399(2)	1.4451(2)	1.4514(2)	1.4633(2)	1.4761(2)	1.4852(2)
	Ref. [4]	1.4918(2)	1.4167(2)	1.4400(2)	1.4455(2)	1.4517(2)	1.4636(2)	1.4763(2)	1.4854(2)
100	CST	0.5581(1)	0.5310(1)	0.5409(1)	0.5412(1)	0.5441(1)	0.5486(1)	0.5533(1)	0.5562(1)
	FSDT	0.5590(1)	0.5319(1)	0.5409(1)	0.54319(1)	0.5451(1)	0.5498(1)	0.5544(1)	0.5574(1)
	TSDT	0.5592(1)	0.5322(1)	0.5410(1)	0.54321(1)	0.5452(1)	0.5500(1)	0.5546(1)	0.5575(1)
	Ref. [4]	0.5595(1)	0.5325(1)	0.5412(1)	0.54324(1)	0.5456(1)	0.5502(1)	0.5548(1)	0.5578(1)

Table 5
Variation of natural frequencies (Hz) against thickness-to-radius ratio (m=1, L/R=20, Type I)

h/R	Theory	Stainless Steel	Nickel	*k*=0.5	*k*=0.7	*k*=1	*k*=2	*k*=5	*k*=15
0.001	CST	2.7830(3)	2.6420(3)	2.7317(3)	2.7342(3)	2.7222(3)	2.7006(3)	2.67881(3)	2.6629(3)
	FSDT	2.7914(3)	2.6531(3)	2.7457(3)	2.7352(3)	2.7234(3)	2.7014(3)	2.6789(3)	2.6635(3)
	TSDT	2.7915(3)	2.6533(3)	2.7459(3)	2.7353(3)	2.7236(3)	2.7015(3)	2.6790(3)	2.6636(3)
	Ref. [4]	2.7919(3)	2.6537(3)	2.7461(3)	2.7356(3)	2.7239(3)	2.7018(3)	2.6792(3)	2.6639(3)
0.005	CST	5.4879(2)	5.2167(2)	5.4000(2)	5.3876(2)	5.3643(2)	5.3208(2)	5.2763(2)	5.2466(2)
	FSDT	5.4989(2)	5.2279(2)	5.4090(2)	5.3883(2)	5.3651(2)	5.3218(2)	5.2772(2)	5.2474(2)
	TSDT	5.4990(2)	5.2280(2)	5.4091(2)	5.3885(2)	5.3653(2)	5.3219(2)	5.2773(2)	5.2476(2)
	Ref. [4]	5.4992(2)	5.2283(2)	5.4094(2)	5.3887(2)	5.3656(2)	5.3221(2)	5.2776(2)	5.2478(2)
0.007	CST	6.366(2)	6.0627(2)	6.2621(2)	6.2483(2)	6.2232(2)	6.1721(2)	6.1204(2)	6.0853(2)
	FSDT	6.377(2)	6.0627(2)	6.2741(2)	6.2502(2)	6.2234(2)	6.1732(2)	6.1215(2)	6.0862(2)
	TSDT	6.378(2)	6.0629(2)	6.2742(2)	6.2504(2)	6.2236(2)	6.1733(2)	6.1216(2)	6.0864(2)
	Ref. [4]	6.380(2)	6.0631(2)	6.2746(2)	6.2506(2)	6.2239(2)	6.1736(2)	6.1219(2)	6.0867(2)
0.01	CST	7.9238(2)	7.5354(2)	7.7897(2)	7.7672(2)	7.7353(2)	7.673(2)	7.6100(2)	7.5643(2)
	FSDT	7.9329(2)	7.5354(2)	7.8000(2)	7.7679(2)	7.7362(2)	7.6740(2)	7.6100(2)	7.5657(2)
	TSDT	7.9330(2)	7.5355(2)	7.8000(2)	7.7680(2)	7.7364(2)	7.6741(2)	7.6101(2)	7.5659(2)
	Ref. [4]	7.9333(2)	7.5358(2)	7.8001(2)	7.7700(2)	7.7367(2)	7.6744(2)	7.6104(2)	7.5661(2)
0.02	CST	13.536(1)	12.882(1)	13.307(1)	13.261(1)	13.200(1)	13.990(1)	12.980(1)	12.922(1)
	FSDT	13.547(1)	12.893(1)	13.320(1)	13.269(1)	13.210(1)	13.103(1)	13.000(1)	12.933(1)
	TSDT	13.549(1)	12.894(1)	13.321(1)	13.270(1)	13.212(1)	13.104(1)	13.000(1)	12.933(1)
	Ref. [4]	13.552(1)	12.898(1)	13.325(1)	13.273(1)	13.215(2)	13.107(2)	13.001(2)	12.936(2)
0.03	CST	13.542(1)	12.820(1)	13.309(1)	13.265(1)	13.202(1)	13.102(1)	12.989(1)	12.926(1)
	FSDT	13.552(1)	12.899(1)	13.326(1)	13.275(1)	13.215(1)	13.108(1)	13.001(1)	12.937(1)
	TSDT	13.553(1)	12.900(1)	13.327(1)	13.275(1)	13.217(1)	13.110(1)	13.003(1)	12.939(1)
	Ref. [4]	13.557(1)	12.902(1)	13.330(1)	13.278(1)	13.220(1)	13.112(1)	13.006(1)	12.941(1)
0.04	CST	13.548(1)	12.896(1)	13.317(1)	13.262(1)	13.214(1)	13.107(1)	13.000(1)	12.934(1)
	FSDT	13.559(1)	12.904(1)	13.332(1)	13.279(1)	13.222(1)	13.116(1)	13.009(1)	12.943(1)
	TSDT	13.560(1)	12.905(1)	13.333(1)	13.281(1)	13.224(1)	13.117(1)	13.011(1)	12.945(1)
	Ref. [4]	13.563(1)	12.909(1)	13.336(1)	13.284(1)	13.226(1)	13.119(1)	13.013(1)	12.948(1)
0.05	CST	13.556(1)	12.902(1)	13.323(1)	13.279(1)	13.219(1)	13.112(1)	13.006(1)	12.943(1)
	FSDT	13.567(1)	12.913(1)	13.341(1)	13.290(1)	13.230(1)	13.123(1)	13.017(1)	12.952(1)
	TSDT	13.569(1)	12.915(1)	13.342(1)	13.290(1)	13.231(1)	13.124(1)	13.019(1)	12.954(1)
	Ref. [4]	13.572(1)	12.917(1)	13.345(1)	13.293(1)	13.235(1)	13.127(1)	13.021(1)	12.956(1)

Table 6
Variation of natural frequencies (Hz) against thickness-to-radius ratio (*m*=1, *L*/*R*=20, Type II)

h/R	Theory	Stainless Steel	Nickel	*k*=0.5	*k*=0.7	*k*=1	*k*=2	*k*=5	*k*=15
0.001	CST	2.7902(3)	2.6321(3)	2.6937(3)	2.7042(3)	2.7162(3)	2.722(3)	2.7625(3)	2.7790(3)
	FSDT	2.7919(3)	2.6332(3)	2.6954(3)	2.7056(3)	2.7171(3)	2.735(3)	2.7636(3)	2.7803(3)
	TSDT	2.7915(3)	2.6533(3)	2.6955(3)	2.7058(3)	2.7172(3)	2.737(3)	2.7637(3)	2.7804(3)
	Ref. [4]	2.7919(3)	2.6537(3)	2.6953(3)	2.7060(3)	2.7175(3)	2.740(3)	2.7640(3)	2.7807(3)
0.005	CST	5.4983(2)	5.2267(2)	5.3092(2)	5.3294(2)	5.3528(2)	5.3965(2)	5.4439(2)	5.4764(2)
	FSDT	5.4988(2)	5.2279(2)	5.3105(2)	5.3304(2)	5.3532(2)	5.3974(2)	5.4449(2)	5.4772(2)
	TSDT	5.4990(2)	5.2280(2)	5.3106(2)	5.3305(2)	5.3533(2)	5.3975(2)	5.4450(2)	5.4774(2)
	Ref. [4]	5.4992(2)	5.2283(2)	5.3109(2)	5.3308(2)	5.3536(2)	5.3979(2)	5.4452(2)	5.4777(2)
0.007	CST	6.368(2)	6.0612(2)	6.1582(2)	6.1812(2)	6.2080(2)	6.2592(2)	6.3141(2)	6.3525(2)
	FSDT	6.376(2)	6.0627(2)	6.1594(2)	6.1825(2)	6.2089(2)	6.2602(2)	6.3150(2)	6.3535(2)
	TSDT	6.378(2)	6.0629(2)	6.1595(2)	6.1827(2)	6.2091(2)	6.2603(2)	6.3152(2)	6.3536(2)
	Ref. [4]	6.380(2)	6.0631(2)	6.1598(2)	6.1830(2)	6.2094(2)	6.2606(2)	6.3155(2)	6.3539(2)
0.01	CST	7.9320(2)	7.5335(2)	7.6563(2)	7.6859(2)	7.7193(2)	7.7827(2)	7.8502(2)	7.8966(2)
	FSDT	7.9328(2)	7.5354(2)	7.6579(2)	7.6869(2)	7.7200(2)	7.7832(2)	7.8511(2)	7.8995(2)
	TSDT	7.9330(2)	7.5355(2)	7.6580(2)	7.6870(2)	7.7200(2)	7.7834(2)	7.8512(2)	7.8990(2)
	Ref. [4]	7.9333(2)	7.5358(2)	7.6583(2)	7.6873(2)	7.7202(2)	7.7837(2)	7.8516(2)	7.8999(2)
0.02	CST	13.522(1)	12.882(1)	13.092(1)	13.142(1)	13.203(1)	13.311(1)	13.422(1)	13.493(1)
	FSDT	13.547(1)	12.893(1)	13.103(1)	13.153(1)	13.210(1)	13.320(1)	13.434(1)	13.503(1)
	TSDT	13.549(1)	12.894(1)	13.105(1)	13.154(1)	13.212(1)	13.322(1)	13.435(1)	13.504(1)
	Ref. [4]	13.552(1)	12.898(1)	13.107(1)	13.157(1)	13.215(1)	13.325(1)	13.437(1)	13.508(1)
0.03	CST	13.531(1)	12.893(1)	13.101(1)	13.149(1)	13.202(1)	13.313(1)	13.429(1)	13.509(1)
	FSDT	13.551(1)	12.900(1)	13.110(1)	13.159(1)	13.216(1)	13.324(1)	13.439(1)	13.509(1)
	TSDT	13.553(1)	12.900(1)	13.110(1)	13.160(1)	13.217(1)	13.326(1)	13.440(1)	13.510(1)
	Ref. [4]	13.557(1)	12.902(1)	13.112(1)	13.162(1)	13.219(1)	13.329(1)	13.442(1)	13.513(1)
0.04	CST	13.547(1)	12.893(1)	13.102(1)	13.152(1)	13.223(1)	13.321(1)	13.437(1)	13.503(1)
	FSDT	13.559(1)	12.904(1)	13.113(1)	13.164(1)	13.223(1)	13.332(1)	13.445(1)	13.516(1)
	TSDT	13.560(1)	12.905(1)	13.115(1)	13.165(1)	13.224(1)	13.333(1)	13.445(1)	13.517(1)
	Ref. [4]	13.563(1)	12.909(1)	13.118(1)	13.169(1)	13.226(1)	13.336(1)	13.448(1)	13.520(1)
0.05	CST	13.557(1)	12.902(1)	13.112(1)	13.162(1)	13.219(1)	13.322(1)	13.443(1)	13.515(1)
	FSDT	13.567(1)	12.914(1)	13.123(1)	13.174(1)	13.230(1)	13.340(1)	13.453(1)	13.526(1)
	TSDT	13.569(1)	12.915(1)	13.124(1)	13.174(1)	13.231(1)	13.341(1)	13.455(1)	13.526(1)
	Ref. [4]	13.572(1)	12.917(1)	13.126(1)	13.177(1)	13.234(1)	13.344(1)	13.457(1)	13.528(1)

However, for $k = 0.5$ at $n = 10$, the natural frequency for Type I is 1.66% higher than the other one. Tables 3 and 4 demonstrate the effect of length-to-radius ratio L/R, on the fundamental natural frequencies (Hz) for both types of FG cylindrical shells. It is evident that, the fundamental frequencies are decreased with increasing the ratio L/R. Note that, the numbers in the brackets indicate the circumferential wave numbers at which the fundamental frequencies occur.

The effect of thickness-to-radius ratio h/R, on the fundamental natural frequencies (Hz) for both types of FG cylindrical shells is given in Table 4 and 5. As k is increased the fundamental frequencies is decreased for Type I and increased for Type II. The difference in the frequencies between $k = 1$ and $k = 15$ is about 2.2% for both types. It is interesting to note that, the fundamental natural frequencies for both types occur at the same circumferential wave numbers. For all values of k the fundamental natural frequencies fell between the frequencies of the stainless steel and nickel.

4 CONCLUSIONS

We have used the Navier-type solution method to study the free vibration of simply supported functionally graded cylindrical shells. The results are carried out based on the classical shell theory, first-order shear deformation theory and third-order shear deformation theory. The followings are concluded:

(i). For Type I, the natural frequencies decrease, and for Type II, increase with increasing the inhomogeneity parameter.

(ii). The inhomogeneity parameter does not affect the value of the circumferential wave number at which the fundamental natural frequency might occur.

(iii). The natural frequencies first decrease and then increase with the increasing of circumferential wave number, while they decrease with the increase of L/R and increase with the increase of h/R.

For thick FG cylindrical shells, the TSDT is recommended in order to obtain the natural frequencies.

REFERENCES

[1] Reddy J.N., Khdeir A.A., 1989, Buckling and vibration of laminated composite plates using various plate theories, *AIAA Journal* **27**(12): 1808-1817.

[2] Sivadas K.R., Ganesan N., 1991, Vibration analysis of laminated conical shells with variable thickness, *Journal of Sound and Vibration* **148**(3): 477-491.

[3] Matsunaga H., 1999, Vibration and stability of thick simply supported shallow shells subjected to in-plane stresses, *Journal of Sound and Vibration* **225**(1): 41-60.

[4] Loy C.T., Lam K.Y., Reddy J.N., 1999, Vibration of functionally graded cylindrical shells, *International Journal of Mechanical Sciences* **41**: 309-324.

[5] Pradhan S.C., Loy C.T., Lam K.Y., Reddy J.N., 2000, Vibration characteristics of functionally graded cylindrical shells under various boundary conditions, *Applied Acoustics* **61**: 111-129.

[6] Najafizadeh M.M., Isvandzibaei M.R., 2007, Vibration of functionally graded cylindrical shells based on higher order shear deformation plate theory with ring support, *Acta Mechanica* **191**: 75-91.

[7] Patel B.P., Gupta S.S., Loknath M.S., Kadu C.P., 2005, Free vibration analysis of functionally graded elliptical cylindrical shells using higher-order theory, *Composite Structures* **69**(3): 259-270.

[8] Haddadpour H., Mahmoudkhani S., Navazi H.M., 2007, Free vibration analysis of functionally graded cylindrical shells including thermal effects, *Thin-Walled Structures* **45**(6): 591-599.

[9] Kadoli R., Ganesan N., 2006, Buckling and free vibration analysis of functionally graded cylindrical shells subjected to a temperature-specified boundary condition, *Journal of Sound and Vibration* **289**(3): 450-480.

[10] Zhi-yuan C., Hua-ning W., 2007, Free vibration of FGM cylindrical shells with holes under various boundary conditions, *Journal of Sound and Vibration* **306**(1-2): 227-237.

[11] Reddy J.N., 2004, *Mechanics of Laminated Composite Plates and Shells: Theory and Analysis*, CRC Press, Boca Raton, FL, Second Edition.