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Reflection From Free Surface of a Rotating Generalized Thermo-Piezoelectric Solid Half Space

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ABSTRACT

The analysis of rotational effect on the characteristics of plane waves propagating in a half space of generalized thermo-piezoelectric medium is presented in context of linear theory of thermo-piezoelectricity including Coriolis and centrifugal forces. The governing equations for a rotating generalized thermo-piezoelectric medium are formulated and solved for plane wave solutions to show the propagation of three quasi plane waves in the medium. A problem on the reflection of these plane waves is considered from a thermally insulated/isothermal boundary of a rotating generalized thermo-piezoelectric solid half space. The expressions for reflection coefficients of three reflected waves are obtained in explicit from. For experimental data of $LiNbO_3$ and $BaTiO_3$, the speeds of various plane waves are computed. The reflection coefficients of various reflected waves are also obtained numerically by using the data of $BaTiO_3$. The dependence of speeds of plane waves and reflection coefficients of various reflected waves is shown graphically on the rotation parameter at each angle of incidence.

Keywords: Thermo-piezoelectric; Plane waves; Reflection; Rotation; Reflection coefficients.

1 INTRODUCTION

PIEZOELECTRICITY is the study of charge gathered in certain solids due to an applied mechanical force. Piezoelectric crystals produce electric field due to an applied mechanical force and vice-versa. Quartz, Rochelle Salt and Tourmaline are widely used natural occurring piezoelectric crystals. For example, Quartz crystals are used in the control of the frequency of oscillators and in the production of very selective filters. Rochelle salt is used in most of low-frequency transducer applications and Tourmaline is used for measuring hydrostatic pressures. Barium Titanate (BaTiO3) is one of widely used piezoelectric ceramics. Due to linear coupling between mechanical and electrical fields in piezoelectric materials, the ceramics are used as transducers, actuators, sensors and filters. Voigt [41] established a linear theory of piezoelectricity. The general formulation of piezoelectricity was developed by Toupin [40]. The classical texts by Cady [7], Tiersten [39], Maugin [21], Ikeda [16], Yang [43] and Eringen and Maugin [13] are referred for detail on the linear theory of piezoelectricity. Wave phenomenon in piezoelectric media has its applications in generation and transmission of disturbances in electro-acoustic devices like transducers and resonators. Reflection and transmission of acoustic energy at a surface plays an important role in the fields of signal

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processing, transduction and frequency control [5, 18, 29, 31]. The characteristics of reflected and refracted waves at such boundaries provide information regarding the resolution characteristics of acoustic transducers. The reflection and refraction of plane waves in piezoelectric anisotropic materials is an important topic of research for last four decades. See for example, Lothe and Barnett [20], Noorbehesht and Wade [25], Alshits *et al.* [3], Every and Neiman [14], Nayfeh and Chien [24], Alshits and Shuvalov [4], Zhang *et al.* [44], Shuvalov and Clezio [35], Clezio and Shuvalov [10], Burkov *et al*., [6], Pang *et al*. [28], Chen *et al*., [9], Darinskii *et al.* [11], Singh [37-38] and Kuang and Yuan [17].

Thermo-piezoelectric materials are being considered for use in the performance of existing aerospace structures. In general, the thermo-piezoelectric materials provide fast response times, good dynamic behavior, the capability to be used as either sensors or actuators, simple integration into a structure, low power requirement, a readily obtainable commercial supply and long familiarity through previous applications in transducers. Thermal effects greatly influence the performance of piezoelectric actuators and sensors, especially when they are required to operate in severe temperature environments. The governing equations of a thermo-piezoelectric material were derived by Mindlin [22-23]. Nowacki [26] established a uniqueness theorem for the solutions of differential equations of thermopiezoelectricity on the basis of energy balance. Chandrasekharaiah [8] obtained the governing equations of a temperature-rate-dependent thermopiezoelectricity theory which predicts a finite speed of propagation for thermal signals. Wave propagation phenomenon in thermo-piezoelectric materials is studied by Pal [27], Sharma and Kumar [33], Singh [36], Sharma and Walia [34], Abd-Alla and Alshaikh [1], Abd-Alla et al. [2] and Ponnusamy [30].

The objective of this paper is to study the wave propagation in a generalized thermo-piezoelectric medium. Problems on plane wave propagation and reflection phenomenon in this medium are not studied yet in literature. The present paper is organized as: In next section, the governing equations of a rotating generalized thermo-piezoelectric medium are formulated in context of generalized theories of thermoelasticity given by Lord and Shulman [19] and Green and Lindsay [15]. In section 3, the medium is assumed to be transversely isotropic with z-axis as the poling direction and the governing equations are obtained in x-z plane. These equations in x-z plane are solved for plane waves to show the propagation three plane waves in the medium. In section 4, a problem on the reflection of plane waves from thermally insulated as well as isothermal boundaries of a rotating generalized thermo piezoelectric solid half space is solved and the expressions for reflection coefficients of various reflected waves are obtained explicitly. In section 5, the numerical values of speeds of plane waves are obtained by using data of $LiNbO_3$ and $BaTiO_3$. The reflection coefficients of various reflected waves are also computed for material parameters of $BaTiO_3$. The speeds and reflection coefficients are depicted graphically to show the effect of rotation at each angle of an incident wave. The numerical results of speeds and reflection coefficients are discussed in detail with concluding remarks.

2 GOVERNING EQUATIONS

Following the theories of Lord and Shulman [19], Green and Lindsay [15] and Schoenberg [32], the governing equations of a rotating generalized thermo-piezoelectric medium in the absence of body force, free charge and inner heat source, are

$$\sigma_{ji,j} = \rho\left[\ddot{u}_i + \left(\vec{\Omega}\times\left(\vec{\Omega}\times\vec{u}\right)\right)_i + \left(2\vec{\Omega}\times\dot{\vec{u}}\right)_i\right], \quad q_{i,i} = -T_0\rho\dot{\eta}, \quad D_{i,i} = 0, \tag{1}$$

$$\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl} - e_{kij}E_k - \gamma_{ij}\left(T + \tau_1\dot{T}\right), \tag{2}$$

$$D_i = e_{ikl}\,\varepsilon_{kl} + p_{ik}E_k + d_i^*\left(T + \tau_1\dot{T}\right), \tag{3}$$

$$q_i = K_{ij}T_{,j} + T_o b_i\dot{T}, \tag{4}$$

$$\rho\eta = \gamma_{kl}\left(\varepsilon_{kl} + \tau_o\Delta\dot{\varepsilon}_{kl}\right) + d_k^*\left(E_k + \tau_o\Delta\dot{E}_k\right) + C'\left(T + \tau_o\dot{T}\right) - b_iT_{,i}, \tag{5}$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \quad E_i = -\phi_{,i}. \tag{6}$$

In the above equations, a comma followed by a suffix denotes material derivative and a superposed dot denotes the derivative with respect to time. σ_{ij} are the components of a stress tensor, ε_{ij} are the components of a strain tensor, E_i are the components of an electric field vector, K_{ij} are the coefficients of thermal conductivity, C_{ijkl} are the elastic constants, ϕ is an electric potential function, γ_{ij} are thermal moduli, e_{ijk}, b_i are the piezoelectric constants, T is temperature increment, D_i are the components of electric displacement, u_i are the components of displacement vector, q_i are the components of heat flux vector, α_{ij} are the coefficients of linear thermal expansion, ρ is mass density, t is time, η is entropy density, T' is absolute temperature, p_{ik} are the dielectric constants, C_E is the specific heat at constant strain, d_i^* is pyroelectric constants, τ_o, τ_1 are thermal relaxation times and T_o is the reference temperature chosen such that $\left|\frac{(T'-T_o)}{T'}\right| << 1$. The use of symbol Δ makes the above equations possible for two different theories of generalized thermopiezoelectric materials. For the *L-S* theory $\tau_1 = 0, \Delta = 1$ and for *G-L* theory $\tau_1 > 0$ and $\Delta = 0$. The thermal relaxation times τ_o and τ_1 satisfy the inequality $\tau_1 \ge \tau_o \ge 0$ for *G-L* theory only.

3 TWO-DIMENSIONAL SOLUTION

The material is assumed to be transversely isotropic with *z*-axis as the poling direction. Making use of Eqs. (2) to (6) into Eq. (1), we obtain the following governing equations in *x-z* plane

$$C_{11}u_{1,11} + \left(C_{44} + C_{13}\right)u_{3,13} + C_{44}u_{1,33} + \left(e_{15} + e_{31}\right)\phi_{,13} - \gamma_1\left(T + \tau_1\dot{T}\right) = \rho\left[\ddot{u}_1 - \Omega^2 u_1 + 2\Omega\dot{u}_3\right], \tag{7}$$

$$C_{44}u_{3,11} + \left(C_{44} + C_{31}\right)u_{1,13} + C_{33}u_{3,33} + e_{15}\phi_{,11} + e_{33}\phi_{,33} - \gamma_3\left(T + \tau_1\dot{T}\right)_{,3} = \rho\left[\ddot{u}_3 - \Omega^2 u_3 - 2\Omega\dot{u}_1\right], \tag{8}$$

$$K_1T_{,11} + K_3T_{,33} - \rho C_E\left(\dot{T} + \tau_o\ddot{T}\right) = T_o\left[\gamma_1\left(\dot{u}_{1,1} + \tau_o\Delta\ddot{u}_{1,1}\right) + \gamma_3\left(\dot{u}_{3,3} + \tau_o\Delta\ddot{u}_{3,3}\right) - d_3^*\left(\dot{\phi}_{,3} + \tau_o\Delta\ddot{\phi}_{,3}\right)\right], \tag{9}$$

$$e_{33}u_{3,33} + e_{15}u_{3,11} + \left(e_{31} + e_{15}\right)u_{1,13} - p_{11}\phi_{,11} - p_{33}\phi_{,33} + d_3^*\left(T + \tau_1\dot{T}\right)_{,3} = 0, \tag{10}$$

where

$$K_1 = K_{11}, K_3 = K_{33},\ \rho C_E = C'T_o,\ \gamma_1 = \gamma_{11},\ \gamma_3 = \gamma_{33},\ e_{33} = e_{333}, e_{31} = e_{311}, e_{15} = e_{131} = e_{113},$$
$$C_{11} = C_{1111}, C_{13} = C_{1133}, C_{44} = C_{1313}, C_{31} = C_{3311}, C_{33} = C_{3333},\ \gamma_1 = (C_{11} + C_{33})\alpha_1 + (C_{13} + e_{31})\alpha_3,$$
$$\gamma_3 = 2C_{13}\,\alpha_1 + (C_{33} + e_{33})\alpha_3.$$

Here α_1 and α_3 are coefficients of thermal expansion. The plane wave solution of Eqs. (7) to (10) are now sought in the following form

$$\left(u_1, u_3, \phi, T\right) = \left(Ad_1, Ad_3, kB, kC\right)e^{\iota k(xp_1 + zp_3 - vt)}, \tag{11}$$

where k is wave number, A, B, C are arbitrary constants and d_1, d_3 are components of unit displacement vector,

p_1 and p_3 are components of unit propagation vector. Making use of Eq. (11) into Eqs. (7) to (10) and after elimination of A, B and C, we obtain following cubic velocity equation

$$A_o\zeta^3 + A_1\zeta^2 + A_2\zeta + A_3 = 0, \tag{12}$$

where $\zeta = \rho v^2$ and

$$\begin{aligned}
&A_o = \Omega_o^{\ 2}(D_4\tau^* - L_5\bar{d}p_3^{\ 2}) + 4\frac{\Omega^2}{\omega^2}(L_5\bar{d}p_3^{\ 2} - D_4\tau^*),\\
&A_1 = \Omega_o(D_1L_5\bar{d}p_3^{\ 2} + D_5L_4\bar{d}p_3^{\ 2} + D_2L_5\bar{d}p_3^{\ 2} + L_2L_3p_1^{\ 2}p_3^{\ 2}\bar{d} - D_5^{\ 2}\tau^* - D_1D_4\tau^* - D_2D_4\tau^* + D_5L_5\bar{\gamma}p_3^{\ 2}\\
&- D_4L_4\bar{\gamma}p_3^{\ 2} - L_2^2\tau^*p_1^{\ 2}p_3^{\ 2} + L_2L_5p_1^{\ 2}p_3^{\ 2} - D_4L_3p_1^{\ 2}) + 2\iota\frac{\Omega}{\omega}(D_4L_4 - D_5L_5)p_1p_3 - D_4D_6\Omega_o^{\ 2} + 4\frac{\Omega^2}{\omega^2}D_4D_6,\\
&A_2 = \Omega_o(D_1D_4D_6 + D_2D_4D_6 + D_5^{\ 2}D_6 + D_6L_2^{\ 2}p_1^{\ 2}p_3^{\ 2}) + D_1D_2D_4\tau^* - D_1D_2L_5\bar{d}p_3^{\ 2} + D_1D_5^{\ 2}\tau^*\\
&- D_1D_5L_4\bar{d}p_3^{\ 2} - D_1D_5L_5\bar{\gamma}p_3^{\ 2} + D_1D_4L_4\bar{\gamma}p_3^{\ 2} - D_4L_1^{\ 2}\tau^*p_1^{\ 2}p_3^{\ 2} + L_1^{\ 2}L_5\bar{d}\,p_1^{\ 2}p_3^{\ 4} - D_5L_1L_2\tau^*p_1^{\ 2}p_3^{\ 2}\\
&+ D_5L_1L_3\bar{d}\,p_1^{\ 2}p_3^{\ 2} + L_1L_2L_5\bar{\gamma}p_1^{\ 2}p_3^{\ 4} - D_4L_1L_3p_1^{\ 2}p_3^{\ 2}\bar{\gamma} - D_5L_1L_2\tau^*p_1^{\ 2}p_3^{\ 2} + L_1L_2L_4\bar{d}\,p_1^{\ 2}p_3^{\ 4}\\
&+ D_2L_2^{\ 2}\tau^*p_1^{\ 2}p_3^{\ 2} - D_2L_3L_2\bar{d}\,p_1^{\ 2}p_3^{\ 2} + L_2^{\ 2}L_4\bar{\gamma}p_1^{\ 2}p_3^{\ 4} - D_5L_2L_3\bar{\gamma}p_1^{\ 2}p_3^{\ 2} + D_5L_1L_5p_1^{\ 2}p_3^{\ 2}\\
&- D_4L_1L_4p_1^{\ 2}p_3^{\ 2} - D_2L_2L_5p_1^{\ 2}p_3^{\ 2} + D_2D_4L_3p_1^{\ 2} - D_5L_2L_4p_1^{\ 2}p_3^{\ 2} + D_5^{\ 2}L_3p_1^{\ 2},\\
&A_3 = -D_1D_2D_4D_6 - D_1D_5^{\ 2}D_6 + D_4D_6L_1^{\ 2}p_1^{\ 2}p_3^{\ 2} - D_2D_6L_2^{\ 2}p_1^{\ 2}p_3^{\ 2} + 2D_5D_6L_1L_2p_1^{\ 2}p_3^{\ 2}.
\end{aligned}$$

And

$$\begin{aligned}
&\Omega_0 = 1 + \frac{\Omega^2}{\omega^2},\\
&D_1 = C_{11}p_1^{\ 2} + C_{44}p_3^{\ 2},\ D_2 = C_{44}p_1^{\ 2} + C_{33}p_3^{\ 2},\ D_3 = K_1p_1^{\ 2} + K_3p_3^{\ 2},\ D_4 = p_{11}p_1^{\ 2} + p_{33}p_3^{\ 2}\\
&D_5 = e_{15}p_1^{\ 2} + e_{33}p_3^{\ 2},\ D_6 = \frac{D_3}{C_E},\ L_1 = C_{44} + C_{13},\ L_2 = e_{15} + e_{31},\ L_3 = \varepsilon_1\tau^*,\ L_4 = \varepsilon_1\bar{\gamma}\bar{\tau},\ L_5 = \varepsilon_1\bar{d}\,\bar{\tau},\\
&\bar{d} = \frac{d_3^*}{\gamma_1},\ \bar{\gamma} = \frac{\gamma_3}{\gamma_1},\ \bar{\tau} = \tau\tau_m v_1^{\ 2},\ \tau_m = 1 - \iota\omega\tau_1,\ \varepsilon_1 = \frac{\gamma_1^{\ 2}T_o}{C_E\rho c_1^{\ 2}},\ c_1^{\ 2} = \frac{C_{11}}{\rho},\ \tau^* = \tau_o + \frac{\iota}{\omega},\ \tau = \tau_o\Delta + \left(\frac{\iota}{\omega}\right).
\end{aligned}$$

Eq. (12) may be solved numerically by Cardan's method to obtain the three values of ζ. The real parts of these three values of ζ corresponds to the phase speeds V_1,V_2 and V_3 of three quasi plane waves, namely, quasi-*P* (*qP*), quasi-Thermal (*qT*) and quasi-*SV* (*qSV*) waves, respectively.

4 REFLECTION FROM A STRESS FREE SURFACE

We consider an incident plane wave (*qP* or *qT* or *qSV*) at the free surface of a rotating generalized thermo-piezoelectric solid half space with free surface along *x*-axis and *z*-axis pointing into the medium. Corresponding to an incident wave making θ_0 with normal, there will be three reflected waves as *qP, qT* and *qSV* waves making angles θ_1, θ_2, and θ_3 with normal to the free surface as shown in Fig. 1. The appropriate displacement components, temperature change and electric potential for incident and reflected waves are given by

$$u_1^{\ \alpha} = A^{(\alpha)}d_1^{\ (\alpha)}e^{\iota k_\alpha\left(p_1^{(\alpha)}x + p_3^{(\alpha)}z - V_\alpha t\right)}, \tag{13}$$

$$u_3^{(\alpha)} = A^{(\alpha)} d_3^{(\alpha)} e^{\iota k_\alpha \left(p_1^{(\alpha)} x + p_3^{(\alpha)} z - V_\alpha t\right)}, \tag{14}$$

$$T^{(\alpha)} = F^{(\alpha)} A^{(\alpha)} e^{\iota k_\alpha \left(p_1^{(\alpha)} x + p_3^{(\alpha)} z - V_\alpha t\right)}, \tag{15}$$

$$\phi^{(\alpha)} = G^{(\alpha)} A^{(\alpha)} e^{\iota k_\alpha \left(p_1^{(\alpha)} x + p_3^{(\alpha)} z - V_\alpha t\right)}, \tag{16}$$

where $V_\alpha = \mathrm{Re}(v_\alpha), (\alpha = 0, 1, 2, 3)$, $F^{(\alpha)}$ and $G^{(\alpha)}$ are given by

$$F^{(\alpha)} = \frac{\left[Q_1^{(\alpha)} - (e_{15} + e_{31}) p_1^{(\alpha)} p_3^{(\alpha)} G^{(\alpha)}\right]}{\gamma_1 V_\alpha\, p_1^{(\alpha)} (\tau_1 + \frac{i}{\omega})},\ G^{(\alpha)} = \frac{\left[\gamma_3 p_3^{(\alpha)} Q_1^{(\alpha)} - \gamma_1 p_1^{(\alpha)} Q_2^{(\alpha)}\right]}{\left[\gamma_3 (e_{15} + e_{31}) p_1^{(\alpha)} p_3^{(\alpha)^2} - \gamma_1 p_1^{(\alpha)} (e_{15} p_1^{(\alpha)^2} + e_{33} p_3^{(\alpha)^2})\right]},$$

And

$$Q_1^{(\alpha)} = \rho V_\alpha^2 [\Omega_0 d_1^{(\alpha)} + 2i \frac{\Omega}{\omega} d_3^{(\alpha)}] - C_{11} p_1^{(\alpha)^2} d_1^{(\alpha)} - C_{44} p_3^{(\alpha)^2} d_1^{(\alpha)} - (C_{13} + C_{44}) p_1^{(\alpha)} p_3^{(\alpha)} d_3^{(\alpha)},$$
$$Q_2^{(\alpha)} = \rho V_\alpha^2 [\Omega_0 d_3^{(\alpha)} - 2i \frac{\Omega}{\omega} d_1^{(\alpha)}] - C_{44} p_1^{(\alpha)^2} d_3^{(\alpha)} - C_{33} p_3^{(\alpha)^2} d_3^{(\alpha)} - (C_{13} + C_{44}) p_1^{(\alpha)} p_3^{(\alpha)} d_1^{(\alpha)},$$
$$p_1^{(\alpha)} = \sin\theta_\alpha,\ d_1^{(\alpha)} = \cos\theta_\alpha,\ (\alpha = 0,1,2,3), p_3^{(0)} = -\cos\theta_0, d_1^{(0)} = \sin\theta_0,\ p_3^{(\beta)} = \cos\theta_\beta, d_3^{(\alpha)} = -\sin\theta_\beta, (\beta = 1,2,3).$$

The required boundary conditions at free surface $z = 0$ are

$$\sigma_{33}^{(\alpha)} = 0, \sigma_{31}^{(\alpha)} = 0, \frac{\partial T^{(\alpha)}}{\partial z} + hT^{(\alpha)} = 0 \tag{17}$$

$$\phi^{(\alpha)} = 0, \text{(electrically shorted)},\ D_3^{(\alpha)} = 0, \text{(charge free)} \tag{18}$$

where $h \to 0$ corresponds to thermally insulated surface, $h \to \infty$ corresponds to isothermal surface and

$$\sigma_{33}^{(\alpha)} = C_{31} u_{1,1}^{(\alpha)} + C_{33} u_{3,3}^{(\alpha)} + e_{33} \phi_{,3}^{(\alpha)} - \gamma_3 \left(T^{(\alpha)} + \tau_1 \dot{T}^{(\alpha)}\right),\ \sigma_{31}^{(\alpha)} = C_{44} \left(u_{1,3}^{(\alpha)} + u_{3,1}^{(\alpha)}\right) + e_{15} \phi_{,1}^{(\alpha)},$$
$$D_3^{(\alpha)} = e_{31} u_{1,1}^{(\alpha)} + e_{33} u_{3,3}^{(\alpha)} - p_{33} \phi_{,3}^{(\alpha)} + d_3 \left(T^{(\alpha)} + \tau_1 \dot{T}^{(\alpha)}\right).$$

The displacement components, temperature change and electric potential given by Eqs. (13) to (16) satisfy the boundary conditions (17) and (18) with following Snell's law

$$k_o p_1^{(0)} = k_1 p_1^{(1)} = k_2 p_1^{(2)} = k_3 p_1^{(3)} \equiv k,\ k_o V_o = k_1 V_1 = k_2 V_2 = k_3 V_3 \equiv \omega. \tag{19}$$

And the reflection coefficients of reflected *qP, qT* and *qSV* waves for thermally insulated case are obtained in explicit from as:

$$\frac{A^{(1)}}{A^{(0)}} = \frac{\Delta_1}{\Delta},\ \frac{A^{(2)}}{A^{(0)}} = \frac{\Delta_2}{\Delta},\ \frac{A^{(3)}}{A^{(0)}} = \frac{\Delta_3}{\Delta}, \tag{20}$$

where

$$\Delta=\begin{vmatrix} a_{10} & a_{20} & a_{30} \\ b_{10} & b_{20} & b_{30} \\ c_{10} & c_{20} & c_{30} \end{vmatrix},\ \Delta_1=\begin{vmatrix} -1 & a_{20} & a_{30} \\ -1 & b_{20} & b_{30} \\ -1 & c_{20} & c_{30} \end{vmatrix},\ \Delta_2=\begin{vmatrix} a_{10} & -1 & a_{30} \\ b_{10} & -1 & b_{30} \\ c_{10} & -1 & c_{30} \end{vmatrix},\ \Delta_3=\begin{vmatrix} a_{10} & a_{20} & -1 \\ b_{10} & b_{20} & -1 \\ c_{10} & c_{20} & -1 \end{vmatrix},$$

$$a_{k0}=\frac{a^{(k)}}{a^{(0)}},\ b_{k0}=\frac{b^{(k)}}{b^{(0)}},\ c_{k0}=\frac{c^{(k)}}{c^{(0)}},\ (k=1,2,3),$$

$$a^{(\alpha)}=[C_{31}p_1^{(\alpha)}d_1^{(\alpha)}+C_{33}p_3^{(\alpha)}d_3^{(\alpha)}+e_{33}G^{(\alpha)}p_3^{(\alpha)}+\gamma_3\left(\tau_1+\frac{i}{\omega}\right)v_\alpha F^{(\alpha)}]k_\alpha,\ (\alpha=0,1,2,3)$$

$$b^{(\alpha)}=[C_{44}(p_3^{(\alpha)}d_1^{(\alpha)}+p_1^{(\alpha)}d_3^{(\alpha)})+e_{15}G^{(\alpha)}]k_\alpha,(\alpha=0,1,2,3),\ c^{(\alpha)}=k_\alpha p_3^{(\alpha)}F^{(\alpha)},(\alpha=0,1,2,3).$$

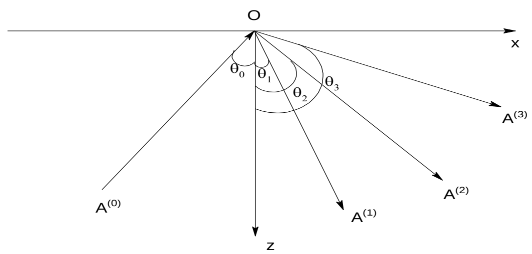


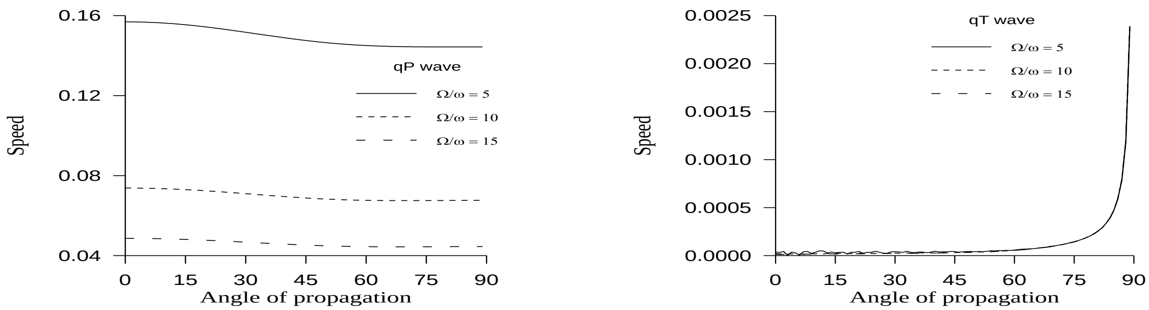
Fig.1
Geometry of the problem showing incident and reflected waves.

5 NUMERICAL RESULTS AND DISCUSSION

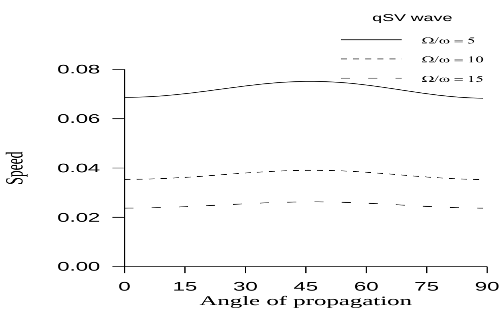
Following material parameters are used for numerical computations of speeds of plane waves and reflection coefficients of various reflected waves:

Material Constants	$LiNbO_3$ (T_0 = 298 K) (Weis and Gaylord [42])	$BaTiO_3$ (T_0 = 293 K) (Dunn [12])
ρ $(kg\ m^{-3})$	4.647×10^3	4.2×10^3
C_{11} $(N\ m^{-2})$	2.03×10^{11}	1.50×10^{11}
C_{33} $(N\ m^{-2})$	2.424×10^{11}	1.46×10^{11}
C_{44} $(N\ m^{-2})$	0.595×10^{11}	0.44×10^{11}
C_{13} $(N\ m^{-2})$	0.752×10^{11}	0.66×10^{11}
e_{31} $(C\ m^{-2})$	0.23	- 4.35
e_{33} $(C\ m^{-2})$	1.33	17.5
e_{15} $(C\ m^{-2})$	3.7	11.4
p_{11} $(C\ N^{-1}m^{-2})$	85.2	$1115 \times 8.85 \times 10^{-12}$
p_{33} $(C\ N^{-1}m^{-2})$	28.7	$1260 \times 8.85 \times 10^{-12}$
C_E $(J\ kg^{-1}K^{-1})$	0.633	0.188
K_1 $(W\ m^{-1}K^{-1})$	4.0	1.0
K_3 $(W\ m^{-1}K^{-1})$	4.5	1.5
$d_3{}^*$ $(N\ C^{-1}K^{-1})$	0.133×10^5	0.133×10^5

The cubic Eq. (12) is solved numerically for a range of angle of propagation varying from 0° to 90°. The phase speeds of *qP, qT* and *qSV* waves are computed for $LiNbO_3$. The speeds of these waves are plotted in Fig. 2 (a-c) against the angle of propagation when $\frac{\Omega}{\omega}=5$, 10, 15 and $\tau_0=\tau_1=0.5\times10^{-9}s$. The speed of *qP* wave is 0.15689 × 10^4 $m.s^{-1}$ at $\theta_0=0^\circ$. It decreases with an increase in angle of propagation and attains a value 0.14435 × 10^4 $m.s^{-1}$ at $\theta_0=89^\circ$. The speed of *qT* wave is 0.36557e-04 × 10^4 $m.s^{-1}$ at $\theta_0=0^\circ$. Initially, it oscillates with an increase in angle of propagation and then increases very sharply to a value 0.23902e-02 × 10^4 $m.s^{-1}$ at $\theta_0=89^\circ$. The speed of *qSV* wave is 0.06864 × 10^4 $m.s^{-1}$ at $\theta_0=0^\circ$. It increases to its maximum value 0.07512 × 10^4 $m.s^{-1}$ at $\theta_0=46^\circ$ and then decreases to a minimum value 0.06827 × 10^4 $m.s^{-1}$ at $\theta_0 = 89^\circ$. These variations are shown by solid curves in Fig.2 (a-c). Comparing solid curves with the dashed curves, it is observed that the speeds of *qP* and *qSV* waves decreases with the increase in rotation rate. The phase speeds of *qP, qT* and *qSV* waves are also computed and plotted in Fig. 3(a-c) for $BaTiO_3$. For $\frac{\Omega}{\omega} = 5$ (solid curves in Figs. 2 and 3), the *qP* and *qSV* waves are observed faster in $BaTiO_3$ than as in $LiNbO_3$. However, the *qT* wave propagate slower in $BaTiO_3$ than as in $LiNbO_3$.

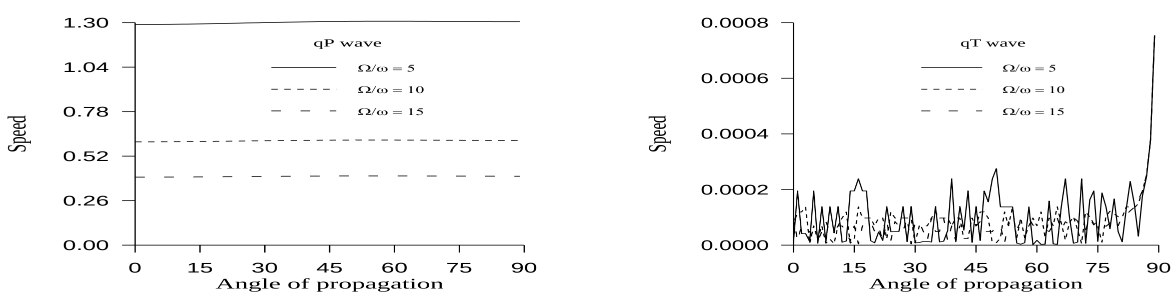


(a) (b)

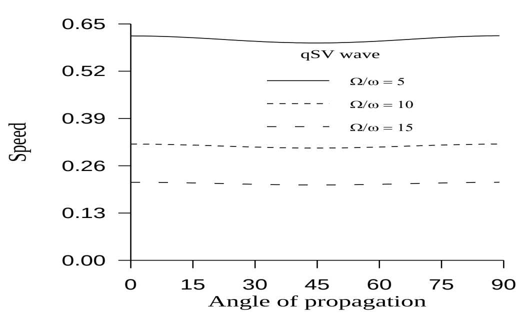


(c)

Fig.2
Variations of speeds of *qP, qT* and *qSV* waves against angle of propagation for $LiNbO_3$.



(a) (b)



(c)

Fig.3
Variations of speeds of *qP, qT* and *qSV* waves against angle of propagation for $BaTiO_3$.

Numerical simulations of reflection coefficients are restricted for an incident *qP* wave on a thermally insulated stress free surface of $BaTiO_3$. With the help of Eq.(20), the reflection coefficients of reflected *qP, qT* and *qSV* waves are obtained numerically against the angle of incidence of *qP* wave when $\frac{\Omega}{\omega}=5$, 10, 15 and. The reflection coefficient of reflected *qP* wave is 0.9997 at $\theta_0=1^\circ$ and it decreases to its minimum value 0.6110 at $\theta_0=46^\circ$. Thereafter, it increases to a value 0.9891 at $\theta_0=89^\circ$. The reflection coefficient of reflected *qT* wave is 0.27323e-12 at $\theta_0=1^\circ$. It oscillates with the increase in angle of incidence. The reflection coefficient of reflected *qSV* wave is 0.1253 at $\theta_0=1^\circ$ and it increases to its maximum value 1.2186 at $\theta_0=37^\circ$. Thereafter, it decreases to value 0.0504 at $\theta_0=89^\circ$. These variations are shown by solid curves in Fig. 4 (a-c). Comparing solid curves with the dashed curves in Fig. 4 (a-c), it is observed that the reflection coefficients of reflected *qP* wave decrease with an increase in rotation rate, whereas the reflection coefficients of reflected *qSV* wave increase. The reflection coefficients of reflected *qT* also change with the change in rotation rate.

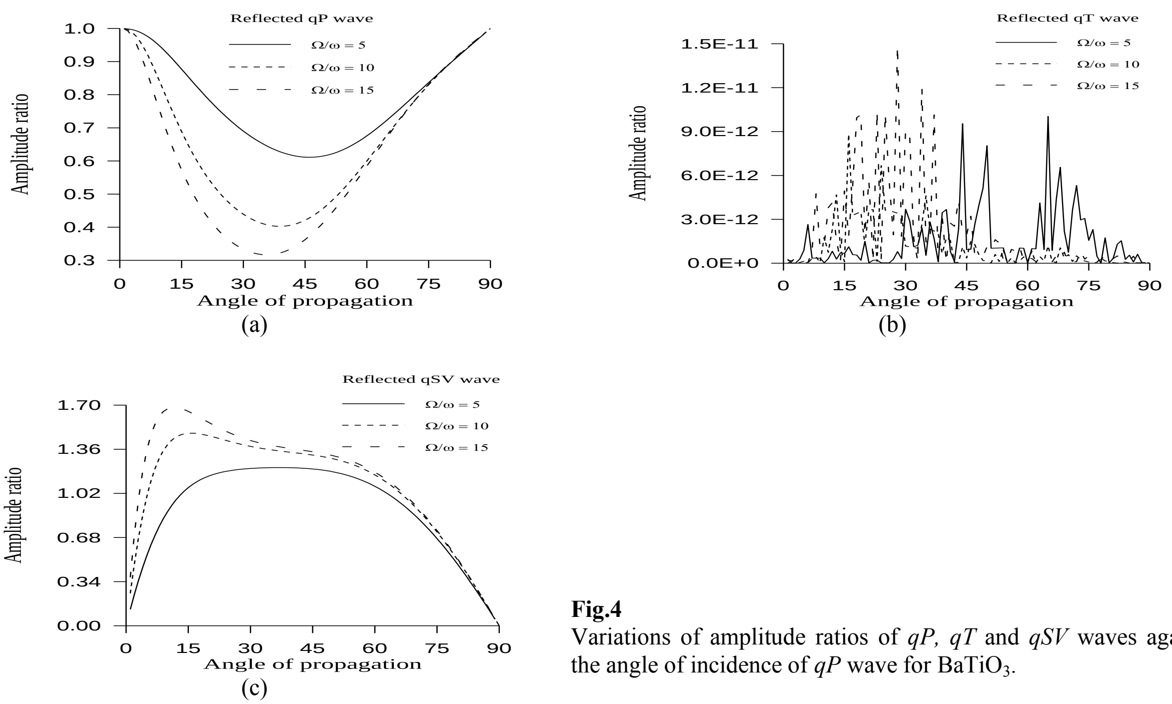


Fig.4
Variations of amplitude ratios of *qP, qT* and *qSV* waves against the angle of incidence of *qP* wave for $BaTiO_3$.

6 CONCLUSIONS

From theory and numerical results, the following points are concluded:

(i) Plane wave solution of governing equations of a rotating generalized thermo-piezoelectric medium results into a cubic velocity Eq. (12) with complex coefficients. The roots of this cubic equation suggests the propagation of three coupled plane waves namely *qP*, *qT* and *qSV* waves in a rotating generalized thermo-piezoelectric medium.

(ii) The expressions for reflection coefficients of reflected *qP*, *qT* and *qSV* waves are obtained in explicit form.

(iii) The speeds of *qP*, *qT* and *qSV* waves are computed numerically for $LiNbO_3$ and $BaTiO_3$ at different values of rotation rate. For $\frac{\Omega}{\omega}=5$, the *qP* and *qSV* waves are observed faster in $BaTiO_3$ than as in $LiNbO_3$. However, the *qT* wave propagate slower in $BaTiO_3$ than as in $LiNbO_3$. The effects of angle of propagation as well as rotation rate are observed significantly on the speeds of plane waves. The nature of dependence of the speeds of *qP, qT* and *qSV* waves on the angle of propagation is different in $LiNbO_3$ and $BaTiO_3$.

(iv) The reflection coefficients of reflected *qP*, *qT* and *qSV* are also computed for $BaTiO_3$ for an incident *qP* wave. The effects of rotation at each angle of incidence are observed on the reflection coefficients of all reflected waves.

The present theoretical derivations and numerical simulations may be of use for further investigation on characteristics of waves in thermo-piezoelectric materials.

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