Journal of Solid Mechanics Vol. 5, No. 1 (2013) pp. 14-21

Double Cracks Identification in Functionally Graded Beams Using Artificial Neural Network

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Received 8 December 2012; accepted 26 January 2013

ABSTRACT

This study presents a new procedure based on Artificial Neural Network (ANN) for identification of double cracks in Functionally Graded Beams (FGBs). A cantilever beam is modeled using Finite Element Method (FEM) for analyzing a double-cracked FGB and evaluation of its first four natural frequencies for different cracks depths and locations. The obtained FEM results are verified against available references. Furthermore, four Multi-Layer Perceptron (MLP) neural networks are employed for identification of locations and depths of both cracks of FGB. Back-Error Propagation (BEP) method is used to train the ANNs. The accuracy of predicted results shows that the proposed procedure is suitable for double cracks identification detection in FGBs.

Keywords: Double cracks; Functionally graded beam; Artificial neural network; Model analysis

1 INTRODUCTION

CRACK is a damage that if develops, may cause catastrophic failure. Therefore identification of cracks is a very important problem. In recent years, many numerical, analytical, as well as experimental studies have been done in the field of crack detection. From non-destructive methods of study of cracked structures, researchers have paid great attention to vibration analysis methods. Crack causes a local flexibility in the structure which has some effects on dynamic behavior. Investigation of these effects can be used for crack detection [1]. Dimarogonas [2] studied methods of investigation of cracked structures. In two different researches, Dimarogonas [3] and also Paipetis and Dimarogonas [4] modeled a crack using local flexibility and evaluated the equivalent stiffness utilizing fracture mechanics. In another study Adams and Cawley [5] developed an experimental technique to estimate crack depth and location using natural frequencies. In another investigation Chan and Dimarogonas [6] presented methods which when the crack location is known it can relate the crack depth to natural frequencies. Goudmunson [7] presented a method for prediction of changes of the natural frequencies caused by different damages. In another paper, Shen and Taylor [8] proposed a method based on minimizing the difference between the measured data and the data obtained from analytical study for identification of cracks in an Euler-Bernoulli beam. Also Masoud et al. [9] studied vibrational characteristics of a fixed-fixed beam which contains a symmetric crack considering coupling effect of axial load and crack depth.

In addition, some studies have been done on vibrational behavior of multi-cracked structures. In a study Sekhar [10] summarized different papers on multiple cracks, the respective influences, and identification methods in some different structures. FEM was used by Lee [11] to solve forward problem in a multi-cracked beam. In another study, Patil and Maiti [12] identified multiple cracks using frequency measurements. Their procedure presented a linear

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relationship explicitly between the changes in natural frequencies and the damage parameters. A vibration analysis of multi-cracked variable cross section beam was performed by Mazanoglu et al. [13] using the Rayleigh–Ritz approximation method. A parametric study on the effect of cracks and axial force levels on the eigenfrequencies was presented by Binici [14]. Meanwhile a new method for natural frequency analysis of beams with an arbitrary number of cracks has been developed by Khiem and Lien [15]. In order to obtain the information about the crack location and depth of beams, Cam et al. [16] studied the vibrations of cracked beam as a result of impact shocks.

Artificial neural network is a new technique that often be used for identification of damage in the recent two decades. In a study Wu et al. [17] used multi-layer feed forward neural network to identify the location of the fault in a simple frame. Also Wang and He [18] developed a numerical simulation and the model experiment upon a hypothetical concrete arch dam for the crack identification based on the reduction of natural frequencies using a statistical neural network. Kao and Hung [19] presented a two-step method for detection of cracks using ANN. Furthermore, ANN was applied by Chen et al. [20] for damage detection in structures when the excitation signal is not available.

Functionally Graded Materials (FGMs) are a new type of materials that are generally made of a mixture of ceramic and metal to satisfy the demand of ultra-high-temperature environment and to eliminate the interface problems. Because of the high cost of productions and importance of applications of FGMs, study the behavior of cracked Functionally Graded (FG) structures is very important. In a study Yu and Chu [21] employed a ρ-version of FEM to evaluate the transverse vibration characteristics of a cracked FGB. In another study Kitipornchai et al. [22] investigated the nonlinear vibration of cracked exponentially FGBs using Timoshenko beam theory and von-Karman geometric nonlinearity. Yang and Chen [23] presented a theoretical investigation of free vibration and elastic buckling of cracked FGBs containing open edge cracks using rotational spring model and the Bernoulli–Euler beam theory. They obtained the analytical solutions of the natural frequencies, critical buckling load, and the corresponding mode shapes for FGBs containing open edge cracks with different end supports. A non-linear dynamic analysis of FGB with pinned–pinned supports due to a moving harmonic load using Timoshenko beam theory with the von-Karma n's non-linear strain–displacement relationships was presented by Simsek [24].

In this study a procedure is presented for the identification of double cracks identification in FGBs. Modal analysis was done on for different cracks conditions. In the proposed procedure of this study BEP algorithm is applied to train the ANNs based on FEM data. The trained ANNs are used for identification of FGB cracks locations and depths. The novelties of this paper can be classified in three aspects, simultaneous identification of more than one crack in FGBs, double cracked FGB modal analysis and applying artificial intelligence techniques for damage detection in FGBs.

2 MATHEMATICAL FORMULATION

The crack of this study is assumed to be perpendicular to the beam surface and always remains open.The considered cracked beam can be treated as two sub-beams which are connected by elastic rotational spring with no mass and length at the crack section according to the rotational spring model. The following relation relates the cracked section bending stiffness K_t to the flexibility G.

$$K_t = \frac{1}{G} \tag{1}$$

The flexibility of the beam due to the presence of edge crack can be calculated from [25]

$$\frac{1-\mu^2}{E(z)} K_1^2 = \frac{M_1^2}{2} \frac{dG}{da}, \tag{2}$$

where M_1 is the cracked section bending moment. Under the first mode of fracture, the Stress Intensity Factor (SIF) is a function of the crack depth, beam geometry, external loading and the material properties. It was derived by Erdogan and Wu [26] for an FGM stripe with an open edge crack under bending as the following equation:

$$K_1 = -\frac{4\sqrt{a}v(z)}{1+\mu^-}\sum_{i=0}^{\infty}\alpha_i T_i(\frac{2z-a}{a}), \tag{3}$$

where T_i is the first kind Chebyshev polynomial and $\mu^- = (3-4\mu)$. By evaluating the boundary integrals using Gaussian quadrature and solving the resulting functional equation by collocation method, the constants α_i can be determined [27].

3 CRACKED BEAM MODAL ANALYSIS

A cantilever FGB is considered in this paper. The elastic modulus *E*, shear modulus μ and mass density ρ are taken to be in the form of Eq. (4). In these equations, E_0, μ_0 and ρ_0 are the values of these parameters in the $y=0$, and $\beta = \ln(E_2 / E_1)/h$ is a constant representing the material gradient, where h is the height of the beam and E_1 and E_2 are the elastic modulus in the top and bottom of the FGB. A schematic view of the FGB has been illustrated in Fig. 1. The material and geometrical characteristic of the FGB are given in Table 1. [27].

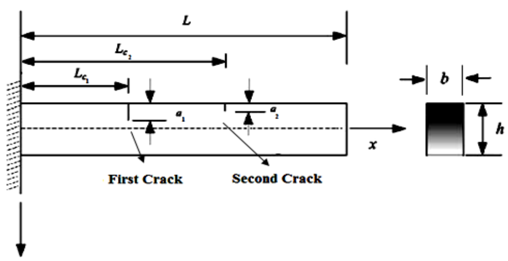


Fig. 1
Geometry of the Cracked FGB.

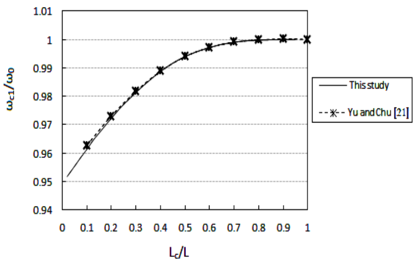
Table 1
Characteristics of the FGB [27]

Length-Height ratio (*L/h*)	20
(E_2/E_1) ratio	0.01
(E_1) (GPa)	70
Density (ρ_1) (kg/m3)	2780
Poisson's ratio	0.33

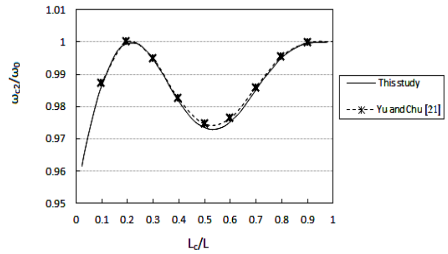
$$E(y) = E_0 e^{\beta y},\ \mu(y) = \mu_0 e^{\beta y},\ \rho(y) = \rho_0 e^{\beta y} \tag{4}$$

In this part of study, 2D FEM of the FGB with and without crack is established using commercial package ANSYS [28]. The FGB is assumed to consist of 30 layers with the same heights containing isotropic homogeneous materials. The material properties are constant in each layer and equal to the value of the middle point of the corresponding layer. The double-layer plate structure is discretized using the 8-node quadrilateral plane stress element (PLANE 183). The element has two degrees of freedom at each node. First four natural frequencies of structure in different conditions of crack are obtained using modal analysis of ANYSY [28]. This number of measured frequencies is adequate for the prediction of location and size of two cracks [27]. For cracked and un-cracked FGB which is shown in Fig. 2 the calculated data is validated with the result of Ref [21]. The properties of the structure used in Ref [21] are presented in Table 2.

The variations of first four non-dimensional frequencies of considered the double cracked FG beam with different locations of second cracks have been illustrated in Fig. 3. In this part the non-dimensional location and depth of first crack and non-dimensional depth of second crack are 0.3, 0.4 and 0.3 respectively and location of second crack is varied from 0.55 to 0.9.

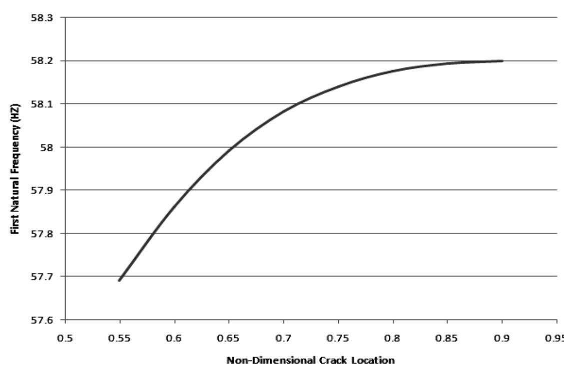


(a)

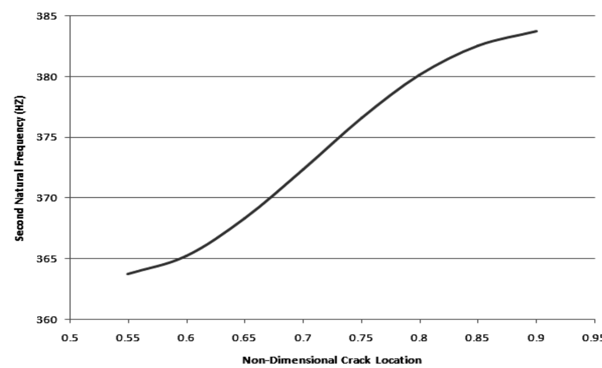


(b)

Fig. 2
Data validation for cracked FGM beam: non-dimensional natural frequency of cracked FGB versus non-dimensional crack location a) First natural frequency b) Second natural frequency.



(a)



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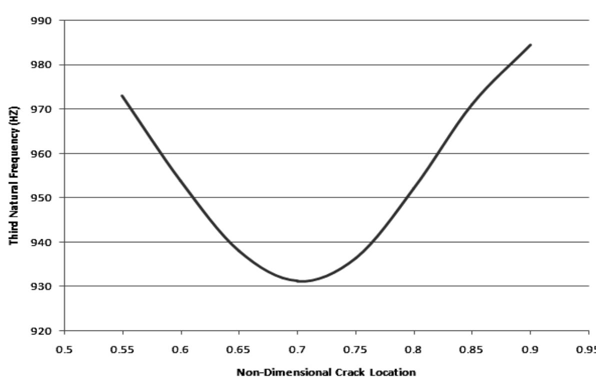
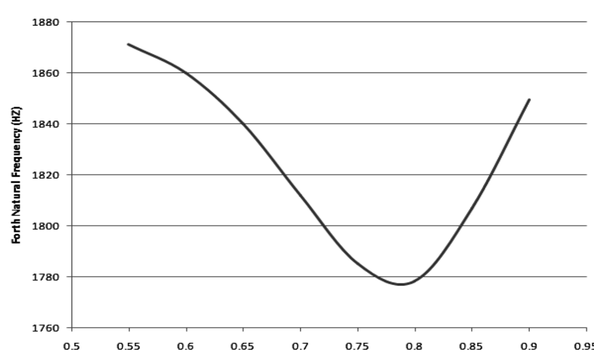


(c)



(d)

Fig.3
The variation of first four fundamental frequencies of the double cracked-FG beam versus the second crack location with assuming the non-dimensional location and depth of first crack and non-dimensional depth of second crack equal to 0.3, 0.4 and 0.3 respectively. a) 1st b) 2nd c) 3rd d) 4th natural frequencies.

4 MULTIPLE CRACKS IDENTIFICATION

4.1 Artificial Neural Network

The ANN used in this study is MLFF neural network consists of an input layer, some hidden layers and an output layer. A schematic of a MLFF neural network is shown in Fig. 4. Usually, knowledge in ANNs is stored as a set of connection weights. The process of modification of the connection weights, using a suitable learning method is called training. In this study four distinct ANNs are employed for prediction of locations and depths of two cracks. The ANNs consist of one input layer with 4 neurons, one hidden layer with 5 neurons and one output layer with one neuron. Transfer functions for neurons of hidden and output layers are defined as Eq. (5), called Tansig function [29].

$$f(s) = \frac{2}{[1+\exp(-2s)]-1}; \tag{5}$$

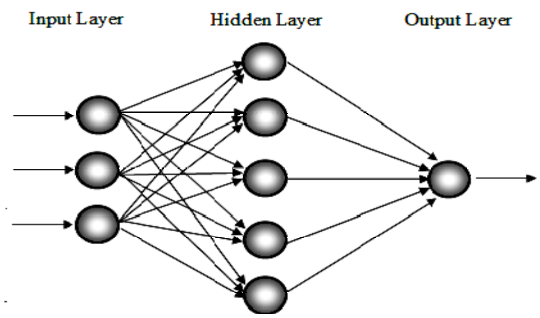


Fig. 4
Schematic diagram of a typical MLFF neural-network architecture.

4.2 Back-Error Propagation Algorithm

The BEP is the most widely used learning algorithm of MLFF neural networks. This learning method was proposed by McClelland and Rumelhart [30] in a ground-breaking study originally focused on cognitive computer science.

In this work the structure of the ANN includes three layers: input, hidden, and output layers. Values of w_{ab} represent the weights between the input and the hidden layer. Values of w_{bc} represent the weights between the hidden and the output layer. The BEP consists of three stages:

1. Feed-forward stage:

$$v = w_{bc}(n).y(n); \tag{6}$$

$$o(n) = \varphi(v(n))) = \frac{2}{1+\exp[-v(2n)]}; \tag{7}$$

where o is the output, u is the input, y is the output of hidden layer and φ is the activation function.

2. Back-propagation stage:

$$\delta(n) = e(n).\varphi[v(n)] = [d(n)-o(n)].[o(n)].[1-o(n)]; \tag{8}$$

where δ represents the local gradient function, e shows the error function, o and d means the actual and desired outputs, respectively.

3.Adjust weighted value:

$$w_{ab}(n+1) = w_{ab}(n) + \Delta w_{ab}(n) = w_{ab}(n) + \eta\delta(n).o(n); \tag{9}$$

where η is the learning rate. Repeating these three stages led to a value of the error function that will be zero or a constant value [29].

4.3 Crack detection procedure

For detection of crack, first of all, MLFF ANNs with 4, 5 and 1 neuron in input, hidden and output layers are created. The first four natural frequencies of the FGB in 74 different crack conditions are applied to ANNs as the input and the corresponding locations of both cracks are applied as targets to first and second networks, and depths of both cracks are applied as targets to third and forth networks, respectively. Number of measured frequencies equal to twice the number of cracks is enough for the prediction of location and size of all the cracks [27].The data that are applied to ANNs are obtained using FEM on the FGB that are modelled by 30 different layers. Training the ANNs is done using BEP algorithm by MATLAB [31]. BEP algorithm is used with 4000 iterations. BEP training procedure for prediction of locations and depths of first and second cracks using MATLAB, are plotted in Figs. 5(a-d), respectively. The trained ANNs were used to prediction the locations and depths of 4 different conditions of cracks as tabulated in Table 2.

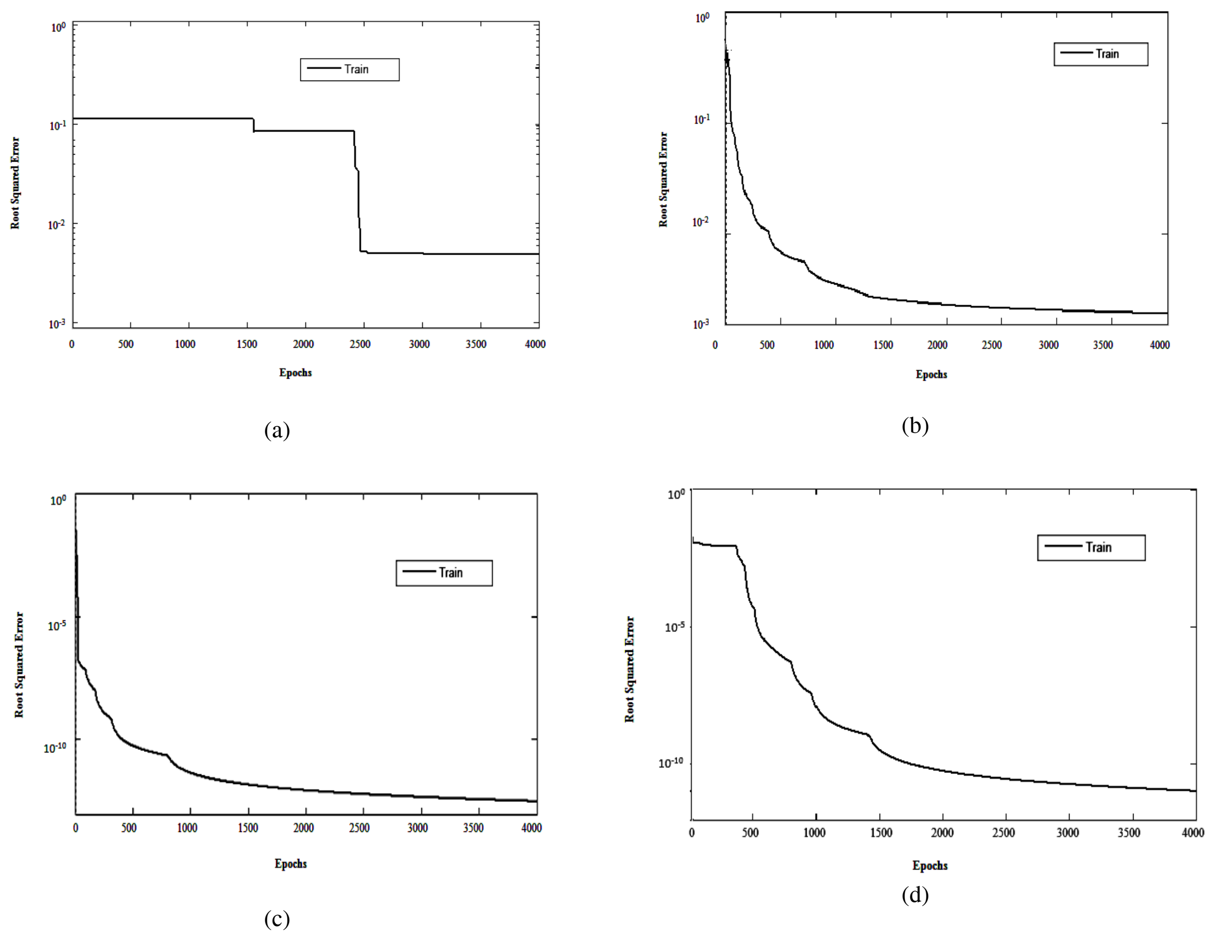


Fig. 5
BEP Training procedures of ANNs for identification of first cracks' a) location and b) depth, and second cracks' c) location and d) depth.

Table 2
The predicted crack location and depth using ANN and different training methods

Case NO	Crack Characteristic	Actual Value	ANN	ANN Error (%)
1	1st Location	0.3000	0.2863	1.368
	1st depth	0.2000	0.2144	1.438
	2nd Location	0.8000	0.7665	3.350
	2nd depth	0.2000	0.2001	0.013
2	1st Location	0.3000	0.2933	0.671
	1st depth	0.3000	0.3138	1.385
	2nd Location	0.7000	0.7178	1.775
	2nd depth	0.3000	0.3134	1.341
3	1st Location	0.4000	0.4000	0.000
	1st depth	0.3000	0.3003	0.026
	2nd Location	0.7000	0.6881	1.187
	2nd depth	0.4000	0.3998	0.018
4	1st Location	0.2000	0.2000	0.000
	1st depth	0.2000	0.2288	2.877
	2nd Location	0.8000	0.7922	0.780
	2nd depth	0.3000	0.2687	3.126

As can be found from this table, the average error in prediction of crack locations is 1.141%. Location Errors were computed as differences between actual and predicted location of crack on length of beam in percent. Also the average error in prediction of cracks depths is 1.278%. Depth Errors were computed as differences between actual and predicted depth of cracks on depth of the beam in percent. Therefore it can be concluded that there is good agreements between actual and predicted results. Also the trained ANNs predicted the crack locations more accurate than the crack depths.

5 CONCLUSIONS

First part of this study concerned with a cantilever FGB modeling using the FEM and validating the obtained results. First four natural frequencies of considered beam were calculated for different crack conditions. Then a procedure based on ANN for detection of crack location and depth in FGB was presented. In the first step of this procedure, four natural frequencies of a FGB for different locations and depths of cracks were calculated using FEM that was obtained in the first part of this study. In the second step, four MLFF neural networks were created and the BEP algorithm was used to train them based on the FEM data. In the third step, trained ANNs were used to predict the characteristics of two cracks of the FGB. The first two ANNs were used to predict the cracks locations and the second pair for the crack depths. Finally it was concluded that the proposed procedure can predict the cracks' locations and depths of double cracked-FGBs accurately. Furthermore, the crack locations were predicted more accurate than the crack depths using ANNs.

ACKNOWLEDGEMENTS

The authors wish to express appreciation to Research Deputy of Ferdowsi University of Mashhad for supporting this project by Grant No.2/24147.

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