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A Problem of Axisymmetric Vibration of Nonlocal Microstretch Thermoelastic Circular Plate with Thermomechanical Sources

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ABSTRACT

In the present manuscript, we investigated a two dimensional axisymmetric problem of nonlocal micro stretch thermoelastic circular plate subjected to thermomechanical sources. An eigenvalue approach is proposed to analyze the problem. Laplace and Hankel transforms are used to obtain the transformed solutions for the displacements, micro rotation, micro stretch, temperature distribution and stresses. The results are obtained in the physical domain by applying the numerical inversion technique of transforms. The results of the physical quantities have been obtained numerically and illustrated graphically. The results show the effect of nonlocal in the cases of Lord Shulman (LS), Green Lindsay (GL) and Coupled Thermoelasticity (CT) on all the physical quantities.

Keywords: Nonlocal micro stretch; Thermoelasticity; Laplace and Hankel transforms; Eigenvalue approach; Circular plate.

1 INTRODUCTION

THE nonlocal elasticity theory can be traced from Kroner [1] who formulated a continuum theory for elastic materials with long range cohesive forces. Eringen [2, 3, 4], Edelen, Green and Laws [5] and Eringen and Edelen [6] developed the nonlocal elasticity theories in the presence of nonlocality residuals of fields like body force, mass, entropy and internal energy and also determined these residuals along with constitutive laws. Eringen and Kim [7], Eringen, Speziale and Kim [8] and Eringen [9, 10] simplified the above mentioned theories for nonlocal elastic solids in such a way that the nonlocal theory differs from the classical one in the stress-strain constitutive relations only and the elastic moduli is the function of the Euclidean distance between the stress and the strain points. Reid and Gooding [11] presented the problem of incorporating an inclusion in a two dimensional linear elastic solid including the nonlocal interactions and strain gradient contributions and also determined an analytical solution for the strain field that minimizes the elastic energy. Gao [12] developed an asymmetric theory of nonlocal elasticity with nonlocal body couple in the context of nonlocal continuum field theory. Sharma and Ganti [13] described the elastic stress state of inclusions having eigenstrains in an infinite nonlocal media. Paola, Failla and Zingales [14] represents the generalization of a three dimensional case of a mechanically based approach for nonlocal elasticity theory. Salehipour, Shahidi and Nahvi [15] presented a modified nonlocal elasticity theory by

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introducing an imaginary nonlocal strain tensor which is used to obtain nonlocal stress tensor. In this modified nonlocal theory, free vibration of functionally graded rectangular micro/Nano plates with simply supported boundary conditions based on the first order plate theory and three dimensional elasticity theory are investigated. Peng, Li, Tang and Shen [16] presented the influence of a nonlocal scale parameter on the deflection of a nonlocal Nano beam and crack growth. Sumelka, Zaera and Fernandez-Saez [17] discussed a theoretical analysis of free axial vibration of rods described in terms of fractional continuum mechanics in the context of Eringen nonlocal elasticity theory. Vasiliev and Lurie [18] presented the nonlocal elasticity theories which are the models of a media with defect fields, gradient elasticity theories and hybrid nonlocal elasticity theories. Chen and Liu [19] established a nonlocal lattice particle method for three dimensional elasticity and fracture simulation of isotropic solids. Singh, Kaur and Tomar [20] investigated the propagation of plane harmonic waves in nonlocal elastic solid with voids. Eringen [21] obtained the constitutive equations of the displacement and the temperature fields of the linear theory of thermoelastic solids together with the balance laws of nonlocal continuum mechanics. Balta and Suhubi [22] developed the nonlocal generalized Thermoelasticity theory within the framework of the nonlocal continuum mechanics. Altan [23] introduced the field equations in a suitable form and defined a class of initial boundary value problems for nonlocal linear Thermoelasticity. Wang and Dhaliwal [24] derived the work and energy equations for generalized nonlocal Thermoelasticity and also proved that the initial boundary value problem has a unique solution. Zenkour and Abouelregal [25] constructed a model of nonlocal Thermoelasticity beam theory with phase lags subjected to a harmonically varying heat. Zenkour and Abouelregal [26] used a unified nonlocal generalized Thermoelasticity model with one relaxation time to study the vibration phenomenon of a Nano beams subjected to a sinusoidal pulse varying heat. Yu, Tian and Xiong [27] established a size dependent thermoelastic model for higher order simple material by adopting the size effect of heat conduction and elasticity with the aids of extended irreversible thermodynamics and generalized free energy.

In the present investigation, we studied the effect of nonlocal in the cases of LS, GL and CT in an isotropic, homogeneous nonlocal micro stretch thermoelastic circular plate. The problem has been solved using eigenvalue approach. The Laplace and Hankel transforms are applied to obtain the results in the transformed domain. The solution is obtained in the physical domain by applying the numerical inversion method. We have presented the numerical results of displacements, micro rotation, micro stretch, temperature distribution and stresses graphically in the presence and absence of nonlocal.

2 BASIC EQUATIONS

Following Eringen [28, 29] and Lord and Shulman [30], the constitutive relations for nonlocal microstretch thermoelastic medium in the absence of body forces, body couples, heat sources and extrinsic equilibrated body force are taken as:

$$\left(1-\varepsilon^2\nabla^2\right)t_{kl}=t^C_{kl}=\left[\lambda_0\psi(x)+\lambda\varepsilon_{rr}(x)\right]\delta_{kl}+\left(\mu+K\right)\varepsilon_{kl}(x)+\mu\,\varepsilon_{lk}(x)-\nu\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla T \tag{1}$$

$$\left(1-\varepsilon^2\nabla^2\right)m_{kl}=m^C_{kl}=\left(b_0\varepsilon_{mlk}\psi_{,m}(x)+\alpha\gamma_{rr}(x)\right)\delta_{kl}+\beta\gamma_{kl}(x)+\gamma\gamma_{lk}(x) \tag{2}$$

$$\left(1-\varepsilon^2\nabla^2\right)\lambda_k=\lambda^C_k=\left(\alpha_0\psi_{,k}(x)+b_0\varepsilon_{klm}\gamma_{lk}(x)\right) \tag{3}$$

$$\left(1-\varepsilon^2\nabla^2\right)\left(s-t\right)=\left(s-t\right)^c=\lambda_1\psi(x) \tag{4}$$

$$\rho T_0\dot{S}=-q_{,ii} \tag{5}$$

$$\rho T_0\dot{S}=\rho C_E\dot{T}(x)+\nu T_0\dot{\varepsilon}_{kk}(x)+mT_0\dot{\psi}(x). \tag{6}$$

The quantities t^{C}_{kl}, m^{C}_{kl}, λ^{C}_{k} and $(s-t)^{c}$ are given by Eringen [29] for classical local micro stretch elastic solid. Equations of motion for a nonlocal isotropic micro stretch solid are given by Eringen [31].

$$t_{kl,k}+\rho\left(f_{l}-\ddot{u}_{1}\right)=0, \tag{7}$$

$$m_{kl,k}+\varepsilon_{lmn}t_{mn}+\rho\left(l_{l}-j\ddot{\phi}_{l}\right)=0, \tag{8}$$

$$\lambda_{k,k}+(t-s)+\rho\left(l-\frac{1}{2}j_{0}\ddot{\psi}\right)=0, \tag{9}$$

$$\left(1+\eta_{0}\tau_{0}\frac{\partial}{\partial t}\right)q_{i}=-KT_{,i}, \tag{10}$$

where f_1 is the applied body force density, l_l is the body moment density and l is applied scalar microstretch tensor. Now using the constitutive relations (1)-(6) into the equations of motion (7)-(10), we obtain

$$(\lambda+\mu)\nabla(\nabla\vec{u})+(\mu+K)\nabla^{2}\vec{u}+K\nabla\times\vec{\phi}$$
$$\lambda_{0}\nabla\psi-\nu\left(1+\tau_{1}\frac{\partial}{\partial t}\right)\nabla T=\rho\left(1-\varepsilon^{2}\nabla^{2}\right)\frac{\partial^{2}\vec{u}}{\partial t^{2}}, \tag{11}$$

$$(\alpha+\beta+\gamma)\nabla\left(\nabla.\vec{\phi}\right)-\gamma\nabla\times\left(\nabla\times\vec{\phi}\right)+K\nabla\times\vec{u}-2K\vec{\phi}=\rho j\left(1-\varepsilon^{2}\nabla^{2}\right)\frac{\partial^{2}\vec{\phi}}{\partial t^{2}}, \tag{12}$$

$$\alpha_{0}\nabla^{2}\psi-\lambda_{0}\left(\nabla\vec{u}\right)-\lambda_{1}\psi+m\left(1+\tau_{1}\frac{\partial}{\partial t}\right)T=\frac{1}{2}\rho j_{0}\left(1-\varepsilon^{2}\nabla^{2}\right)\frac{\partial^{2}\psi}{\partial t^{2}}, \tag{13}$$

$$K_{1}^{*}\nabla^{2}T-\nu T_{0}\left(\frac{\partial}{\partial t}+\eta_{0}\tau_{0}\frac{\partial^{2}}{\partial t^{2}}\right)(\nabla\vec{u})-mT_{0}\left(\frac{\partial}{\partial t}+\eta_{0}\tau_{0}\frac{\partial^{2}}{\partial t^{2}}\right)\psi=\rho C^{*}\left(\frac{\partial}{\partial t}+\tau_{0}\frac{\partial^{2}}{\partial t^{2}}\right)T, \tag{14}$$

$$\lambda_{1}^{*}=\alpha_{0}\psi_{,i}+b_{0}\varepsilon_{ijk}+\phi_{j,k}, \tag{15}$$

For L-S theory, $\tau_1=0, \tau_0\succ 0$, and $\eta_0=1$.

For G-L theory, $\tau_1\geq\tau_0\succ 0$ and $\eta_0=0$,

where λ,μ are Lame's constants, α,β,γ,K are constants of local micro polarity, $m,\alpha_0,b_0,\lambda_0,\lambda_1,j_0$ are constants of local micro stretch elasticity, $\vec{u}$ is the displacement vector, $\vec{\phi}$ is the micro rotation vector, ψ is the scalar micro stretch, $\varepsilon=e_0 a$ is a nonlocal parameter, e_0 is a material constant and a being the internal characteristic length. The internal characteristic length a is the interatomic distance or lattice distance, $\nu=(3\lambda+2\mu+K)\alpha_t$, α_t is the coefficient of linear thermal expansion, C^* is the specific heat at constant strain, K_1^* is the coefficient of thermal conductivity, τ_0,τ_1 are thermal relaxation times, T is the change in temperature of the medium at any time, t_{ij}, m_{ij} and δ_{ij} are the stress tensor, couple stress tensor and Kronecker delta and $\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}+\frac{\partial^2}{\partial z^2}$ is the Laplacian operator.

3 FORMULATION OF THE PROBLEM

A homogeneous and isotropic nonlocal micro stretch thermoelastic circular plate of thickness $2d$ is considered. The plate occupied the region $0 \le r \le \infty, -d \le z \le d$. We consider cylindrical polar coordinate system (r,θ,z) with symmetry about z-axis. The origin of the coordinate system (r,θ,z) is taken as the middle surface of the plate. We assume that the z-axis is normal to the plate along its thickness. The thick circular plate initially has constant temperature T_0.

For two dimensional problem, let

$$\vec{u} = (u_r, 0, u_z),\ \vec{\phi} = (0, \phi_\theta, 0). \tag{16}$$

To facilitate the solution, we take

$$r' = \frac{\omega^* r}{c_1},\ Z' = \frac{\omega^* r}{c_1},\ u_r' = \frac{\rho c_1 \omega^* u_r}{\nu T_0},\ u_Z' = \frac{\rho c_1 \omega^* u_z}{\nu T_0},\ \phi_\theta' = \frac{\rho c_1^2 \phi_\theta}{\nu T_0},\ \psi' = \frac{\rho c_1^2 \psi}{\nu T_0},\ T' = \frac{T}{T_0},$$
$$t' = \omega^* t,\ \tau_0' = \omega^* \tau_0,\ \tau_1' = \omega^* \tau_1,\ t_{ij}' = \frac{t_{ij}}{\nu T_0},\ m_{ij}' = \frac{\omega^*}{c_1 \nu T_0} m_{ij}, \tag{17}$$

where $c_1^2 = \frac{\lambda + 2\mu + K}{\rho},\ \omega^* = \frac{\rho C^* c_1^2}{K_1^*}$.

Define the Laplace and Hankel transforms as:

$$\bar{f}(r,z,s) = L\{\bar{f}(r,z,t)\} = \int_0^\infty f(r,z,t) e^{-st} dt, \tag{18}$$

$$\tilde{f}(\xi,z,s) = H\{\bar{f}(r,z,s)\} = \int_0^\infty r\bar{f}(r,z,s) J_n(\xi r) dr. \tag{19}$$

with the aid of (16) -(19), Eqs. (11) -(14), becomes

$$\tilde{u}_r'' = a_{11}\tilde{u}_r + a_{14}\tilde{\psi} + a_{15}\tilde{T} + b_{12}\tilde{u}_z' + b_{13}\tilde{\phi}_\theta', \tag{20}$$

$$\tilde{u}_z'' = a_{22}\tilde{u}_z + a_{23}\tilde{\phi}_\theta + b_{21}\tilde{u}_r' + b_{24}\tilde{\psi}' + b_{25}\tilde{T}', \tag{21}$$

$$\tilde{\phi}_\theta'' = a_{32}\tilde{u}_z + a_{33}\tilde{\phi}_\theta + b_{31}\tilde{u}_r', \tag{22}$$

$$\tilde{\psi}'' = a_{41}\tilde{u}_r + a_{44}\tilde{\psi} + a_{45}\tilde{T} + b_{42}\tilde{u}_z', \tag{23}$$

$$\tilde{T}'' = a_{51}\tilde{u}_r + a_{54}\tilde{\psi} + a_{55}\tilde{T} + b_{52}\tilde{u}_z', \tag{24}$$

where

$$a_{11} = \left(\frac{\xi^2 + s^2 + s_1 \xi^2 s^2}{\delta^2 + s_1 s^2}\right),\quad a_{14} = \frac{p_0 \xi}{\delta^2 + s_1 s^2},\quad a_{15} = -\frac{\xi(1+\tau_1 s)}{\delta^2 + s_1 s^2},\quad a_{22} = \left(\frac{\xi^2 \delta^2 + s^2 + s_1 \xi^2 s^2}{1 + s_1 s^2}\right),$$

$$a_{23}=-\frac{p\xi}{1+s_1 s^2},\quad a_{32}=-\frac{\xi\delta^{*^2}}{1+\frac{s_1 s^2}{\delta_1^2}},\quad a_{33}=\left(\frac{\xi^2+\frac{s^2}{\delta_1^2}+2\delta^{*^2}+\frac{s_1\xi^2 s^2}{\delta_1^2}}{1+\frac{s_1 s^2}{\delta_1^2}}\right),\quad a_{41}=\frac{p_0\delta_1^*\xi}{1+\delta_2^* s_1 s^2},$$

$$a_{44}=\left(\frac{\xi^2+p_1\delta_1^*+\delta_2^* s^2\left(1+s_1\xi^2\right)}{1+\delta_2^* s_1 s^2}\right),\quad a_{45}=-\frac{\bar{\nu}\delta_1^*(1+\tau_1 s)}{1+\delta_2^* s_1 s^2},\quad a_{51}=\varepsilon\xi\left(s+\eta_0\tau_0 s^2\right),$$

$$a_{54}=\varepsilon\bar{\nu}\left(s+\eta_0\tau_0 s^2\right),\quad a_{55}=\left(\xi^2+Q^*\left(s+\tau_0 s^2\right)\right),\quad b_{12}=\frac{\xi\left(1-\delta^2\right)}{\delta^2+s_1 s^2},\quad b_{13}=\frac{p}{\delta^2+s_1 s^2},\quad b_{21}=-\frac{\xi\left(1-\delta^2\right)}{1+s_1 s^2},$$

$$b_{24}=-\frac{p_0}{1+s_1 s^2},\quad b_{25}=\frac{\left(1+\tau_1 s\right)}{1+s_1 s^2},\quad b_{31}=-\frac{\delta^{*^2}}{1+\frac{s_1 s^2}{\delta_1^2}},\quad b_{42}=\frac{p_0\delta_1^*}{1+\delta_2^* s_1 s^2},\quad a_{52}=\varepsilon\left(s+\eta_0\tau_0 s^2\right),\quad c_2^2=\frac{\mu+K}{\rho},$$

$$\delta^2=\frac{c_2^2}{c_1^2},\quad p=\frac{K}{\rho c_1^2},\quad p_0=\frac{\lambda_0}{\rho c_1^2},\quad s_1=\frac{\varepsilon^2\omega^{*^2}}{c_1^2},\quad \delta^{*^2}=\frac{Kc_1^2}{\gamma\omega^{*^2}},\quad \delta_1^2=\frac{c_3^2}{c_1^2},\quad c_3^2=\frac{\gamma}{\rho j},\quad \delta_1^*=\frac{\rho c_1^4}{\alpha_0\omega^{*^2}},$$

$$\bar{\nu}=\frac{m}{\nu},\quad p_1=\frac{\lambda_1}{\rho c_1^2},\quad \delta_2^*=\frac{\rho j_0 c_1^2}{2\alpha_0},\quad Q^*=\frac{\rho C^* c_1^2}{K_1^*\omega^*},\quad \varepsilon=\frac{\nu^2 T_0}{\rho K_1^*\omega^*}.$$

The system of Eqs. (20) -(24) can be written as:

$$\frac{d}{dz}W(\xi,z,s)=A(\xi,s)W(\xi,z,s), \tag{25}$$

where

$$W=\begin{bmatrix}U\\ DU\end{bmatrix},\quad A=\begin{bmatrix}O & I\\ A_2 & A_1\end{bmatrix},\quad U=\begin{bmatrix}\tilde{u}_r\\ \tilde{u}_z\\ \tilde{\phi}_\theta\\ \tilde{\psi}\\ \tilde{T}\end{bmatrix},\quad D=\frac{d}{dz},\quad O=\begin{bmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix},\quad I=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix},$$

$$A_2=\begin{bmatrix}a_{11}&0&0&a_{14}&a_{15}\\0&a_{22}&a_{23}&0&0\\0&a_{32}&a_{33}&0&0\\a_{41}&0&0&a_{44}&a_{45}\\a_{51}&0&0&a_{54}&a_{55}\end{bmatrix},\quad A_1=\begin{bmatrix}0&b_{12}&b_{13}&0&0\\b_{21}&0&0&b_{24}&b_{25}\\b_{31}&0&0&0&0\\0&b_{42}&0&0&0\\0&b_{52}&0&0&0\end{bmatrix}.$$

We take the solution of (25) as:

$$W(\xi,z,s)=X(\xi,s)e^{qz}, \tag{26}$$

such that $A(\xi,s)W(\xi,z,s)=qW(\xi,z,s)$, which leads to the eigen value problem. The characteristic equation corresponding to the matrix A is given by $\det(A-qI)=0$, which on expansion gives

$$q^{10} - \lambda_1 q^8 + \lambda_2 q^6 - \lambda_3 q^4 + \lambda_4 q^2 - \lambda_5 = 0, \tag{27}$$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ and λ_5 are given in Appendix I.

The roots of Eq. (27) are $\pm q_i, 1,2,3,4,5$. The eigenvectors $X_i(\xi,S)$ corresponding to the eigenvalues q_i may be obtained by solving $[A - qI]X_i(\xi,S) = 0$. The set of Eigen vector $X_i(\xi,S)$ can be written as:

$$X_i(\xi,s) = \begin{bmatrix} X_{i1}(\xi,s) \\ X_{i2}(\xi,s) \end{bmatrix},$$

where

$$X_{i1}(\xi,s) = \begin{bmatrix} a_i q_i \\ b_i \\ -\xi \\ d_i \\ e_i \end{bmatrix}, \quad X_{i2}(\xi,s) = \begin{bmatrix} a_i q_i^2 \\ b_i q_i \\ -\xi q_i \\ d_i q_i \\ e_i q_i \end{bmatrix}, \quad q = q_i; \quad i = 1,2,3,4,5,$$

$$X_{j1}(\xi,s) = \begin{bmatrix} -a_i q_i \\ b_i \\ -\xi \\ d_i \\ e_i \end{bmatrix}, \quad X_{j2}(\xi,s) = \begin{bmatrix} a_i q_i^2 \\ -b_i q_i \\ \xi q_i \\ -d_i q_i \\ -e_i q_i \end{bmatrix}, \quad j = i + 5, q = -q_i; \quad i = 1,2,3,4,5,$$

where a_i, b_i, d_i and e_i are given in Appendix II.

We take the solution of Eq. (25) as:

$$W(\xi,z,s) = \sum_{i=1}^{5} N_i X_i(\xi,s) \cosh(q_i z), \tag{28}$$

where N_1, N_2, N_3, N_4 and N_5 are arbitrary constants.

4 BOUNDARY CONDITIONS

The boundary conditions at the surface $z = \pm d$ of the plate is given by

(i) A thermal Source

$$\frac{dT}{dz} = \pm g_0 F(r,z), \tag{29}$$

where $F(r,z) = z^2 e^{-\omega r}$, $\omega \succ 0$, $F(r,z)$ is a function that increases in the axial direction symmetrically and falls off exponentially as one moves away from the centre of the plate along the radial direction. g_0 is the constant temperature applied on the boundary.

(ii) A Concentrated normal force.

$$t_{zz}^{C}=\delta(r)\delta(t), \tag{30}$$

where $\delta()$ is the Dirac delta function.

(iii) Vanishing of shear stress component

$$t_{zr}^{C}=0. \tag{31}$$

(iv) Vanishing of couple shear stress component

$$m_{z\theta}^{C}=0. \tag{32}$$

(v) Vanishing of micro stress component

$$\lambda_{z}^{*C}=0, \tag{33}$$

where $t_{zz}^{C}=\left(1-\varepsilon^{2}\nabla^{2}\right)t_{zz}$, $t_{zr}^{C}=\left(1-\varepsilon^{2}\nabla^{2}\right)t_{zr}$, $m_{z\theta}^{C}=\left(1-\varepsilon^{2}\nabla^{2}\right)m_{z\theta}$, $\lambda_{z}^{*C}=\left(1-\varepsilon^{2}\nabla^{2}\right)\lambda_{z}^{*}$ are given by

$$t_{zz}^{C}=(\lambda+2\mu+K)\frac{\partial u_{z}}{\partial z}+\lambda\left(\frac{\partial u_{r}}{\partial r}+\frac{u_{r}}{r}\right)-\nu\left(1+\tau_{1}\frac{\partial}{\partial t}\right)T+\lambda_{0}\psi, \tag{34}$$

$$t_{zr}^{C}=(\mu+K)\frac{\partial u_{r}}{\partial z}+\mu\frac{\partial u_{z}}{\partial r}-K\phi_{\theta}, \tag{35}$$

$$m_{z\theta}^{C}=b_{0}\frac{\partial\psi}{\partial r}+\gamma\frac{\partial\phi_{\theta}}{\partial z}, \tag{36}$$

$$\lambda_{z}^{*C}=\alpha_{0}\frac{\partial\psi}{\partial z}-b\frac{\partial\phi_{\theta}}{\partial z}, \tag{37}$$

The expressions of displacements, micro rotation, micro stretch, temperature distribution and stresses are obtained in the transformed domain with the aid of (1) -(3), (16) -(19) and (28) -(37) as:

$$\left(\tilde{u}_{r},\tilde{u}_{z},\tilde{\phi}_{\theta},\tilde{\psi},\tilde{T}\right)=\frac{1}{\Delta}\sum_{1=1}^{5}\left(a_{i}q_{i},b_{i},-\xi,d_{i},e_{i}\right)\Delta_{i}\cosh(q_{i}z), \tag{38}$$

$$\left(\tilde{t}_{zz},\tilde{t}_{zr},\tilde{m}_{z\theta}\right)=\frac{1}{\Delta}\sum_{1=1}^{5}\left(L_{i},M_{i},P_{i}\right)\Delta_{i}\cosh(q_{i}z), \tag{39}$$

where

$$\Delta=\begin{vmatrix} S_{1} & S_{2} & S_{3} & S_{4} & S_{5}\\ T_{1} & T_{2} & T_{3} & T_{4} & T_{5}\\ U_{1} & U_{2} & U_{3} & U_{4} & U_{5}\\ V_{1} & V_{2} & V_{3} & V_{4} & V_{5}\\ W_{1} & W_{2} & W_{3} & W_{4} & W_{5}\end{vmatrix},$$

and $\Delta_i(1,2,3,4,5)$ are obtained from Δ by replacing i^{th} column of Δ with $\left|Q, R, 0,0,0\right|^{tr}$, also

$$S_i = e_i q_i \sinh(q_i d),\ T_i = L_i \cosh(q_i d),\ U_i = M_i \cosh(q_i d),\quad i = 1,2,3,4,5,$$
$$\mathrm{V}_i = P_i \cosh(q_i d),\ W_i = Q_i \sinh(q_i d),\quad i = 1,2,3,4,5,$$
$$Q = \pm g_0 \frac{z^2\omega}{\left(z^2+\omega^2\right)^{\frac{3}{2}}},\quad R = J_0(\xi),$$
$$L_i = \left(1+s^2\left(\xi^2-q_i^2\right)\right)\left(\frac{\lambda\xi a_i q_i}{\rho c_1^2}+p_0 d_i-(1+\tau_1 s)e_i+b_i q_i\right),\quad i = 1,2,3,4,5,$$
$$M_i = \left(-\frac{\mu\xi b_i}{\rho c_1^2}+\frac{\xi K}{\rho c_1^2}+\left(\frac{\mu+\kappa}{\rho c_1^2}\right)a_i q_i^2\right),\quad i = 1,2,3,4,5,$$
$$P_i = \left(-\frac{\gamma\xi\omega^{*^2}}{\rho c_1^4}\right)\left(p_0 d_i + q_i\right),\quad i = 1,2,3,4,5,$$
$$Q_i = \frac{\omega^{*^2} q_i}{\rho c_1^4}(\alpha_0 d_i + b_0\xi),\quad i = 1,2,3,4,5,$$

5 PARTICULAR CASES

In the absence of nonlocality, i.e., we take $\varepsilon = 0$, then the results are obtained for local micro stretch thermoelastic elastic solid. In the manuscript, we have discussed LS, GL and CT theories in the absence and presence of nonlocal elasticity.

In the absence of nonlocal and micro stretch parameter, we obtain the corresponding results for axisymmetric thermoelastic circular plate with thermal sources (without mechanical). These results are similar as obtained by Tripathi et al. [32].

6 INVERSION OF THE TRANSFORMS

We have to obtain the transformed displacements, micro rotation, micro stretch, temperature distribution and stresses in the physical domain, so, we invert the transforms in the resulting expressions (38) -(39). All these expressions are functions of the form $\tilde{f}(\xi,z,s)$. Therefore, we get the function $f(r,z,t)$ by using the inversion of the Hankel and Laplace transforms are defined by

$$\tilde{f}(\xi,z,s) = \int_0^\infty \xi\bar{f}(\xi,z,s) J_n(\xi r)\,d\xi, \tag{40}$$

$$f(r,z,t) = \frac{1}{2\iota\pi}\int_{c-\iota\infty}^{c+\iota\infty} f(r,z,s)e^{-st}\,ds, \tag{41}$$

where c is an arbitrary constant greater than all real parts of the singularities of $\bar{f}(r,z,t)$.

7 NUMERICAL RESULTS AND DISCUSSIONS

The analysis is conducted for aluminum epoxy materials. Following Kiris and Inan [33]and Tomar and Khurana [34], the values for aluminum epoxy materials as nonlocal micro stretch elastic solid are given by

$$\lambda = 7.59\times10^{9} Nm^{-2},\ \mu = 41.90\times10^{9} Nm^{-2},\ \ K = 1.3234\times10^{5} Nm^{-2},\ \ \rho = 2192 kgm^{-3},$$
$$j, j_0 = 0.196\times10^{-6} m^2,\ e_0 = 0.39,\ \ a = 0.5\times10^{-9} m,\ \ \alpha = 8.3255\times10N,\ \beta = 0.10282\times10^{3} N,$$
$$\gamma = 0.779\times10^{-9} N,\ \alpha_0 = 15.947\times10^{3} N,\ b_0 = 0.096\times10^{6} N,\ \lambda_0 = 0.57702\times10^{3} N,\ \lambda_1 = 34.650\times10^{3} N.$$

Following Dhaliwal and Singh [35] give the values for thermal parameters as:

$$C^* = 1.04\times10^{3} JKg^{-1}K^{-1},\ K_1^* = 1.7\times10^{6} Jm^{-1}s^{-1}K^{-1},\ \alpha_t = 2.33\times10^{-5} K^{-1},\ \tau_0 = 6.131\times10^{-13}\ \text{sec},$$
$$\tau_1 = 8.765\times10^{-13}\ \text{sec},\ T_0 = 0.298\times10^{3} K,\ m = 1.13849\times10^{10} Nm^{-2}K^{-1},\ t = 0.01\text{sec}.$$

Figs. 1-5 represent the variations of normal stress, shear stress, couple shear stress, micro stretch and temperature distribution with distance *r* in case of nonlocal micro stretch thermoelastic for Lord Shulman theory (NMLS), micro stretch thermoelastic for Lord Shulman theory (MLS), nonlocal micro stretch thermoelastic for Green Lindsay theory (NMGL), micro stretch thermoelastic for Green Lindsay theory (MGL), nonlocal micro stretch coupled Thermoelasticity (NMCT) and Micro stretch Coupled Thermoelasticity (MCT). In all these figures, NMLS, MLS, NMGL, MGL, NMCT and MCT corresponding to solid line (——), solid line with centered symbol $(-*-*-)$, dash line(-------), dash line with centered symbol (-∗-∗-∗-∗-), dash line (– – – -) and dash line with centered symbol $(--*--*)$ respectively.

Fig. 1 displays that the values of t_{zz} initially increasing for $1 \le r \le 1.4$, decreasing for $1.4 \le r \le 3.3$ and again increasing for $3.3 \le r \le 4$ for NMLS and NMCT. Its values are decreasing for $1 \le r \le 1.7$ increasing for $1.7 \le r \le 3.5$ and become stationary for $3.5 \le r \le 4$ for MLS, MGL and MCT. Also, the variation is small in case of NMGL. However, the variation of t_{zz} for NMLS, NMGL and NMCT is opposite to the variation for MLS, MGL and MCT for $1 \le r \le 4$.

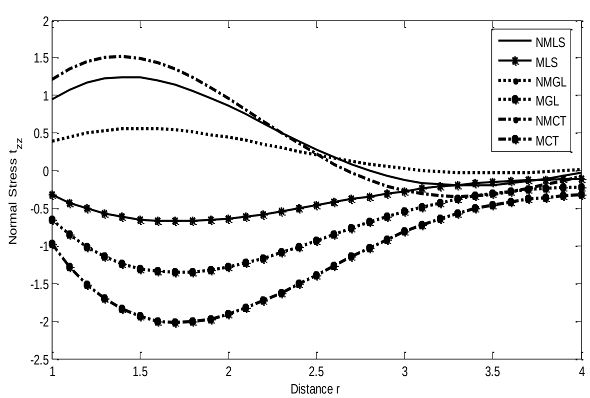

Fig.1
Variations of Normal Stress t_{zz} .

Fig.2 exhibits that the value of t_{zr} initially decreasing for $1 \le r \le 2.5$ and increasing for $2.5 \le r \le 4$ for NMLS, MLS, MGL, NMCT and MCT. The reverse behaviour is noticed in the case of NMGL. The maximum values are obtained for MGL and minimum values are obtained for NMGL near the application of the source. The values are similar for MLS and MCT for $1 \le r \le 4$ with slightly different magnitude.

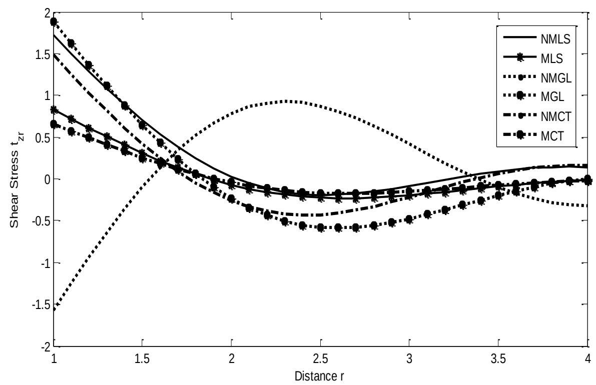

Fig.2
Variations of Shear Stress t_{zr} .

Fig. 3 declares that the values of $m_{z\theta}$ initially deceasing for $1 \le r \le 2.5$ and then increasing $2.5 \le r \le 4$ for MLS, MGL and NMCT. A decreasing trend of variation is exhibited in the case of NMLS. However, the variation is small for MCT. The values of $m_{z\theta}$ initially increase and then decrease smoothly for NMGL. The variation of NMGL is opposite to MLS, MGL and NMCT.

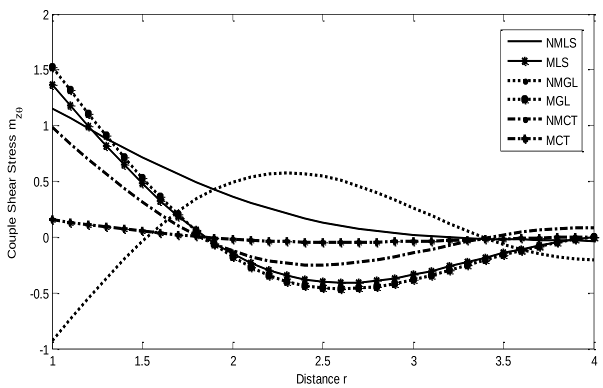

Fig.3
Variations of couple shear stress $m_{z\theta}$.

Fig. 4 shows that the values of ψ initially increasing for $1 \le r \le 1.6$, decreasing sharply for $1.6 \le r \le 3.5$, and then again increasing for $3.5 \le r \le 4$ for NMLS, NMGL, NMCT. For MLS and MGL, the values of ψ are initially increasing for $1 \le r \le 1.7$, decreasing for $1.7 \le r \le 3.7$ and its value become stationary for $3.7 \le r \le 4$. For MCT, its value initially increasing for $1 \le r \le 1.3$, decreasing for $1.3 \le r \le 3$ and again increasing for $3 \le r \le 4$. Away from the sources, all the quantities have similar behavior.

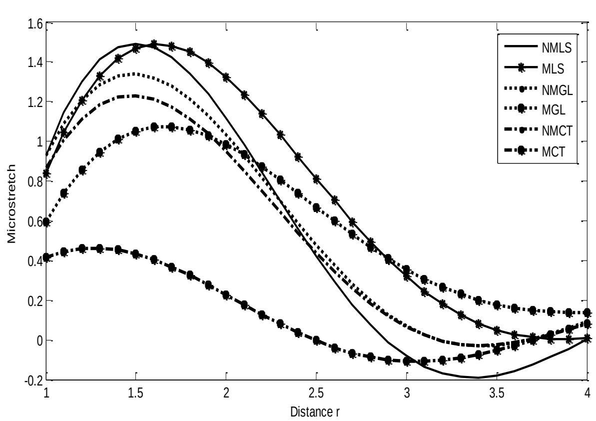

Fig.4
Variations of micro stretch ψ .

Fig. 5 exhibits that the values of *T* initially increasing for $1 \le r \le 1.7$, decreasing for $1.7 \le r \le 3.6$ and its values become stationary for $3.6 \le r \le 4$ for MLS, MGL, NMCT and MCT. It is noticed that the variation of T corresponding to the case NMGL remains the small over the whole range in comparison to the other cases. Its value initially decreasing for $1 \le r \le 1.5$, increasing for $1.5 \le r \le 3.4$ and again decreasing for $3.4 \le r \le 4$.

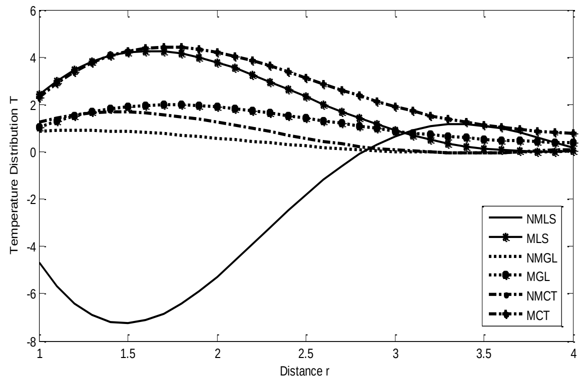

Fig.5
Variations of temperature distribution *T*.

8 CONCLUSIONS

In the present investigation, we studied a two dimensional axisymmetric problem of nonlocal micro stretch thermoelastic circular plate due to thermomechanical sources. Generalized theories of Thermoelasticity with one relaxation time [30] and two relaxation times [36] are used to investigate the problem. An eigenvalue approach is used to analyze the problem. Laplace and Hankel transforms are used to obtain the solutions in the transformed domain. The results are obtained in the physical domain by applying the numerical inversion of transforms. The effect of nonlocal for LS, GL and CT theories on normal stress, shear stress, couple shear stress, micro stretch and temperature distribution are shown graphically. The main conclusions of the paper are given below:

1. The variations of all the physical quantities are more uniform in nature.
2. The behaviour of temperature distribution is similar for all the cases except for NMLS. For normal stress, the variation is similar for NMLS, NMGL and NMCT and reversed behaviour is observed for MLS, MGL and MCT. The similar behaviour is also observed for couple shear stress for MLS, MGL and NMCT whereas a small variation is noticed for MCT. All the quantities have similar behaviour for micro stretch.
3. The presence of nonlocal in micro stretch thermoelastic solid plays an important role on the resulting quantities.
4. The nonlocal has significant effect on all the field quantities for LS, GL and CT theories.
5. The results obtained in this paper can be used to design various homogeneous thermoelastic elements to meet special engineering requirements. Such types of problems in nonlocal micro stretch thermoelastic medium will find great applications in many dynamical systems and industries.

APPENDIX I

$$\lambda_1 = -\left(a_{11}+a_{22}+a_{33}+a_{44}+a_{55}+b_{12}b_{21}+b_{13}b_{31}+b_{25}b_{52}+b_{24}b_{42}\right),$$

$$\lambda_2 = -a_{14}a_{41}+a_{33}a_{55}+a_{44}a_{55}+a_{11}a_{55}+a_{22}a_{55}+a_{33}a_{44}+a_{11}a_{33}+a_{22}a_{33}$$
$$+a_{11}a_{44}+a_{22}a_{44}+a_{11}a_{22}-a_{15}a_{51}-a_{45}a_{54}-a_{23}a_{32}+\left(a_{33}+a_{44}+a_{55}\right)b_{12}b_{21}$$
$$-\left(a_{14}b_{42}+a_{15}b_{52}+a_{32}b_{13}\right)b_{21}+\left(a_{11}+a_{33}+a_{55}\right)b_{42}b_{24}+\left(a_{11}+a_{33}+a_{44}\right)b_{25}b_{52}$$
$$-\left(a_{41}b_{24}+a_{23}b_{31}+a_{51}b_{25}\right)b_{12}+\left(a_{22}+a_{44}+a_{55}+b_{42}b_{24}+b_{25}b_{52}\right)b_{31}b_{13}-a_{45}b_{52}b_{24}-a_{54}b_{42}b_{25},$$

$$\lambda_3 = \left(a_{11}a_{22}+a_{22}a_{55}\right)\left(a_{33}+a_{44}\right)-a_{23}a_{32}\left(a_{11}+a_{44}+a_{55}\right)$$
$$+a_{11}a_{55}\left(a_{22}+a_{33}+a_{44}\right)+a_{33}a_{44}\left(a_{11}+a_{22}+a_{55}\right)-a_{45}a_{54}\left(a_{11}+a_{22}+a_{33}\right)$$

$$
\begin{aligned}
&-a_{14}a_{41}(a_{22}+a_{33}+a_{55})-a_{15}a_{51}(a_{22}+a_{33}+a_{44})+b_{42}b_{25}\\
&(a_{14}a_{51}-a_{11}a_{54}-a_{33}a_{54})+b_{52}b_{25}(-a_{14}a_{41}+a_{11}a_{33}+a_{14}a_{44}+a_{33}a_{44})\\
&+b_{52}b_{24}(a_{15}a_{41}-a_{11}a_{45}-a_{33}a_{45})-b_{12}b_{25}(a_{33}a_{51}+a_{44}a_{51}-a_{41}a_{54})\\
&-b_{12}b_{24}(a_{33}a_{41}-a_{45}a_{51}+a_{41}a_{55})+b_{42}b_{24}(-a_{15}a_{51}+a_{11}a_{33}+a_{11}a_{55}+a_{33}a_{55})\\
&\lambda_4=(a_{44}a_{55}-a_{45}a_{54})(a_{33}b_{12}b_{21}-a_{23}b_{12}b_{31})+(a_{15}a_{54}-a_{14}a_{55})\\
&(a_{33}b_{21}b_{42}-a_{23}b_{31}b_{42})+(a_{14}a_{45}-a_{15}a_{44})(a_{33}b_{21}b_{52}-a_{23}b_{31}b_{52})\\
&+(a_{45}a_{54}-a_{44}a_{55})(a_{32}b_{21}b_{13}-a_{22}b_{31}b_{13})+a_{33}b_{12}b_{24}(a_{45}a_{51}-a_{41}a_{55})\\
&+a_{33}b_{42}b_{24}(-a_{15}a_{51}+a_{11}a_{55})+a_{32}b_{13}b_{24}(a_{41}a_{55}-a_{45}a_{51})\\
&+(a_{41}a_{54}-a_{44}a_{51})(a_{33}b_{12}b_{25}-a_{32}b_{13}b_{25})+a_{33}b_{42}b_{25}(a_{14}a_{51}-a_{11}a_{54})\\
&+a_{33}b_{52}b_{25}(a_{11}a_{44}-a_{14}a_{41})+a_{15}a_{51}(a_{23}a_{32}-a_{22}a_{33}-a_{22}a_{44}-a_{33}a_{44})\\
&+a_{14}a_{45}a_{51}(a_{22}+a_{33})+a_{15}a_{41}a_{54}(a_{22}+a_{33})+a_{45}a_{54}(a_{23}a_{32}-a_{11}a_{22}-a_{11}a_{33}\\
&-a_{22}a_{33})+a_{14}a_{41}(a_{23}a_{32}-a_{22}a_{33}-a_{22}a_{55}-a_{33}a_{55})-a_{11}a_{23}a_{32}(a_{44}+a_{55})\\
&+a_{55}(a_{11}a_{22}a_{33}-a_{23}a_{32}a_{44})+a_{11}a_{22}a_{44}(a_{33}+a_{55})+a_{33}a_{44}a_{55}(a_{11}+a_{22})+(a_{15}a_{41}-a_{11}a_{45})a_{33}b_{24}b_{52},\\
&\lambda_5=(a_{22}a_{33}-a_{23}a_{32})(a_{11}a_{44}a_{55}-a_{11}a_{45}a_{54}+a_{14}a_{45}a_{51})\\
&+(a_{15}a_{54}-a_{14}a_{55})(a_{22}a_{33}a_{41}-a_{23}a_{32}a_{41})+(a_{23}a_{32}-a_{22}a_{33})a_{15}a_{44}a_{51}.
\end{aligned}
$$

APPENDIX II

$$
\begin{aligned}
&a_i=\frac{\xi}{\Delta_i}\Big[r_1^2\big\{r_2\big(r_3(1-\delta^2)+p\delta^{*^2}\big)-p_0^2\delta_1^*r_3\big\}+\varepsilon s^2r_1r_5\big\{r_3\big(r_2+\bar{\nu}^2\delta_1^*(1-\delta^2)-2\bar{\nu}p_0\delta_1^*-\bar{\nu}\delta_1^*\big)+p\bar{\nu}^2\delta_1^*\delta^{*^2}\big\}\Big],\\
&b_i=\frac{-1}{\Delta_i}\Big[r_1^2\big\{r_2\big(r_3r_4+p\delta^{*^2}q_i^2\big)-p_0^2\delta_1^*\xi^2r_3\big\}+\varepsilon s^2r_1r_3r_5\big(\xi^2r_2+\bar{\nu}^2\delta_1^*r_4-2\bar{\nu}p_0\delta_1^*\xi^2\big)+\varepsilon s^2p\bar{\nu}^2\delta_1^*\delta^{*^2}q_i^2r_1r_5\Big],\\
&d_i=\delta_1^*(p_0r_1+\varepsilon S^2\bar{\nu}r_5)(\xi a_i+b_i)q_i/\big(-r_2r_1-\varepsilon s^2\bar{\nu}^2\delta_1^*r_5\big),\\
&e_i=\Big[\varepsilon r_5\Big\{S^2\big(r_2r_1+\varepsilon s^2\bar{\nu}^2\delta_1^*r_5\big)_1^*-\bar{\nu}\delta_1^*\big(p_0r_1+\varepsilon\bar{\nu}r_5\big)\Big\}\Big](\xi a_i+b_i)q_i/\big\{r_1\big(-r_2r_1-\varepsilon s^2\bar{\nu}^2\delta_1^*r_5\big)\big\},\\
&\Delta_i=\delta^{*^2}\Big[r_1^2\big\{r_2\big(\xi^2+s^2-q_i^2\big)+p_0^2\delta_1^*\big(q_i^2-\xi^2\big)\big\}+\varepsilon s^2r_1r_2r_5\big(\xi^2-q_i^2\big)+\varepsilon s^2\delta_1^*r_1r_5\big\{\bar{\nu}^2\big(\xi^2+s^2-q_i^2\big)-2p_0\bar{\nu}\big(\xi^2-q_i^2\big)\big\}\Big],\\
&r_1=\big(\xi^2+Q^*(s+\tau_0s^2)-q_i^2\big),\\
&r_2=\big(\xi^2+p_1\delta_1^*+\delta_3^*s^2(1+s_1\xi^2)-q_i^2(1+\delta_2^*s_1s^2)\big),\\
&r_3=\left(\xi^2+\frac{s^2}{\delta_1^2}+2\delta^{*^2}+\frac{s_1\xi^2s^2}{\delta_1^2}-q_i^2\left(1+\frac{s_1s^2}{\delta_1^2}\right)\right),\\
&r_4=\big(\xi^2+s^2+s_1\xi^2s^2-q_i^2(\delta^2+s_1s^2)\big),\\
&r_5=(1+\tau_1s)(s+\eta_0\tau_0s^2).
\end{aligned}
$$

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