Journal of Solid Mechanics Vol. 12, No. 4 (2020) pp. 883-901
DOI: 10.22034/jsm.2020.1898267.1583

Thermoelastic Behaviour in a Multilayer Composite Hollow Sphere with Heat Source

S.P. Pawar [1], J.J. Bikram [2,*], G.D. Kedar [2]

[1]*Department of Mathematics, S. N. Mor College, Tumsar (MS), India*
[2]*Department of Mathematics, R. T. M. Nagpur University, Nagpur (MS), India*

Received 26 July 2020; accepted 1 October 2020

ABSTRACT
This paper deals with the mathematical approach to discuss the radially varying transient temperature distribution in a multilayer composite hollow sphere subjected to the time independent volumetric generation of heat in each layer. Initially the layers are at arbitrary temperature and the analysis assumes all the layers of the body are thermally isotropic and having a perfect thermal contact. It is novel to obtain the exact solution for temperature field by the separation of variables by splitting the problem into two parts homogeneous transient and non-homogeneous steady state. The set of equations obtained are solved by using the rigorous applications of analytic techniques with the help of eigen value expansion method. The thermoelastic response is studied in the context of uncoupled Thermoelasticity. The results obtained pointed out that the magnitude and distribution of the temperature and thermal stresses are greatly influenced by the layered heat generation parameter. The accuracy and feasibility of the proposed model is demonstrated by an example of three layered hollow sphere of Aluminium, Copper and Iron subjected to given conditions. The results presented in this article could be found hardly in an open literature despite of extensive search.

Keywords: Heat conduction; Internal heat generation; Multilayer; Thermal stresses.

1 INTRODUCTION

THE heat conduction in multilayer composites solids has a variety of engineering applications. The research in industrial furnaces, nuclear reactors, chemical industry, turbines, space crafts and instruments are the topics of continued research where these multilayer materials are highly employed. In early nineteenth century, the manufacturing industry had to face critical problems of designing advanced materials with the different types of geometries of multilayer having variety of boundary conditions. Many of these applications require a detailed knowledge of transient temperature, heat flux and thereby stress distribution within the component layers. Both analytical and numerical techniques are used to sort out the problems. The analytical solution is useful to gain better

*Corresponding author.
E-mail address: jayabikram@gmail.com (J.J. Bikram).

insight through the mathematical form of solution compared to other one. The series solution of one-dimensional problem, using separation of variable were obtained several decades ago, Bulavin and Kashcheev [1] used the method of separation of variables and orthogonal expansion and orthogonal expansion of function over a one dimensional multilayer region to obtain transient heat conduction problem involving distributed volume heat source. However, since computation of eigen values needed for this methodology is difficult. Yener and Ozisik [2] discussed the solution of unsteady heat conduction in multi region media with time dependent heat transfer coefficient, Lu et al. [3] obtained the analytical solution for the problem of transient heat conduction in multidimensional composite cylinder slab is developed for a time dependent boundary conditions. They discussed the problem by the application of the method of the Laplace transform and separation of variable together with variable transformation, Jain et al. [4] presented an analytical double-series solution for the time-dependent asymmetric heat conduction in a multilayer annulus. Recently, Kukla and Siedlecka [5] considered the heat conduction in radial direction while time dependent boundary conditions are assumed. Chen and Yang [6] discussed the thermal response one dimensional quasi-static coupled thermoelastic problem of an infinite long cylinder composed of two different materials. They applied the Laplace transform with respect to time and used the Fourier series and matrix operation to obtain the solution, Jen and Lee [7] considered the solution by using the Laplace transform and the finite difference method. They obtained the solution for temperature and thermal stress distribution, Lee [8] used Laplace transform and finite difference method to obtain the solution of wide range of transient thermal stresses, Ootao [9] presented transient thermoelastic analysis for a multilayer hollow cylinder with piecewise power law non-homogeneity. Recently, Koo and Valgur [10] discussed the thermoelastic effects in deformation of plates with arbitrary changing elastic parameters and temperature through thickness. Using the semi inverse method, a simple analytic solution is obtained for a thermoelastic problem of a nonhomogeneous plate with arbitrary contour. Zamani Nejad et al. [11] introduced an analysis of displacements and stresses of FGM thick spherical pressure vessels with exponential varying material properties using the semi-analytical solution. Pawar et al. [12] presented the exact analytical solution for thermal stresses in a hollow thick sphere of functionally graded material subjected to non-uniform internal heat generation using theory of elasticity. The distribution of thermal stresses for different values of powers of the module of elasticity and varying power of index of heat generation is studied, Pawar et al. [13] discussed the thermoelastic analysis of the functionally graded solid sphere due to non-uniform heat source inside it .The implicit finite difference scheme is used to determine the transient temperature and stress field inside the sphere. Guerrache and Kebli [14] investigated an analytical solution of an axisymmetric frictionless contact problem developed on a rigid circular base with penetration of rigid punch into an elastic layer.
This investigation is concerned with the mathematical approach to obtain the transient temperature distribution for a composite multilayer by the method discussed by Ozisik [15]. That method is applied to spherical geometry and radially varying time independent volumetric generation of heat inside the each layer is introduced. Then solution is obtained for thermal stress fields as Noda [16].

The aim of the work is to obtain the mathematical model for predicting the temperature and stress field inside multilayer composite hollow sphere experiencing internal heat source. The analysis is made on the basis of uncoupled thermoelasticity. We have used the known temperature field obtained earlier to determine stress field inside the body. The results are used in illustrative example involving three layered hollow sphere of Aluminium, Copper and Iron subjected to given conditions and it is illustrated numerically and graphically. The results presented here could not be found in an open literature despite of extensive search.

2 FORMULATION OF THE PROBLEM

This Works deals with uncoupled problem of thermoelasticity of multi-layered sphere using quasi static approach. The problem possess the spherical symmetry with (i) material of each layer is assumed to be homogeneous, isotropic and linearly thermoelastic (ii) the results are discussed for small variation in temperature (iii) the composite sphere is constructed of multilayer with perfect material properties (iv) tll physical quantities are the assumed to be functions of the radial coordinate and time only (v) the medium is initially undisturbed, traction free and without body force. The time independent volumetric heat source $g_i\left(r\right)$ is actuated in each layer for $t \geq 0$.

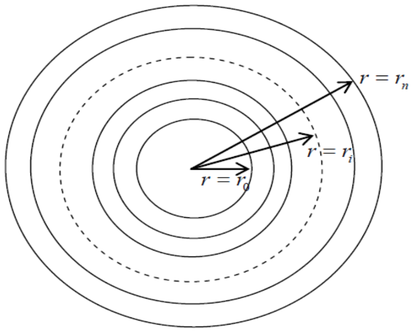


Fig.1
Geometry of the composite multilayer hollow sphere.

The radial heat conduction in the i^{th} layer is governed by the differential equation as Ozisik [15]

$$\frac{\partial^2 T_i}{\partial r^2}+\frac{2}{r}\frac{\partial T_i}{\partial r}+\frac{g_i(r)}{k_i}=\frac{1}{\alpha_i}\frac{\partial T_i}{\partial t}$$
$$T_i=T_i(r,t),\ a\le r\le b\ ,\ r_{i-1}\le r\le r_i\ ,\ i=1,2,3,4..........n\ \ \text{(Layers)} \tag{1}$$

where $T_i\,(K)$ is the temperature distribution in i^{th} layer, $r(m)$ is a radial coordinate, $t(s)$ is time, $k_i\,(W/mK)$ and $\alpha_i\left(m^2/s\right)$ are thermal conductivity and thermal diffusivity of i^{th} layer, $\alpha_i=\frac{k_i}{\rho_i c_i}$ where $\rho_i\left(Kg/m^3\right)$ and $c_i\,(J/Kg\,K)$ are density and specific heat of material of i^{th} layer. Without any generality the temperature at the inner surface is assumed to be zero.

$$T_i(r,t)=0 \quad \text{at } r_0=a \text{ inner surface of hollow sphere} \tag{2}$$

$$k_n\frac{\partial T_n(r_n,t)}{\partial r}+h_n T_n(r_n,t)=0 \quad \text{at } r=r_n=b \text{ outer surface} \tag{3}$$

Inner surface of the layers $(i=2,3,4.......n)$

$$T_i(r_{i-1},t)=T_{i-1}(r_{i-1},t)$$
$$k_i\frac{\partial T_i(r_{i-1},t)}{\partial r}=k_{i-1}\frac{\partial T_{i-1}(r_{i-1},t)}{\partial r} \tag{4}$$

Outer surface of the layers $(i=1,2,3,4.......n)$

$$T_i(r_i,t)=T_{i+1}(r_i,t)$$
$$k_i\frac{\partial T_i(r_i,t)}{\partial r}=k_{i+1}\frac{\partial T_{i+1}(r_i,t)}{\partial r} \tag{5}$$

Initially

$$T_i(r,0)=f_i(r) \tag{6}$$

h_n is the heat transfer coefficient of outer surface. The following dimensionless variables defined and used as follows:

$$\begin{aligned}&(\vartheta,\vartheta_i,\vartheta_n)=(T,T_i,T_n)/T_0,\\&(R,R_i,R_0,R_n)=(r,r_i,r_0,r_n)/r_n,\\&\overline{\alpha}_i=\frac{\alpha_i}{\alpha_0},\overline{k}_i=\frac{k_i}{k_0},\ \tau=\frac{\alpha_0 t}{r_n^{\,2}},\ H_n=\frac{h_n r_n}{k_n}\end{aligned} \tag{7}$$

where T_0, k_0 and α_0 are the typical values of temperature, thermal conductivity and thermal diffusivity respectively. Introducing these new variables into the governing and auxiliary Eqs. (1-6) the problem of heat conduction will transformed into more concise form as:

$$\frac{\partial^2\vartheta_i}{\partial\eta^2}+\frac{2}{\eta}\frac{\partial\vartheta_i}{\partial\eta}+\frac{Q_i(\eta)}{\overline{k}_i}=\frac{1}{\overline{\alpha}_i}\frac{\partial\vartheta_i}{\partial\tau},\quad \frac{r_0}{r_n}=\eta_0\le\eta\le 1,\quad \eta_{i-1}\le\eta\le\eta_i,\quad t\ge 0 \tag{8}$$

Subjected to conditions

$$\eta=\eta_0,\ \vartheta_i(\eta,t)=0, \tag{9a}$$

$$\eta=\eta_n=1,\ \frac{\partial\vartheta_n(\eta_n,\tau)}{\partial\eta}+H_n\,\vartheta_n(\eta_n,\tau)=0 \tag{9b}$$

Inner surface of the layers $(i=2,3,4.......n)$

$$\eta=\eta_{i-1};\quad \vartheta_i(\eta_{i-1},\tau)=\vartheta_{i-1}(\eta_{i-1},\tau);\quad \frac{\partial\vartheta_i(\eta_{i-1},\tau)}{\partial\eta}=\kappa_{i-1}\frac{\partial\vartheta_{i-1}(\eta_{i-1},\tau)}{\partial\eta} \tag{9c}$$

Outer surface of the layers $(i=1,2,3,4.......n-1)$

$$\eta=\eta_i;\ \vartheta_i(\eta_i,\tau)=\vartheta_{i+1}(\eta_i,\vartheta);\quad \kappa_i\frac{\partial\vartheta_i(\eta_i,\tau)}{\partial\eta}=\frac{\partial\vartheta_{i+1}(\eta_i,\tau)}{\partial\eta} \tag{9d}$$

$$\tau=0;\ \vartheta_i(\eta,0)=\frac{f_i(\eta)}{T_0},\ (i=1,2,3,4.......n) \tag{9e}$$

where $\kappa_{i-1}=\dfrac{k_{i-1}}{k_i}, i=2,3....n$ and $\kappa_i=\dfrac{k_i}{k_{i+1}}, i=1,2,..n-1$

Surface heat transfer coefficient $H_n=\dfrac{h_n r_n}{k_n}$ and heat transfer parameter $Q_i(\eta)=\dfrac{g_i(r)r_n^2}{k_0 T_0}$

3 THERMOELASTICITY PROBLEM

In this the temperature determined from Eq. (1) as the known temperature function to obtain the temperature (thermal) stresses in a given body. We consider the multilayer hollow spherical body is free of external mechanical loads, in which the inner $(r_0=a)$ and the outer surfaces $(r_n=b)$ are under given thermal condition and ambient medium is at zero temperature. We assume that an ideal thermos-mechanical contact exists between the layers and that the material properties are different but constant in each layer.

One dimensional problem in a spherical coordinates which means spherically symmetric problem, in which shearing stress and strain components vanish, stress and strain components along θ and ϕ directions are identical. The only nonzero components of the displacement are the radial component $u_{r,i}(r)$ which can be denoted by $u_{r,i}$ for i^{th} layer. The stress displacement relations for the isotropic and homogeneous material layer may be expressed as Noda [16]

$$\varepsilon_{rr,i} = \frac{\partial u_{r,i}}{\partial r}, \ \varepsilon_{\theta\theta,i} = \varepsilon_{\phi\phi,i} = \frac{u_{r,i}}{r} \tag{10}$$

The corresponding thermoelastic stress strain relation or Hook's law are

$$\begin{aligned} \sigma_{rr,i} &= 2\mu_i \varepsilon_{rr,i} + \lambda_i e_i - \beta_i \Delta T_i \\ \sigma_{\theta\theta,i} &= \sigma_{\phi\phi,i} = 2\mu_i \varepsilon_{\theta\theta,i} + \lambda_i e_i - \beta_i \Delta T_i \end{aligned} \tag{11}$$

where $\sigma_{rr,i}$,$\sigma_{\theta\theta,i}$ and $\sigma_{\phi\phi,i}$ (GPa) the component of stress in radial and tangential direction, $\varepsilon_{rr,i}$,$\varepsilon_{\theta\theta,i}$ is strain components in radial and tangential direction for the i^{th} layer of the composite hollow sphere. $\Delta T_i = T_i - T_i(r,0)$, $e_i = \varepsilon_{rr,i} + 2\varepsilon_{\theta\theta,i}$ is dilatation and λ_i and μ_i are the lame constants related with the modulus of elasticity E_i (GPa) and the Poisson's ratio ν_i as,

$$\lambda_i = \frac{\nu_i E_i}{(1+\nu_i)(1-2\nu_i)}, \ \mu_i = \frac{E_i}{2(1+\nu_i)} \tag{12}$$

The equilibrium equation in radial direction excluding the body forces and inertia term as,

$$\frac{d\sigma_{rr,i}}{dr} + \frac{2}{r}\left(\sigma_{rr,i} - \sigma_{\theta\theta,i}\right) = 0 \tag{13}$$

Assuming the traction free condition i.e. the boundary conditions of inner and outer surfaces

$$\sigma_{rr,r_0}(r) = 0 \ \text{ at } \ r = r_0, \ \sigma_{rr,r_n}(r) = 0 \ \text{ at } \ r = r_n \tag{14}$$

The stress components are obtained can be expressed for multilayer composite hollow sphere as [16]

$$\begin{aligned} \sigma_{rr,i} &= \frac{a_{ti} E_i}{(1-\nu_i)} \frac{2}{r^3}\left[\frac{(r^3 - r_0^3)}{(r_n^3 - r_0^3)} \int_{r_{i-1}}^{r_i} \Delta T_i \, r^2 dr - \int_{r_{i-1}}^{r} \Delta T_i \, r^2 dr\right] \\ \sigma_{\theta\theta,i} &= \sigma_{\phi\phi,i} = \frac{a_{ti} E_i}{(1-\nu_i)} \frac{1}{r^3}\left[\frac{(2r^3 + r_0^3)}{(r_n^3 - r_0^3)} \int_{r_{i-1}}^{r_i} \Delta T_i \, r^2 dr + \int_{r_{i-1}}^{r} \Delta T_i \, r^2 dr - \Delta T_i \, r^3\right] \\ i &= 1,2,3,4 \ldots\ldots\ldots n \qquad (n \text{, layers}) \end{aligned} \tag{15}$$

where a_{ti} $\left(\frac{1}{K}\right)$ is the coefficient of linear expansion of i^{th} layer.

Assuming the interface conditions i.e. continuity on the interfaces

$$r = r_i \ ; \ \sigma_{rr,i}(r_i) = \sigma_{rr,i-1}(r_i), \ i = 1,2,\ldots\ldots,n \tag{16}$$

Using the dimensionless coordinates defined as (7) one can obtain the following stress functions in dimensionless form as,

$$\begin{aligned}
&\bar{\sigma}_{\eta\eta,i} = \frac{\sigma_{rr,i}(1-\nu_i)}{2a_{ti}E_iT_0} = \frac{1}{\eta^3}\left[\frac{(\eta^3-\eta_0^3)}{(\eta_n^3-\eta_0^3)}\int_{\eta_{i-1}}^{\eta_i}\Delta\vartheta_i(\eta,\tau)\eta^2 d\eta - \int_{\eta_{i-1}}^{\eta}\Delta\vartheta_i(\eta,\tau)\eta^2 d\eta\right] \\
&\bar{\sigma}_{\theta\theta,i} = \bar{\sigma}_{\phi\phi,i} = \frac{\sigma_{\theta\theta,i}(1-\nu_i)}{2a_{ti}E_iT_0} = \frac{1}{\eta^3}\left[\frac{(2\eta^3+\eta_0^3)}{(\eta_n^3-\eta_0^3)}\int_{\eta_{i-1}}^{\eta_i}\Delta\vartheta_i(\eta,\tau)\eta^2 d\eta + \int_{\eta_{i-1}}^{\eta}\Delta\vartheta_i(\eta,\tau)\eta^2 d\eta - \Delta\vartheta_i(\eta,\tau)\eta^3\right] \\
&\bar{\sigma}_{\eta\eta,\eta_0}(\eta) = 0 \ \text{ at } \eta = \eta_0, \ \bar{\sigma}_{\eta\eta,\eta_n}(\eta) = 0 \ \text{ at } \eta = \eta_n \\
&\eta = \eta_i \ ; \ \bar{\sigma}_{\eta\eta,i}(\eta_i) = \bar{\sigma}_{\eta\eta,i-1}(\eta_i), \ i = 2,.......,n, \ \text{ where } \Delta\vartheta_i(\eta,\tau) = \vartheta_i(\eta,\tau) - \vartheta_i(\eta,0)
\end{aligned} \tag{17}$$

The Eqs. (1-9, 17) constitutes the mathematical formulation of the heat conduction and thermoelasticity problem in dimensionless variables.

4 SOLUTION

Defining new dependent variable $U_i(\eta,\tau)$ as:

$$U_i(\eta,\tau) = \eta\vartheta_i(\eta,\tau) \tag{18}$$

The Eqs. (8-9) will take the form as:

$$\begin{aligned}
&\frac{\partial^2 U_i}{\partial\eta^2} + \frac{\eta Q_i(\eta)}{\bar{k}_i} = \frac{1}{\bar{\alpha}_i}\frac{\partial U_i}{\partial\tau} \\
&U_i = U_i(\eta,\tau), \ \eta_0 \le \eta \le \eta_n, \ \eta_{i-1} \le \eta \le \eta_i
\end{aligned} \tag{19}$$

Subjected to the conditions

$$\begin{aligned}
&\eta = \eta_0, \quad U_i(\eta,\tau) = 0 \\
&\eta = \eta_n = 1, \quad \frac{\partial U_n(\eta_n,\tau)}{\partial\eta} + \left(H_n - \frac{1}{\eta}\right)U_n(\eta_n,\tau) = 0
\end{aligned}$$

Inner surface of the i^{th} layer $(i = 2,3,4....n)$

$$\eta = \eta_{i-1}, \ U_i(\eta_{i-1},\tau) = U_{i-1}(\eta_{i-1},\tau) \ \frac{\partial U_i(\eta_{i-1},\tau)}{\partial\eta} - \frac{U_i(\eta_{i-1},\tau)}{\eta} = \kappa_{i-1}\left(\frac{\partial U_{i-1}(\eta_{i-1},\tau)}{\partial\eta} - \frac{U_{i-1}(\eta_{i-1},\tau)}{\eta}\right)$$

Outer surface of the i^{th} layer $(i = 1,2,3,4....n-1)$

$$\eta = \eta_i, \ U_i(\eta_i,\tau) = U_{i+1}(\eta_i,\tau) \ \kappa_i\left(\frac{\partial U_i(\eta_i,\tau)}{\partial\eta} - \frac{U_i(\eta_i,\tau)}{\eta}\right) = \frac{\partial U_{i+1}(\eta_i,\tau)}{\partial\eta} - \frac{U_{i+1}(\eta_i,\tau)}{\eta}$$

Initially

$$\tau = 0\,,\; U_i\left(\eta,t\right) = \frac{\eta f_i\left(\eta\right)}{T_0} \tag{20}$$

The problem of heat conduction defined by (19-20) is a heat conduction is with non-homogeneity due to dimensionless heat generation parameter $Q_i\left(\eta\right)$ and initial temperature $U_i\left(\eta,0\right)$ are the function of space variable η and hence the problem as Ozisik [15] can be solved by splitting the problem (19-20) into nonhomogeneous steady state with heat generation and homogeneous transient problem then solution written as

$$U_i\left(\eta,\tau\right) = U_{s,i}\left(\eta\right) + U_{h,i}\left(\eta,\tau\right) \tag{21}$$

4.1 Nonhomogeneous steady state

$$\frac{\partial^2 U_{s,i}\left(\eta\right)}{\partial\eta^2} + \frac{\eta\, Q_i\left(\eta\right)}{\bar{k}_i} = 0\,,\; \eta_0 \le \eta \le \eta_n\,,\; \eta_{i-1} \le \eta \le \eta_i \tag{22}$$

Subjected to the conditions

$$\eta = \eta_0\,, \quad U_{s,i}\left(\eta\right) = 0$$

$$\eta = \eta_n = 1\,, \quad \frac{\partial U_{s,n}\left(\eta_n\right)}{\partial\eta} + \left(H_n - \frac{1}{\eta}\right)U_{s,n}\left(\eta_n\right) = 0$$

Inner surface of the i^{th} layer $\left(i = 2,3,4....n\right)$

$$\eta = \eta_{i-1}\,,\; U_{s,i}\left(\eta_{i-1}\right) = U_{s,i-1}\left(\eta_{i-1}\right)$$

$$\frac{\partial U_{s,i}\left(\eta_{i-1}\right)}{\partial\eta} - \frac{U_{s,i}\left(\eta_{i-1}\right)}{\eta_{i-1}} = \kappa_{i-1}\left(\frac{\partial U_{i-1}\left(\eta_{i-1}\right)}{\partial\eta} - \frac{U_{i-1}\left(\eta_{i-1}\right)}{\eta}\right)$$

Outer surface of the i^{th} layer $\left(i = 1,2,3,4....n-1\right)$

$$\eta = \eta_i\,,\; U_{s,i}\left(\eta_i\right) = U_{s,i+1}\left(\eta_i\right)$$

$$\kappa_i\left(\frac{\partial U_{s,ii}\left(\eta_i\right)}{\partial\eta} - \frac{U_{s,i}\left(\eta_i\right)}{\eta}\right) = \frac{\partial U_{s,i+1}\left(\eta_i\right)}{\partial\eta} - \frac{U_{s,i+1}\left(\eta_i\right)}{\eta} \tag{23}$$

The problem (22-23) can be solved by using general method of higher order i.e. variation of parameter with constant coefficients, writing the solution of (22) as:

$$U_{s,i}\left(\eta\right) = \text{C.F.} + \text{P.I.}$$

$$\text{C.F.} = \left(c_1 + c_2\eta\right)e^{0\eta} = c_1 + c_2\eta \tag{24}$$

Let the primitive be $c_1 + c_2\eta$. Assume that

$$U_{s,i}\left(\eta\right) = v_1 U_1 + v_2 U_2 \tag{25}$$

From primitive $U_1 = 1, U_2 = \eta$ and $U_1' = 0, U_2' = 1$, $W = U_1 U_2' - U_2 U_1' = 1$

Hence, $v_1 = \int -U_2 \left(-\frac{\eta Q_i(\eta)}{\bar{k}_i} \right) d\eta$, $v_2 = \int U_1 \left(-\frac{\eta Q_i(\eta)}{\bar{k}_i} \right) d\eta$

Therefore, P.I. $= \frac{1}{\bar{k}_i} \int \eta^2 Q_i(\eta) d\eta - \frac{\eta}{\bar{k}_i} \int \eta Q_i(\eta) d\eta$

Hence,

$$U_{s,i}(\eta) = a_{s,i} + b_{s,i}\eta + \frac{1}{\bar{k}_i} \int \eta^2 Q_i(\eta) d\eta - \frac{\eta}{\bar{k}_i} \int \eta Q_i(\eta) d\eta \tag{26}$$

4.2 Solution of homogeneous transient problem

$$\frac{\partial^2 U_{h,i}(\eta,\tau)}{\partial \eta^2} = \frac{1}{\bar{\alpha}_i} \frac{\partial U_{h,i}(\eta,\tau)}{\partial \tau} \;,\; \eta_0 \le \eta \le \eta_n \;,\; \eta_{i-1} \le \eta \le \eta_i \tag{27}$$

Subjected to the conditions

$$\eta = \eta_0 \;,\; U_{h,i}(\eta,\tau) = 0$$

$$\eta = \eta_n = 1 \,,\; \frac{\partial U_{h,n}(\eta_n)}{\partial \eta} + \left(H_n - \frac{1}{\eta} \right) U_{h,n}(\eta_n) = 0$$

Inner surface of the i^{th} layer $(i = 2,3,4....n)$

$$\eta = \eta_{i-1} \;,\; U_{h,i}(\eta_{i-1},\tau) = U_{h,i-1}(\eta_{i-1},\tau)$$

$$\frac{\partial U_{h,i}(\eta_{i-1},\tau)}{\partial R} - \frac{U_{h,i}(\eta_{i-1},\tau)}{\eta_{i-1}} = \kappa_{i-1} \left(\frac{\partial U_{h,i-1}(\eta_{i-1},\tau)}{\partial \eta} - \frac{U_{h,i-1}(\eta_{i-1},\tau)}{\eta_{i-1}} \right)$$

$$\kappa_{i-1} = \frac{\bar{k}_{i-1}}{\bar{k}_i}$$

Outer surface of the i^{th} layer $(i = 1,2,3,4....n-1)$

$$\eta = \eta_i \;,\; U_{h,i}(\eta_i,\tau) = U_{h,i+1}(\eta_i,\tau)$$

$$\kappa_i \left(\frac{\partial U_{h,i}(\eta_i,\tau)}{\partial \eta} - \frac{U_{h,i}(\eta_i,\tau)}{\eta_i} \right) = \frac{\partial U_{h,i+1}(\eta_i,\tau)}{\partial \eta} - \frac{U_{h,i+1}(\eta_i,\tau)}{\eta_i}$$

$$\kappa_i = \frac{\bar{k}_i}{\bar{k}_{i+1}}$$

$$\tau = 0 \,,\; U_{h,i}(\eta,\tau) = \frac{\eta f_i(\eta)}{T_0} - U_{s,i}(\eta) = f_i^*(\eta)$$

Using the value of $U_{s,i}(\eta)$ from Eq. (26) one gets

$$U_{h,i}(\eta,0) = \frac{\eta f_i(\eta)}{T_0} - a_{s,i} - b_{s,i}\,\eta - \frac{1}{\bar{k}_i} \int \eta^2 Q_i(\eta) d\eta + \frac{\eta}{\bar{k}_i} \int \eta Q_i(\eta) d\eta = f_i^*(\eta) \tag{28}$$

Using the method of separation of variable differential Eq. (27) will be separated as:

$$\frac{d\Gamma_i\left(\tau\right)}{d\tau}+\lambda_{im}^2\Gamma_i\left(\tau\right)=0 \tag{29}$$

$$\frac{dR_{im}\left(\eta\right)}{d\eta}+\frac{\lambda_{im}^2}{\alpha_i}\eta_{im}\left(\eta\right)=0 \tag{30}$$

λ_{im} is a separation constant, the subscript m is induced to imply there are infinite number of discrete values of Eigen λ of corresponding eigen function η_{im} and subscript i for layers. The solution for the time variable function $\Gamma\left(\tau\right)$ is immediate from Eq. (29) as:

$$\Gamma\left(\tau\right)=e^{-\lambda_{im}^2}\tau \tag{31}$$

and the general solution for $U_{h,i}\left(\eta,\tau\right)$ in any region i is constructed as [15]

$$U_{h,i}\left(\eta,\tau\right)=\sum_{m=1}^{\infty}C_{im}\eta_{im}\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta\right)e^{-\lambda_{im}^2\tau} \tag{32}$$

where the summation is taken over all Eigen values λ_{im}, $i=1,2,3,......n$ and $\eta_{im}\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta\right)$ is the solution of (30). Using the initial condition (28) of the problem (27-28) one gets

$$C_{im}=\frac{1}{N}\sum_{i=1}^{n}\frac{\overline{k}_i}{\alpha_i}\int_{\eta_{i-1}}^{\eta_i}\eta_{im}\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta\right)f_i^*\left(\eta\right)d\eta \tag{33}$$

Hence one gets the solution $U_{h,i}\left(\eta,\tau\right)$ as:

$$U_{h,i}\left(\eta,\tau\right)=\sum_{m=1}^{\infty}\frac{e^{-\lambda_{im}^2\tau}}{N\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\right)}\eta_{im}\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\right)\sum_{j=1}^{n}\frac{\overline{k}_j}{\alpha_j}\int_{\eta'=\eta_{j-1}}^{\eta'=\eta_j}\eta'_{jn}\left(\frac{\lambda_{jn}}{\sqrt{\alpha_j}}\eta'\right)f_j^*\left(\eta'\right)d\eta' \qquad \eta_{i-1}\prec\eta\prec\eta_i,\quad i=1,2,3,...n \tag{34}$$

$$N\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\right)=\sum_{i=1}^{n}\frac{\overline{k}_i}{\alpha_i}\int_{\eta_{i-1}}^{\eta}\eta_{im}^2\left(\frac{\lambda_{im}}{\sqrt{\alpha_{i\,i}}}\eta\right)d\eta \tag{35}$$

Hence the temperature distribution $U_i\left(\eta,\tau\right)$ is obtained by adding (26) and (34) as:

$$\begin{aligned}U_i\left(\eta,\tau\right)=&a_{s,i}+b_{s,i}\eta+\frac{1}{k_i}\int\eta^2Q_i\left(\eta\right)d\eta-\frac{\eta}{k_i}\int\eta Q_i\left(\eta\right)d\eta\\&+\sum_{m=1}^{\infty}\frac{e^{-\lambda_{im}^2\tau}}{N\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\right)}\eta_{im}\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\right)\sum_{j=1}^{n}\frac{\overline{k}_j}{\alpha_j}\int_{\eta'=\eta_{j-1}}^{\eta'=\eta_j}\eta'_{jn}\left(\frac{\lambda_{jn}}{\sqrt{\alpha_j}}\eta'\right)f_j^*\left(\eta'\right)d\eta'\end{aligned} \tag{36}$$

4.3 To find the coeefficents $a_{s,i}$ and $b_{s,i}$

Applying the conditions (23) to (26) one gets the equations in $a_{s,i}$ *and* $b_{s,i}$

$$a_{s,1}+b_{s,1}\eta_0=-\frac{1}{\bar{k}_1}\int\eta^2Q_1(\eta)d\eta+\frac{\eta_0}{\bar{k}_1}\int\eta Q_1(\eta)d\eta=G_1$$

$$a_{s,1}+b_{s,1}\eta_1-a_{s,2}-b_{s,2}\eta_1=-\frac{1}{\bar{k}_1}\int\eta^2Q_1(\eta)d\eta+\frac{\eta_1}{\bar{k}_1}\int\eta Q_1(\eta)d\eta+\frac{1}{\bar{k}_2}\int\eta^2Q_2(\eta)d\eta-\frac{\eta_1}{\bar{k}_2}\int\eta Q_2(\eta)d\eta=G_2$$

$$-\frac{\kappa_1}{\eta_1}a_{s,1}+\frac{1}{\eta_1}a_{s,2}=\frac{1}{\eta_1\bar{k}_2}\left[\int\eta^2Q_1(\eta)d\eta-\int\eta^2Q_2(\eta)d\eta\right]=G_3$$

$$a_{s,2}+b_{s,2}\eta_2-a_{s,3}-b_{s,3}\eta_2=-\frac{1}{\bar{k}_2}\int\eta^2Q_2(\eta)d\eta+\frac{\eta_2}{\bar{k}_2}\int\eta Q_2(\eta)d\eta+\frac{1}{\bar{k}_3}\int\eta^2Q_3(\eta)d\eta-\frac{\eta_2}{\bar{k}_3}\int\eta Q_3(\eta)d\eta=G_4$$

$$-\frac{\kappa_2}{\eta_2}a_{s,2}+\frac{1}{\eta_2}a_{s,3}=\frac{1}{\eta_2k_3'}\left[\int\eta^2Q_2(\eta)d\eta-\int\eta^2Q_3(\eta)d\eta\right]=G_5$$

In general

$$(H_n-1)a_{s,n}+H_n b_{s,n}=\frac{1}{\bar{k}_n}\left[(H_n-1)\eta_n-1\right]\int\eta Q_n(\eta)d\eta-\frac{(H_n-1)}{\bar{k}_n}\int\eta^2Q_n(\eta)d\eta=G_n \tag{37}$$

The system of equations in unknowns (37) is expressed in a matrix form as:

$$\begin{pmatrix} 1 & \eta_0 & 0 & 0 & 0 & . & . & . & 0 & 0 \\ 1 & \eta_1 & -1 & -\eta_1 & 0 & 0 & . & . & 0 & 0 \\ -\frac{\kappa_1}{\eta_1} & 0 & \frac{1}{\eta_1} & 0 & 0 & . & . & . & 0 & 0 \\ 0 & 0 & 1 & \eta_2 & -1 & -\eta_2 & 0 & . & 0 & 0 \\ 0 & 0 & -\frac{\kappa_2}{\eta_2} & 0 & \frac{1}{\eta_2} & . & . & . & 0 & 0 \\ . & . & . & . & . & . & . & . & . & . \\ & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & . & . & . & . & . & . & H_n-1 & H_n \end{pmatrix}\begin{pmatrix} a_{s,1} \\ b_{s,1} \\ a_{s,2} \\ b_{s,2} \\ s_{s,3} \\ b_{s,3} \\ . \\ . \\ a_{s,n} \\ b_{s,n} \end{pmatrix}=\begin{pmatrix} G_1 \\ G_2 \\ G_3. \\ G_4 \\ G_5 \\ . \\ . \\ . \\ . \\ G_n \end{pmatrix} \tag{38}$$

From (38) one gets the values of $a_{s,1},b_{s,1},a_{s,2},b_{s,2},a_{s,3},b_{s,3}....$ $a_{s,n}$ and $b_{s,n}$ and then $U_{s,1}(\eta),U_{s,2}(\eta),U_{s,3}(\eta)...U_{s,n}(\eta)$, for layers $i=1,2,3,......n$. We follow the method given as Ozisik [15] to solve eigen value problem. The homogeneous system of equations to obtain A_{im} and B_{im}, assuming $A_{1m}=1$ and for heat flux to be continuous at the layer interfaces for all values of time τ, $\lambda_{im}=\lambda_{1m}\sqrt{\frac{\bar{\alpha}_1}{\bar{\alpha}_i}}$ *for* $i=2,3,4....n$ is expressed in a matrix form as,

$$\begin{pmatrix} c_{1in} & c_{2in} & 0 & 0 & 0 & .. & .. & .. & 0 & 0 \\ x_{11} & x_{12} & x_{13} & x_{14} & 0 & .. & .. & .. & 0 & 0 \\ y_{11} & y_{12} & y_{13} & y_{14} & 0 & .. & .. & .. & 0 & 0 \\ .. & .. & .. & .. & .. & .. & .. & .. & 0 & 0 \\ 0 & 0 & 0 & .. & x_{i1} & x_{i2} & x_{13} & x_{i4} & .. & 0 \\ 0 & 0 & 0 & .. & y_{i1} & y_{i2} & y_{i3} & y_{i4} & .. & 0 \\ .. & .. & .. & .. & .. & .. & .. & .. & .. & 0 \\ .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\ 0 & 0 & .. & .. & .. & .. & y_{n-1,1} & y_{n-1,2} & y_{n-1,3} & y_{n-1,4} \\ 0 & 0 & .. & .. & .. & .. & .. & .. & c_{1out} & c_{2out} \end{pmatrix}\begin{pmatrix} 1 \\ B_{1m} \\ A_{2m} \\ B_{2m} \\ A_{3m} \\ . \\ . \\ . \\ A_{nm} \\ B_{nm} \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ .. \\ .. \\ .. \\ 0 \\ 0 \end{pmatrix} \tag{39}$$

The $2n$ equations of above matrix can be used to find the coefficients A_{im} and B_{im}

$$\begin{pmatrix} c_{2in} & 0 & 0 & 0 & 0 & .. & .. & .. & 0 & 0 \\ x_{12} & x_{13} & x_{14} & 0 & 0 & .. & .. & .. & 0 & 0 \\ y_{12} & y_{13} & y_{14} & 0 & 0 & .. & .. & .. & 0 & 0 \\ .. & .. & .. & .. & .. & .. & .. & .. & 0 & 0 \\ 0 & 0 & 0 & .. & x_{i1} & x_{i2} & x_{13} & x_{i4} & .. & 0 \\ 0 & 0 & 0 & .. & y_{i1} & y_{i2} & y_{i3} & y_{i4} & .. & 0 \\ .. & .. & .. & .. & .. & .. & .. & .. & .. & 0 \\ .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\ 0 & 0 & .. & .. & .. & .. & y_{n-1,1} & y_{n-1,2} & y_{n-1,3} & y_{n-1,4} \\ 0 & 0 & .. & .. & .. & .. & .. & .. & c_{1out} & c_{2out} \end{pmatrix} \begin{pmatrix} B_{1m} \\ A_{2m} \\ B_{2m} \\ A_{3m} \\ B_{3m} \\ . \\ . \\ . \\ A_{nm} \\ B_{nm} \end{pmatrix} = \begin{pmatrix} -c_{1in} \\ -x_{11} \\ -y_{11} \\ .. \\ .. \\ .. \\ .. \\ .. \\ .. \\ .. \end{pmatrix} \tag{40}$$

The solution of above matrix gives coefficients $B_{1m}, A_{2m}, B_{2m}, A_{3m}, B_{3m}$ ..... A_{nm} and B_{nm} .The transcendental equation to find positive roots i.e. the eigen values λ_{1m} $i.e. \lambda_{11} < \lambda_{12} < \lambda_{13} < ...... < \lambda_{1m} < ...$, which is obtained from the determinant of the $(2n \times 2n)$ coefficient matrix in (39) should vanish. This is the condition leads to the following transcendental equation for determination of the eigen values λ_{1m} and hence eigen functions will be obtained.

$$\begin{vmatrix} c_{1in} & c_{2in} & 0 & 0 & 0 & .. & .. & .. & 0 & 0 \\ x_{11} & x_{12} & x_{13} & x_{14} & 0 & .. & .. & 0 & 0 & 0 \\ y_{11} & y_{12} & y_{13} & y_{14} & 0 & .. & .. & 0 & 0 & 0 \\ .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\ 0 & .. & x_{i1} & x_{i2} & x_{i3} & x_{i4} & .. & .. & 0 & 0 \\ 0 & .. & y_{i1} & y_{i2} & y_{i3} & y_{i4} & 0 & .. & 0 & 0 \\ .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\ 0 & .. & .. & .. & 0 & 0 & y_{n-1,1} & y_{n-1,2} & y_{n-1,3} & y_{n-1,4} \\ 0 & 0 & .. & .. & .. & .. & .. & 0 & c_{1out} & c_{2out} \end{vmatrix} = 0 \tag{41}$$

where

$$c_{1in} = \cos\left(\frac{\lambda_{1m}}{\sqrt{\alpha_1'}} R_0\right), \; c_{2in} = \sin\left(\frac{\lambda_{1m}}{\sqrt{\alpha_1'}} R_0\right)$$

$$x_{11} = \cos\left(\frac{\lambda_{1m}}{\sqrt{\alpha_1'}} R_1\right), \; x_{12} = \sin\left(\frac{\lambda_{1m}}{\sqrt{\alpha_1'}} R_1\right), \; x_{13} = -\cos\left(\frac{\lambda_{2m}}{\sqrt{\alpha_2'}} R_1\right), \; x_{14} = -\sin\left(\frac{\lambda_{2m}}{\sqrt{\alpha_2'}} R_1\right)$$

$$y_{11} = -\frac{\kappa_1 \lambda_{1m}}{\sqrt{\alpha_1'}} \sin\left(\frac{\lambda_{1m}}{\sqrt{\alpha_1'}} R_1\right) - \frac{\kappa_1}{R_1} \cos\left(\frac{\lambda_{1m}}{\sqrt{\alpha_1'}} R_1\right), \; y_{12} = \frac{\kappa_1 \lambda_{1m}}{\sqrt{\alpha_1'}} \cos\left(\frac{\lambda_{1m}}{\sqrt{\alpha_1'}} R_1\right) - \frac{\kappa_1}{R_1} \sin\left(\frac{\lambda_{1m}}{\sqrt{\alpha_1'}} R_1\right)$$

$$y_{13} = \frac{\lambda_{2m}}{\sqrt{\alpha_2'}} \sin\left(\frac{\lambda_{2m}}{\sqrt{\alpha_2'}} R_1\right) + \frac{1}{R_1} \cos\left(\frac{\lambda_{2m}}{\sqrt{\alpha_2'}} R_1\right), y_{14} = -\frac{\lambda_{2m}}{\sqrt{\alpha_2'}} \cos\left(\frac{\lambda_{2m}}{\sqrt{\alpha_2'}} R_1\right) + \frac{1}{R_1} \sin\left(\frac{\lambda_{2m}}{\sqrt{\alpha_2'}} R_1\right)$$

$$x_{21} = \cos\left(\frac{\lambda_{2m}}{\sqrt{\alpha_2'}} R_2\right), \; x_{22} = \sin\left(\frac{\lambda_{2m}}{\sqrt{\alpha_2'}} R_2\right), \; x_{23} = -\cos\left(\frac{\lambda_{3m}}{\sqrt{\alpha_3'}} R_2\right), \; x_{24} = -\sin\left(\frac{\lambda_{3m}}{\sqrt{\alpha_3'}} R_2\right)$$

$$y_{21} = -\frac{\kappa_2 \lambda_{2m}}{\sqrt{\alpha_2'}} \sin\left(\frac{\lambda_{2m}}{\sqrt{\alpha_2'}} R_2\right) - \frac{\kappa_2}{R_2} \cos\left(\frac{\lambda_{2m}}{\sqrt{\alpha_2'}} R_2\right), y_{22} = \frac{\kappa_2 \lambda_{2m}}{\sqrt{\alpha_2'}} \cos\left(\frac{\lambda_{2m}}{\sqrt{\alpha_2'}} R_2\right) - \frac{\kappa_2}{R_2} \sin\left(\frac{\lambda_{2m}}{\sqrt{\alpha_2'}} R_2\right)$$

$$y_{23} = \frac{\lambda_{3m}}{\sqrt{\alpha_3'}} \sin\left(\frac{\lambda_{3m}}{\sqrt{\alpha_3'}} R_2\right) + \frac{1}{R_2} \cos\left(\frac{\lambda_{3m}}{\sqrt{\alpha_3'}} R_2\right), \; y_{24} = -\frac{\lambda_{3m}}{\sqrt{\alpha_3'}} \cos\left(\frac{\lambda_{3m}}{\sqrt{\alpha_3'}} R_2\right) + \frac{1}{R_2} \sin\left(\frac{\lambda_{3m}}{\sqrt{\alpha_3'}} R_2\right)$$

$$c_{1out}=-\frac{\lambda_{nm}}{\sqrt{\alpha'_n}}\sin\left(\frac{\lambda_{nm}}{\sqrt{\alpha'_n}}\right)+\left(H_n-1\right)\cos\left(\frac{\lambda_{nm}}{\sqrt{\alpha'_n}}\right),\ c_{2out}=\frac{\lambda_{nm}}{\sqrt{\alpha'_n}}\cos\left(\frac{\lambda_{nm}}{\sqrt{\alpha'_n}}\right)+\left(H_n-1\right)\sin\left(\frac{\lambda_{nm}}{\sqrt{\alpha'_n}}\right)$$

5 NUMERICAL RESULTS AND DISCUSSION

In this work, we carried out some numerical results for temperature distribution in a multilayer composite hollow sphere and the resulting thermal stresses. For this a three layer composite multilayer hollow sphere was selected to demonstrate the numerical calculations. For the multilayer the geometry is shown in Fig.1 and the dimensionless physical and mechanical quantities are given in tables. We considered the multilayer composed of Aluminium, copper and Iron. The inner and outer radii of the sphere are assumed to be 0.02 to 0.30 (*m*). Each layer assumed to have different thickness. The numerical results are illustrated in terms of the dimensionless Temperature distribution $U_i\left(\eta,\tau\right)$ and components of thermal stresses $\overline{\sigma}_{\eta\eta,i}$ and $\overline{\sigma}_{\theta\theta,i}$. The results are represented graphically. The initial temperature in multilayer sphere is assumed as $f_i\left(r\right)=T_0=293.5K$ and it is also assumed as reference temperature T_0 and internal heat source is taken as $Q_i\left(\eta\right)=Q_0\,\eta$.

Table 1
Physical parameters of the composite hollow sphere η.

Layer	η_{i-1} to η_i , $i=1,2,3$	Width of each layer
$i=1$	η_0 to η_1 , 0.06 to 0.33	0.027
$i=2$	η_1 to η_2 , 0.33 to 0.66	0.33
$i=3$	η_2 to η_3 0.66 to 1	0.34
η varies as	η_0 to η_3 0.06 to 1 inner & outer surface	0.94

Table 2
Material properties of layers of the composite hollow sphere.

Layer → Material properties ↓	$i=1$ (Aluminium)	2 (Copper)	3 (Iron)
Thermal conductivity $k_i\ (W/mK)$	204.2	386	72.7
Thermal diffusivity $\alpha_i\ \left(m^2/s\right)$	84.18×10^{-6}	112.34×10^{-6}	20.34×10^{-6}
Poisson's ratio v_i	0.35	0.33	0.30
Modulus of Elasticity $E_i\ (GPa)$	70	117	100
Coefficient of thermal expansion $a_{ti}\ (1/K)$	2.3×10^{-6}	16.5×10^{-6}	
Average conductivity $k_0\ (W/mK)$	221		
Average thermal diffusivity $\alpha_0\left(m^2/s\right)$	72.29×10^{-6}		

Table 3
The dimensionless mechanical material properties of the layers of composite.

Layer → Material Properties ↓	$i=1$	$i=2$	$i=3$
Thermal Conductivity $\overline{k}_i=k_i/k_0$	0.92	1.74	0.33
Thermal diffusivity $\overline{\alpha}_i=\alpha_i/\alpha_0$	1.16	1.56	0.28
Ratio of conductivity parameter $\kappa_i=\dfrac{\overline{k}_i}{\overline{k}_{i+1}}$	$\kappa_1=0.528$	$\kappa_2=5.27$	

Table 4
First 15 Eigen values obtained layer wise.

λ_{im}	m=1	2	3	4	5	6	7	8	9	10	11
λ_{1m}	1.4978	3,7256	8.7587	11.0230	17.0512	18.5606	19.9434	21.8516	23.5826	25.4256	27.2574
λ_{2m}	1.2916	3.2126	7.5528	9.5053	14.7035	16.0051	17.1975	18.8430	20.3357	21.9249	23.5045
λ_{3m}	3.0486	7.5830	17.8276	22.4362	34.7060	37.7782	40.5928	44.4768	48.0001	51.7513	55.4797
λ_{im}	m=12	13	14	15							
λ_{1m}	28.6724	30.4880	35.1396	37.7836							
λ_{2m}	24.7247	26.2903	30.3015	32.5814							
λ_{3m}	58.3599	62.0563	71.5232	76.9048							

Temperature distribution

$$U_i(\eta,\tau)=a_{s,i}+b_{s,i}\,\eta-\frac{Q_0\eta^4}{12k_i}+\sum_{m=1}^{\infty}\frac{e^{-\lambda_{1m}^2\tau}\left[A_{1m}\cos\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta\right)+B_{im}\sin\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta\right)\right]}{N\left(\lambda_{im}/\sqrt{\alpha_i}\right)}\times$$

$$\sum_{i=1}^{3}\frac{k_i}{\alpha_i}\left\{\begin{aligned}&A_{im}\left[\begin{aligned}&(1-b_{s,i})\left\{\frac{\sqrt{\alpha_i}}{\lambda_{im}}\left[\eta_i\sin\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right)-\eta_{i-1}\sin\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\right]+\frac{\alpha_i}{\lambda_{im}^2}\left[\cos\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right)-\cos\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\right]\right\}\\&-a_{s,i}\frac{\sqrt{\alpha_i}}{\lambda_{im}}\left\{\sin\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right)-\sin\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\right\}+\\&\frac{Q_0}{12k_i}\left\{\begin{aligned}&\left(\frac{\sqrt{\alpha_i}\,\eta_i^4}{\lambda_{im}}-\frac{12\alpha_i^{\frac{3}{2}}\eta_i^2}{\lambda_{im}^3}+\frac{24\alpha_i^{\frac{5}{2}}}{\lambda_{im}^5}\right)\sin\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right)-\left(\frac{\sqrt{\alpha_i}\,\eta_{i-1}^4}{\lambda_{im}}-\frac{12\alpha_i^{\frac{3}{2}}\eta_{i-1}^2}{\lambda_{im}^3}+\frac{24\alpha_i^{\frac{5}{2}}}{\lambda_{im}^5}\right)\sin\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\\&+\left(\frac{4\alpha_i\eta_i^3}{\lambda_{im}^2}-\frac{24\eta_i^2}{\lambda_{im}^4}\right)\cos\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right)-\left(\frac{4\alpha_i\eta_{i-1}^3}{\lambda_{im}^2}-\frac{24\eta_{i-1}}{\lambda_{im}^4}\right)\cos\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\end{aligned}\right\}\end{aligned}\right]\\&+B_{im}\left[\begin{aligned}&(1-b_{s,i})\left\{\frac{-\sqrt{\alpha_i}}{\lambda_{im}}\left[\eta_i\cos\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right)-\eta_{i-1}\cos\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\right)\right]+\frac{\alpha_i}{\lambda_{im}^2}\left[\sin\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right)-\sin\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\right]\right\}\\&+a_{s,i}\frac{\sqrt{\alpha_i}}{\lambda_{im}}\left\{\cos\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right)-\cos\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\right\}+\\&\frac{Q_0}{12k_i}\left\{\begin{aligned}&\left(-\frac{\sqrt{\alpha_i}\eta_i^4}{\lambda_{im}}+\frac{12\eta_i^2}{\lambda_{im}^3}-\frac{24}{\lambda_{im}^5}\right)\cos\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right)-\left(-\frac{\sqrt{\alpha_i}\eta_{i-1}^4}{\lambda_{im}}+\frac{12\eta_{i-1}^2}{\lambda_{im}^3}-\frac{24}{\lambda_{im}^5}\right)\cos\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\\&+\left(\frac{4\alpha_i\eta_i^3}{\lambda_{im}^2}-\frac{24\eta_i}{\lambda_{im}^4}\right)\sin\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right)-\left(\frac{4\alpha_i\eta_{i-1}^3}{\lambda_{im}^2}-\frac{24\eta_{i-1}}{\lambda_{im}^4}\right)\sin\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\end{aligned}\right\}\end{aligned}\right]\end{aligned}\right\} \qquad (42)$$

where,

$$N\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\right)=\sum_{i=1}^{3}\frac{k_i}{\alpha_i}\left[\begin{aligned}&\left(\frac{A_{im}^2+B_{im}^2}{2}\right)(\eta_i-\eta_{i-1})+\left(\frac{A_{im}^2-B_{im}^2}{4\lambda_{im}}\right)\left\{\sin\left(2\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right)-\sin\left(2\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\right\}\\&-\frac{A_{im}B_{im}\sqrt{\alpha_i}}{2\lambda_{im}}\left\{\cos\left(2\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right)-\cos\left(2\frac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\right\}\end{aligned}\right]$$

Thermal stresses

$$\sigma_{\eta\eta,i}=\frac{1}{\eta^3}\left[\begin{array}{l}\dfrac{(\eta^3-\eta_0^3)}{(\eta_3^3-\eta_0^3)}\left\{\begin{array}{l}\dfrac{a_{s,i}}{3}(\eta_i^3-\eta_{i-1}^3)+\dfrac{(b_{s,i}-1)}{4}(\eta_i^4-\eta_{i-1}^4)-\dfrac{Q_0}{72k_i}(\eta_i^6-\eta_{i-1}^6)+\\ \displaystyle\sum_{m=1}^{\infty}\frac{e^{-\lambda_{im}^2\tau}}{N\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\right)}C_{im}\sum_{i=1}^{3}D_i\end{array}\right\}\\ -\left\{\begin{array}{l}\dfrac{a_{s,i}}{3}(\eta^3-\eta_{i-1}^3)+\dfrac{(b_{s,i}-1)}{4}(\eta^4-\eta_{i-1}^4)-\dfrac{Q_0}{72k_i}(\eta^6-\eta_{i-1}^6)+\\ \displaystyle\sum_{m=1}^{\infty}\frac{e^{-\lambda_{im}^2\tau}}{N\left(\frac{\lambda_{im}}{\sqrt{\alpha_i}}\right)}F_{im}\sum_{i=1}^{3}D_i\end{array}\right\}\end{array}\right] \tag{43}$$

where, $i=1,2,3$

$$C_{im}=A_{im}\left[\begin{array}{l}\dfrac{\sqrt{\alpha_i}}{\lambda_{im}}\left\{\eta_i^2\sin\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right)-\eta_{i-1}^2\sin\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\right\}+\dfrac{2\alpha_i}{\lambda_{im}^2}\left\{\eta_i\cos\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right)-\eta_{i-1}\cos\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\right\}\\ +\dfrac{2\alpha_i^{\frac{3}{2}}}{\lambda_{im}^3}\left\{\sin\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right)-\sin\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\right\}\end{array}\right]$$

$$+B_{im}\left[\begin{array}{l}-\dfrac{\sqrt{\alpha_i}}{\lambda_{im}}\left\{\eta_i^2\cos\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right)-\eta_{i-1}^2\cos\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\right\}+\dfrac{2\alpha_i}{\lambda_{im}^2}\left\{\eta_i\sin\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right)-\eta_{i-1}\sin\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\right\}\\ -\dfrac{2\alpha_i^{\frac{3}{2}}}{\lambda_{im}^3}\left\{\cos\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right)-\cos\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\right\}\end{array}\right]$$

$$F_{im}=A_{im}\left[\begin{array}{l}\dfrac{\sqrt{\alpha_i}}{\lambda_{im}}\left\{\eta^2\sin\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta\right)-\eta_{i-1}^2\sin\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\right\}+\dfrac{2\alpha_i}{\lambda_{im}^2}\left\{\eta\cos\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta\right)-\eta_{i-1}\cos\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\right\}\\ +\dfrac{2\alpha_i^{\frac{3}{2}}}{\lambda_{im}^3}\left\{\sin\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta\right)-\sin\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\right\}\end{array}\right]$$

$$+B_{im}\left[\begin{array}{l}-\dfrac{\sqrt{\alpha_i}}{\lambda_{im}}\left\{\eta^2\cos\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta\right)-\eta_{i-1}^2\cos\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\right\}+\dfrac{2\alpha_i}{\lambda_{im}^2}\left\{\eta\sin\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta\right)-\eta_{i-1}\sin\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\right\}\\ -\dfrac{2\alpha_i^{\frac{3}{2}}}{\lambda_{im}^3}\left\{\cos\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta\right)-\cos\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\right\}\end{array}\right]$$

$$D_i = \left\{ \begin{array}{l} A_{im}\left[\begin{array}{l} \left(1-b_{s,i}\right)\left\{ \dfrac{\sqrt{\alpha_i}}{\lambda_{im}}\left[\eta_i \sin\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right) - \eta_{i-1}\sin\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\right] + \dfrac{\alpha_i}{\lambda_{im}^2}\left[\cos\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right) - \cos\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\right]\right\} \\ -a_{s,i}\dfrac{\sqrt{\alpha_i}}{\lambda_{im}}\left\{\sin\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right) - \sin\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\right\} + \\ \dfrac{Q_0}{12k_i}\left\{ \begin{array}{l} \left(\dfrac{\sqrt{\alpha_i}\,\eta_i^4}{\lambda_{im}} - \dfrac{12\alpha_i^{\frac{3}{2}}\eta_i^2}{\lambda_{im}^3} + \dfrac{24\alpha_i^{\frac{5}{2}}}{\lambda_{im}^5}\right)\sin\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right) - \left(\dfrac{\sqrt{\alpha_i}\,\eta_{i-1}^4}{\lambda_{im}} - \dfrac{12\alpha_i^{\frac{3}{2}}\eta_{i-1}^2}{\lambda_{im}^3} + \dfrac{24\alpha_i^{\frac{5}{2}}}{\lambda_{im}^5}\right)\sin\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right) \\ + \left(\dfrac{4\alpha_i\eta_i^3}{\lambda_{im}^2} - \dfrac{24\eta_i^2}{\lambda_{im}^4}\right)\cos\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right) - \left(\dfrac{4\alpha_i\eta_{i-1}^3}{\lambda_{im}^2} - \dfrac{24\eta_{i-1}}{\lambda_{im}^4}\right)\cos\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right) \end{array}\right\} \end{array}\right] \\ + B_{im}\left[\begin{array}{l} \left(1-b_{s,i}\right)\left\{ \dfrac{-\sqrt{\alpha_i}}{\lambda_{im}}\left[\eta_i \cos\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right) - \eta_{i-1}\cos\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{n-1}\right)\right] + \dfrac{\alpha_i}{\lambda_{im}^2}\left[\sin\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right) - \sin\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\right]\right\} \\ +a_{s,i}\dfrac{\sqrt{\alpha_i}}{\lambda_{im}}\left\{\cos\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right) - \cos\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right)\right\} + \\ \dfrac{Q_0}{12k_i}\left\{ \begin{array}{l} \left(-\dfrac{\sqrt{\alpha_i}\,\eta_i^4}{\lambda_{im}} + \dfrac{12\eta_i^2}{\lambda_{im}^3} - \dfrac{24}{\lambda_{im}^5}\right)\cos\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right) - \left(-\dfrac{\sqrt{\alpha_i}\,\eta_{i-1}^4}{\lambda_{im}} + \dfrac{12\eta_{i-1}^2}{\lambda_{im}^3} - \dfrac{24}{\lambda_{im}^5}\right)\cos\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right) \\ + \left(\dfrac{4\alpha_i\eta_i^3}{\lambda_{im}^2} - \dfrac{24\eta_i}{\lambda_{im}^4}\right)\sin\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_i\right) - \left(\dfrac{4\alpha_i\eta_{i-1}^3}{\lambda_{im}^2} - \dfrac{24\eta_{i-1}}{\lambda_{im}^4}\right)\sin\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta_{i-1}\right) \end{array}\right\} \end{array}\right] \end{array}\right\}$$

Subjected to the conditions

$$\sigma_{\eta\eta,i}\left(\eta\right)=0 \text{ at } \eta=\eta_0 \text{ and } \eta=\eta_3=1\ ,\ i=1,3\ ;\ \sigma_{\eta\eta,i}\left(\eta_i\right)=\sigma_{\eta\eta,i-1}\left(\eta_i\right),\quad i=2,3 \tag{44}$$

$$\sigma_{\theta\theta,i}=\frac{1}{\eta^3}\left[\begin{array}{l} \dfrac{\left(2\eta^3-\eta_{i-1}^3\right)}{\left(\eta_i^3-\eta_{i-1}^3\right)}\left\{ \begin{array}{l} \dfrac{a_{s,i}}{3}\left(\eta_i^3-\eta_{i-1}^3\right)+\dfrac{\left(b_{s,i}-1\right)}{4}\left(\eta_i^4-\eta_{i-1}^4\right)-\dfrac{Q_0}{72k_i}\left(\eta_i^6-\eta_{i-1}^6\right)+ \\ \displaystyle\sum_{m=1}^{\infty}\dfrac{e^{-\lambda_{im}^2\tau}}{N\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\right)}C_{im}\sum_{i=1}^{3}D_i \end{array}\right\} \\ +\left\{ \begin{array}{l} \dfrac{a_{s,i}}{3}\left(\eta^3-\eta_{i-1}^3\right)+\dfrac{\left(b_{s,i}-1\right)}{4}\left(\eta^4-\eta_{i-1}^4\right)-\dfrac{Q_0}{72k_i}\left(\eta^6-\eta_{i-1}^6\right)+ \\ \displaystyle\sum_{m=1}^{\infty}\dfrac{e^{-\lambda_{im}^2\tau}}{N\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\right)}F_{im}\sum_{i=1}^{3}D_i \end{array}\right\} \\ -\eta^3\left\{a_{s,i}+\left(b_{s,i}-1\right)\eta-\dfrac{Q_0}{12k_i}\eta^4+\displaystyle\sum_{m=1}^{\infty}\dfrac{e^{-\lambda_{im}^2\tau}}{N\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\right)}\left[A_{im}\cos\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta\right)+B_{im}\sin\left(\dfrac{\lambda_{im}}{\sqrt{\alpha_i}}\eta\right)\right]\sum_{i=1}^{3}D_i\right\} \end{array}\right] \tag{45}$$

Table 5
Layer wise 15 Values of A_{im} and B_{im} .

A_{im}	*m*=1	2	3	4	5	6	7	8	9	10	11
A_{1m}	1	1	1	1	1	1	1	1	1	1	1
A_{2m}	-2.5154	-1.5685	-0.6153	3.3577	0.2204	-0.6682	-1.1711	-2.0915	-2.2473	-0.6599	-0.5753
A_{3m}	10.4452	4.1164	-2.9170	0-0794	-1.8274	0.4104	0.3430	1.9040	-2.9699	1.6762	-0.3901
A_{im}	*m*=12	13	14	15							
A_{1m}	1	1	1	1							
A_{2m}	-0.6126	-0.2947	-0.0983	-1.3637							
A_{3m}	0.8751	-1.1074	0.7178	1.3113							
B_{im}	*m*=1	2	3	4	5	6	7	8	9	10	11
B_{1m}	-11.9567	-4.7488	-1.8842	-1.4184	-0.7153	-0.5951	-0.4952	-0.3690	-0.2629	-0.1556	-0.0524
B_{2m}	-6.0651	-3.8177	-2.1769	3.5159	0.2204	-0.6682	-1.1711	-2.0915	-2.2473	-0.6599	-0.5753
B_{3m}	-4.0210	-15.7357	3.2544	-7.8540	0.7120	-0.4672	1.3765	-1.8062	1.2021	0.7498	-0.9541
B_{im}	*m*=12	13	14	15							
B_{1m}	0.0265	0.1283	0.4073	0.5914							
B_{2m}	-0.6126	-0.2947	-0.0983	-1.3637							
B_{3m}	0.3117	-1.8962	0.0606	1.4317							

Table 6
Layer wise values of $a_{s,i}$ $\&b_{s,i}$

i	1	2	3
$a_{s,i}$	-0.1756	-0.0927	-0.4886
$b_{s,i}$	2.9265	2.6738	3.3325

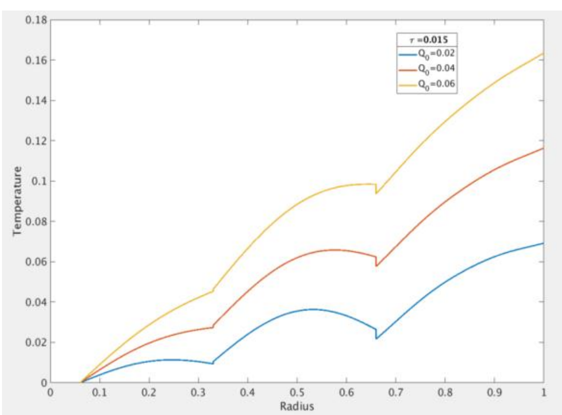


Fig.2
Distribution of temperature versus radius for varying Q_0.

In Fig. 2 the temperature distribution is shown along radial locations for Q_0=0.02, 0.04, 0.06 and a fixed time τ =0.015. The heat source is a function of radius R & hence the source parameter increases along radius and accordingly the variation in the temperature distributions is observed. The temperature gradient varies in each layer because of the difference in the thermal conductivity coefficients which has been seen at radial positions. Internal Source is the only means by which the body getting heated. Since the inner boundary of multilayer is kept at zero temperature and convection at outer boundary to ambient at zero temperature. The graphs shows change in temperature from inner to outer. In Fig. 3 the variation is shown for varying time τ = 0.005, 0.015 and 0.025 for fixed source Q_0=0.04. This graph shows that the layer has spectacular temperature variation with respect to time and satisfies boundary conditions.

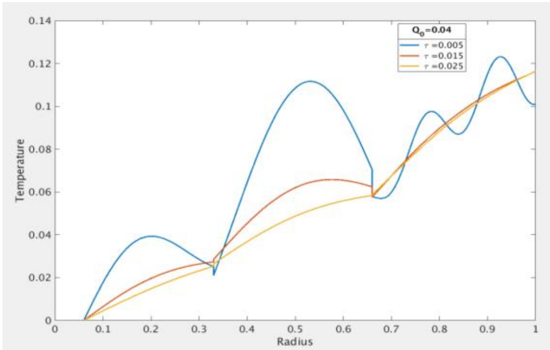
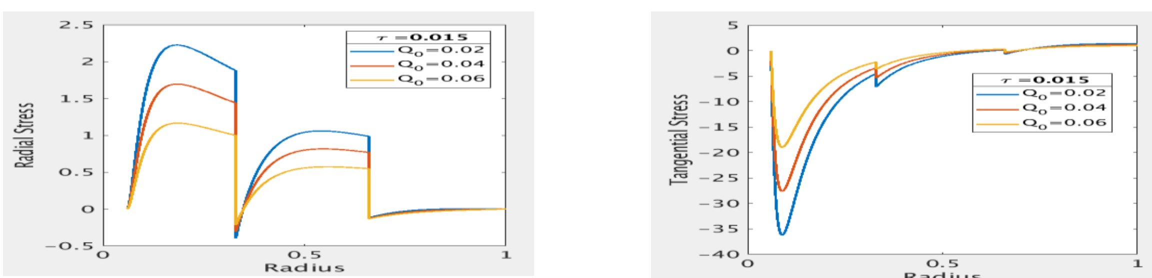


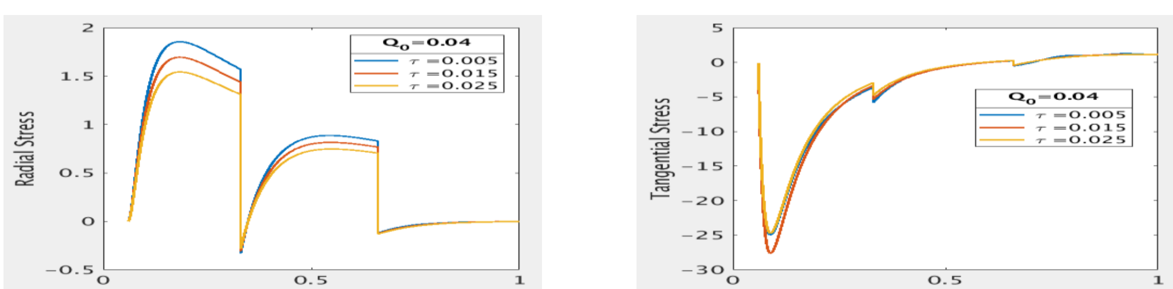
Fig.3
Distribution of Temperature versus radius with varying time τ.

In Fig. 4(a) and 4(b) the transient, radial and tangential stress distributions are shown along radial direction with varying values of heat source and time respectively. Fig. 4(a) shows that the transient radial stresses vanish on the inner and outer boundary surfaces of the multilayer hollow sphere as per induced mechanical boundary conditions. In observations it is found clearly that the stress increases along radial direction and spectacular changes are seen for different layers. The first layer shows compression which decreases on interface. The compression is large for lesser source parameter. Interior of the first layer is under compression and it decreases on its outer interface. There is a decrease with respect to source parameter. In second layer the nature of the stress function is same but it changes its sign from negative to positive and compression decreases while in third layer the variation for considered source parameter again decreases becomes zero on outer surface of multilayer as per assumption. It is observed that the inner surface of the Multilayer hollow sphere is under compression while outer surface shows tension. Fig. 4(b) shows the variation of Tangential stress distribution along radial direction. It is observed that the inner surface of the first layer means inner surface of the multilayer is under compression and changes to tension with respect to radius and this nature continues to tension on outer surface and changes from negative to positive. In Fig. 5(a) and 5 (b) the variation of radial and Tangential stress fields is shown foe different time parameter τ for fixed value of Q_0 and the nature is found to be same as 4(a) and 4(b). As expected the temperature and stress distribution exhibits significant jumps at all interfaces and these are due to the differences in a material properties.



(a) (b)

Fig.4
Radial and Tangential stress distribution versus radius with varying heat source $Q_0 = 0.02, 0.04, 0.06$.



(a) (b)

Fig.5
Radial and Tangential stress distribution versus radius with $\tau = 0.005, 0.015, 0.025$.

6 CONCLUSIONS

The results and the calculations in the analysis can be summarised as follows;

1. The transient thermoelastic problem involving a multilayered hollow sphere with internal heat generation is analyzed in this work. The analytical solutions are obtained for temperature and stress functions.
2. The exact analytic solution for temperature distribution is obtained using separation of variable method by splitting the problem into homogeneous transient and nonhomogeneous steady state. The solution of homogeneous transient part is obtained in a series solution by eigen function expansion. In this inhomogeniety is due to internal heat source and initial temperature which function of radius.
3. The transient thermoelastic response of a multilayer experiencing internal heat source is studied by using temperature function which is obtained earlier.
4. The results were discussed numerically and graphically layer wise and observations are presented. The mathematical software MATLAB is used for the purpose.

ACKNOWLEDGEMENTS

The authors are thankful to Prof. K.C. Deshmukh, Department of Mathematics, Rashtrasant Tukadoji Maharaj Nagpur University, Nagpur (India) for his valuable suggestions while preparing the problem. We are grateful to Mr Ajinkya More, former Graduate student, Department of ECE, UC, Davis (USA) for his valuable cooperation in Numerical and Graphical work.

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