

Structural and Crack Parameter Identification on Structures Using Observer Kalman Filter Identification/Eigen System Realization Algorithm

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ABSTRACT

Structural and crack parameters in a continuous mass model are identified using Observer Kalman filter Identification (OKID) and Eigen Realization Algorithm (ERA). Markov parameters are extracted from the input and out responses from which the state space model of the structural system is determined using Hankel matrix and singular value decomposition by Eigen Realization algorithm. The structural parameters are identified from the state space model. This method is applied to a lumped mass system and a cantilever which are excited with a harmonic excitation at its free end and the acceleration responses at all nodes are measured. The stiffness and damping parameters are identified from the extracted matrices using Newton-Raphson method on the structure. Later, cracks are introduced in the cantilever and all structural parameters are assumed as known priori, the unknown crack parameters such as normalized crack depth and its location are identified using OKID/ERA. The parameters extracted by using this algorithm are compared with other structural identification methods available in the literature. The main advantage of this algorithm is good accuracy of identified structural parameters.

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Keywords : Observer Kalman filter identification; Eigen realization; Markov parameters; Newton-Raphson; Structural identification.

1 INTRODUCTION

CRACKS in a structural member lead to a change in the stiffness and consequently its dynamic characteristics. Hence, it is mandatory that such structures must undergo a Structural Health Monitoring (SHM) process periodically in which the magnitude and location of the crack can be identified and the remaining life of the structure can also be predicted. Gounaris and Dimarogonas [3] developed elemental stiffness and mass matrices for a cracked beam element with open crack and the dynamic behavior of the cracked cantilever was studied. Doebling et al. [2] summarized the vibration-based damage identification methods to detect, locate and characterize crack damage in structural and mechanical systems by examining changes in measured structural vibration response. In the real engineering structures such as bridge structure, wings of an aircraft, the inertial and stiffness properties are

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distributed continuously throughout the structures. These structures can be modelled as continuous mass model. Cracks developed in such structures provide more flexibility and may cause even complete failure due to crack propagation. Hence, health of these structures must be verified periodically by accurate crack identification method. Some of the structures are assumed to be a lumped mass model with reduced degrees of freedom (DOF) where the mass is concentrated at certain points only [7]. In parameter identification of such structures, the accuracy of the results is moderate. In order to improve the accuracy, it is better to consider the system as a continuous mass model with all possible DOFs. In case of the crack damage identification problem, the accurate crack location and magnitude of crack depth may be identified if the structure is modelled as continuous mass model. Several researches have been done on detection of cracks in beam like structures [8, 18, 19, 9, 5], but still there is a lag in the development of a rigorous cracked beam vibration theory as well as a method to detect the multiple cracks with good accuracy and reduced computational effort. Qian and Jiang [13] derived the element stiffness matrix for a beam with a crack by integrating stress intensity factors. Krawczuk et al. [8] developed a finite element model for the cantilever beam with a crack at its mid-point using the theory of elasto plastic fracture mechanics. Viola et al. [19] developed the finite element model for a cracked Timoshenko beam with open crack and derived the stiffness and consistent mass matrices. The crack effect on the stiffness matrix, as well as on mass matrix, is investigated. Lee and Chung [10] used Eigen frequency data for identifying crack parameters in a cantilever with a single open crack. Parhi and Das [12] have performed the analytical studies on fuzzy inference system for detection of crack depth and location of a cracked cantilever beam structure using six input parameters to the fuzzy membership functions. The six input parameters are percentage deviation of first three natural frequencies and first three mode shapes of the cantilever beam. The two output parameters of the fuzzy inference system are relative crack depth and relative crack location. Experimental setup has been developed for verifying the robustness of the developed fuzzy inference system. The developed fuzzy inference system can predict the location and depth of the crack in a close proximity to the real results. Baghmisheh et al. [16] have proposed a method in which damage in a cracked structure was identified using Genetic Algorithm (GA). In a cracked cantilever beam, the natural frequencies were obtained for the first four modes. The identification of the crack location and depth in the cantilever beam was formulated as an optimization problem. GA is used to minimize the error in the measured and calculated natural frequencies of the structure. Varghese and Shankar [17] identified crack parameters in a sub-structure of a beam using combined power flow and acceleration response matching using Particle Swarm Optimization (PSO) algorithm. Suh et al. [14] have presented a method to identify the location and depth of a crack on a structure by using hybrid Neuro-Genetic technique. Feed-forward multi-layer neural networks trained by back-propagation are used to learn the input (the location and depth of a crack)–output (the structural Eigen frequencies) relation of the structural system. With this trained neural network, genetic algorithm is used to identify the crack location and depth minimizing the difference from the measured frequencies. Chou and Ghaboussi [1] defined the structural damage detection problem as an optimization problem, which was solved using Genetic Algorithm (GA). Static measurements of displacements at few degrees of freedom (DOFs) are used to identify the changes of the characteristic properties of structural members such as the Young's modulus and cross-sectional area, which are indicated by the difference of measured and computed responses. In order to avoid structural analyses in fitness evaluation, the displacements at unmeasured DOFs are also determined by GA. The proposed method is able to detect the approximate location of the damage. Juang and Pappa [6] presented a deterministic SI algorithm based on the state space model of second order systems using OKID/ERA by which all the structural properties such as mass, damping coefficient, stiffness can be identified. Based on this theory, Jacob and Nandakumar [4] successfully identified the crack parameters in a cantilever with single crack using OKID/ER Algorithm.

Crack or local defect in a structural member introduces local flexibility that affects the dynamic response of the structure. To the best of Authors' knowledge, it is a novel attempt to identify structural crack parameters of a cantilever using OKID/ERA. A finite element model was developed for cantilever beam with a crack at any location based on elasto plastic fracture mechanics. In general, damage detection methods have a limitation in that there is only one mid-crack in an element. In this paper, a finite element model for cracked beam element with crack at any location has been developed.

2 METHODOLOGY

The parameter identification methodology consists of two phases. First, the system parameter matrices such as mass, damping, stiffness matrices are identified using OKID/ERA and later the structural/ crack parameters are identified using Newton-Raphson method.

2.1 OKID/ER algorithm

The equations of motion of a structural system is given in the form of second order differential equation [6]

$$[M]\{\ddot{q}(t)\} + [L]\{\dot{q}(t)\} + [K]\{q(t)\} = [B]u(t) \quad (1)$$

where $[M]$, $[L]$ and $[K]$ are the mass, damping and stiffness matrices of the structure respectively. $[B]$ is the input matrix ($N \times r$) containing r external excitations acting on the structure, $u(t)$ is input excitation vector ($r \times 1$), $q(t)$ is the displacement vector ($N \times 1$). Eq.(1) is written in state space model [6], taking the state vector as:

$$\begin{aligned} x(t) &= [q(t) \quad \dot{q}(t)]^T \\ \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \quad (2)$$

where A , B , C and D are continuous time system matrices, $y(t)$ is output vector. The OKID algorithm is used to obtain the Markov parameters from the input and output responses, state space model and structural parameters are identified from the Markov parameters using ERA. The detailed OKID/ERA algorithm is explained by Juang and Pappa [6].

2.2 Newton-Raphson method

The Newton-Raphson method is used to identify the structural parameters of continuous mass model from the elements of the identified structural parameter matrices. In case of a cracked beam element, normalized crack depth (ξ) and crack location (λ) of cracked element are the unknowns and all other parameters such as the Young's modulus (E), area moment of inertia (J) are assumed to be known. Using the theory of fracture mechanics, the matrices are derived using Finite Element procedure and the elements of those matrices are equated with the corresponding elements of identified matrices using OKID/ERA. As many numbers of unknowns are to be identified, the same number of equations are formed and they are written as follows.

$$\begin{aligned} f_1(x_1, x_2, x_3, \dots, x_n) &= 0 \\ f_2(x_1, x_2, x_3, \dots, x_n) &= 0 \\ f_3(x_1, x_2, x_3, \dots, x_n) &= 0 \\ &\vdots \\ &\vdots \\ &\vdots \\ f_n(x_1, x_2, x_3, \dots, x_n) &= 0 \end{aligned} \quad (3)$$

where $x_1, x_2, x_3, \dots, x_n$ are the unknown parameters. An initial guess for each unknown is given, let $\{X_0\}$ be the vector contains all initial values of the unknowns, then $\{X_0\} = [x_1^0, x_2^0, x_3^0, \dots, x_n^0]^T$. The next better-unknown value is calculated by the Newton-Raphson formula. The unknown parameters are obtained by successive iterations by using the relation,

$$\{X_{i+1}\} = \{X_i\} - J^{-1}(\{X_i\})f(\{X_i\}) \quad (4)$$

where J is Jacobian matrix, which is given by

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \dots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \dots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \dots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \quad (5)$$

3 THE CRACKED BEAM ELEMENT MODEL

An Euler-Bernoulli beam element of length (l_e) with an open crack of depth (a) at a distance (l_c) from the left node is considered. A cracked beam finite element model developed by Viola et.al [18] is considered in the present study to model the crack damage. Three different segments can be distinguished in the element as shown in Fig.1. The left and right segments are intact beam, the crack is modelled by a massless torsional spring. The torsional rigidity of the spring is $K = \frac{EH^3\beta}{6f(\xi)}$ and $\lambda = \frac{l_c}{l_e}$; $0 \leq \lambda \leq 1$, where λ is normalized crack location, the normalized crack depth $\xi = \frac{a}{H}$, E is the Young's Modulus and $\beta = \frac{B}{H}$, B is the breadth and H is the height of rectangular beam cross-section. The function $f(\xi)$ depending on the dimensionless crack ratio $\xi = \frac{a}{H}$ can be expressed as Viola et.al [18],

$$f(\xi) = \xi^2 \begin{bmatrix} 12 - 19.5\xi + 70.1\xi^2 - 97.6\xi^3 + 142\xi^4 - 138\xi^5 + 128\xi^6 \\ -132\xi^7 + 379\xi^8 - 417\xi^9 + 131\xi^{10} + 313\xi^{12} - 357\xi^{13} + 102\xi^{14} \end{bmatrix}, \quad 0 \leq \xi \leq 0.5 \quad (6)$$

and

$$f(\xi) = \frac{1.32}{(1-\xi)^2} - 1.78, \quad 0.5 \leq \xi \leq 1. \quad (7)$$

The derivation of element stiffness matrix for a cracked beam element is explained in Appendix in detail. The mass matrix for the cracked element is considered as same as that of non-cracked beam element because the mass matrix derived from the cracked beam element do not affect the natural frequencies and mode shapes much [8]. The element mass matrix $[M^e]$ of the beam element is as follows:

$$[M^e] = \frac{\rho A l_e}{420} \begin{bmatrix} 156 & 22l_e & 54 & -13l_e \\ 22l_e & 4l_e^2 & 13l_e & -3l_e^2 \\ 54 & 13l_e & 156 & -22l_e \\ -13l_e & -3l_e^2 & -22l_e & 4l_e^2 \end{bmatrix} \quad (8)$$

where ρ is the mass density of the material and A is the cross section area of the beam element. The corresponding element stiffness matrix $[K^e]$ and mass matrix $[M^e]$ of each structural member respectively, are assembled in the global stiffness matrix $[K]$ and mass matrix $[M]$. The damping property of the beam structure is modelled with Rayleigh's proportional damping. Damping is expressed as the linear combination of the mass and stiffness matrices,

$$[C] = \alpha[M] + \beta[K] \quad (9)$$

where α and β are constants.

$$\begin{Bmatrix} \alpha \\ \beta \end{Bmatrix} = \begin{bmatrix} \frac{1}{2\omega_1} & \frac{\omega_1}{2} \\ \frac{1}{2\omega_2} & \frac{\omega_2}{2} \end{bmatrix}^{-1} \begin{Bmatrix} \zeta_1 \\ \zeta_2 \end{Bmatrix} \quad (10)$$

where ω_1 and ω_2 be the first and second natural frequency and ζ_1 and ζ_2 the corresponding damping ratio.

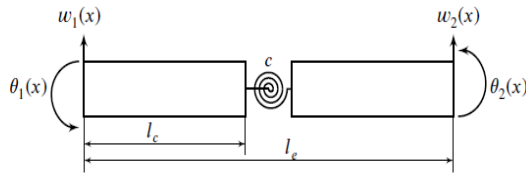


Fig.1
Cracked beam element model.

4 PARAMETER IDENTIFICATION AND CRACK DETECTION

The structure is discretized into many finite elements and is excited by a known harmonic force at a node. The acceleration responses are measured at each node on the structure. The system matrices such as mass, damping and stiffness of the structure are identified from the measured acceleration response by using OKID/ER algorithm as explained in Section 2.1. In the parameter identification problem, the stiffness parameters of each element are the unknown parameter and mass and damping parameters are known. In the crack damage detection problem, EI of each element is known priori; the unknown parameters are the normalized crack depth (ξ) and its location (λ) in the beam. Each element is assumed as cracked one and the parameters are extracted from the identified stiffness matrix by using the Newton Raphson method. The corresponding elements of extracted global stiffness matrix and the assembled global stiffness matrix are equated. Number of equations required is same as the number of unknown parameters in the structure. Since the crack parameters are ranging from zero to one, initial value is given within the range. Zero value of the identified normalized crack depth shows the undamaged state of an element. The unknown parameters are extracted by solving the equations formed by using Newton Raphson method as explained in Section. 2.2.

4.1 Numerical example 1: 12 DOF lumped mass model

A 12 DOF lumped mass system which was used by Tee et.al [15] is considered to identify structural parameters as shown in Fig.2 . The stiffness of the springs are $k_1 - k_6 = 1000MN / m$ and $k_7 - k_{12} = 800MN / m$. The masses are $m_1 = 600kg$, $m_2 - m_5 = 400kg$ and $m_6 - m_{12} = 300kg$. The first natural frequency of the structure is 34.89 Hz. The structure is excited with a harmonic excitation of $F_7 = 10\sin(2\pi \times 38t)N$ at the seventh DOF. Rayleigh's damping model is adopted with a modal damping ratio of 1% for the first two modes. Acceleration responses are measured at all DOF of the model and are used for parameter identification. They are simulated by Newmark constant acceleration method for a time period of 1s with a time step of 0.0005s. The Markov parameters are identified from the measured acceleration responses and input force response. From the identified Markov parameters Hankel matrix is formed, from which the discrete state space model of the structural system is determined using singular value decomposition of Hankel matrix. The discrete state space model is converted into continuous state space model. From the determined state space model, the mass, stiffness and damping matrices are extracted by using Eigen Realization algorithm. From these matrices, the mass, stiffness for each element and damping ratio are directly obtained. The identified stiffness parameters of each level and the percentage of error in each identified stiffness is tabulated in Table1.

The stiffness parameters are identified with a mean absolute error of 0.019% which is very accurate than the stiffness identified by any other method. This problem was solved by Complete Structural Identification (CSI) method proposed by Koh et. al [7], the stiffness parameters were identified with a mean absolute error of 10.75%. The same problem was also solved by Successive identification method using transfer matrix [11] with a mean absolute error of 0.91%. The damping ratio is identified as 1.14% and the absolute error in identification of damping ratio is 14.71%. Hence, it is clearly proved that the OKID/ERA method is very accurate in parameter identification.

Table 1
Exact and Identified Structural parameters.

Element	Exact stiffness (MN/m)	Identified Stiffness (MN/m)	Error (%)
1	1000	1000.017	0.0017
2	1000	1000.17	0.017
3	1000	1000.21	0.021
4	1000	1000.21	0.021
5	1000	1000.21	0.021
6	1000	1000.23	0.023
7	800	800.19	0.026
8	800	800.18	0.022
9	800	800.18	0.022
10	800	800.18	0.022
11	800	800.18	0.022
12	800	800.13	0.017
% of MAE			0.019

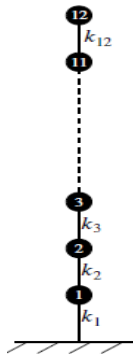


Fig.2
Twelve DOF shear model.

4.2 Numerical example2: parameter identification of cantilever

The OKID/ER algorithm is now applied to an intact cantilever, which has fixed boundary at its left end, to identify the stiffness parameter EI of each element. The uniform slender cantilever of cross section $20 \times 8 \text{ mm}$ and length of 680 mm is considered. The material of the cantilever beam is steel with the Young's modulus (E) of 206 GPa and its mass density (ρ) of 7850 kg/m^3 . The entire beam is divided into five elements each of length 136 mm . The Rayleigh's damping is assumed with modal damping ratio of 1%. The first and second natural frequencies of the cantilever are 14.46 Hz and 90.69 Hz . The free end of the cantilever is excited with a harmonic excitation of $F(t) = 10 \sin(2\pi \times 10t) \text{ N}$. All the measured responses are numerically simulated by the Newmark's constant acceleration method using MATLAB, for a time period of 1 s with time step of 0.0005 s . The mass, stiffness and damping parameters are identified from the acceleration responses using OKID/ER Algorithm and Newton Raphson method as explained above. The identified and exact value of stiffness parameter EI for the intact beam is given in Table 2.

The EI parameters are identified with a maximum percentage error of 1.7% and the mean absolute error 0.83%. The same problem was also solved using the CSI method proposed by Koh et.al [7], EI parameters are identified with complete measurement of acceleration at all DOF with a mean absolute error of 1.57%. It shows that OKID/ERA is better than the other parameter identification methods available in the literature.

Table 2
Exact and Identified EI .

Element	Exact $EI (Nm^2)$	Identified $EI (Nm^2)$	Error (%)
1	175.8	174.51	-0.7
2	175.8	176.88	0.6
3	175.8	177.63	1.05
4	175.8	178.78	1.7
5	175.8	175.99	0.1
		% of MAE	0.83

4.3 Numerical example 3: crack damage detection in a cantilever

4.3.1 Single crack

The OKID/ER algorithm is now applied on a cantilever with single crack to identify the crack parameters such as crack depth and its location. A cantilever of length 0.68 m with rectangular cross section of 8 mm thickness (H) and 20 mm width is considered. The left end of the cantilever has fixed boundary. The Young's modulus of the beam material (E) is 210 GPa and its density is 7850 kg/m^3 . A crack with various depth (a) of 0.5 mm , 2.6 mm and 3.5 mm , each located at 359.3 mm and 390.8 mm from the fixed end of the cantilever is assumed as shown in Fig. 3. The

normalized crack depth for the cracks are $\xi = \frac{a}{H} = 0.0625$, 0.3251 and 0.4375 and crack locations in global

structure are $\lambda_1 = \frac{l_1}{L} = 0.5283$ and $\lambda_2 = 0.5747$. The cantilever is divided into five elements of equal length. The

crack lies in the element 3 in the finite element model. The element wise normalized crack location for the location mentioned above is $\lambda_{e1} = 0.642$ and $\lambda_{e2} = 0.874$. The first natural frequency of the cracked beam for different crack parameters is tabulated in Table.3. The cantilever is excited with a harmonic excitation of $F_0 = 10\sin(2\pi 10t)\text{ N}$ at its free end node. The acceleration and angular accelerations are assumed to be measured at each node by fixing two accelerometers very closed to each other at each node. The acceleration response is the mean of the response measured by two accelerometers fixed at a node. Angular acceleration at each node is obtained by dividing the difference in the acceleration by the distance between the centers of the accelerometers fixed at that node. All the acceleration responses are numerically simulated by Newmark's constant acceleration method with a time step of 0.001 s for a period of 2 s . From the measured acceleration responses, by using OKID/ER Algorithm mass, stiffness and damping matrices are identified for the cracked cantilever. From the identified stiffness matrix, the crack parameters for each element is extracted using Newton-Raphson method. For example, to identify the crack parameters in i th element, the elements correspond to the i th element (sub matrix) from the identified global stiffness matrix is considered as follows.

$$[K^i] = \begin{bmatrix} k_{11}^* & k_{12}^* & k_{13}^* & k_{14}^* \\ k_{21}^* & k_{22}^* & k_{23}^* & k_{24}^* \\ k_{31}^* & k_{32}^* & k_{33}^* & k_{34}^* \\ k_{41}^* & k_{42}^* & k_{43}^* & k_{44}^* \end{bmatrix}$$

In the above sub-matrix, the principal diagonal elements have connection with preceding and succeeding elements, they are the function of crack parameters of i^{th} and its preceding or succeeding elements which is given below.

$$\begin{aligned} k_{11}^* &= f(K_{i-1}, \lambda_{i-1}, K_i, \lambda_i) = k_{33}^{\{i-1\}} + k_{11}^i \\ k_{22}^* &= f(K_{i-1}, \lambda_{i-1}, K_i, \lambda_i) = k_{44}^{\{i-1\}} + k_{22}^i \\ k_{33}^* &= f(K_i, \lambda_i, K_{i+1}, \lambda_{i+1}) = k_{33}^i + k_{11}^{i+1} \\ k_{44}^* &= f(K_i, \lambda_i, K_{i+1}, \lambda_{i+1}) = k_{44}^i + k_{22}^{i+1} \end{aligned} \quad (11)$$

where $k_{i,j}, i, j = 1, 2, 3, 4$ are defined by Eq.(12). Further, the elements k_{12}^* and k_{24}^* are also has the connectivity with other elements. Hence the elements $k_{13}^*, k_{14}^*, k_{23}^*$ and k_{24}^* do not have any coupling with other neighbor elements, any two of them are selected to identify the two crack parameters (k_i and λ_i) of the i^{th} element.

$$\begin{aligned}
 k_{\{13\}}^* &= f(K_i, \lambda_i) = k_{\{13\}}^i \\
 k_{\{14\}}^* &= f(K_i, \lambda_i) = k_{\{14\}}^i \\
 k_{\{23\}}^* &= f(K_i, \lambda_i) = k_{\{23\}}^i \\
 k_{\{24\}}^* &= f(K_i, \lambda_i) = k_{\{24\}}^i
 \end{aligned}
 \tag{12}$$

An initial value for both unknown parameters is set as zero and above equations are solved using Newton-Raphson method as explained in Section 2.2. From identified K the normalized crack depth ξ is obtained by solving Eq. (6). The identified results for a typical crack depth of $\xi = 0.0625$ and its location of $\lambda_e = 0.642$ is shown in Fig.4. The identified crack parameters in the element three for different crack parameters are tabulated in Table 3. The crack with least depth of $\xi = 0.0625$ and $\lambda_e = 0.5283$ is identified with an error of 15.84% in depth and 1.16% in its global location. Also the crack with large depth of $\xi = 0.4375$ and $\lambda = 0.5283$ is identified with an error of 2.3% in depth and -0.18% in its global location. From the result, it is understood that the crack with small depth is identified with less accuracy and the accuracy improves when the crack depth increases.

Table 3
Identified crack parameters with single crack cantilever.

Actual Parameters		Natural frequency		Identified Parameters		
Location (λ)	Depth (ξ)	$f_n(Hz)$	Location (λ)	Error (%)	Depth (ξ)	Error (%)
0.5283	0.4375	14.45	0.5274	-0.18	0.4477	2.3
	0.3251	14.44	0.5298	0.29	0.3429	5.2
	0.0625	14.42	0.5344	1.16	0.0724	15.84
0.5747	0.4375	14.45	0.5759	0.21	0.4301	-1.7
	0.3251	14.44	0.5773	0.44	0.3315	1.9
	0.0625	14.43	0.5825	1.35	0.0735	17.6

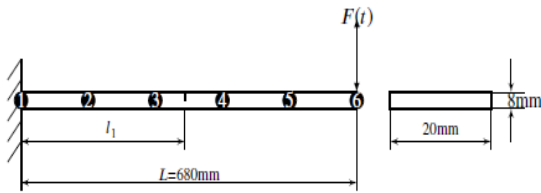


Fig.3
Finite Element model of cantilever with single crack.

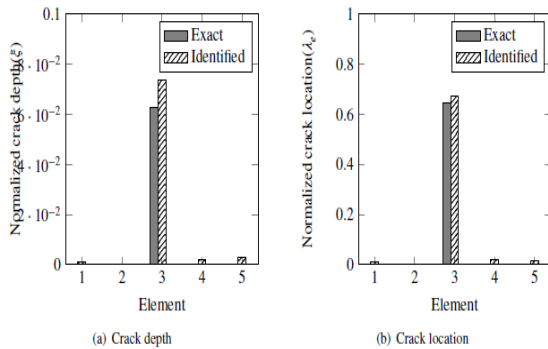


Fig.4
Identified crack parameters in each element of cantilever for $\xi = 0.0625$ and $\lambda_e = 0.642$.

4.3.2 Two crack

Two cracks of various depth at two locations at $l_1=375 \text{ mm}$ and $l_2=516.8 \text{ mm}$ from the fixed end of the cantilever are considered as shown in Fig. 5. The different crack depth (a) 0.5 mm , 2.5 mm and 4 mm are considered at each crack location. The dimensionless normalized crack depths (ξ) corresponds to these crack depths are 0.0625 , 0.3125 and 0.5 . The dimensionless normalized crack location from the fixed end of the cantilever is $\lambda_1 = l_1/L = 0.55$ and is $\lambda_2 = l_2/L = 0.76$. The two cracks lie on third and fourth elements respectively. The element wise crack locations for each crack are is ($\lambda_{e1} = 0.7574$) and ($\lambda_{e2} = 0.8$)

The cantilever is excited with a harmonic excitation of $F_0(t) = 10\sin(2\pi 50t)N$ at its free end node. Acceleration response at all nodes are simulated numerically by Newmark's constant acceleration method. The crack parameters are identified for each element using the OKID/ER algorithm and Newton-Raphson method as explained in single crack case. The first natural frequency of the cantilever is shown in Table.4. The identified results of a typical crack in each location is shown in Fig.6.

The crack with the smallest depth $\xi = 0.0625$ and $\lambda = 0.55$ is identified with an error of 14.24% in depth and 1.93% . The crack with the largest depth $\xi = 0.5$ and $\lambda = 0.76$ is identified with minimum error of -0.82% in depth and 0.3% in location. Hence, it is understood that the crack with small depth is identified with less accuracy. It is due to the small change in its dynamic characteristics due to the presence of the crack in the structure. The error in identified depth decreases gradually when the crack depth increases.

Table 4
Identified crack parameters with two-crack cantilever.

Cracked Element	Actual Parameters		Natural frequency	Identified Parameters			
	Location (λ)	Depth (ξ)	$f_n(\text{Hz})$	Location (λ)	Error (%)	Depth (ξ)	Error (%)
3 rd	0.55	0.5	14.43	0.5534	0.35	0.4937	-1.26
		0.3125	14.44	0.554	0.46	0.32	2.4
		0.0625	14.45	0.5621	1.93	0.0714	14.24
4 th	0.76	0.5		0.7623	0.3	0.4959	-0.82
		0.3125		0.7703	1.34	0.3181	1.79
		0.0625		0.7794	2.55	0.0693	10.88

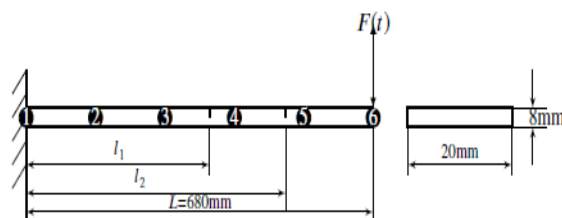


Fig.5
Finite Element model of cantilever with two cracks.

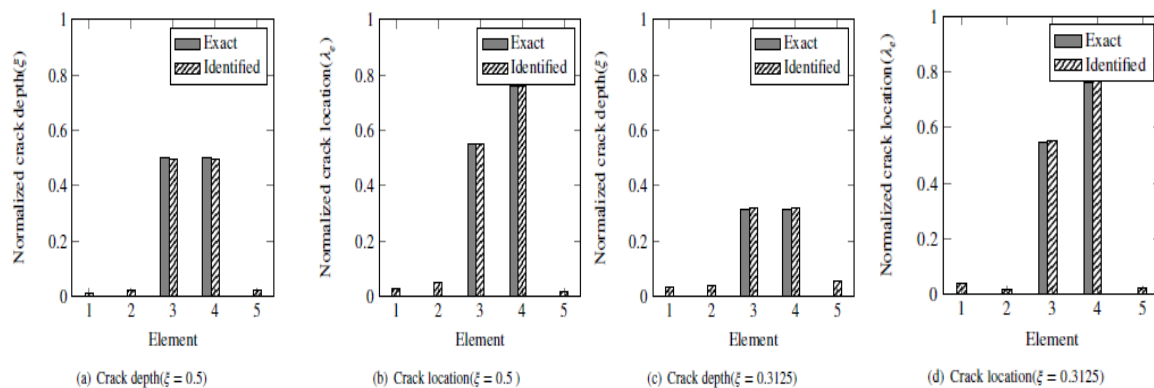


Fig.6
Identified crack parameters in each element of cantilever.

5 CONCLUSIONS

Structural parameters and crack damage parameters in a beam structure is detected using OKID/ER Algorithm and Newton Raphson method. The crack element was modeled using the theory of Linear Elastic Fracture Mechanics (LEFM) and finite element method. Acceleration responses at all nodes are numerically simulated. Structural parameters such as mass, stiffness and damping are extracted from the measured acceleration and force responses using OKID/ERA and Newton Raphson method. The structural parameters were identified for an intact cantilever using OKID/ERA. Later this algorithm was applied for the structure with crack. Crack damage parameters are identified for a cantilever with single crack. Normalized crack depth and its location are the parameters to be identified. Further, this algorithm was applied on a cantilever with two cracks with variable depths. The maximum percentage of error in identified smallest crack depth is 17.6% and the minimum percentage of error in identified largest crack depth is -0.82%. Viola et.al [18] identified the crack parameters in a cantilever using frequency domain damage detection method. It is reported that a crack of depth $\xi = 0.5$ and $\lambda = 0.76$ is identified with an absolute percentage error of 2.5% in depth and 0.53% in location. Using ER algorithm, the same crack depth ($\xi = 0.5$) is identified with a percentage error of 0.82% in depth and 0.3% in location. Hence, it is proved that this ER algorithm is better than the other method available in the literature. This algorithm is capable of identifying single crack and multiple cracks in the structure at any location with good accuracy.

APPENDIX A

Stiffness matrix for a cracked beam finite element. The displacement for the left and right segments of the element is given by

$$\begin{aligned}
 w_1(x) &= a_1 + a_2x + a_3x^2 + a_4x^3; 0 \leq x \leq l_c \\
 \Phi_1(x) &= \frac{dw_1(x)}{dx} = a_2 + 2a_3x + 3a_4x^2; 0 \leq x \leq l_c \\
 w_2(x) &= a_5 + a_6x + a_7x^2 + a_8x^3; l_c \leq x \leq l_e \\
 \Phi_2(x) &= \frac{dw_2(x)}{dx} = a_6 + 2a_7x + 3a_8x^2; l_c \leq x \leq l_e
 \end{aligned} \tag{A.1}$$

The constants $a_1 - a_8$ are evaluated using the following conditions.

$$\begin{aligned}
 w_1(0) &= W_1; \Phi_1(0) = \phi_1 \\
 w_2(l_e) &= W_2; \Phi_2(l_e) = \phi_2 \\
 w_1(l_1) &= w_2(l_1); [\Phi_2(l_1) - \Phi_1(l_1)]K = EIw_2''(l_1) \\
 w_1''(l_1) &= w_2''(l_1); w_1'''(l_1) = w_2'''(l_1)
 \end{aligned} \tag{A.2}$$

where $W_1; W_2; \Phi_1$ and ϕ_2 are the displacement and rotation at the nodes 1 and 2 respectively. The shape function matrix $[N(x)]$ relates the nodal DOF with the field variables as follows:

$$\begin{Bmatrix} w_1(x) \\ \Phi_1(x) \\ w_2(x) \\ \Phi_2(x) \end{Bmatrix} = \begin{bmatrix} N_{11} & N_{12} & N_{13} & N_{14} \\ N_{21} & N_{22} & N_{23} & N_{24} \\ N_{31} & N_{32} & N_{33} & N_{34} \\ N_{41} & N_{42} & N_{43} & N_{44} \end{bmatrix} \begin{Bmatrix} W_1 \\ \phi_1 \\ W_2 \\ \phi_2 \end{Bmatrix} \tag{A.3}$$

The stiffness matrix for the cracked beam element is derived as follows.

$$[K^e] = EI \left(\int_0^{l_c} B_1^T B_1 dx + \int_{l_c}^l B_2^T B_2 dx \right) \quad (A.4)$$

where $B_1 = \frac{d^2 [N_1(x)]}{dx^2}$ and $B_2 = \frac{d^2 [N_3(x)]}{dx^2}$, Eq.(A.4) gives the stiffness matrix for a beam element with crack at any location on the element. The elements of the stiffness matrix are as follows.

$$k_{11} = \frac{12(l_e^4 + 2l_e^3K + 12l_e^2K^2\lambda^2 - 12l_e^2K^2\lambda + 4l_e^2K^2)}{l_e(l_e^3 + 12Kl_e^2\lambda^2 - 12Kl_e^2\lambda + 4Kl_e^2)^2}$$

$$k_{12} = \frac{6l_e^5 - 144l_e^3K^2\lambda^2 + 144l_e^3K^2\lambda^3 + l_e\lambda(24l_e^3K + 48l_e^2K^2)}{l_e(l_e^3 + 12Kl_e^2\lambda^2 - 12Kl_e^2\lambda + 4Kl_e^2)^2}$$

$$k_{13} = -\frac{12(l_e^4 + 2l_e^3K + 12l_e^2K^2\lambda^2 - 12l_e^2K^2\lambda + 4l_e^2K^2)}{l_e(l_e^3 + 12Kl_e^2\lambda^2 - 12Kl_e^2\lambda + 4Kl_e^2)^2}$$

$$k_{14} = \frac{24l_e^4K + 6l_e^5 + l_e^3(48K^2 - 24LK\lambda) - 192l_e^3K^2\lambda + 288l_e^3K^2\lambda^2 - 144l_e^3K^2\lambda^3}{l_e(l_e^3 + 12Kl_e^2\lambda^2 - 12Kl_e^2\lambda + 4Kl_e^2)^2}$$

$$k_{21} = \frac{6l_e^5 - 144l_e^3K^2\lambda^2 + 144l_e^3K^2\lambda^3 + l_e\lambda(24l_e^3K + 48l_e^2K^2)}{l_e(l_e^3 + 12Kl_e^2\lambda^2 - 12Kl_e^2\lambda + 4Kl_e^2)^2}$$

$$k_{22} = \frac{4(l_e^6 + 6l_e^5K\lambda^2 + 36l_e^4K^2\lambda^4 - 36l_e^4K^2\lambda^3 + 12l_e^4K^2\lambda^2)}{l_e(l_e^3 + 12Kl_e^2\lambda^2 - 12Kl_e^2\lambda + 4Kl_e^2)^2}$$

$$k_{23} = -\frac{6l_e^5 - 144l_e^3K^2\lambda^2 + 144l_e^3K^2\lambda^3 + l_e\lambda(24l_e^3K + 48l_e^2K^2)}{l_e(l_e^3 + 12Kl_e^2\lambda^2 - 12Kl_e^2\lambda + 4Kl_e^2)^2}$$

$$k_{24} = -\frac{l_e^3(24l_e^2K\lambda^2 - 48l_eK^2\lambda) - 2l_e^6 + 192l_e^4K^2\lambda^2 - 288l_e^4K^2\lambda^3 + 144l_e^4K^2\lambda^4 - 24l_e^5K\lambda}{l_e(l_e^3 + 12Kl_e^2\lambda^2 - 12Kl_e^2\lambda + 4Kl_e^2)^2}$$

$$k_{31} = -\frac{12(l_e^4 + 2l_e^3K + 12l_e^2K^2\lambda^2 - 12l_e^2K^2\lambda + 4l_e^2K^2)}{l_e(l_e^3 + 12Kl_e^2\lambda^2 - 12Kl_e^2\lambda + 4Kl_e^2)^2}$$

$$k_{32} = -\frac{6l_e^5 - 144l_e^3K^2\lambda^2 + 144l_e^3K^2\lambda^3 + l_e\lambda(24l_e^3K + 48l_e^2K^2)}{l_e(l_e^3 + 12Kl_e^2\lambda^2 - 12Kl_e^2\lambda + 4Kl_e^2)^2}$$

$$k_{33} = \frac{12(l_e^4 + 2l_e^3K + 12l_e^2K^2\lambda^2 - 12l_e^2K^2\lambda + 4l_e^2K^2)}{l_e(l_e^3 + 12Kl_e^2\lambda^2 - 12Kl_e^2\lambda + 4Kl_e^2)^2}$$

$$k_{34} = -\frac{24l_e^4K + 6l_e^5 + l_e^3(48K^2 - 24LK\lambda) - 192l_e^3K^2\lambda + 288l_e^3K^2\lambda^2 - 144l_e^3K^2\lambda^3}{l_e(l_e^3 + 12Kl_e^2\lambda^2 - 12Kl_e^2\lambda + 4Kl_e^2)^2}$$

$$k_{41} = \frac{24l_e^4K + 6l_e^5 + l_e^3(48K^2 - 24LK\lambda) - 192l_e^3K^2\lambda + 288l_e^3K^2\lambda^2 - 144l_e^3K^2\lambda^3}{l_e(l_e^3 + 12Kl_e^2\lambda^2 - 12Kl_e^2\lambda + 4Kl_e^2)^2}$$

$$k_{42} = -\frac{l_e^3(24l_e^2K\lambda^2 - 48l_eK^2\lambda) - 2l_e^6 + 192l_e^4K^2\lambda^2 - 288l_e^4K^2\lambda^3 + 144l_e^4K^2\lambda^4 - 24l_e^5K\lambda}{l_e(l_e^3 + 12Kl_e^2\lambda^2 - 12Kl_e^2\lambda + 4Kl_e^2)^2}$$

$$k_{43} = -\frac{24l_e^4K + 6l_e^5 + l_e^3(48K^2 - 24LK\lambda) - 192l_e^3K^2\lambda + 288l_e^3K^2\lambda^2 - 144l_e^3K^2\lambda^3}{l_e(l_e^3 + 12Kl_e^2\lambda^2 - 12Kl_e^2\lambda + 4Kl_e^2)^2}$$

$$k_{44} = \frac{l_e^3 (24l_e^2 K \lambda^2 - 240l_e K^2 \lambda) + 24l_e^5 K + 4l_e^6 + l_e^4 (48K^2 - 48l_e \lambda K) + 480l_e^4 K^2 \lambda^2 - 432l_e^4 K^2 \lambda^3 + 144l_e^4 K^2 \lambda^4}{l_e (l_e^3 + 12Kl_e^2 \lambda^2 - 12Kl_e^2 \lambda + 4Kl_e^2)^2}$$

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