Journal of Solid Mechanics Vol. 12, No. 4 (2020) pp. 744-751
DOI: 10.22034/jsm.2020.574250.1330

Determination of Interaction Force Between Single Core Cable Elements under Deformation

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Received 1 July 2020; accepted 2 September 2020

ABSTRACT

The purpose of this paper is to investigate friction forces in the design of a single-core flexible cable in the deformation zone and the practical applicability of the results obtained for these purposes. The power interaction of the design of a single-core flexible cable during its deformation is considered. For the first time, formulas were obtained for determining the interaction forces between the constituent parts of a single-core cable as a composite multilayer beam. A calculation technique has been developed and shear force values have been determined for some types of single-core flexible cables. The nature of the change in these forces along the length of the cable is investigated. At the beginning of the cable deformation zone, the force value can fluctuate within its constant value. In the remaining part, the shear forces along the section are constant and only at the end of the deformation zone is zero. Practically, the formulas work for the tasks. The resulting expressions for shearing forces allow one to evaluate the tribological interaction of the constituent parts of each cable element and take into account their influence when creating multicore cables. The results of the research can be used to improve the reliability of the design of flexible cables at the design stage.

Keywords: Tribological interaction; Flexible cable; Displacements; Wear; Shifting efforts; Current-carrying conductor.

1 INTRODUCTION

FLEXIBLE cables, in contrast to fixed installation cables, have a short service life due to severe operating conditions [1]. Under external mechanical loads during the operation, destruction occurs both on the outer sheath of the cable and on its individual elements due to the tribological interaction between them [2]. It is known that copper and polymeric materials are the main materials for the production of flexible cables. Studies [3, 4] on polymer tribology are promising due to an increase in their use in the cable industry. This raises the need to study the tribological interaction between the structure elements of both single core and multicore cables. The interaction

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forces between the contacting surfaces of the elements cause the wear of copper wires and polymer insulation. In some cases, especially in multicore cables, wear debris accelerates the wear of current-carrying conductor insulation. The work [5] features the characteristics of a tribological interaction of cable elements in contact. Element shift causes a tearing-up of surfaces at different levels. It is the tangential stresses on the contact surface that determine the strength of the constituent elements of the cable. For example, pairs “insulation of conductors – copper wires”, or “sheath insulation – insulation of elements”, etc. [6]. The interaction of the pair “copper wire – insulation of current-carrying conductor (CCC)” may lead to the insulation wear out and also to short-circuit of copper wires of the adjacent conductor strands or external metal wires. All this may result in a cessation of the energy transfer to consumers of electric energy from mobile mechanisms and machines. It is unacceptable to operate mine cables under conditions of gas accumulation in mines as any spark may cause a gas explosion during the operation. Broken cables can lead to catastrophic consequences both for people’s lives and for operational objects. On the other hand, failures related to wear were a serious problem for any production, and their prevention is seen as a basis of saving in operation. The possibility to control friction allows to control the process of tribological interaction of contacting surfaces and becomes an important direction in the creation of reliable products [7–10]. To increase the safety of flexible cable operation without sacrificing its tribological properties is a relevant problem.

The study [6] demonstrates experimental research results on determination of the force between the elements of a multicore cable under deformation. However, there has been no theoretical approach to determine the force for a single core flexible cable in real working conditions. The purpose of the present research is to determine the friction forces for a deformable single core flexible cable section from the point of view of practical applicability. In the present work, the forces between internal surfaces of the insulation and the copper wire are determined.

2 DETERMINATION OF THE INTERACTION FORCES BETWEEN THE CABLE ELEMENTS

The following technical conditions are normally set to operate a cable: a nominal voltage, the minimum cable-bending radius R, expressed in terms of cable diameters; a temperature range and the cable durability. However, the warranty period of the cable operation is much lower than its life [1]. Therefore, cable product manufacturers cannot guarantee the reliability of cables under unforeseen situations during the cable operation since it depends on many factors: the nature of the load interaction, the physical and mechanical characteristics of the materials used some technological factors in the production of cables.

Most researchers do not consider the electrical strength of cables and the effects of temperature in their papers. To simplify the problem, the cable is regarded as a composite beam, and its copper wire as a homogeneous solid rod. Winding the cable onto the drum, forming a loop of a certain radius, etc. are the most common cases in research. The system can be represented as a multilayer plate under the conditions of planar deformation. To transit to an axisymmetric problem, the authors have applied transformations used in problems for cables as regarded in [11]. When the cable is deformed as a monolithic beam, the bending moment can be determined by the known formulae: through the bend radius and the flexural stiffness of the cable [12]. The single core cable represents a symmetrical three-layer composite beam relative to its axis. We assume that the transverse connections of an equivalent beam are rigid. Fig. 1 shows the cable deformation scheme and its geometric dimensions. Let us introduce the following notations: r_1 is the radius of insulation, r_2 is the radius of the conductor (copper wires), *and E1* and E_2 are the elasticity moduli of the materials of each element.

In the calculations, we assume that the bending stiffness of rod 1 is equal to the rigidity of the semi-cylinder of the insulation; the bending stiffness of rod 2 is the rigidity of the current-conducting copper core. The longitudinal connection of the conductor and insulation joint is indicated by reference numeral 3.

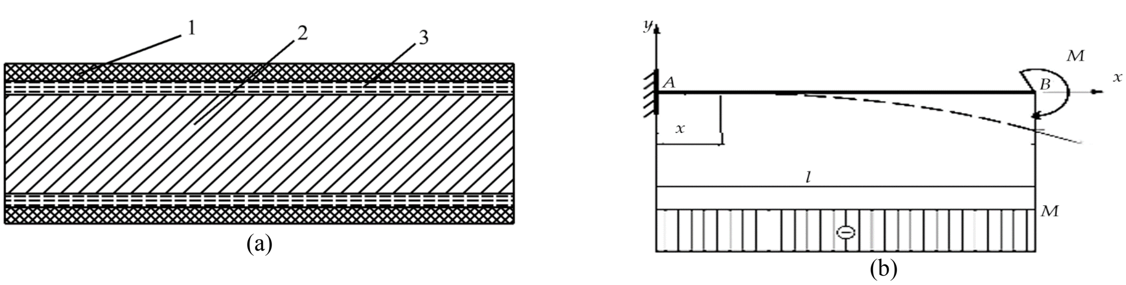

Fig.1
Cable deformation scheme and its geometric dimensions: (a) cable as a three-layer composite beam; (b) design scheme.

Let us write the equilibrium equations for a three-layer beam according to the equilibrium conditions for structural mechanics [13]

$$\begin{aligned} \frac{T_1''}{\xi_1} &= \Delta_{11}T_1 + \Delta_{12}T_2 + \Delta_{10}; \\ \frac{T_2''}{\xi_2} &= \Delta_{21}T_1 + \Delta_{22}T_2 + \Delta_{20}, \end{aligned} \tag{1}$$

where Δ_{ij} are the coefficients; ξ_i is the stiffness coefficient of the seam, which is determined experimentally for the problem being solved; T_i is the total shearing force in the *i-th* seam, accumulated along the length of the rod from its origin to the section under consideration, $T_i = \int_0^x \tau_i \, dx$.

Here, τ_i is the shearing force, per unit length of the *i-th* seam; x is the coordinate of the section under consideration.

The system (1) can be reduced to two independent equations:

$$\begin{aligned} \bar{T}_1'' &= \lambda_1^2 \bar{T}_1 + \bar{R}_1; \\ \bar{T}_2'' &= \lambda_2^2 \bar{T}_2 + R_2'', \end{aligned} \tag{2}$$

where $\lambda_{1,2}^2 = \frac{1}{2}\left(\xi_1\Delta_{11} + \xi_2\Delta_{22}\right) \pm \sqrt{\frac{1}{4}\left(\xi_1\Delta_{11} - \xi_2\Delta_{22}\right)^2 + \xi_1\xi_2\Delta_{12}^2}$;

$\bar{R}_1, \bar{R}_2$ – generalized loads;

$$\begin{aligned} \bar{R}_1 &= \sqrt{\xi_1}\left(\Delta_{10}\cos\varphi - \Delta_{20}\sin\varphi\right); \\ \bar{R}_2 &= \sqrt{\xi_2}\left(\Delta_{10}\sin\varphi + \Delta_{20}\cos\varphi\right); \end{aligned}$$

$\bar{T}_1, \bar{T}_2$ – generalized unknowns;

$$\begin{aligned} \bar{T}_1 &= \frac{1}{\sqrt{\xi_1}}\cos\varphi T_1 + \frac{1}{\sqrt{\xi_2}}\sin\varphi T_2; \\ \bar{T}_2 &= -\frac{1}{\sqrt{\xi_1}}\sin\varphi T_1 + \frac{1}{\sqrt{\xi_2}}\cos\varphi T_2. \end{aligned}$$

The angle φ is determined by its tan:

$$tg\,\varphi = \frac{\lambda_1^2 - \xi_1\Delta_{11}}{\sqrt{\xi_1\xi_2}\Delta_{12}} = \frac{\sqrt{\xi_1\xi_2}\Delta_{21}}{\lambda_1^2 - \xi_2\Delta_{22}}.$$

The independent Eqs. (2) have the following solutions:

$$\begin{aligned} \bar{T}_1 &= A_1 sh\,\lambda_1 x + B_1 ch\,\lambda_1 x + \frac{1}{\lambda_1}\int_0^x \bar{R}_1(t)\, sh\left[\lambda_1(x-t)\right]dt, \\ \bar{T}_2 &= A_2 sh\,\lambda_2 x + B_2 ch\,\lambda_2 x + \frac{1}{\lambda_2}\int_0^x \bar{R}_2(t)\, sh\left[\lambda_2(x-t)\right]dt. \end{aligned} \tag{3}$$

The coefficients A_i, B_i are determined from borderline conditions. The reversed inversion to shearing forces T_1, T_2 can be realized as follows:

$$T_1 = \sqrt{\xi_1}\, cos\, \varphi \overline{T}_1 - \sqrt{\xi_1}\, sin\, \varphi \overline{T}_2;$$
$$T_1 = \sqrt{\xi_2}\, sin\, \varphi \overline{T}_1 + \sqrt{\xi_2}\, cos\, \varphi \overline{T}_2. \quad (4)$$

Determine the parameters, included in formula (1) for this three-layer composite beam. Considering the fact, that equivalent three-layer beam is placed symmetrically to the relative centerline [14], then

$$\xi_1 = \xi_2 = \xi;\ c_1 = c_2 = c;\ E_1 = E_3 = E_u;\ E_2 = E_c;\ F_1 = F_3;\ J_1 = J_3.$$

The formulae to determine the cross-section area and moments of inertia ratio of the composite beam components are given in Table 1.

Table 1
Formulae to determine the coefficients Δ_{ij} and the geometrical characteristics of the beam section.

Coefficients	
$\Delta_{10} = \Delta_{20}$	$-\frac{M^0 c}{\sum EJ} = -\frac{1}{R}c$
$\Delta_{10} = \Delta_{20}$	$-\frac{c}{R} = \frac{4}{3\pi R} \cdot \frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}$
$\Delta_{12} = \Delta_{21}$	$-\frac{1}{E_2 F_2} + \frac{c^2}{\sum EJ}$
$\Delta_{12} = \Delta_{21}$	$-\frac{1}{E_2 \pi r_2^2} + \frac{c^2}{\Sigma EJ}$
$\Delta_{11} = \Delta_{22}$	$\frac{1}{E_1 F_1} + \frac{1}{E_2 F_2} + \frac{c^2}{\sum EJ}$
$\Delta_{11} = \Delta_{22}$	$\frac{2}{E_1 \pi \left(r_1^2 - r_2^2\right)} + \frac{1}{E_2 \pi r_2^2} + \frac{c^2}{\Sigma EJ}$
The cross-sectional areas and the moment of inertia of the beam elements	
F_1	$\frac{\pi}{2}\left(r_1^2 - r_2^2\right)$
F_2	πr_2^2
J_1	$0.11\left(r_1^4 - r_2^4\right) - 0.238 r_1^2 r_2^2 \frac{r_1 - r_2}{r_1 + r_2}$
J_2	$\frac{\pi}{4} r_2^4$

Now determine the coefficients $\lambda_{1,2}$:

$$\lambda_1^2 = \xi\left(\Delta_{11} + \Delta_{12}\right) = \frac{\xi}{E_1 F_1} + \frac{2\xi c^2}{\sum EJ};$$
$$\lambda_2^2 = \xi\left(\Delta_{11} - \Delta_{12}\right) = \frac{\xi}{E_1 F_1} + \frac{\xi}{E_2 F_2}.$$

Also, determine $tg\varphi$:

$$tg\, \varphi = \frac{\lambda_1^2 - \xi_1 \Delta_{11}}{\sqrt{\xi_1 \xi_2}\Delta_{12}} = 1,$$

Then

$$sin\ \varphi = cos\ \varphi = \frac{1}{\sqrt{2}},$$

$$\bar{R}_1 = 0;\ \ \bar{R}_2 = \frac{\sqrt{\xi c}\sqrt{2}}{R}.$$

The solutions of Eqs. (3) are

$$\bar{T}_1 = A_1 sh\,\lambda_1 x + B_1 ch\,\lambda_1 x\,,$$

$$\bar{T}_2 = A_2 sh\,\lambda_2 x + B_2 ch\,\lambda_2 x + \frac{1}{\lambda_2^2}\frac{\sqrt{\xi c}\sqrt{2}}{R}\left(ch\,\lambda_2 x - 1\right).$$

Passing to the required forces T_1, T_2 by (4), determine the constants A_1, A_2, B_1, B_2 from the following borderline conditions:

$$x = 0;\quad T_1' = 0;\quad T_2' = 0;$$

$$x = l;\quad T_1 = 0;\quad T_2 = 0.$$

Determine the shearing forces by the formula

$$T_1 = T_2 = \frac{1}{\lambda_2^2}\frac{\xi c}{R}\left(1 - \frac{ch\lambda_2 x}{ch\lambda_2 l}\right). \tag{5}$$

The obtained formulae (5) allow us to determine the shearing forces in single core flexible cables, taking into account the properties of materials, cable geometry and operating conditions.

3 RANGE OF PRACTICE AND DISCUSSION

To test the practical application of the formula (5), flexible power cables have been chosen, intended for connecting mobile mechanisms to electrical networks with an alternating voltage of 660 V or a constant voltage of 1,000 V. The cables have been operated at ambient temperature. The minimum cable bend radius is not less than $R = 8d_1$.The warranty period for the operation of the cables is set at 6 months from the date of commissioning. The service life of the cables is 4 years. Current-carrying conductors are made of twisted copper wires (strands), and the cable sheath is made of rubber of the RS type. Table 2 shows the geometric dimensions of flexible cables KG [1]. The elastic moduli of insulation and conductors have been used for calculation: for rubber $E_1 = 8\ MPa$ and copper $E_2 = 1.08\times10^5\ MPa$ [6].

Table 2
Geometric dimensions of KG cables.

Number of conductors and nominal cross-section, mm^2	Diameter of conductors d_2, *mm*	Thickness of insulation, *mm*	Shell thickness *t*, *mm*	Outer cable diameter d_1, *mm*
1×2.5	2.1	–	2.3	6.7
1×4	2.6	–	2.5	7.6
1×6	3.3	–	2.6	8.5
1×10	4.0	–	3.0	10.0
1×16	5.2	–	3.1	11.4
1×25	6.8	–	3.4	13.6
1×35	7.8	–	3.6	15.0
1×50	9.8	–	4.0	17.8
1×70	11.5	–	4.2	19.9
1×95	13.8	–	4.6	23.0

In [15], the tribological characteristics of a friction pair of insulation "current-carrying conductor – current-carrying conductor" have been determined experimentally. For such pairs with shear $\Delta = 6\times10^{-4}$ *m* tangential stresses are $\tau = 1\times10^{5}$ N/m^2. Then the stiffness coefficient for the pair "CCC – CCC" is determined by the known formula [6]

$$\tau = \frac{F}{l} = \frac{10}{100\cdot10^{-3}} = 100\ N/m.$$
$$\xi_1 = \frac{\tau}{\Delta} = \frac{100}{0.6\cdot10^{-3}} = 167\cdot10^{3}\ N/m^{2}.$$

However, for the pair "copper wire – CCC insulation" these parameters are unknown. Therefore, the stiffness coefficient of the seam has been determined from the ratio of the friction coefficients for this pair

$$\frac{\xi_2}{\xi_1} = \frac{f_2}{f_1},$$

where f_1 is the coefficient of friction of the insulation materials of the "rubber – rubber" CCC shells; f_2 is the coefficient of friction "current-carrying copper conductor – shell insulation".

From the directories [16, 17] we have found the corresponding friction coefficients $f_1 = 0.5$ и $f_2 = 0.4$ and determined the stiffness coefficient of the copper-rubber seam as:

$$\xi = \xi_2 = \frac{f_2}{f_1}\cdot\xi_1 = \frac{0.4}{0.5}\cdot167 = 134\ N/m^{2}.$$

The geometrical characteristics of the cable and the coefficients of these formulae have been calculated for the single core cables. Table 3 summarizes all the parameters of the coefficients of Eq. (5) for the selected single core cables.

Table 3
Equation coefficients for single core cables.

Number of conductors and nominal cross-section, mm^2	$c\times10^{-3}$, *m*	$\Delta_{10} = \Delta_{20}$, $\times10^{-3}$	$\Delta_{11} = \Delta_{22}$, $\times10^{-3}$, N^{-1}	$\Delta_{12} = \Delta_{21}$, $\times10^{-6}$, N^{-1}	λ_1^2, m^{-2}	λ_2^2, m^{-2}
1×2.5	1.754	2.751	7.866	–2.673	1054.00	1054.00
1×4	1.998	3.156	6.243	–1.744	836.44	836.81
1×6	2.250	3.599	5.189	–1.083	695.14	695.43
1×10	2.651	4.256	3.790	–0.737	507.78	507.88
1×16	3.047	4.977	3.093	–0.436	414.43	414.55
1×25	3.657	6.061	2.295	–0.255	307.48	307.55
1×35	4.195	6.952	1.744	–0.194	233.69	233.74
1×50	4.820	8.128	1.442	–0.123	193.18	193.21
1×70	5.410	9.210	1.207	–0.089	161.71	161.74
1×95	6.272	11.000	0.940	–0.062	125.99	126.00

Table 3 shows the coefficients are equal to each other $\lambda = \lambda_1 = \lambda_2$. The formula (5) is given in the most convenient form. To do this, we denote the factor in front of the brackets $T_0 = \frac{\xi\cdot c}{\lambda^2\cdot R}$ – the constant component of the shearing force. Then the formula can be written as:

$$T = T_0\cdot\left(1 - \frac{ch\lambda x}{ch\lambda l}\right). \qquad (6)$$

Fig. 2 shows the change in the shearing force for KG cables 1×4, 1×16, 1×50, 1×95 at a length $l = 0.3$ *m*. For cables with a small diameter, the force in this section is constant. For cables with a larger diameter, the change is more intense, the force value decreases on a decline curve. For all cables before the end of the bending deformation zone, the shearing force tends to zero according to the initial conditions. Fig. 3 shows the variation in shear force for a 1×70 KG cable for different lengths of the deformation zone. At the beginning of the deformation zone, the force value oscillates about its constant value. Then the shearing force is constant along the length of the cable, and at the end the force it also tends to zero.

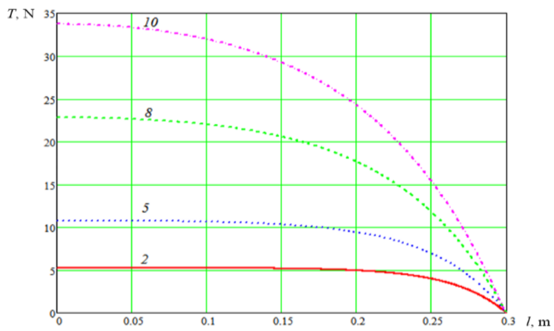

Fig.2
Dependence of the shearing force on the length of the KG cable: *2* – 1×4; *5* – 1×16; *8* – 1×50; *10* – 1×95.

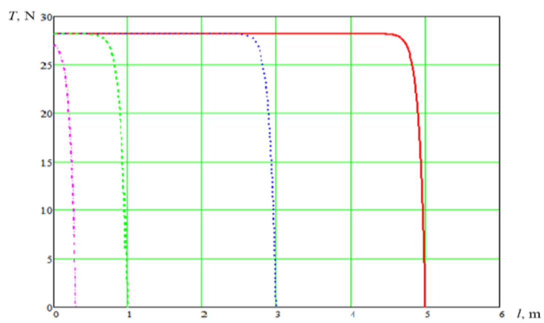

Fig.3
Dependence of shearing force variation for different length of KG cable 1×70.

To determine the shearing stress, we differentiate the Eq. (6) with respect to *x*

$$\tau = \frac{dT}{dx} = \lambda \cdot T_0 \cdot \frac{sh\lambda x}{ch\lambda l}. \tag{7}$$

Based on the calculated data, make the shifting voltage plot for a 1×16 KG cable at the deformation length of $l = 0.3$ *m*. The diagram represents a hyperbolic sinusoid with its own parameters $\lambda \times T_0$ for each cable (Fig. 4). The value of the maximum shear stress will be at the ends of the rod and is determined as:

$$\tau_{\max} = \lambda \cdot T_0 \cdot th\lambda l. \tag{8}$$

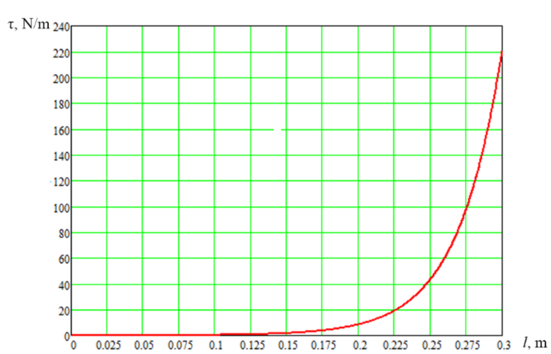

Fig.4
Dependence of the shear stress on the deformation zone of the KG cable 1×16.

The obtained formulae (6), (7) allow us to determine the interaction forces between copper wires and cable insulation under bending deformations. However, the values of the obtained forces can be used to calculate a multicore cable for the tribological interaction estimation of both individual elements and constituents of each element. The relative shifts (movement) are also known to occur not only between the elements of the cable, but also with respect to its insulation. Therefore, it is necessary to consider any tribological interaction as an organized construction with a hierarchical structure for a full-fledged calculation. It is specifically important in cases when CCC is destroyed not least because of copper wire attrition.

4 CONCLUSION

Formulae (6) and (7) for an approximate determination of the interaction forces between the constituent parts of a single core cable as a composite multilayer beam have been obtained for the first time. The values of the shearing force for certain types of single core flexible cables have been determined. The shearing forces along the section are constant. At the beginning of the cable deformation zone, the force value can fluctuate within its constant value. At the end of the zone, the shearing force is zero. In practice, the formulae work for the tasks set. The resulting expressions for shearing forces allow us to evaluate the tribological interaction of the constituent parts of each cable element and take into account their influence when creating multicore cables [18, 19].

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