Modelling of Random Geometrical Imperfections and Reliability Calculations for Thin Cylindrical Shell Subjected to Lateral Pressure

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ABSTRACT

It is well known that it is very difficult to manufacture perfect thin cylindrical shell. Initial geometrical imperfections existing in the shell structure is one of the main determining factor for load bearing capacity of thin cylindrical shell under uniform lateral pressure. As these imperfections are random, the strength of same size cylindrical shell will also random and a statistical method can be preferred to find the allowable load of these shell structures and therefore a In this work the cylindrical shell of size $R/t = 228$, $L/R = 2$ and $t=1mm$ is taken for study. The random geometrical imperfections are modeled by linearly adding the first 10 eigen mode shapes using 2^k full factorial design matrix of DoE. By adopting this method 1024 FE random imperfect cylindrical shell models are generated with tolerance limit of ± 1 *mm*. Nonlinear static FE analysis of ANSYS is used to find the buckling strength of these 1024 models. FE results of 1024 models are used to predict the reliability based on MVFOSM method.

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Keywords : Design of experiments; Thin cylindrical shell; Random geometrical imperfections; Reliability; Mvfosm.

1 INTRODUCTION

THIN cylindrical shell structures are one among the important structural parts which have wide applications in many engineering fields such as in mechanical, civil, aerospace, nuclear and marine structures etc., owing to high effective specific load bearing ability. In operation these shells are subjected to external pressure loading and may fail due to buckling is uncontrollable in nature. Hence accurate prediction of ultimate load carrying capacity is significant for safe design of these structures. Generally, the load bearing capacity of plain cylindrical shell structures under pressure loading alone can be increased by stiffening them along circumferential direction using stiffeners called ring stiffeners. These ring stiffened cylindrical structures under external lateral pressure loading fail with any one or combination of the following modes of buckling failure: a) shell-stiffener combined failure called

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general instability failure mode, b) Failure of bare cylindrical shell portion in between stiffeners called local shell instability failure mode, and c) Stiffener tripping. Since the collapse of bare cylindrical shell in between stiffeners is a basic failure mode, it has been taken for the present study.

Thin cylindrical structures acquire different types of imperfections either during manufacturing or in the course of their service life. Out of the different types of imperfections (namely geometrical, material and structural imperfections), geometrical imperfections plays a leading role in determining the load bearing capacity of thin cylindrical structures, which in turn depends on the form and amplitude of the imperfections [1, 2]. Hence for the accurate determination of the ultimate strength of thin cylindrical structures, it is imperative to model the geometrical imperfections accurately. Also a probabilistic approach is required to arrive at a safe load bearing capacity of shell structure. Therefore reliable method is to be adopted to find the collapse pressure and modelling of initial imperfections which is highly random in nature. As given in reference Ranganathan [3] the structural reliability calculation methods can be categorized into level 1, 2 and 3 methods. Since in present case collapse load of the cylindrical shells are scattered widely from the theoretical value level2 method is adopted.

2 LITERATURE REVIEW

Two approaches are followed for modelling distributed geometrical imperfections namely deterministic approach and random approach. In deterministic approach there are two ways of computing the imperfections such as by (i) assuming imperfection pattern and by (ii) actual measurement (for example Paor et al [4]; Sadovsky et al, [5,6]; Athiannan and Palaninathan, [7]; Singer, [8]; Scheneider, [9]; Arbocz and Hol, [10]; and Kirkpatrick and Holmes,[11]). The assumed imperfection pattern may be first eigen mode shape pattern (Featherston, [12]; Kim and Kim, [13]; Khelil, [14]; Teng and Song, [15]) or harmonic pattern (Ikeda et al, [16]; Khamlichi et al, []17]; and Pircher et al,[18]). There are two methods of obtaining random modeling of imperfections. One way is to vary the nodal locations of the model randomly and the other way is the stochastic FE approach. Each manufacturing method possess unique characteristic imperfection shapes of imperfection and these shapes can be represented by 2D random surface defined by double Fourier series. In earlier studies (for example Paor et al, [4]; Athiannan and Palaninathan, [7]; Chrysanthopoulos, [19]) coefficient of Fourier series were randomly varied to generate more number of initial random geometrical imperfection models. Monte Carlo simulation technique based reliability method was proposed by Elishakoff [20] and illustrated the method taking finite column buckling problem assuming geometrical imperfections as Gaussian random fields. Elishaoff et al [21] used mean and standard deviation of the measured geometrical imperfections to generate random geometrical imperfections and compared the safe load obtained from MVFOSM (Mean Value First Order Second Moment) method with Monte Carlo simulation results. Chryssanthopoulos et al [22] adopted RSM – Response Surface Methodology to evaluate the safe load of axially compressed stiffened plate and cylindrical shells considering the weld induced residual stresses and initial geometrical imperfections. Sadovsky and Bulaz [23] proposed inverse reliability method based on FORM. One important conclusion arrived was that by considering the real imperfections, marginally conservative design can be achieved and proved this by applying the reliability calculations for unstiffened thin plates and girders under compression and bending. In reference Warren [24] random geometrical imperfections were generated by adding eigen buckling mode shapes following design matrix of 2^k factorial design. Taking the framed structure as an example random models are generated keeping the variance of nodes within manufacturing tolerance. The stochastic finite element method was proposed by Náprstek [25] considering the larger displacements as cause for non-linearities and analyzed to find the response of the structures with random imperfections. A simulation method was used by Bielewicz and Gorski [26] to generate random geometrical imperfections using random nonhomogeneous random regular nets of 2D fields. Schenk and Schueller [27] developed geometrical imperfection models using Karhunen-Lo`eve expansion method with the help of imperfection databank available at Delft University of Technology. Papadopoulos and Papadrakakis [28] carried out structural stability analysis of composite shell structure using the developed triangular element. The initial imperfections are described as uni-variate 2D homogeneous random field. Craig and Roux [29] used the Karhunen–Lo`eve expansion method to include random general geometrical imperfections in FE nonlinear analysis and verified the buckling analysis result with published experimental and numerical results. In the work of Sadovsky et al [30] safe laod was determined as a function of two variables namely shape factor and integral energy measure. Applying this variable for a rectangular plate under longitudinally compressed plate and showed that this approach leads to less conservative design. Papadopoulos et al [31] in their work assumed distribution of modulus of elasticity and shell thickness variations as 2D-IV homogeneous non-Gaussian stochastic fields based on mean and standard deviation obtained from actual

measurement and it was illustrated that the selection of material and thickness variability distributions were critical for reliability calculations.. Rzeszut and Garstecki [32] in their work modeled the initial geometrical imperfection with summation of eigen modes with scale factor computed from actual measurement and carried out the stability analysis of column made of cold formed thin walled steel structures. Jalal et al [33], in their work determine the reliability of cylindrical shell with interacting localized geometric imperfections (either a triangular or a wavelet form) subject to axial compression by adopting FORM method. Brar et al [34], in their work concluded that in the negligence of initial geometric imperfections, thickness variation would be the important factor in predicting buckling load reduction. Hence in their work, they studied about safe buckling load reduction factor of externally pressurized cylindrical shell assuming random shell thickness employing the Monte Carlo technique. In the present work initial random imperfections are generated using first 10 eigen modes of cylindrical shell as recommended by Chryssanthopoulos and Poggi [35], Arbocz and Hol [10] etc., and by adding linearly he mode shapes considered following the design matrix of 2^k factorial design [24]. All FE imperfect cylindrical shell models are generated such that the maximum nodal deviations are kept within the manufacturing tolerance of ± 1 *mm*. Through deterministic FE analysis, distribution of strength is obtained and safe load of the structure is determined using MVFOSM method.

3   FE MODELLING

Since it is required to model both shell and dent accurately, a 8 noded SHELL281 is selected for analysis. It has capability to compute effect of membrane, bending and transverse shear effect. It can also support plasticity, stress stiffening effect, greater strain effects and large deflection in addition.

3.1 Thin cylindrical shell

The cylindrical shell structures under external loading are often used in the offshore construction. The thin steel cylindrical shell considered for analysis is [36]: Length /Radius (L/R) = 2, Radius (R) = 228 *mm*, Radius /Shell thickness (R/t) = 228, Density (ρ) = 8000 *kg/m*^{3} , Yield stress (σ_Y) = 240 *N/mm*^{2}, Young's modulus (E) = 2.1×10^5 *N/mm*^{2}, Poisson's ratio (γ) = 0.3. In analysis zero strain hardening effect is assumed.

3.2 Loading condition (LC) and boundary condition (BC)

Since rigid ring type boundary condition is to be applied at both ends, nodes at both ends are restrained to move radially at both end planes. Further, in order to prevent rigid body motion of cylindrical shell axial displacement of all the nodes at one end is restraint in addition.

3.3 Model validation

FE model taken for study is validated for both linear (perfect cylindrical shell model) and non-linear (imperfect cylindrical shell model) buckling analysis by comparing with experimental and other published numerical results for the boundary and loading conditions taken for study.

3.3.1 Eigen buckling analysis

To verify the boundary conditions effect and eigen buckling analysis, results published in Ref. Combescure and Gusic [37] are considered and present FE result comparison is shown in Table 1., with corresponding first eigen mode shape of shell *C* in Fig. 1.
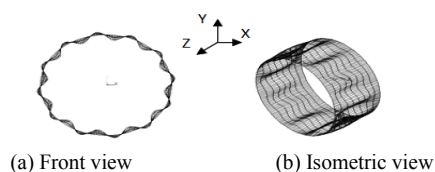

(a) Front view   (b) Isometric view

Fig.1
Mode shape of the cylindrical shell *C* (Combescure and Gusic [37]).
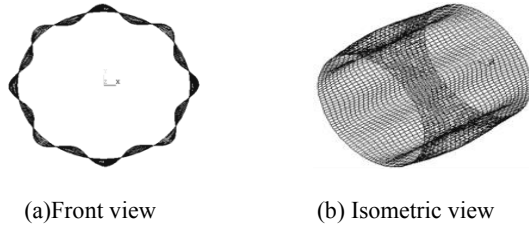
(a)Front view   (b) Isometric view

Fig.2
Mode shape of model 53 (Windenburg and Trilling [38]).

To verify the present FE results, numerical results are validated with published experimental results of model 53 and 57 (suitable to the present work) given in Ref. Windenburg and Trilling [38] and the results are presented in Table 2. From this it is clear that both number of circumferential lobes and critical buckling pressure are matches with each other. Fig. 2 presents the first eigen mode shape of model 53.

Table 1
Validation of critical buckling pressure of the cylindrical shells with published results in Ref. Combescure and Gusic [37].

Cylindrical shell	t, mm	L, mm	R, mm	Z	Poisson's ratio (γ)	$E \times 10^5$ N/mm^2	FE eigen buckling pressure (MPa)	
							Present study	Ref. [37]
A	0.15	20	50	51	0.3	2	0.2927	0.2936
C	0.247	113.8	100	500	0.3	2	0.0619	0.0624
D	0.247	508.5	100	5000	0.3	2	0.0135	0.0136

Table 2
Validation of experimental critical buckling pressure with published results in Ref. Windenburg and Trilling [38].

Model number	t, mm	L, mm	R, mm	Z	Poisson's ratio (γ)	$E \times 10^5$ N/mm^2	Critical Buckling Pressure (MPa)	
							Present study	Ref. [38]
53	0.8	406.4	203.2	969	0.3	1.93	0.108(8)	0.096(8)
57	0.78	406.4	203.2	994	0.3	2.06	0.109(8)	0.103(8)

Bracketed number indicates number of circumferential lobes and Z - Batdorf parameter $Z = \sqrt{1-v^2}\left(\dfrac{L}{R}\right)^2\left(\dfrac{R}{t}\right)$

3.3.2 Nonlinear FE analysis

For non linear FE analysis validation, the results obtained from eigen buckling analysis (first eigen mode shape) of the cylindrical shell $t = 1$ mm, $R = 228$ mm, $L = 456$ mm, n (circumferential lobe) = 8 and m (longitudinal lobe) =1, critical pressure = 153412.73 N/m^2 is added on the surface nodes of perfect cylinder model by varying maximum amplitude of imperfection between 0.001 mm and 1 mm to form different imperfect cylinder models. These models are analyzed using non-linear static FE analysis including both material and geometrical non-linearities. Snap through non-linear FE analysis approach with arc tangent option is employed to find the highest load bearing capacity of the structure (Forde and Stiemer [39]). Maximum and minimum arc lengths adopted for the analysis are 1 and 10E-6 respectively. Default force and moment convergence tolerance are adopted. Fig. 3 shows the results calculated from this nonlinear analysis. Buckling Strength Ratio (ratio between collapse pressure of imperfect shell to eigen buckling pressure of perfect shell) represented as BSR is used to denote the critical buckling pressure. From the Fig. 3 it can be observed that as the imperfection amplitude reduces, collapse pressure approaches to eigen buckling pressure. Thereby non linear analysis is validated for accuracy and boundary conditions.
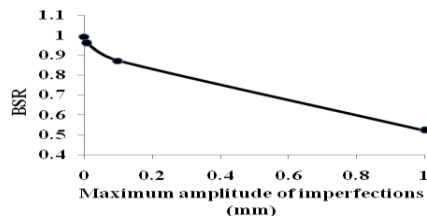

Fig.3
BSR vs Maximum amplitude of imperfections.

4   MODELING OF IMPERFECT THIN CYLINDRICAL SHELLS

To incorporate randomness in the model i.e., variation in imperfection at a point in FE cylindrical model except the boundary edge node, the first ten eigen buckling modes of cylindrical shell are added linearly following design matrix of 2^k factorial design. Ten linear buckling modes obtained from FE cylindrical shell model having 32 elements along longitudinal direction and 100 elements along circumferential directions respectively are used to develop random imperfection models as shown in Fig. 4 with their critical pressure.

The assumptions incorporated in achieving the modeling of the initial random imperfections is listed below.

- Δ – Nodal imperfection amplitudes of all nodes except boundary nodes need to be normally distributed.
- Mean value of nodal imperfection amplitude from all the models must be equal to zero.
- All eigen mode shapes are given equal chance in analyzing random imperfection modeling.

Implementing above assumptions, the nodal imperfection amplitude vector of entire model is given by

$$\Delta_{i\times 1} = \varphi_{i\times j} \times M_{j\times 1} \tag{1}$$

where,

Δ - Nodal amplitude of imperfection vector

φ - The matrix of normalized mode shape vectors containing the modal imperfection amplitudes of all nodal points.

M - Modal vector of model imperfection magnitude

i – node number

j - eigen mode shape number

If the nodal imperfection amplitude vector is known, the modal vector of modal imperfection values can be arrived using relationship (2)

$$M_{j\times 1} = \varphi^*_{j\times i} \times \Delta_{i\times 1} \tag{2}$$

where, φ^* is the pseudo-inverse matrix of φ and it can be calculated using the method of least squares as the equation given below

$$\varphi^* = \left(\varphi^T \varphi\right)^{-1} \times \varphi^T \tag{3}$$

If nodal imperfections Δ_i are considered as variables having independent normal distribution random then the mean and variance of all modal magnitudes are given by

$$\mu_{Mj} = \sum_1^i \varphi^*_{ji} \mu_{\Delta i} \tag{4}$$

$$\sigma^2_{Mj} = \sum_1^i \left(\varphi^*_{ji}\right)^2 \sigma^2_{\Delta i} \tag{5}$$

where,

$\sigma^2_{\Delta i}$ & $\mu_{\Delta i}$ - variance & mean of the nodal imperfection amplitude respectively.

σ^2_{Mj} & μ_{Mj} - variance & mean of the modal imperfection value respectively.

Similarly, variance & mean of all the nodal amplitudes can be obtained from Eqs. (6)-(7).

$$\mu_{\Delta i} = \sum_1^j \varphi_{ij} \mu_{Mj} \tag{6}$$

$$\sigma_{\Delta i}^2 = \sum_1^j \left(\varphi_{ij}\right)^2 \sigma_{Mj}^2 \qquad (7)$$

when the nodal amplitude (Δ_i) of any node i of the structure follows normal distribution so that the mean of modal imperfection amplitude (μ_Δ) becomes zero, as per Eq.(4) the value of mean of modal imperfection magnitude (μ_M) also becomes zero. The next step is to obtain the amplitude of imperfections of all nodes for each model for which the model magnitude of every model has been calculated utilizing the Eq. (5). With the results of these modal magnitudes calculated as above, the nodal amplitudes of imperfections can be computed with the help of Eq.(1). Using these values it is possible to generate random geometrical imperfection models by varying the modal magnitudes of imperfections in a random manner by making use of 2^k factorial design matrix of DoE.
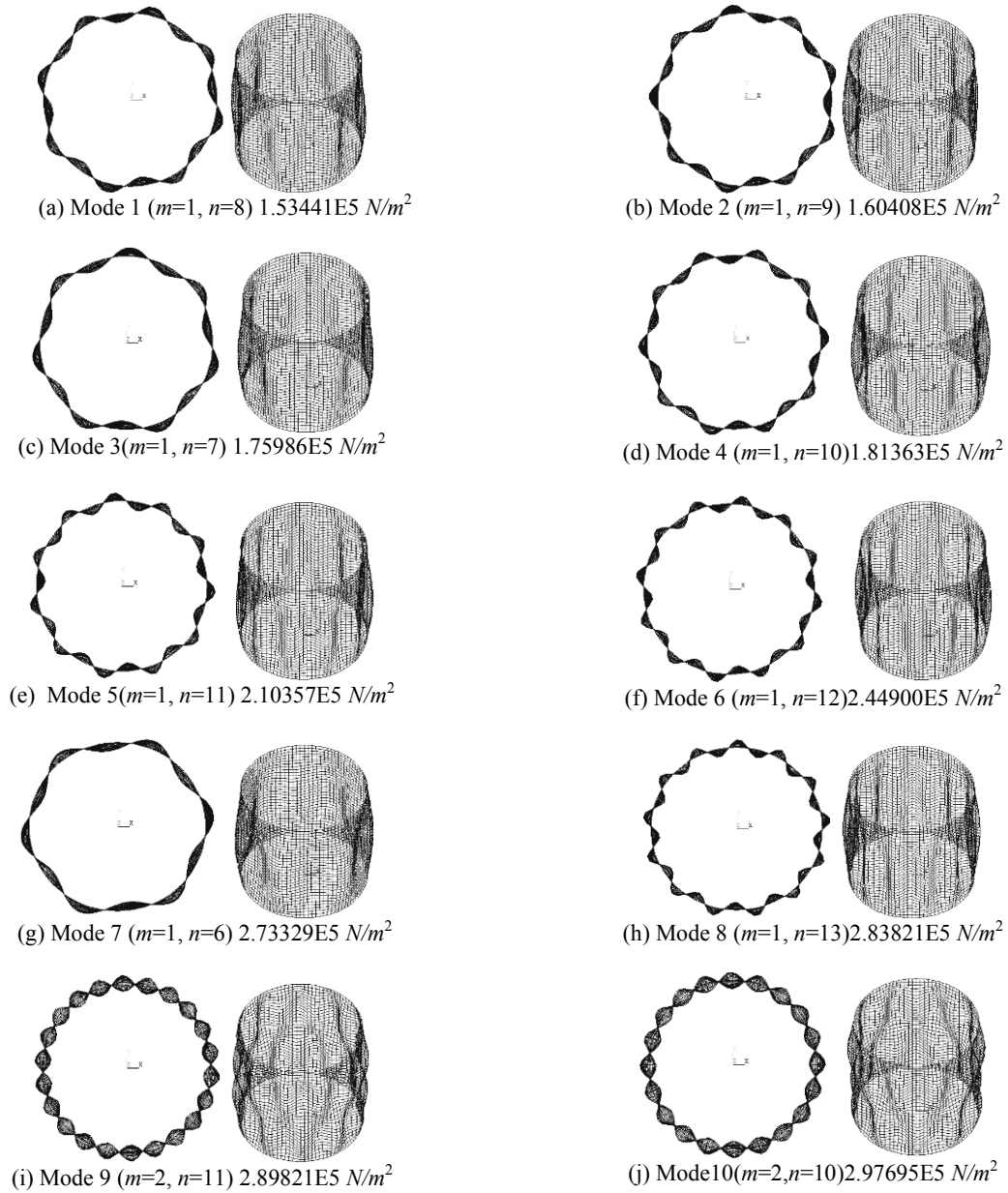
(a) Mode 1 (*m*=1, *n*=8) 1.53441E5 *N/m²*

(b) Mode 2 (*m*=1, *n*=9) 1.60408E5 *N/m²*

(c) Mode 3(*m*=1, *n*=7) 1.75986E5 *N/m²*

(d) Mode 4 (*m*=1, *n*=10)1.81363E5 *N/m²*

(e) Mode 5(*m*=1, *n*=11) 2.10357E5 *N/m²*

(f) Mode 6 (*m*=1, *n*=12)2.44900E5 *N/m²*

(g) Mode 7 (*m*=1, *n*=6) 2.73329E5 *N/m²*

(h) Mode 8 (*m*=1, *n*=13)2.83821E5 *N/m²*

(i) Mode 9 (*m*=2, *n*=11) 2.89821E5 *N/m²*

(j) Mode10(*m*=2,*n*=10)2.97695E5 *N/m²*

Fig.4
Ten eigen affine modes of perfect cylindrical shell considered for random modeling of imperfections (amplitudes enlarged by 50 times).

5   PROCEDURE FOLLOWED TO GENERATE RANDOM GEOMETRICAL IMPERFECTION

In first step variance of vector of modal imperfection magnitude was assumed as:

$$\sigma_M^2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{8}$$

In second step utilizing the Eq. (7), vector of σ_Δ^2 was calculated.

In third step every element of σ_Δ^2 vector attained from first step was normalized with the highest value in that vector and σ_{tol}^2 is multiplied with that value.

In fourth step with the value of σ_Δ^2 vector arrived from third step, revised σ_M^2 vector was found with the help of Eq. (5).

Modal imperfection magnitude vector *M*, was calculated utilizing revised vector σ_M^2 with the condition $\mu_\Delta = 0$, $\mu_M = 0$ such that $M = \pm\sigma_M$.

In final step by utilizing 2^k factorial design, every column in design matrix was chosen and corresponding element in the *M* vector arrived from earlier step is multiplied. With the help of new design matrix, 1024 ($2^{10} = 1024$) random geometrical imperfection models are generated.

$$(\ \Delta = \varphi x \ \text{new design matrix}\ ) \tag{9}$$

(a)   Initial amplitude of imperfection in *m*   (b) Initial amplitude of imperfection in *m*
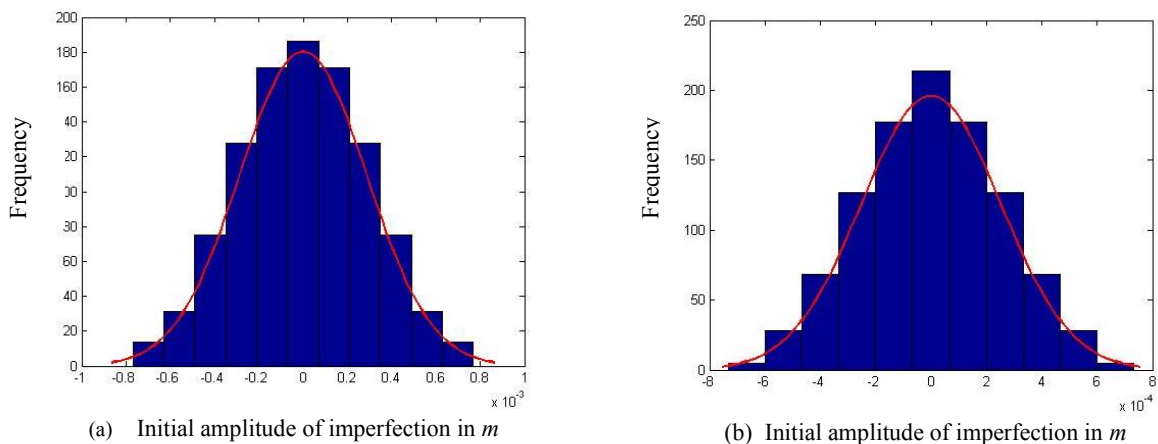
Fig.5
Normal distribution of radial amplitudes of imperfections from all 1024 random imperfection models (keeping RMS = 0.33 mm) of a node (a) 1000 (x = 227.9 mm, y = -7.2 mm and z = 57 mm) and (b) 4000 (x = 227.9 mm, y = -7.2 mm and z = 270.8 mm) on the surface of the cylindrical shell.

By substituting the value of *M* in Eq.(1), **Δ** is calculated. The value of modal imperfection is determined by element value in the design matrix (+1 or -1). The Δ matrix arrived from Eq.(1) has 1024 rows. Each row corresponds to imperfect nodal displacements of all nodes of a random model. By applying the steps given above 1024 random geometrical imperfect cylindrical shell models are generated maintaining SD (Standard Deviation) or RMS (Root Mean Square) of imperfections = 0.33 *mm*. Fig. 5 shows nodal initial displacement distribution of some sample nodes taken from all 1024 models and it indicates that nodal initial displacement is random by following normal distribution. This modeling procedure results in 512 pairs of mirror image random imperfect models. Fig. 6 shows some pairs of imperfect cylindrical shell models.
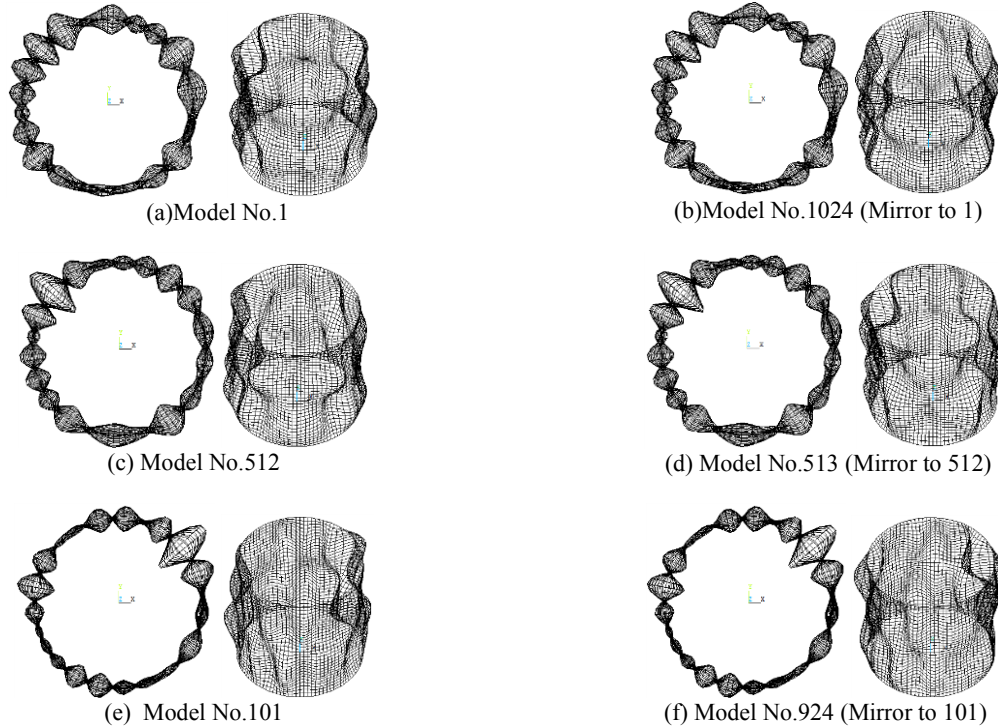

(a)Model No.1

(b)Model No.1024 (Mirror to 1)

(c) Model No.512

(d) Model No.513 (Mirror to 512)

(e) Model No.101

(f) Model No.924 (Mirror to 101)

Fig.6
Samples of random imperfection cylindrical shell models (amplitude enlarged by 50 times).

6   RESULTS AND DISCUSSION

Non-linear FE analysis is used to determine BSR. As a sample, von Mises stress contour and deformations on cylindrical shells at various LSS (Load Sub Step is ratio of between pressure applied and eigen buckling pressure) for the cylindrical shell of FE model no.696 are presented in Fig. 7. From this figure it can be noted that the stress value and deformations are different at different locations of cylindrical shell and it can also be noted that these stress value and deformations enhances with increase in pressure on the cylindrical shell. Fig. 7(d) shows formation of lobes and von Mises stress contours at the limit load condition and this further indicates that cylindrical shell fails before reaching elastic limit. Fig. 8 shows the curve of radial displacement vs LSS for node 62 (Location of this node is $x = 0$ *mm*, $y = -228$ *mm* and $z = 228$ *mm*). Fig.8 also reveals that when the LSS is 0.81845 (at 'a' in Fig. 8) the cylindrical shell reaches the limit load condition and the slope of that point on the curve becomes zero. Fig. 9 shows the scatter of BSR values of the cylindrical shell taken for study obtained from 1024 model.

From Fig. 10(a) it is understood that the distribution is not exact normal distribution but it is a skewed one. Since the normal distribution is simple, well developed and well known [40] the skewed strength distribution is converted into equal normal distribution by following the procedure of Verderaime [40]. As per this method the mode of actual distribution is taken as mean of equivalent distribution i.e., right side of skewed distribution mirrored about the mode to get equivalent normal distribution (Fig. 10(b)).
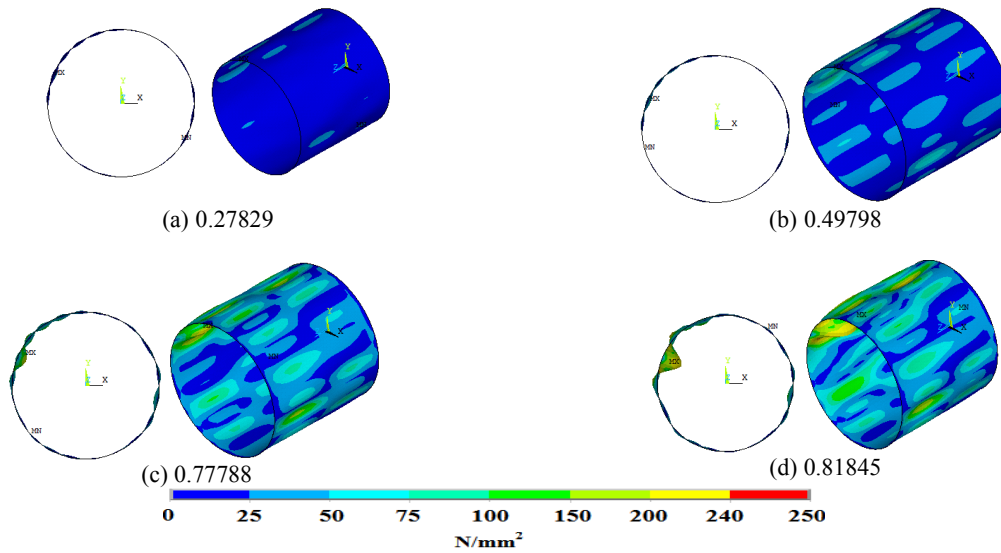
(a) 0.27829   (b) 0.49798

(c) 0.77788   (d) 0.81845

Fig.7
Front and pictorial views of von Mises stress contours of thin cylindrical shell taken for study for various LSS (with enlargement scale of 15 times).
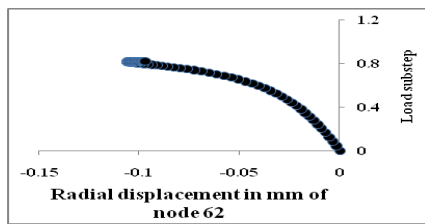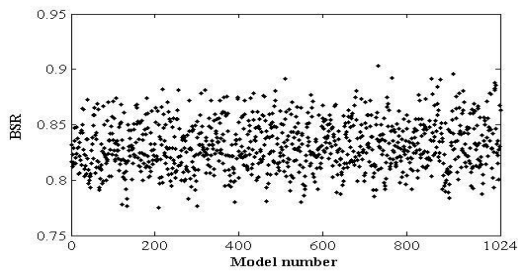

Fig.8
Radial displacement in mm vs LSS.

Fig.9
Model number vs BSR.

Mode of distribution =0.82369
Mean of distribution =0.83124

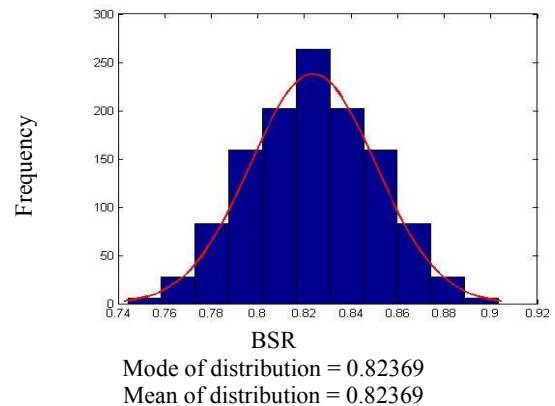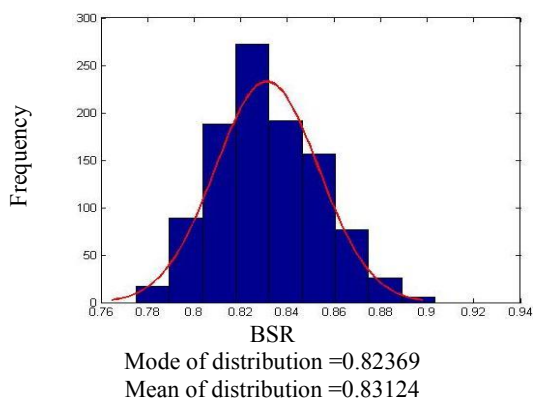Mode of distribution = 0.82369
Mean of distribution = 0.82369

Fig.10
Actual skewed strength distribution (b) Equivalent normal distribution of strength of the cylindrical shell.

As per MVFOSM the index of reliability is given by,

$$\beta = \frac{\mu_s - \mu_L}{\sqrt{\sigma_s^2 + \sigma_L^2}}$$ (10)

μ_s = strength distribution mean μ_L = load distribution Mean σ_s = strength distribution S.D, σ_L = load distribution S.D.

Therefore, the failure probability can be calculated as below,

$$Pf = \varphi(-\beta)$$ (11)

where, φ = function of cumulative normal distribution.

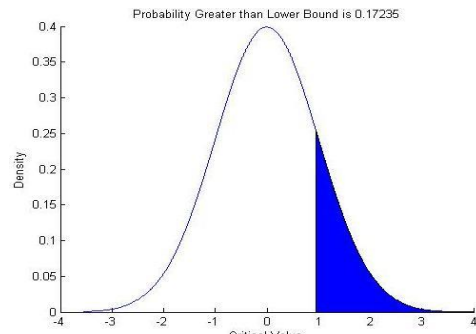The structural reliability can be calculated by,

$$R = 1 - P_f$$ (12)

In the present work, applied load is considered as deterministic single value. Therefore, $\sigma_L = 0$ and then β can

be modified as, $\beta = \dfrac{\mu_s - loadinBSR}{\sigma_s}$

The reliability of the structure is calculated by varying the applied load. Fig. 11 shows the survival probability for various loads. The change in reliability vs BSR is shown in Fig. 12. From this figure it is noticed that reliability is maximum of 100% for BSR of 0.74 and minimum of 0 % for BSR of 0.9 if the random imperfect cylindrical shell models having tolerance value of imperfection= ± 1 *mm*.
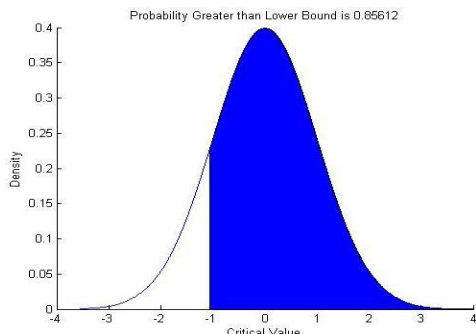
(a) Probability of survival = 0.85612 for load = 0.7951   (b) Probability of survival = 0.17235 for load = 0.8491
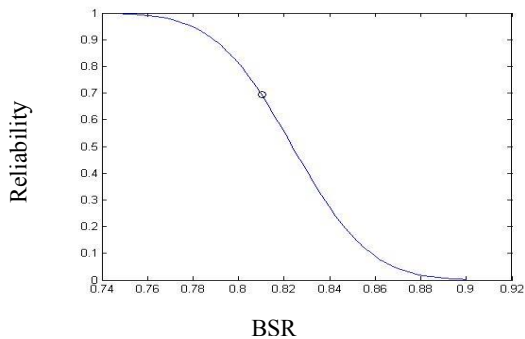
Fig.11
Survival probability at different loads.

BSR

Fig.12
Reliability vs BSR.

7 CONCLUSIONS

The following conclusions are arrived from the work carried out for thin cylindrical shell taken for study.

1. It has been demonstrated that design matrix of 2^k factorial design can be used to generate random imperfect cylindrical shell model giving equal importance to all the eigen mode shapes of cylindrical shell taken for study, maintaining the mean of nodal imperfections of a model equal to zero and nodal imperfection value within the tolerance limit.
2. The mirror image of random imperfect cylindrical shell models have different buckling pressure value.
3. By adopting MVFOSM reliability method recommended by Verderaime [40], it is found that the reliability of shell considered for analysis is 100% below 0.74 times the eigen buckling pressure of thin cylindrical shell and it reduces to 0% when the applied pressure exceeds 0.9 times the eigen buckling pressure of thin cylindrical shell under external pressure loading.

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