Journal of Solid Mechanics Vol. 11, No. 1 (2019) pp. 210-221
DOI: 10.22034/JSM.2019.664230

Influence of Addendum Modification Factor on Root Stresses in Normal Contact Ratio Asymmetric Spur Gears

R. Prabhu Sekar [1,*], R. Ravivarman [2]

[1]*Mechanical Engineering Department, Motilal Nehru National Institute of Technology, Allahabad, India*
[2]*Research Scholar Department of Mechanical Engineering, Pondicherry Engineering college, Pondicherry, India*

Received 2 January 2019; accepted 5 March 2019

ABSTRACT

Tooth root crack is considered as one of the crucial causes of failure in the gearing system and it occurs at the tooth root due to an excessive bending stress developed in the root region. The modern power transmission gear drives demand high bending load capacity, increased contact load capacity, low weight, reduced noise and longer life. These subsequent conditions are satisfied by the aid of precisely designed asymmetric tooth profile which turns out to be a suitable alternate for symmetric spur gears in applications like aerospace, automotive, gear pump and wind turbine industries. In all step up and step down gear drives (gear ratio > 1), the pinion (smaller in size) is treated as a vulnerable one than gear (larger in size) which is primarily due to the development of maximum root stress in the pinion tooth. This paper presents an idea to improve the bending load capacity of asymmetric spur gear drive system by achieving the same stresses between the asymmetric pinion and gear fillet regions which can be accomplished by providing an appropriate addendum modification. For this modified addendum the pinion and gear teeth proportion equations have been derived. In addition, the addendum modification factors required for a balanced maximum fillet stress condition has been determined through FEM for different parameters like drive side pressure angle, number of teeth and gear ratio. The bending load capacity of the simulated addendum modified asymmetric spur gear drives were observed to be prevalent (very nearly 7%) to that of uncorrected asymmetric gear drives.

Keywords: Asymmetric gear, Addendum modification factor, Finite element model, Fillet stress factor.

1 INTRODUCTION

GEARS are rotating machine elements used to transmit the torque and motion between the two parallel and non-parallel shafts with conjugate action. In general, during transmission the gear tooth is expected to fail by

[*]Corresponding author. Tel.: +91 9894986396.
E-mail address: prabhusekar.r@gmail.com (R.P.Sekar).

fracture at the root fillet region due to inadequate bending load capacity of the tooth. The shape of the gear tooth profiles (involute and trochoidal) plays a major role in bending strength of the tooth. Buckingham [1] developed the mathematical expressions for generating the involute and trochoidal curves which form the shape of the symmetric gear tooth. There is a great deal of investigates and studies on asymmetric gear analysis published by several authors. Asymmetric spur gear is one whose drive and coast sides pressure angle are not equal and it was developed through direct design approach by Kapelevich [2]. The bending strength of the direct designed gear is higher than that of the conventional designed gears when the gear pair is loaded with high pressure angle side [2-3]. Yang [4] proposed a mathematical model for tooth contact analysis on internal asymmetric spur gear. In that work, the internal gear and pinion with asymmetric tooth were manufactured by using rapid prototyping and the Von-Mises stresses was evaluated in that proposed internal gear. The increase in asymmetry with the stub tooth was suggested to improve the bending load carrying capacity of the asymmetric internal spur gear pair [5]. Asymmetric gear with higher addendum reduces the dynamic factor and static transmission error considerably than symmetric gear [6]. Costopoulos and Spitas [7] developed a one-sided involute asymmetric gear tooth to improve the bending load capacity. And, reported that the increase in load carrying capacity could reach up to 28% compared to the standard 20^{o} involute symmetric gear. Alipiev [8] developed the geometric design procedure for symmetric and asymmetric spur gears using realized potential method. This method was considered as an appropriate one for gear drives with small number of teeth. Prabhu Sekar and Muthuveerappan [9] studied the influence of different gear parameters on load sharing based maximum fillet stresses in asymmetric helical gear drives. In that work, a sizable decrease in the highest maximum fillet stress was observed in the asymmetric helical gear due to an increase in asymmetric factor which was always lesser than that of symmetric helical gear. An experimental work was conducted on composite asymmetric gears to evaluate the bending fatigue performance by Anand and Senthilvelan [10]. They demonstrated that the load carrying capacity of 20°/34° asymmetric gears is found to be superior to that of 20°/20° symmetric gears due to the increased gear tooth width at the root region. Marimuthu and Muthuveerappan [11-12] determined the maximum fillet and contact stresses on asymmetric normal and high contact ratio spur gears based on load sharing ratio concept using finite element method. It was evident that an increase in drive side pressure angle improves the contact load capacity of the asymmetric spur gear drive. Many of the works done on the asymmetric spur gear dealt with the determination of bending and contact stress using finite element method. Recently, Benny Thomas et al. [13] developed an analytical relation for evaluating bending strength of the asymmetric spur gear using Search method. The maximum gear tooth bending stresses decreases with increase in the number of gear tooth due to increase in contact ratio. Mo Shuai et al. [14] established the 3D model of asymmetric internal helical gear and analyzed the relationship between the maximum pressure angle, number of tooth, contact ratio and related parameters. The limit range of the pressure angle of the asymmetric internal helical gear was obviously larger than that of the conventional symmetric gears. The researchers above mostly deal the designing procedure of external and internal asymmetric spur, helical gears. The major concerns of those researches are determination of maximum root stresses on symmetric and asymmetric spur gears using analytical and finite element methods.

In general, the bending load capacity of an asymmetric spur gear tooth is evaluated based on the maximum root stress developed in the tooth fillet region. Any kind attempt made is only to reduce the maximum root stress by means of improving the bending strength of the gear. In step down and step up gear drives (gear ratio i>1), the smaller gear is called a pinion and the larger one is called as wheel. In these gear drives, the root stresses developed in the pinion is higher than that of wheel (hereafter called as gear) so that the pinion is more vulnerable one than gear. This difference is called as inequality of maximum fillet stress**.** The bending load capacity of asymmetric gear drive can be further improved by removing the inequality of maximum root stress between the pinion and gear. In this work, we have carried out some analysis on asymmetric spur gear drive to achieve the balanced root stress (the pinion and gear are equally stronger) by giving an appropriate addendum modification factor (profile shift x_p and x_g) to the pinion and gear which will be very valuable for optimum design asymmetric gear pair. As a study on the balanced maximum root stress on asymmetric gears with addendum modification is not found in any literature, an attempt has been made in this regard.

2 DESIGN OF ASYMMETRIC SPUR GEAR

The transverse cross-sectional shape of the basic asymmetric rack cutter used for generating the respective asymmetric pinion and gear tooth profiles is shown in Fig. 1. In this rack cutter, the rack tooth thickness equals the rack tooth space at the tool reference line ($t_{0r} = \pi m - t_{0r} = 0.5\pi m$) and the sum of the rack tooth thickness and the rack tooth space of the rack cutter is always equal to one pitch. The rack tooth thickness at the generating pitch line is expressed by

$$t_{wr} = 0.5\pi m - xm(\tan\alpha_{0d} + \tan\alpha_{0c}) \tag{1}$$

where, m – Module of the gear (mm), α_{0d} – Drive side pressure angle in degree, α_{0c} – Coast side pressure angle in degree, h_{fr} – Dedendum height of the rack cutters ($h_{fr} = 1m$) (mm), t_{0r} – Rack tooth thickness at the cutter reference line (mm), h_{ar} – Addendum height of the rack cutter ($h_{ar} = 1.25m$) (mm).

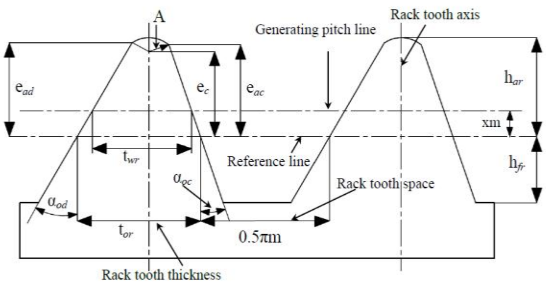


Fig.1
A symbolic transverse cross-sectional shape of asymmetric rack cutter.

The fully rounded cutter tip radius (A) for asymmetric rack cutter is defined by Muni et.al [3] as:

$$A = \left(\frac{(0.5\pi m) - \left(h_{ar}\left(\tan\alpha_{0d} + \tan\alpha_{0c}\right)\right)}{\left(\frac{1}{\cos\alpha_{0d}} + \frac{1}{\cos\alpha_{0c}}\right) - \left(\tan\alpha_{0d} + \tan\alpha_{0c}\right)} \right) \tag{2}$$

The drive and coast side involute profiles for asymmetric spur gear tooth are shown in Fig. 2. The tip angles (v_{dp} and v_{cp}) at the intersection point of two involutes are (Fig. 2)

$$\cos v_{dp} = \frac{r_{bdp}}{r_{\Delta p}} \qquad \cos v_{cp} = \frac{r_{bcp}}{r_{\Delta p}}$$

The coefficient of asymmetry (u) is defined by Kapelevich [2] as:

$$u = \frac{r_{bcp}}{r_{bdp}} = \frac{\cos v_{cp}}{\cos v_{dp}} = \frac{\cos\alpha_{0c}}{\cos\alpha_{0d}} \tag{3}$$

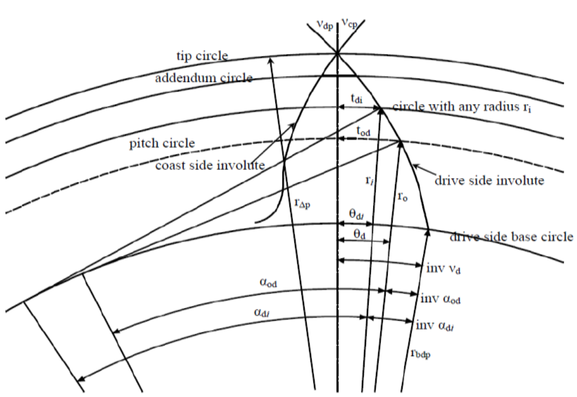
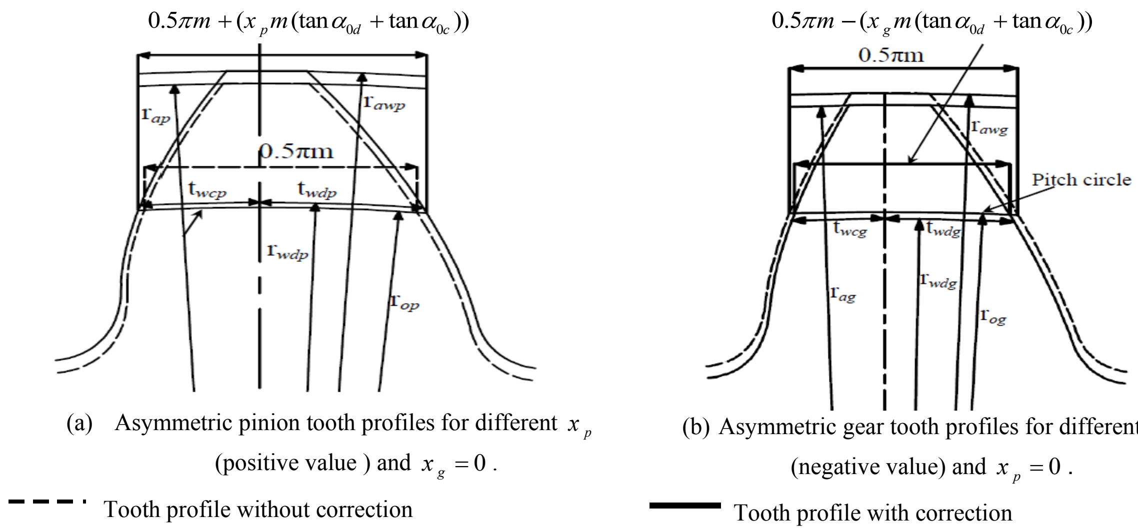


Fig.2
Asymmetric spur gear tooth.

where, $r_{\Delta p}$ – Tip circle radius for pinion (*mm*), r_{bdp} and r_{bcp} – Base circle radius for pinion at the drive and coast sides (*mm*). The tooth space of the rack cutter equals the thickness of the pinion tooth and the tooth thickness of the rack cutter equals the tooth space of the pinion tooth respectively. In a zero backlash gear drive, the tooth space of the pinion tooth equals the tooth thickness of the gear tooth. The tooth profile variation for the asymmetric pinion and gear generated by the rack cutter with different values of x_p and x_g are shown in Figs. 3(a) and (b). The tooth thickness increases by the amount of $x_p m\,(\tan\alpha_{0d} + \tan\alpha_{0c})$ at the pitch circle in the positive addendum modified pinion and it decreases by an amount of $x_g m\,(\tan\alpha_{0d} + \tan\alpha_{0c})$ in the gear, if the addendum modification of the gear tooth is negative.



(a) Asymmetric pinion tooth profiles for different x_p (positive value) and $x_g = 0$.

(b) Asymmetric gear tooth profiles for different x_g (negative value) and $x_p = 0$.

Tooth profile without correction

Tooth profile with correction

Fig.3
Tooth profile variation for the asymmetric pinion and gear with respect to x_p and x_g (for *m*=1, z_p =20, *i*=1.5, α_{0d} =30° and α_{0c} =20°).

The thickness of the tooth for addendum modified pinion and gear at the standard pitch circle are defined by (Figs. 3 (a, b)).

$$t_{0p} = 0.5\pi m + \left(x_p m\left(\tan\alpha_{0d} + \tan\alpha_{0c}\right)\right) \tag{4}$$

$$t_{0g} = 0.5\pi m - \left(x_g m\left(\tan\alpha_{0d} + \tan\alpha_{0c}\right)\right) \tag{5}$$

The thickness of the tooth at the working circle for the asymmetric pinion and gear are defined by (detailed derivation given in Appendix A)

$$t_{wp} = r_{wdp}\left(\frac{t_{0p}}{r_{0dp}} + \left(inv\,\alpha_{0d} + inv\,\alpha_{0c}\right) - \left(inv\,\alpha_{wd} + inv\,\alpha_{wc}\right)\right) \tag{6}$$

$$t_{wg} = r_{wdg}\left(\frac{t_{0g}}{r_{0dg}} + \left(inv\,\alpha_{0d} + inv\,\alpha_{0c}\right) - \left(inv\,\alpha_{wd} + inv\,\alpha_{wc}\right)\right) \tag{7}$$

where, r_{wdp} and r_{wdg} – working circle radius for the respective pinion and gear (*mm*).

3 FINITE ELEMENT MODEL

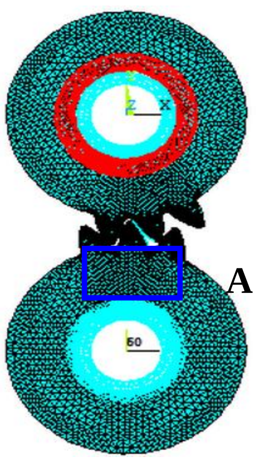
An optimum design of high-quality gears requires an accurate estimation of fillet stress. There are many numerical techniques such as FEM, complex potential method and boundary element method widely used in practice to estimate the stresses developed in machine element components. In the present context, FEM is deliberated as one of the most appropriate numerical methods used for gear stress analysis. Here, the involute tooth profile is developed by using the asymmetric rack cutters from the given basic gear parameters such as the drive and coast sides pressure angle, module, teeth number, gear ratio and addendum modifications [1-2]. In the present study, longer face width of the gears is been considered has plane strain formulation for FE analysis.

3.1 Material model

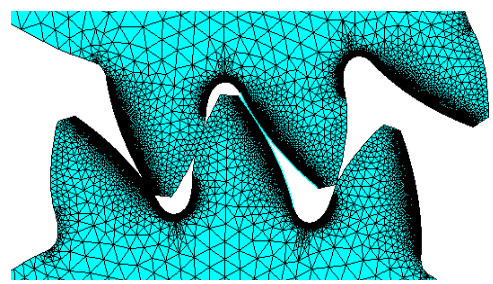
In this study, the material model is assumed as linear elastic with isotropic material. The following material (steel) properties are taken in this analysis such as Young’s modulus (E) = 200 *GPa*, Poisson's ratio =0.3 and density of steel = 7800 kg/m^3.

3.2 Loading condition

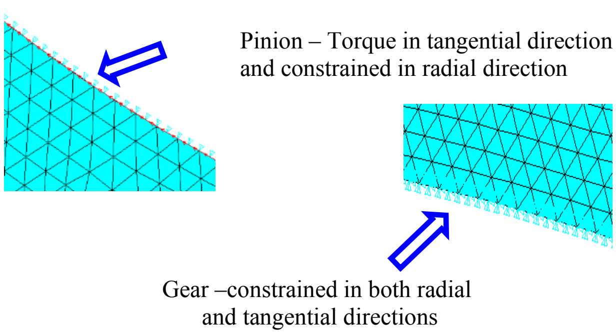
In the multi pair contact model (Figs. 4 (a and c)), the pinion is constrained only in the radial direction along the rim of the inner radius whereas the torque T (($T = F_n^* r_{bdp}$) *N-mm*) is uniformly distributed in the tangential direction along the rim of the inner radius. Gear is constrained both in x and y directions (radial and tangential) at the rim of the inner radius.



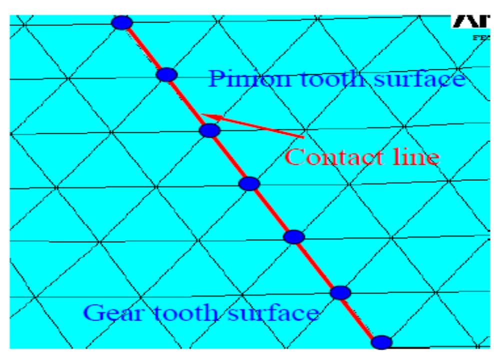
(a) Asymmetric gear drive



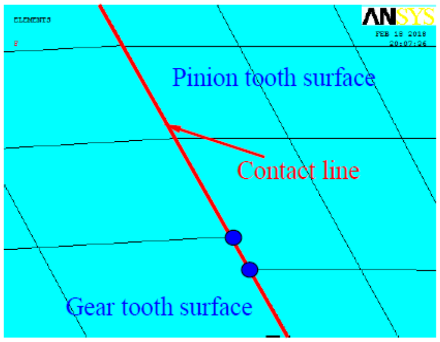
(b) Magnification of A



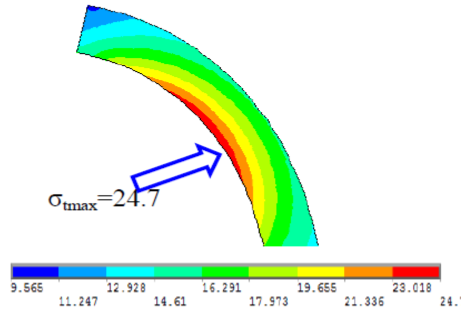
(c)Loading condition for pinion and gear



(d)Mesh pattern with triangular element



(e)Mesh pattern with quadrilateral element



(f)Maximum root stresses at fillet region

Fig.4
Finite element model of the asymmetric spur gear.

3.3 Element type

ANSYS-12 recommends the 2D-PLANE 42 elements having two degrees of freedom per node. From Figs. 4 (d and e), it is observed that an accurate node to node connectivity between the pinion and gear teeth elements is obtained when the gear pairs mesh with lower order triangular elements. This type of mesh pattern transfers the entire load from pinion to gear tooth surfaces which gives a better result. But, this kind of node connectivity cannot be obtained when the pinion and gear teeth surfaces are discretized using lower order quadrilateral elements. Hence, in this work, the lower order plane triangular element is used to discretize the asymmetric pinion and gear teeth. To establish the contact between the pinion and gear, the TARGE 169 and CONTAC 172 elements are used in this analysis. The value of friction coefficient is taken as 0.1 to conduct the contact analysis.

3.4 Convergence study

A convergence study with respect to the fine mesh of the element at the critical fillet region is carried out to assess the approximate number of elements requirement. The maximum root stresses are evaluated for different element numbers at the root region and the same are plotted in Fig. 5. It is found that the convergence occurs when the number of elements used in the root region is 3000 and above. The total number of elements used in the asymmetric gear drive model is 215756 elements.

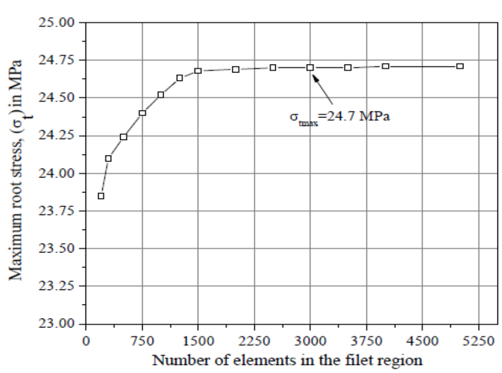


Fig.5
Convergence study for maximum root stress.

4 RESULT AND DISCUSSIONS

Certain standard procedures like ISO and AGMA are available for evaluating the maximum fillet stress developed in the symmetric spur gear tooth. However, the available standard procedures are not suitable for asymmetric gear tooth. Hence, in the present work, the maximum fillet stress $(\sigma_t)_{max}$ for the given gear parameters (Table 1) is determined for conventionally designed asymmetric spur gears through FEM and it is normalized in terms of fillet stress factor (σ) for unit load and unit face width.

The fillet stress factor is expressed by

$$\sigma = \frac{(\sigma_t)_{\max}}{\left(\dfrac{F_n}{bm}\right)} \tag{8}$$

where, m – Module (*mm*), b – Face width (*mm*), F_n – Normal load (*N*), σ – Fillet stress factor (no unit), $(\sigma_t)_{\max}$ – Maximum fillet stress (*MPa* or N/mm^2).

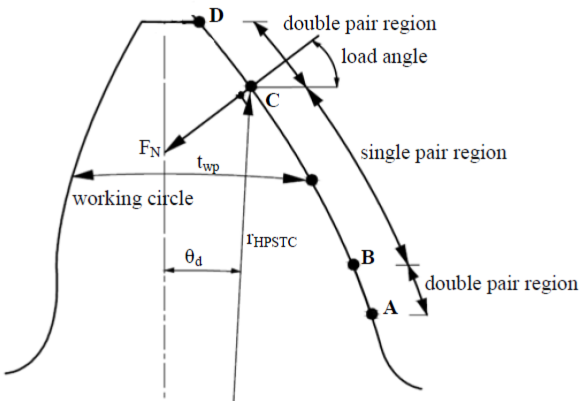
Table 1
Gear parameters for symmetric and asymmetric gears.

S.NO	Parameter Name	Asymmetric gears
1.	Module m (*mm*)	1
2.	Number of teeth in pinion z_p	20
3.	Gear ratio i	1.5
4.	Drive side pressure angle α_{0d} (degree)	30^{o}
5.	Coast side pressure angle α_{0c} (degree)	20^{o}
6.	Addendum height h_a (*mm*)	1m
7.	Dedendum height h_f (*mm*)	1.25m
8.	Normal load F_n (*N*)	10
9.	Face width b (*mm*)	Unit width
10.	Rack cutter type	Full round rack cutter
11.	Friction coefficient	0.1

4.1 LSR and the respective fillet stresses with and without addendum modified asymmetric spur gear drives

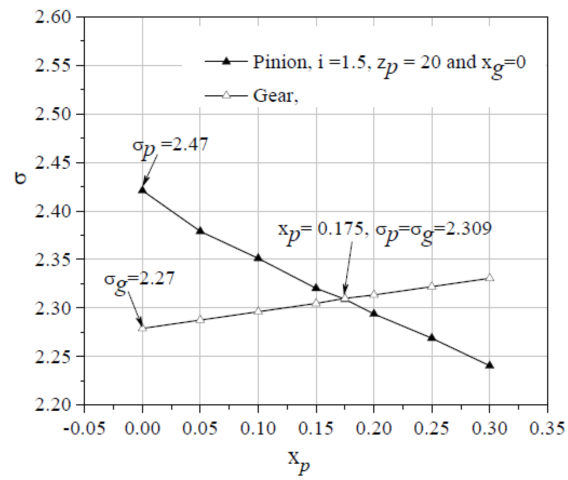
In the normal contact ratio (NCR) asymmetric spur gear drives (1 < ε_d < 2), contact begins at the lowest point of tooth contact (LPTC) and ends at the highest point of tooth contact (HPTC). In Fig. 6 (a), the regions AB and CD are the double pair contact regions and the region BC is the single pair contact region. In the double pair contact regions, the total normal load is shared by two pairs and hence the stress developed at the fillet region is lower even though the moment arm is higher when the tooth is loaded at D. The stress developed at the fillet region is maximum, when the tooth of the gear is loaded at the highest point of single tooth contact (HPSTC) because of full load (LSR=1.0) reasonably with a highest moment arm. So this position is regarded as the critically loaded position by researchers for a relatively fine finished gear.

For asymmetric spur gear drives, the balanced fillet stress factor (σ) is determined by a suitable selection of addendum modifications for the pinion and gear (x_p and x_g) and the same are shown in Fig. 6 (b). As x_p increases, the critical tooth thickness increases in the pinion fillet region which tends to decrease the stress level in the pinion whereas the tooth thickness at the gear fillet region decreases which tends to increase the stress level in the gear. The same root stress is obtained in the asymmetric pinion and gear for $x_p = 0.175$ and $x_g = 0$. The balanced σ of addendum modified asymmetric pinion is still lesser than that of unbalanced σ developed in the uncorrected asymmetric pinion ($x_p = x_g = 0$, without addendum modification) (2.309 < 2.47). Figs. 6 (c and d) show the influence of x_p on LSR and the respective σ for one mesh cycle. Due to an increase in x_p, the position of LPTC (A) and LPSTC (B) of the pinion are shifted towards the pitch point and the position of HPSTC (C) and HPTC (D) of the pinion are shifted away from the pitch point. A small increase in LSR is observed for the pair in contact between A and B and a proportional decrease in the LSR are also observed for the simultaneous meshing pair between C and D. It is found from Fig.6 (d) that the maximum root stresses decrease in the asymmetric pinion and it increases in gear throughout the meshing cycle due to increase in positive values of x_p given to the asymmetric pinion. Finally, it is concluded that the bending load capacity of the addendum modified asymmetric spur gear drives were found to be superior (almost 7%) to that of uncorrected asymmetric gear drives due to the increased tooth thickness at the pinion root region.

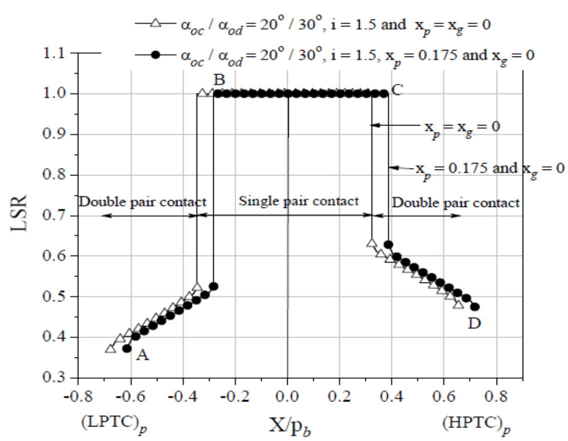


A – Lowest point of tooth contact (LPTC), B – Lowest point of single tooth contact (LPSTC) C – Highest point of single tooth contact (HPSTC), D – Highest point of tooth contact (HPTC)

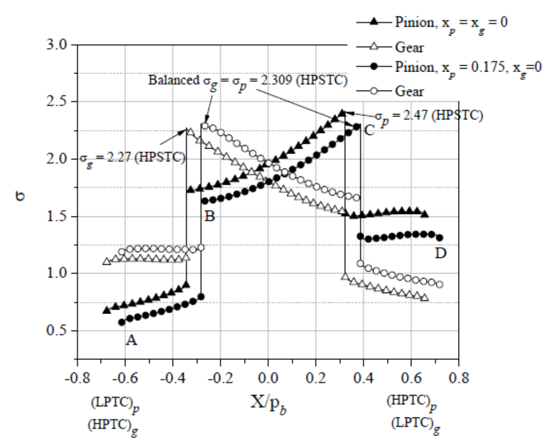
(a) Critical loading points in normal contact ratio (NCR) asymmetric spur gear.



(b) Variation of σ with respect to x_p for asymmetric pinion and gear.



(c) Variation of LSR in one mesh cycle for different x_p (i=1.5, z_p =20 , α_{0c} = 20° and α_{0d} =30°).



(d) Variations of σ in one mesh cycle for different x_p (without and with balanced stress).

Fig.6
Influence of addendum modification on LSR and the respective σ (for m=1, z_p = 20, i=1.5, α_{0c} =20°, α_{0d} =30° and x_g = 0).

4.2 Influence of addendum modification to the gear on balanced fillet stress factor

The respective x_p and the balanced σ are determined by giving a negative addendum modification to the gear (x_g) and the same is shown in Fig. 7. As x_g increases (magnitude values 0, 0.1 and 0.2) for a given gear ratio (i=1.5), x_p (0.175, 0.1125 and 0.04) required to achieve a balanced root stresses decreases and the respective balanced σ (2.309, 2.34 and 2.37) increases. It is also inferred that the unbalanced maximum root stresses between the pinion and gear decrease significantly at $x_p = 0$. From this study, it is concluded that the larger values of addendum modification for pinion (x_p>0) with keeping $x_g = 0$ is a suitable suggestion for achieving the minimum balanced σ between the asymmetric pinion and gear. (For example $x_p = 0.175$, $x_g = 0$, and $\sigma = 2.309$).

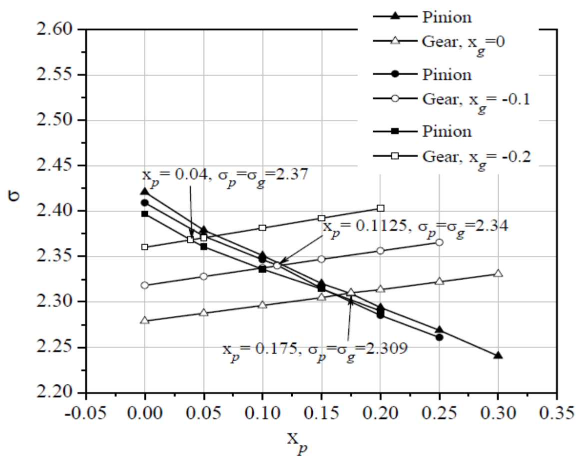


Fig.7
Influence of addendum modification to the gear on balanced σ (for *m*=1, $z_p = 20$, *i*=1.5, α_{0d} =30°, α_{0c} =20° and load at HPSTC).

4.3 Influence of drive side pressure angle on balanced fillet stress factor

For asymmetric spur gear drive with gear ratio equal to 1.5, the addendum modification factors (x_p) required to achieve the balanced σ are determined for different drive side pressure angles and the same is shown in Fig. 8. It is found that as α_{0d} increases the optimum values of x_p increase (0.1075, 0.132 and 0.175) and the respective balanced σ decreases (2.457, 2.373 and 2.309). It is also inferred that the unbalanced root stresses between the pinion and gear increase at $x_p = 0$ due to the increase in α_{0d}. Hence, a larger value of x_p is required to achieve the balanced fillet stresses between the asymmetric pinion and gear which explain the trend of x_p *vs.* α_{0d} as remarked here.

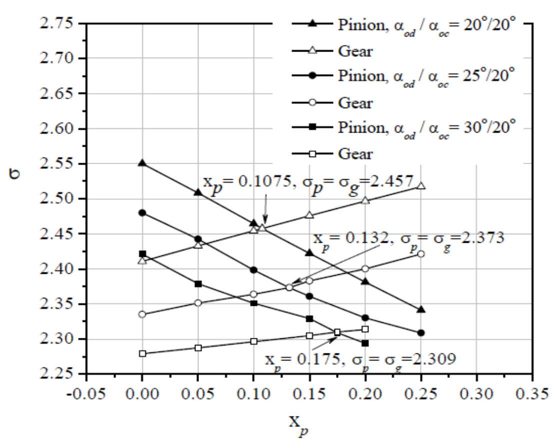


Fig.8
Influence of drive side pressure angle on balanced σ (for *m*=1, $x_g = 0$, $z_p = 20$, *i*=1.5 and load at HPSTC).

4.4 Influence of number of teeth on balanced fillet stress factor

For asymmetric spur gear drive with gear ratio equal to 1.5, the addendum modification factors (x_p) required to achieve the balanced σ are determined for different teeth number in pinion (z_p) and the same is shown in Fig. 9. It is found that as z_p increases the optimum values of x_p (0.175, 0.16 and 0.13) and the respective balanced σ decrease (2.309, 2.167 and 2.0988). It is also inferred that the unbalanced root stresses between the pinion and gear decrease at $x_p = 0$ due to the increase in z_p. Thus, a lower value of x_p is required to achieve the balanced fillet stresses between the asymmetric pinion and gear which explain the trend of x_p *vs*. z_p as mentioned here.

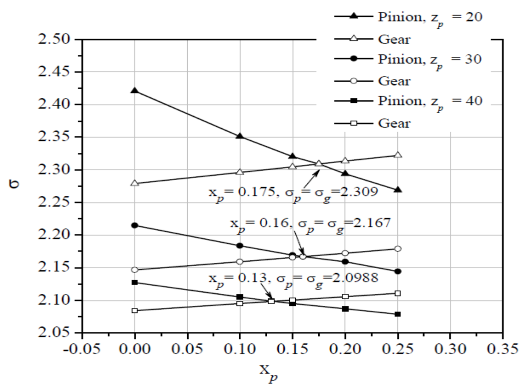


Fig.9
Influence of number of teeth on balanced σ (for *m*=1, $x_g = 0$, *i*=1.5, α_{0d} =30°, α_{0c} =20° and load at HPSTC).

4.5 Influence of gear ratio on balanced fillet stress factor

For asymmetric gear drives with gear ratio equal to 1, 1.5 and 2, the respective addendum modifications to the pinion (x_p) are determined for the balanced σ and the same is shown in Fig. 10. It is found that as gear ratio increases the optimum values of x_p (0, 0.175 and 0.2465) increases and the respective balanced σ decreases (2.452, 2.309 and 2.25). It is also inferred that the unbalanced root stresses between the pinion and gear increase at $x_p = 0$ due to the increase in gear ratio. Hence, a larger value of x_p is required to achieve the balanced fillet stresses between the asymmetric pinion and gear which explain the trend of x_p *vs*. *i* as remarked here.

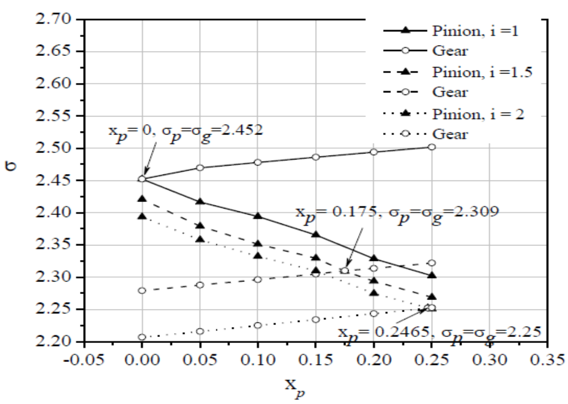


Fig.10
Influence of gear ratio on balanced σ (for *m*=1, $x_g = 0$, z_p = 20, α_{0d} =30° and α_{0c} =20° and load at HPSTC).

5 CONCLUSIONS

If the asymmetric spur gears are designed accurately, it becomes a suitable alternate for symmetric spur gears by providing high bending strength, increased contact load capacity, low weight, reduced noise and longer life. In the present study, the methods for enhancement of bending strength of the asymmetric spur gears of the same size have been investigated and the following observations are made,

1. The tooth thickness relations for addendum modified asymmetric pinion and gear have been derived in this research work.
2. The bending load capacity of the asymmetric spur gear is improved by giving appropriate addendum modification factors to the pinion and gear. The balanced maximum root stress ($\sigma_{tp} = \sigma_{tg} = 23.09\,MPa$)

between the asymmetric pinion and gear is achieved at $x_p = 0.175$ and $x_g = 0$ for the given gear parameters $m = 1$, $i = 1.5$, $\alpha_{0d} = 30^{o}$, $\alpha_{0c} = 20^{o}$ and $z_p = 20$ which is lesser than the unbalanced root stress developed in the asymmetric pinion at $x_p = 0$ ($\sigma_{tp} = 24.7\,MPa$). Hence, the bending load capacity of the addendum modified asymmetric spur gear drives is found to be superior (almost 7%) to that of uncorrected asymmetric gear drives due to the increased tooth thickness at the pinion root region.

3. As the magnitude of x_g increases (0 to 0.2), the bending load capacity of asymmetric spur gear drive decreases approximately 3%. Hence, an increase in x_g is not a suitable suggestion to enhance the balanced bending strength of the asymmetric pinion.
4. The balanced root stresses developed in the asymmetric spur gear drives can be reduced further by increasing the gear and drive parameters. As the drive side pressure angle (α_{0d}) and gear ratio (*i*) increases, the balanced σ decreases. But the magnitude of x_p increases with the respective increase in α_{0d} and *i*, whereas the magnitude of x_p and the respective balanced fillet stress factor decreases with the increase in z_p.
5. As the drive pressure angle (20^{o} to 30^{o}), gear ratio (1.5 to 3) and number of teeth (20 to 40) in pinion increases the bending load capacity of addendum modified asymmetric spur gear drives is further improved by 6%, 8% and 9 %.

APPENDIX A

Determination of tooth thickness at the working circle in the asymmetric spur gear. With the reference from Fig. A, the tooth thickness at the pitch circle of the corrected gear is expressed by

$$t_{0p} = r_{0p}\theta_{0p} \tag{A.1}$$

The involute angles of v_{dp} and v_{cp} can be defined in terms of drive and coast side pressure angles at pitch circle which is given as (Fig. A)

$$inv\,\upsilon_{dp} + inv\,\upsilon_{cp} = \theta_{0p} + inv\,\alpha_{0d} + inv\,\alpha_{0c} \tag{A.2}$$

In a similar way, the involute angles of v_{dp} and v_{cp} can be defined in terms of drive and coast side working pressure angles at the working circle which is given as:

$$inv\,\upsilon_{dp} + inv\,\upsilon_{cp} = \theta_{wp} + inv\,\alpha_{wd} + inv\,\alpha_{wc} \tag{A.3}$$

Equating Eqs. (A.2) and (A.3) we have

$$\theta_{wp} = \theta_{0p} + inv\,\alpha_{0d} + inv\,\alpha_{0c} - inv\,\alpha_{wd} - inv\,\alpha_{wc} \tag{A.4}$$

Substitute the θ_{0p} value (from Eq.(A.1)) in (A.4). Eq. (A.4) is then rewritten as:

$$\theta_{wp} = \left(\frac{t_{0p}}{r_{0p}}\right) + (inv\,\alpha_{0d} + inv\,\alpha_{0c}) - (inv\,\alpha_{wd} + inv\,\alpha_{wc}) \tag{A.5}$$

The thickness of the pinion tooth at the working circle is defined by

$$t_{wp} = r_{wdp}\theta_{wp} \qquad t_{wp} = r_{wdp}\left(\left(\frac{t_{0p}}{r_{0p}}\right) + (inv\,\alpha_{0d} + inv\,\alpha_{0c}) - (inv\,\alpha_{wd} + inv\,\alpha_{wc})\right) \tag{A.6}$$

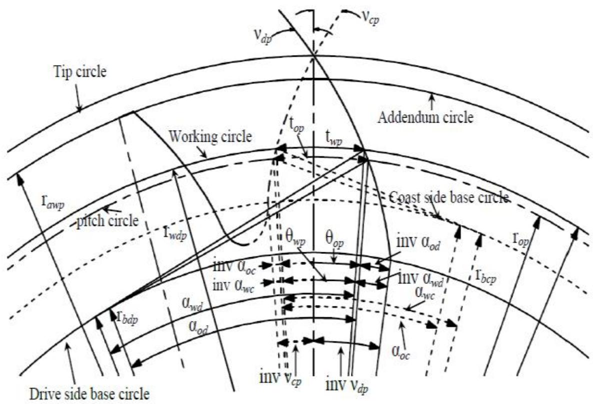


Fig.A
Determination of the tooth thickness at the working circle.

In a similar way, the tooth thickness of the gear at the working circle can also be defined by

$$t_{wg} = r_{wdg}\left(\left(\frac{t_{0g}}{r_{0d}}\right) + (inv\,\alpha_{0d} + inv\,\alpha_{0c}) - (inv\,\alpha_{wd} + inv\,\alpha_{wc})\right) \tag{A.7}$$

REFERENCES

[1] Buckingham E., 1988, *Analytical Mechanics of Gears*, Dover Publications, Inc.

[2] Kapelevich A., 2000, Geometry and design of involute spur gears with asymmetric teeth, *Mechanism and Machine Theory* **35**: 117-130.

[3] Muni D.V., kumar V.S., Muthuveerappan G., 2007, Optimization of asymmetric spur gear drives for maximum bending strength using direct gear design method, *Mechanics based design of structures and machines* **35**: 127-145.

[4] Yang S.C.**,** 2007, Study on internal gear with asymmetric involute teeth, *Mechanism and Machine Theory* **42**: 974-994.

[5] Muni D.V., Muthuveerappan G., 2009, A comprehensive study on the asymmetric internal spur gear drives through direct and conventional gear design, *Mechanics Based Design of Structures and Machines* **37**: 431-461.

[6] Karat F., Ekwaro-Osire S., Cavdar K., Babalik F.C., 2008, Dynamic analysis of involute spur gears with asymmetric teeth, *International Journal of Mechanical Sciences* **50**: 1598-1610.

[7] Costopoulos Th., Spitas V., 2009, Reduction of gear fillet stresses using one side asymmetric teeth, *Mechanism and Machine Theory* **44**: 1524-1534.

[8] Alipiev O., 2011, Geometric design of involute spur gear drives with symmetric and asymmetric teeth using the realized potential method, *Mechanism and Machine Theory* **46**: 10-32.

[9] Sekar P., Muthuveerappan G., 2014, Load sharing based maximum fillet stress analysis of asymmetric helical gear designed through direct design method, *Mechanism and Machine Theory* **80**: 84-102.

[10] Mohan N.A., Senthilvelan S., 2014, Preliminary bending fatigue performance evaluation of asymmetric composite gears, *Mechanism and Machine Theory* **78**: 92-104.

[11] Marimuthu P., Muthuveerappan G., 2016, Design of asymmetric normal contact ratio spur gear drive through direct design to enhance the load carrying capacity, *Mechanism and Machine Theory* **95**: 22-34.

[12] Marimuthu P., Muthuveerappan G., 2016, Investigation of load carrying capacity of asymmetric high contact ratio spur gear based on load sharing using direct gear design approach, *Mechanism and Machine Theory* **96**: 52-74.

[13] Thomas B., Sankaranarayanasamy K., Ramachandra S., Kumar S., 2018, Search method applied for gear tooth bending stress prediction in normal contact ratio asymmetric spur gears, *Proceedings of the Institution of Mechanical Engineers Part C Journal of Mechanical Engineering Science*.

[14] Shuai M., Shuai M., Geoguang J., Jiabai G., 2018, Design principle and modeling method of asymmetric involute internal helical gears, *Proceedings of the Institution of Mechanical Engineers Part C Journal of Mechanical Engineering Science.*