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Effect of Micropolarity on the Propagation of Shear Waves in a Piezoelectric Layered Structure

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ABSTRACT

This paper studies the propagation of shear waves in a composite structure consisting of a piezoelectric layer perfectly bonded over a micro polar elastic half space. The general dispersion equations for the existence of shear waves are obtained analytically in the closed form. Some particular cases have been discussed and in one special case the relation obtained is in agreement with existing results of the classical – Love wave equation. The micro polar and piezoelectric effects on the phase velocity are obtained for electrically open and mechanically free structure. To illustrate the utility of the problem numerical computations are carried out by considering PZT-4 as a piezoelectric and aluminum epoxy as micro polar elastic material. It is observed that the micro polarity present in the half space influence the phase velocity significantly in a particular region. The micro polar effects on the phase velocity in the piezoelectric coupled structure can be used to design high performance acoustic wave devices.

Keywords : Shear wave, Micro polar, Piezoelectric, Dispersion, Phase velocity.

1 INTRODUCTION

A piezoelectric material has the property to generate an electric field when subjected to strain fields and it deforms under an electric field. Due to this electromechanical property, these materials are widely used in many devices like sensors, actuators and transducers. The combination of a piezoelectric member with another material is the study of interest nowadays, because effective control of electromechanical coupling can be achieved by optimal combination of control elements in piezoelectric structure. The use of transducer consisting of thin piezoelectric layer coupled with an elastic substrate is more prevalent in signal processing, low cost and low energy consumption technology. The micro-sensors constituted of piezoelectric composite based upon surface acoustic waves (SAW) have numerous applications in many fields due to their high sensitivity and enhanced electromechanical responses. The dynamic response of these sensors is evaluated by analyzing vibration and wave propagation pattern in the composite structure based on piezoelectric material. Numerous researchers have investigated the propagation

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behavior of shear waves in piezoelectric layered structure in combination with elastic material based on classical models. Keeping in view the micro structure of modern sensing devices this paper tries to present the propagation behavior of shear waves by using micro polar theory instead of classical model.

Many investigations related to surface waves in piezoelectric material are available in literature due to their vast applications in sensing devices. Bleustein [1] investigated that there exist a new type surface waves in piezoelectric materials with no counterpart in purely elastic homogeneous material and derived the expression for velocity of these waves. Mindlin [2] and Tiersten [3] worked on pure piezoelectric materials and studied about the thickness vibrations in these materials. Curtis and Redwood [4] developed the general conditions for the existence of various modes for both Love waves and Bleustein-Gulyaev waves in a piezoelectric composite material. Wang, Quek and Varadan [5] investigated the propagation of Love waves in piezoelectric layer bonded onto a semi-infinite solid medium. The publications Qian et al. [6] and Qian et al. [7] studied the surface wave propagation in piezoelectric material in combination with different type of media. Qian, Jin and Hirose [8] presented the study of transverse surface waves in piezoelectric material with multiple hard metal interlayers. Liu and Wang [9] discussed the propagation of Love waves in a functionally graded piezoelectric layered structure. Liu, Cao and Wang [10] used functionally graded piezoelectric layer to analyze the shear wave propagation in the composite structure. Son and Kang [11] studied surface wave propagation in a piezoelectric coupled plate perfectly bonded with elastic material. Saroj and Sahu [12] investigated reflection of plane waves at traction free surface of pre stressed functionally graded piezoelectric material half space. Arefi [13-14] discussed the wave propagation in a functionally graded piezoelectric nano-rod loaded under electric potential and electro-magnetic potential using non local elasticity model. Arefi and Zenkour recently used piezo materials in sandwich structure to study wave propagation under electric and magnetic effects in the articles [15-20].

The micropolar theory is preferred for describing the behavior of complex media due to its ability to explain size effects on small length scale by taking into consideration the additional degree of freedoms. This theory successfully explains the behavior of materials such as cellular solids, platelet composites, aluminum epoxy, bones, masonry, granular materials, polymers, crystals and many other have complex microstructures. The classical theory of elasticity found to be inadequate to explain the behavior such materials as it considered the material to be continuum in the mathematical sense. In particular, the classical theory does not explain some discrepancies that take place in the case of elastic vibration of high frequency and small wavelength. Voigt [21] tried to remove these shortcomings by introducing additional couple vector to describe the interaction between two particles in a body, which introduced the concept of couple stresses in elasticity. Eringen and Suhubi [22] initiated the general theory linear and nonlinear micro polar elastic continua and Eringen's [23] generalized the classical theory of elasticity by considering three extra rotational degrees of freedom in addition to classical displacement degrees of freedom. In the micro polar theory of elasticity, each element or grain of microstructure is not only translated but also rotated about its center of gravity.

A comprehensive study is available on the phenomenon of wave propagation in micro polar solid due to their practical applicability in the various fields of science and technology such as, seismology, acoustics, aerospace and submarine structures. Eringen [24] discussed about the existence of Rayleigh waves in homogeneous micro polar medium. Singh and Kumar [25] considered an interface between a micro polar elastic and viscoelastic solid and evaluated the amplitude ratio for reflection and refraction of plane waves. Tomar [26] derived the frequency equations for Rayleigh–Lamb wave propagation in a plate of micro polar elastic material with voids. Kumar and Deswal [27] have studied some problems on wave propagation in micro polar media with voids. Midya [28] discussed about the propagation of Love waves in homogeneous micro polar isotropic elastic media consisting of a layer of finite thickness lying over a semi-infinite medium. Kumar, Kaur and Rajvanshi [29] investigated the propagation of Lamb waves in micro polar-generalized thermoelastic solid with two temperatures bordered with layer of inviscid liquid. Kaur, Sharma and Singh [30] investigate the shear wave propagation in vertically heterogeneous viscoelastic layer over a micro polar elastic half-space. Singh, Kumar, Dharmender and Mahto [31] investigated the propagation behavior of transverse waves in the presence of parabolic and rectangular irregularity on the surface of a piezoelectric layer. Kumar and Kaur [32] studied the problem of reflection and transmission of Plane Waves at Micro polar Piezothermoelastic Solids. Recently Kundu et al. [33] studied the propagation of Love waves in heterogeneous micro polar layer over an elastic inhomogeneous media.

In the present article, the composite structure consisting of a homogeneous micro polar elastic material and a piezoelectric layer is considered for surface wave propagation problem. The effects of various parameters of both the materials on the phase velocity of shear waves are studied. The dispersion relation of shear waves in closed form is obtained analytically. Numerical computation for phase velocity is carried out and the results are illustrated graphically.

2 FORMULATION AND SOLUTION OF THE PROBLEM

Consider a layer of piezoelectric material of thickness *h* lying over a micro polar elastic half space as shown in Fig.1. The rectangular Cartesian coordinate system is considered such that the piezoelectric material is polarized along x_3 direction perpendicular to $x_1 - x_2$ plane. It is assumed that the shear wave propagates in the x_1 direction and x_2- axis is positive vertically downward. For Shear wave propagating in the $x_1 - x_2$ plane, the displacement components will be independent of x_3 coordinate.

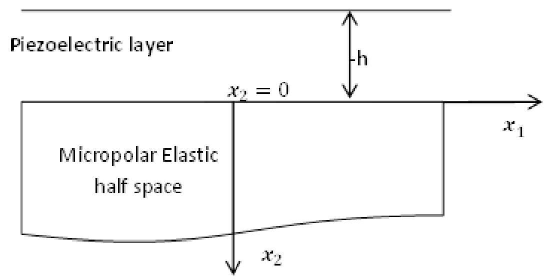

Fig.1
Geometry of the problem.

2.1 Dynamics of Piezoelectric layer

Following Qian et al. [6], the governing constitutive relations and equations of motion for a piezoelectric medium are given as:

$$\begin{cases} \tau_{ij} = c_{ijkl} S_{kl} - e_{kij} E_k \\ D_j = e_{jkl} S_{kl} + \varepsilon_{jk} E_k \end{cases} \tag{1}$$

$$\begin{cases} \tau_{ij,j} = \rho \ddot{u}_i \\ D_{i,i} = 0 \end{cases} \tag{2}$$

where τ_{ij} is the stress tensor, D_j represents electric displacement, c_{ijkl} , e_{kij} and ε_{jk} are the elastic, piezoelectric and dielectric constants respectively. ρ is the density of the piezoelectric material and u_i is the mechanical displacement. S_{kl} and E_k represents the strain tensor and electric field intensity respectively which can be expressed in terms mechanical displacement and electric field potential φ as:

$$\begin{cases} S_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \\ E_i = -\frac{\partial \varphi}{\partial x_i} \end{cases} \tag{3}$$

As expressed by Bleustein [1], shear waves propagates in the x_1 direction and causing displacement in x_3 direction, the components of mechanical displacement (u_1, u_2, u_3) and electrical field potential φ in the upper piezoelectric layer can be written as:

$$u_1 = u_3 = 0,\ u_3 = u_3(x_1, x_2, t),\ \varphi = \varphi(x_1, x_2, t) \tag{4}$$

Using of Eqs.(1)-(4) , we obtained the following equations of motion and constitutive relation for propagation of shear waves in piezoelectric layer

$$c_{44}\left(\frac{\partial^2 u_3}{\partial x_1^{\,2}}+\frac{\partial^2 u_3}{\partial x_2^{\,2}}\right)+e_{15}\left(\frac{\partial^2 \varphi}{\partial x_1^{\,2}}+\frac{\partial^2 \varphi}{\partial x_2^{\,2}}\right)=\rho\frac{\partial^2 u_3}{\partial t^2} \tag{5}$$

$$e_{15}\left(\frac{\partial^2 u_3}{\partial x_1^{\,2}}+\frac{\partial^2 u_3}{\partial x_2^{\,2}}\right)-\varepsilon_{11}\left(\frac{\partial^2 \varphi}{\partial x_1^{\,2}}+\frac{\partial^2 \varphi}{\partial x_2^{\,2}}\right)=0 \tag{6}$$

$$\begin{cases}\tau_{11}=\tau_{22}=\tau_{33}=\tau_{12}=0\\ \tau_{23}=c_{44}\dfrac{\partial u_3}{\partial x_2}+e_{15}\dfrac{\partial \varphi}{\partial x_2}\\ \tau_{31}=c_{44}\dfrac{\partial u_3}{\partial x_1}+e_{15}\dfrac{\partial \varphi}{\partial x_1}\end{cases} \tag{7}$$

$$\begin{cases}D_1=e_{15}\dfrac{\partial u_3}{\partial x_1}-\varepsilon_{11}\dfrac{\partial \varphi}{\partial x_1}\\ D_2=e_{15}\dfrac{\partial u_3}{\partial x_2}-\varepsilon_{11}\dfrac{\partial \varphi}{\partial x_2}\\ D_3=0\end{cases} \tag{8}$$

where, c_{44} , e_{15} and ε_{11} are the elastic , piezoelectric and dielectric coefficients for the piezoelectric medium respectively. The solution of equations for the wave propagation in the x_1 direction can be taken as:

$$\begin{cases}u_3=u_3(x_2)e^{i(kx_1-\omega t)}\\ \varphi=\varphi(x_2)e^{i(kx_1-\omega t)}\end{cases} \tag{9}$$

where k is the wave number and $\omega=kc$ is the circular frequency. Substituting the values of u_3 and φ from Eq. (9) into Eqs. (5) and (6), we obtain

$$\begin{cases}u_3=(A_1\cos\lambda_1 x_2+A_2\sin\lambda_1 x_2)e^{i(kx_1-\omega t)}\\ \varphi=\left[B_1e^{kx_2}+B_2e^{-kx_2}+\dfrac{e_{15}}{\varepsilon_{11}}\left(A_1\cos\lambda_1 x_2+A_2\sin\lambda_1 x_2\right)\right]e^{i(kx_1-\omega t)}\end{cases} \tag{10}$$

where, $\lambda_1^2=k^2\left(\dfrac{c^2}{c_{sh}^2}-1\right)$, $c_{sh}=\sqrt{\dfrac{c_{44}\varepsilon_{11}+(e_{15})^2}{\rho\varepsilon_{11}}}$ is consider as bulk shear wave velocity in the piezoelectric layer. A_1, A_2, B_1 and B_2 are arbitrary constants. The solution (10) of Eqs. (5) and (6) is obtained under the assumption that $c>c_{sh}$.

2.2 Dynamics of micro polar elastic half space

The micro polar theory of elasticity is one of the simplest extension of classical theory of elasticity to analyses the behavior of materials with microstructure. It includes a local rotation of points in addition to translation assumed in classical elasticity. Following Eringen [23] the equation of motion and constitutive relation for a micro polar elastic space are given by

$$\left(\mu+\kappa\right)\nabla^2\vec{u}+\left(\lambda+\mu\right)\nabla(\nabla.\vec{u})+\kappa(\nabla\times\vec{\phi})=\rho^m\frac{\partial^2\vec{u}}{\partial t^2} \tag{11}$$

$$(\alpha+\beta+\gamma)\nabla(\nabla.\vec{\phi})-\gamma\nabla\times(\nabla\times\vec{\phi})+\kappa(\nabla\times\vec{u})-2\kappa\vec{\phi}=\rho^m j\frac{\partial^2\vec{\phi}}{\partial t^2} \tag{12}$$

$$\sigma_{ij}=\lambda u_{l,l}\delta_{ij}+\mu(u_{i,j}+u_{j,i})+\kappa(u_{j,i}-\varepsilon_{ijl}\phi_l) \tag{13}$$

$$m_{ij}=\alpha\phi_{l,l}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}\ ,\ (i,j,l=1,2,3) \tag{14}$$

where $\vec{u}=(u_1^m,u_2^m,u_3^m)$ is the components mechanical displacement in the micro polar elastic medium, ρ^m is the density of the micro polar elastic material, j is the micro inertia, $\vec{\phi}=(\phi_1^m,\phi_2^m,\phi_3^m)$ is the micro rotation vector, λ,μ are Lame's constants. $\kappa,\alpha,\beta,\gamma$ are the additional micro polar material constants, σ_{ij} and m_{ij} are the stress tensor and couple stress tensor respectively. ε_{ijl} is the permutation tensor and δ_{ij} is the Kronecker delta.

The components of displacement and micro rotation of shear wave propagation in x_1-direction, causing displacement in x_3-direction for micro polar elastic half space can be expressed as:

$$\begin{aligned}&u_1^m=u_2^m=0,u_3^m=u_3^m(x_1,x_2,t)\\&\phi_1=\phi_1(x_1,x_2,t),\phi_2=\phi_2(x_1,x_2,t),\phi_3=0\end{aligned} \tag{15}$$

Let us introduced the potential functions ψ and ξ as:

$$\phi_1=\frac{\partial\psi}{\partial x_1}+\frac{\partial\xi}{\partial x_2},\phi_2=\frac{\partial\psi}{\partial x_2}-\frac{\partial\xi}{\partial x_1} \tag{16}$$

with the aid of Eqs. (15)- (16) and Eqs. (11) -(12), we obtain wave equations of shear waves in micro polar elastic half space as follows:

$$\nabla^2u_3^m-c_1\nabla^2\xi=\frac{1}{c_2^2}\frac{\partial^2u_3^m}{\partial t^2} \tag{17}$$

$$\nabla^2\psi-\frac{2c_5^2}{c_3^2+c_4^2}\psi=\frac{1}{c_3^2+c_4^2}\frac{\partial^2\psi}{\partial t^2} \tag{18}$$

$$\nabla^2\xi-\frac{2c_5^2}{c_3^2}\xi+\frac{c_5^2}{c_3^2}u_3^m=\frac{1}{c_3^2}\frac{\partial^2\xi}{\partial t^2} \tag{19}$$

where $c_1=\dfrac{\kappa}{\mu+\kappa},c_2=\sqrt{\dfrac{\mu+\kappa}{\rho^m}},c_3=\sqrt{\dfrac{\gamma}{\rho^m j}},c_4=\sqrt{\dfrac{\alpha+\beta}{\rho^m j}},c_5=\sqrt{\dfrac{\kappa}{\rho^m j}}$

The solutions of Eqs. (17)- (19) are taken as:

$$(\psi,\xi,u_3^m)(x_1,x_2,t)=(\psi,\xi,u_3^m)(x_2)e^{i(kx_1-\omega t)} \tag{20}$$

Putting the values ψ,ξ and u_3^m from (20) in the Eqs. (17)-(19), we get

$$(D^2-r^2)\psi(x_2)=0 \tag{21}$$

$$(D^4 - PD^2 + Q)(\xi, u_3^m)(x_2) = 0 \quad (22)$$

where, $D = \frac{d}{dx_2}, r^2 = k^2 - \frac{\omega^2}{c_3^2 + c_4^2} + \frac{2c_5^2}{c_3^2 + c_4^2}, P = (k^2 - \frac{\omega^2}{c_2^2}) + (k^2 - \frac{\omega^2}{c_3^2}) + \frac{c_5^2}{c_3^2}(2 - c_1)\ , Q = \left(k^2 - \frac{\omega^2}{c_2^2}\right)\left(k^2 - \frac{\omega^2}{c_3^2} + \frac{2c_5^2}{c_3^2}\right) - \frac{c_1 c_5^2 k^2}{c_3^2}$

Using Eq. (20) and the radiation conditions $\psi(x_2), \xi(x_2), u_3^m(x_2) \to 0$ as $x_2 \to \infty$ on the general solutions of the Eqs. (21) and (22), we obtain

$$\psi = (De^{-rx_2})e^{i(kx_1 - \omega t)} \quad (23)$$

$$\xi = (Ee^{-px_2} + Fe^{-qx_2})e^{i(kx_1 - \omega t)} \quad (24)$$

$$u_3^m = (Es_1 e^{-px_2} + Fs_2 e^{-qx_2})e^{i(kx_1 - \omega t)} \quad (25)$$

where

$$p, q = \sqrt{\frac{P \pm \sqrt{P^2 - 4Q}}{2}}\ ,\ \ p^2 + q^2 = P, p^2 q^2 = Q \quad (26)$$

The components of micro rotation obtained from Eq. (16) and Eqs. (23) -(24) are given by

$$\phi_1 = (ikDe^{-rx_2} - pEe^{-px_2} - qFe^{-qx_2})e^{i(kx_1 - \omega t)} \quad (27)$$

$$\phi_2 = (-rDe^{-rx_2} - ik\,(Ee^{-px_2} + Fe^{-qx_2})e^{i(kx_1 - \omega t)} \quad (28)$$

The derivations in Eqs. (23)-(28) are valid under the assumption that $c < c_2$ and $c < c_3$. The wave corresponding to $c > c_2$ represents refracted waves carrying energy away from the layer. These types of waves are not significant as they lose their energy very quickly.

3 BOUNDARY CONDITIONS

For the propagation of shear waves in piezoelectric layer with its surface bonded with a micro polar half space, following boundary conditions should be satisfied

The upper most surface of piezoelectric layer is considered mechanically free and electrically open i.e. at $x_2 = -h$

$$\tau_{23} = 0, D_2 = 0 \quad (29)$$

Continuity condition at the common interface $x_2 = 0$ between the layer and half space can be written as:

$$u_3 = u_3^m, \tau_{23} = \sigma_{23} \quad (30)$$

At the common interface $x_2 = 0$ between piezoelectric layer and micro polar elastic half space, piezoelectric layered medium does not exhibit micro polar property so couple stress must vanishes and consider the piezoelectric layer is electrically shorted at the common interface.

$$m_{21} = 0, m_{22} = 0, \varphi = 0 \quad (31)$$

Using Eqs. (10), (13)-(14) and (23)-(29) in the boundary conditions (30)–(31), we obtain the following equations with seven unknown constants $A_1, A_2, B_1, B_2, D, E, F$.

$$\begin{aligned}
&\bar{c}_{44}\lambda_1(A_1\sin\lambda_1 h+A_2\cos\lambda_1 h)+e_{15}k\,(B_1e^{-kh}-B_2e^{kh})=0\\
&B_1e^{-kh}-B_2e^{kh}=0\\
&A_1=Es_1+Fs_2\\
&\bar{c}_{44}\lambda_1A_2+e_{15}k\,(B_1-B_2)=s_3E+s_4F+is_5D\\
&\frac{e_{15}}{\varepsilon_{11}}A_1+B_1+B_2=0\\
&is_6D+s_7E+s_8F=0\\
&s_9D+is_{10}E+is_{11}F=0
\end{aligned} \tag{32}$$

where , $\overline{c_{44}}=c_{44}+\dfrac{e_{15}^2}{\varepsilon_{11}}$ being the piezo electrically stiffened elastic constant as defined by Curtis and Redwood [4]

$$s_1=\frac{c_3^2}{c_5^2}(k^2-\frac{\omega^2}{c_3^2}+\frac{2c_5^2}{c_3^2}-p^2)\ ,\ s_2=\frac{c_3^2}{c_5^2}(k^2-\frac{\omega^2}{c_3^2}+\frac{2c_5^2}{c_3^2}-q^2)\ ,\ s_3=p(-\mu s_1+\kappa(1-s_1)),$$
$$s_4=q(-\mu s_2+\kappa(1-s_2)),\ s_5=-k\kappa\ ,\ s_6=-(\beta+\gamma)kr\ ,\ s_7=\beta k^2+\gamma p^2,\ s_8=\beta k^2+\gamma q^2,$$
$$s_9=-\alpha k^2+(\alpha+\beta+\gamma)r^2,\ s_{10}=(\beta+\gamma)kp\ ,\ s_{11}=(\beta+\gamma)kq$$

For the nontrivial solutions of system of Eqs. (32), determinant of the coefficient matrix must be equal to zero, so we obtained the following dispersive relation

$$\frac{e_{15}^2k\ \tanh(kh)L_1-\varepsilon_{11}L_2}{\varepsilon_{11}L_1}=\overline{c_{44}}\lambda_1\tan(\lambda_1 h) \tag{33}$$

where, $L_1=s_1s_6s_{11}-s_2s_6s_{10}+s_1s_8s_9-s_2s_7s_9$, $L_2=s_5s_7s_{11}-s_5s_8s_{10}-s_3s_6s_{11}+s_4s_6s_{10}-s_3s_8s_9+s_4s_7s_9$

4 PARTICULAR CASE

In the absence of micro polar constants α,β,γ,j and $\kappa\to 0$ in Eq. (33) we get the following changed values of c_2 and $\dfrac{L_2}{L_1}$ as $c_2=\sqrt{\dfrac{\mu+\kappa}{\rho^m}}\to\sqrt{\dfrac{\mu}{\rho^m}}=\beta_2$, using Eq. (26) the term $\dfrac{L_2}{L_1}\to-\mu k\sqrt{1-\dfrac{c^2}{\beta_2^2}}$. Substituting in (33) we obtain

$$\frac{e_{15}^2tahh(kh)}{\varepsilon_{11}}+\mu\sqrt{1-\frac{c^2}{\beta_2^2}}=\overline{c_{44}}\sqrt{\frac{c^2}{c_{sh}^2}-1}\tan\left(kh\sqrt{\frac{c^2}{c_{sh}^2}-1}\right) \tag{34}$$

Eq. (34) is the dispersion equation for the propagation of shear waves in piezoelectric layer bonded over an elastic half space.

In the absence of piezoelectric parameter e_{15}, $\overline{c_{44}}\to c_{44}$ and let $c_{sh}^2=\dfrac{c_{44}}{\rho}=\beta_1^2$, the Eq. (34) reduces to

$$c_{44}\sqrt{\frac{c^2}{\beta_1^2}-1}\tan\left(kh\sqrt{\frac{c^2}{\beta_1^2}-1}\right)=\mu\sqrt{1-\frac{c^2}{\beta_2^2}} \tag{35}$$

Eq. (35) is the well-known classical equation given by Love [34]

5 NUMERICAL CALCULATIONS AND DISCUSSIONS

To study the behavior of shear wave based upon the dispersion relation (33) numerical computations have been carried out and results are depicted graphically. Values of relevant physical constants used in numerical calculations are listed below, where aluminum epoxy is taken as a micro polar solid and PZT-4 is the piezoelectric material.

For micro polar elastic half space following Gauthier [35] the material properties are given in Table1.

Table1
The physical data for micropolar elastic material.

	$\rho^m (kg/m^3)$	$\lambda (N/m^2)$	$\mu (N/m^2)$	$\kappa (N/m^2)$
	2.19×10^3	7.59×10^3	1.89×10^3	0.0149×10^{10}
Aluminium Epoxy	$\alpha (N)$	$\beta (N)$	$\gamma (N)$	$j(m^2)$
	0.01×10^6	0.015×10^6	0.268×10^6	0.196×10^4

The material constant for piezoelectric material (PZT-4) are taken from Liu, Wang and Wang [36] as

Table 2
The physical data for piezoelectric material.

	$\rho (kg/m^3)$	$e_{15} (C/m^2)$	$c_{44} (N/m^2)$	$\varepsilon_{11} (F/m)$
PZT-4	7.5×10^3	12.7	2.56×10^{10}	6.46×10^{-9}

Fig.2 compares the non-dimensional phase velocity in piezo-micro polar elastic structure with piezo-elastic (without micro polar effects). The non-dimensional phase velocity is taken as c/c_{sh} and it has been plotted against non-dimensional wave number $K_1 = kh/2\pi$. The piezoelectric layer is considering to be of fixed thickness h =5m. The first three modes of phase velocity have been compared and it is readily seen from the plot that the micro polarity has a significant effect on the dispersion curve on first and second mode. In the first mode as clear from the figure, the phase velocity (curve-1) is on higher side due the presence the micro polarity effect as compared to the phase velocity in Piezo-elastic system (curve-2) for $K_1 > 0.2$ and it slowly approach to the phase velocity in piezo-elastic structure with increasing wave number. Similarly, as visible from the plot, the micro polarity effect is quite pertinent on the second mode (curve-3 and curve- 4), while this effect is not observed on third mode in the given range. From the plot, we can say that the internal micro structure of the material as consider in micro polar theory is dominate in a certain range of wave number which is due extra couple stress and rotation of the particle at microscale level. The micro polarity effect favored the phase velocity is in good agreement with reference [30], where the micro polar half space is investigated for shear wave propagation bonded with a viscoelastic layer.

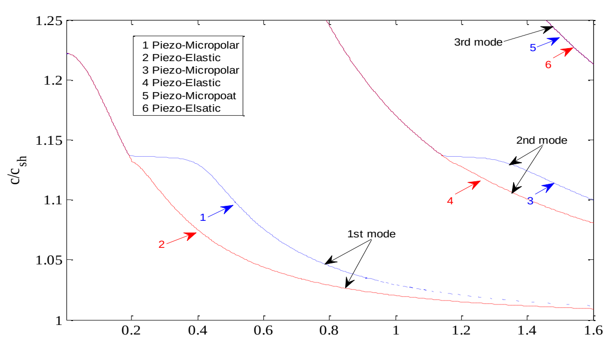

Fig.2
Dispersive curves of different mode for piezoelectric layer and half space with micro polar and without micro polar effects.

Fig. 3 and Fig. 4 represent the effects of non-dimensional micro polar parameter (κ/μ) on the non-dimensional phase velocity in piezo-micro polar elastic structure. In order to show the effect of micro polarity clearly, the fundamental and higher modes are drawn separately in Fig. 3 and Fig. 4 respectively. As obvious from Fig. 3, the first mode phase velocity increases even with small increment in non-dimensional parameter value. As wave number rises, the phase velocity approaches to the bulk shear wave velocity of piezoelectric material. The same effect is observed on second and third mode phase velocity as shown in Fig. 4. Fig.3 and Fig.4 validate that micropolarity plays important role in guiding the behavior of shear waves and this result is also accordance with the obtained results of the reference [30]

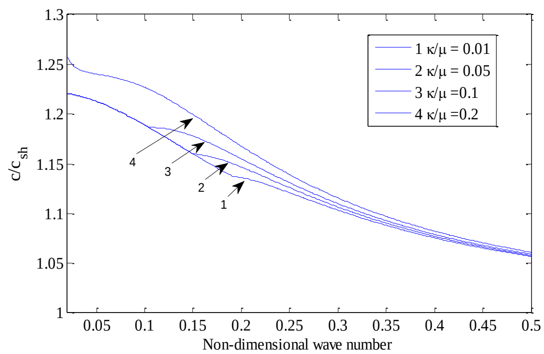

Fig.3
Dimensionless phase velocity c / c_{sh} of first mode against the dimensionless wave number $kh / 2\pi$ in Piezo-Micro polar elastic system for selected values of non-dimensional micropolar parameter (κ / μ) .

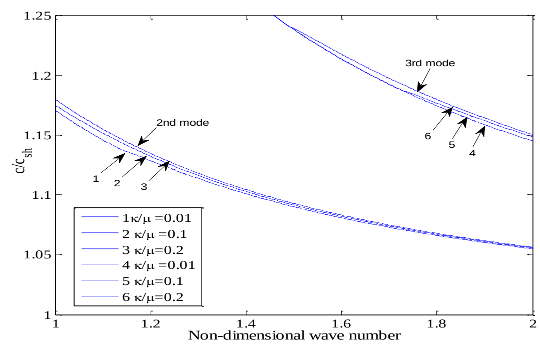

Fig.4
Dimensionless phase velocity c / c_{sh} of second and third mode against the dimensionless wave number $kh / 2\pi$ in Piezo-Micro polar elastic system for selected values of non-dimensional micro polar parameter (κ / μ) .

Fig. 5 present the piezoelectric parameter effects on the non-dimensional phase velocity in piezo-micro polar elastic and piezo-elastic structure. The difference in phase velocities due to different piezoelectric parameter values is quite visible for initial small wave number as shown in Fig. 5. For small wave number the first mode phase velocity is on higher side with increasing piezoelectric parameter values in both piezo-elastic as well as piezo-micro polar structure but opposite behavior is observed as wave number increases (curve-3 and curve-4) in case of piezo-micro polar structure and this effect die out with further increment in wave number. The phase velocity in piezo-micro polar structure is on higher side as compared to piezo-elastic structure up to a certain wave number. Similar effects of piezoelectric coefficient are noticed on second mode but this influence decreases successively on higher modes. Fig.6 represents the effects of dielectric coefficient on phase velocity in piezo-elastic and piezo-micro polar elastic structure. As clear from plot, the phase velocity of first and second mode decreases with increase in the values of dielectric coefficient for initial small wave number and then increases with increase in wave number values. The change in dielectric coefficient values affects phase velocity in second modes in the same way as in case of first mode phase velocity. The effects of piezoelectric and dielectric parameters on phase velocity are also explored in the reference [31], which are quite similar for higher wave number values.

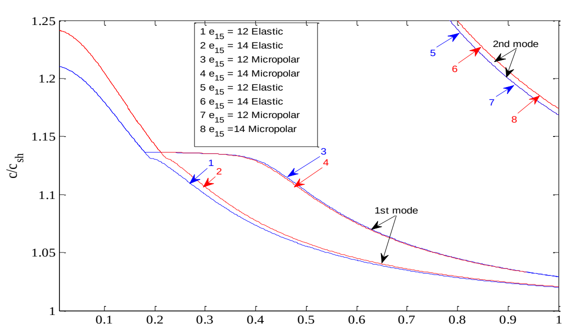

Fig.5
Phase velocity c / c_{sh} of first mode against $kh / 2\pi$ in Piezo-Micro polar elastic and Piezo-Elastic system for different values of piezoelectric parameter e_{15} .

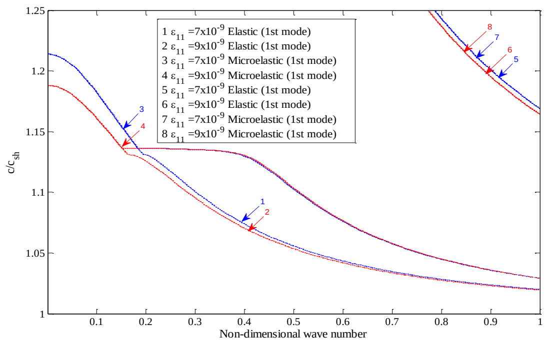

Fig.6
Phase velocity c/c_{sh} of first and second mode against $kh/2\pi$ in Piezo-Micro polar and Piezo-Elastic system for different values of dielectric coefficient ε_{11} .

6 CONCLUSIONS

This paper presents the study of SH wave propagation in piezoelectric layer lying over a micro polar elastic solid. A general dispersion equation of the wave is derived and the relation obtained as a particular case is in agreement with the classical equation given by Love. Numerical computations are performed to evaluate the phase velocity as a function of wave number and it is observed that the micro polar and piezoelectric effects influence the phase velocity. From the graphical analysis, we conclude that

- Shear waves exists in the composite structure consisting of a piezoelectric layer perfectly bonded over a micro polar elastic material and there is significant effect of micro polarity on the propagation of shear waves.
- The presence of micro polarity in the piezo-elastic structure increases the phase velocity in certain range when calculated as a function of non-dimensional wave number. It is observed that the micro polarity effects dominated in a particular region and then vanishes with increasing wave number.
- The phase velocity is appreciably depending upon the layer of piezoelectric material in the assumed model as significant variations in non-dimensional wave speed is observed for different values of piezoelectric and dielectric coefficient.

The micro polar materials find its applications in modern engineering smart material structures. The results obtained in this study may be useful for developing the new class of sensors with higher sensitivity and improved response under certain boundary conditions and suitably selection of micro polar and piezoelectric parameters.

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