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Analysis of Thermal-Bending Stresses in a Simply Supported Annular Sector Plate

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ABSTRACT

The present article deals with the analysis of thermal-bending stresses in a heated thin annular sector plate with simply supported boundary condition under transient temperature distribution using Berger's approximate methods. The sectional heat supply is on the top face of the plate whereas the bottom face is kept at zero temperature. In this study, the solution of heat conduction is obtained by the classical method. The thermal moment is derived on the basis of temperature distribution, and its stresses are obtained using thermally induce resultant moment and resultant forces. The numerical calculations are obtained for the aluminium plate in the form of an infinite series involving Bessel functions, and the results for temperature, deflection, resultant bending moments and thermal stresses have been illustrated graphically with the help of MATHEMATICA software.

Keywords: Heat conduction; Annular sector; Thin plate; Thermal deflection; Stresses.

1 INTRODUCTION

IN structural mechanics and many branches of engineering mechanics and aeronautics, the widespread use of plates and shells are made to deal with thermal stresses, deformations, buckling and vibrations of plates and shells for which there arises the need for reliance upon different methods of analysis. Analytical techniques in dealing with such problems have severe limitations because of difficulties in deriving closed-form solutions of nonlinear differential equations. As far as theoretical mechanics is concerned, the solution methods for nonlinear differential equations play a very crucial role in many problems using such equations. Many sector plates are subjected to high pressure and thermal load may results in nonlinear loaded deflection relationship due to large deformations of the plate. A short history of the research work associated with the significant deflection insights various approximate methods like the Ritz energy method, Galerkin Method, finite element models and perturbation theory to solve the system. For the analysis of large deflections of plates von Karman's coupled nonlinear partial differential equations have extensively been employed by many earlier researchers. Von Karman's equations are difficult to deal with because of its coupled nonlinearity and yet no general solutions of these equations are known. However,

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approximate and different numerical and computational methods have been adopted for the solution of such large deflection analysis of plates and shells. Few authors [1-3] employed von Karman equations to the nonlinear analysis behaviour of thin plates, isotropic and orthotropic under mechanical and other kinds of loading with the inclusion of curvature and thermal loading parameter. Due to the mathematical complexities, many researchers have resorted to the use of different computational and numerical techniques to deal with problems of nonlinear dynamic analysis of plates and shells under thermal stresses. To simplify the nonlinear differential equations, Berger [4] proposed quasi-linear partial differential equations in the decoupled form with extension to thermal stresses for plates and shallow shells. This method is based on the neglect of e_2, the second invariant of the middle surface strains, in the expression corresponding to the total potential energy of the system. Although there doesn't seem to be any physical justification for this approximation, comparisons of the outcomes with known solutions indicate that for a broad scope of problems, Berger's approach yields adequately accurate results [5]. Basuli [6] has obtained the equation of equilibrium for the large deflection by Berger's approximation under uniform pressure load and thermal heating under steady-state temperature distribution. Some authors [7, 8] have obtained a significant deflection based on the total strain energy concept devised by Berger. In all, the results obtained by the Berger's method which is not referred the present manuscript was well describes by Nowinski [9] for the edges of plates restrained from in-plane motion. An application of this technique to the cases of orthotropic plates has been offered by Iwinski and Nowinski [10] and further boundary value problems associated with circular and rectangular plates have been investigated by Nowinski [11-13]. Okumura et al. [14] investigated thermal-bending stresses in an annular sector plate with the moderate thickness is carried out by the theory of moderately thick plates, together with the thermoelastic displacement potential using classical methods. Wang and Lim [15] established the exact relationships between the bending solutions of sectorial plates based on the Kirchhoff (or classical thin) plate theory and the Mindlin plate theory. Golmakani [16] has performed a study on stiffened annular functionally graded sector plates and ring-stiffened circular plates using dynamic relaxation numerical method combined with the finite difference discretization technique. Eren [17] investigated both horizontal and most considerable vertical deflections for simply supported beams under uniformly distributed load. Sitar et al. [18] discussed a relatively simple method for determining significant deflection using Euler– Bernoulli and large displacement theory and solved it numerically by Runge–Kutta–Fehlberg integration and Newton method. Bakker [19] describes an approximate analytical method to determine the large-deflection behaviour of rectangular simply supported thin plates under transverse loading. The method is based on a simple trial function to describe the shape of the initial and total deflections of the plate, corresponding to the first buckling mode shape of the plate when subjected to uniform in-plane compression. Jang [20] discussed the semi-analytical procedure for moderately large deflections of an infinite non-uniform static beam resting on a nonlinear elastic foundation. Very recently, Choi [21] developed a geometrically nonlinear finite element program to study the induced cylinder stress due to internal pressure by considering shear deformation theory of a doubly curved shell, von Karman's large deflection theory, as well as a newly proposed strain–displacement relation including initial strain terms. Bhad [22] investigate the thermoelastic effect on the elliptical plate during large deflection while heating with non-stationary temperature distribution. In previously referred papers, their objective was to maximize the load carrying capacity of the structural support either by locating the supports or by changing stiffness. Even few reports have been based on the calculus of variation, Hamilton's principle, and many others performed large deflection studies with different objects. Thin plates and shells of regular polygonal and irregular shapes made of isotropic, orthotropic and sandwich materials are often subjected to different kinds of mechanical and thermal loading. Therefore, as a result, such structural components are prone to deformations, bucking and vibrations for which proper analysis are required to be made and of great interest to designers, engineers, scientists and researchers. Hence, to the best of authors' knowledge, there exists scarce concentration on this topic of research in which large deflection is taken into consideration the thermal moments in the strain energy equation. Owing to the gap in existing literature, the authors have been motivated to conduct this study. In this paper, an attempt is made to obtain the large deflection of a thin annular sector plate subjected to arbitrary temperature on the upper face with the lower face and simply supported edges at zero temperature using the method of Berger and Basuli. In the formulations, the equation of equilibrium is modified by introducing a thermally induced resultant moment showing little difference with the previous one [6]. The theoretical calculation has been considered using the dimensional parameter, whereas graphical calculations are carried out using the dimensionless parameter. The success of this novel research mainly lies on the new mathematical procedures that present a much more straightforward approach for optimization of the design in terms of material usage and performance in engineering problem, particularly in the determination of thermoelastic behaviour in annular sector plate engaged as the foundation of pressure vessels, furnaces, etc.

2 FORMULATION OF THE PROBLEM

Consider a thin annular sector bounded in cylindrical coordinates (r,θ,z), occupying the space $D=\{(r,\theta,z)\in R^3: a<r<b,\ -\gamma/2\le\theta\le\gamma/2,\ -\ell/2<z<\ell/2\}$. The center of the plate in the middle surface is taken as the origin.

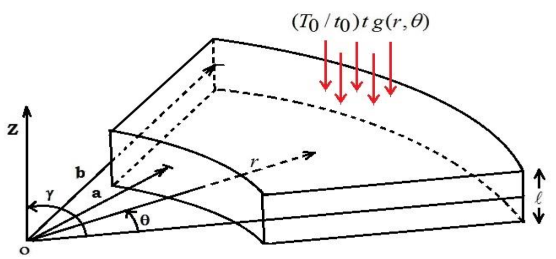


Fig.1
Sector geometry of the problem.

We assume that the temperature distribution is given by

$$T(r,\theta,z,t)=\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}f_q(t)C_m(\alpha_{mq}r)\cos m\theta[\sinh\alpha_{mq}(z+\ell/2)] \tag{1}$$

In which $C_m(\alpha_{mq}r)$ denotes a cylinder function of order *m* as defined in the form

$$C_m(\alpha_{mq}r)=J_m(\alpha_{mq}r)-\varepsilon_{mq}Y_m(\alpha_{mq}r) \tag{2}$$

$$\varepsilon_{mq}=\frac{J_m(\alpha_{mq}a)}{Y_m(\alpha_{mq}a)}\qquad m=p\pi/\gamma\quad(p=1,3,5..) \tag{3}$$

The heat conduction of the circular sector is described as:

$$T_{,t}/\kappa=T_{,r}(rT_{,r})/r+T_{,\theta\theta}/r^2+T_{,zz} \tag{4}$$

The boundary conditions can be defined as:

$$T(r,\theta,z,t)\big|_{r=a}=0\quad,\ T(r,\theta,z,t)\big|_{r=b}=0 \tag{5}$$

$$T(r,\theta,z,t)\big|_{\theta=\pm\gamma/2}=0 \tag{6}$$

$$T(r,\theta,z,t)\big|_{z=-\ell/2}=0 \tag{7}$$

$$\begin{aligned}T(r,\theta,z,t)\big|_{z=\ell/2}&=(T_0/t_0)t\,g(r,\theta)\qquad\text{for}\quad 0\le t\le t_0\\&=T_0\,g(r,\theta)\qquad\text{for}\quad t\ge t_0\end{aligned} \tag{8}$$

In which $T(r,\theta,z,t)$ is the temperature distribution at any time parameter t, T_0 is the temperature at $t=0$, $g(r,\theta)$ is the sectional heat supply on the upper face, thermal diffusivity is taken as $\kappa=\lambda/\rho C_V$, in which λ is the thermal conductivity of the material, ρ is the density and C_V is the calorific capacity. From the second equation of boundary condition (5), we have α_{mq} as the roots of the transcendental equation $J_m(\alpha_{mq}b)Y_m(\alpha_{mq}a)-J_m(\alpha_{mq}a)Y_m(\alpha_{mq}b)=0$.

We assume the Fourier-Bessel series as [14]

$$g(r,\theta)=\sum_{p=1}^{\infty}\sum_{q=1}^{\infty} s_{pq}\, C_m(\alpha_{mq} r)\cos m\theta \tag{9}$$

In which

$$s_{pq}=\frac{\int_{-\gamma/2}^{\gamma/2}\int_a^b g(r,\theta)\, r\, C_m(\alpha_{mq} r)\cos m\theta\, dr\, d\theta}{\int_{-\gamma/2}^{\gamma/2}(\cos m\theta)^2\, d\theta \int_a^b r\,(C_m(\alpha_{mq} r))^2\, dr} \tag{10}$$

Using Eq. (1) in Eq. (4), one obtains

$$\frac{f_q(t)_{,t}}{f_q(t)}=-\kappa(1-\alpha_{mq}^2)m^2/r^2 \tag{11}$$

On integrating Eq. (11), one obtains

$$f_q(t)=A_q \exp\{[-\kappa(1-\alpha_{mq}^2)\, m^2/r^2]\, t\} \tag{12}$$

In which the constant A_q is to be determined by the nature of the temperature prescribed on the upper face and prime comma denotes differentiation with respect to the variable.

Now initially we assume

$$(T_0/t_0)t\;\; g(r,\theta)=\gamma\sum_{q=1}^{\infty} A_q\, C_m(\alpha_{mq}\, r) \tag{13}$$

Then from the theory of Bessel function,

$$\begin{aligned}\gamma A_q\int_a^b r\,[C_m(\alpha_{mq}\, r)]^2 d\theta dr &= (T_0/t_0)\, t\int_a^b g(r,\theta)\; r\,[C_m(\alpha_{mq}\, r)]dr\\ &= (T_0/t_0)\, t\int_a^{r_0} r\,[C_m(\alpha_{mq}\, r)]dr\end{aligned}$$

Since $g(r,\theta)=\begin{cases}1, & a<r<r_0\\ 0, & \text{otherwise}\end{cases}$ hence,

$$A_q=(T_0/t_0)\, t\int_a^{r_0} r\;[C_m(\alpha_{mq} r)]dr/N_q \tag{14}$$

In which $N_q=\gamma\int_a^b r\;[C_m(\alpha_{mq} r)]^2 dr$. By substituting the Eq. (12) in Eq. (1) with the value of constant A_q derived in Eq. (14), obtained the required expression for temperature distribution.

2.1 Thermal deflection

The equation of equilibrium for the large deflection of a heated plate as derived by Basuli [3] on the basis of Berger's approximations is given under uniform load and stationary distribution [i.e. $T(r,\theta,z)=T_0(r,\theta)+g(z)T(r,\theta)$] as:

$$D\,\nabla^2(\nabla^2-\beta_1^2)w(r,\theta,t)=-\frac{E\alpha f(\ell)}{1-\upsilon}\nabla^2 T \tag{15}$$

where $f(\ell) = \int_{-\ell/2}^{\ell/2} z\, g(z) dz$ and β_1^2 is a normalising constant of integration.

In deriving the large deflection equation of a heated plate, we have generalized the aforesaid equation of equilibrium for the transient temperature distribution, as:

$$\nabla^2(\nabla^2 - \beta_1^2) w(r,\theta,t) = -\frac{\nabla^2 M_T}{D(1-\upsilon)} \tag{16}$$

where $w(r,\theta,t)$ is the normal transverse deflection along the z-direction, ∇^2 indicates the two-dimensional Laplacian operator in (r, θ), the constant υ denotes the Poisson's ratio of the plate, $D = E\ell^3/12(1-\upsilon^2)$ is the flexural stiffness of the plate and β_1^2 is to be determined from

$$u,_r + u/r + [w(r,\theta,t),_r]^2/2 + v,_\theta/r + [w(r,\theta,t),_\theta]^2/2r^2 = \beta_1^2\,\ell^2/12 + (1+\upsilon)\alpha N_T \tag{17}$$

The result of the above heat conduction gives thermally induced resultant moment and resultant force as: [23]

$$M_T = \alpha E \int_{-\ell/2}^{\ell/2} z\, T(r,\theta,z,t) dz \tag{18}$$

$$N_T = \alpha E \int_{-\ell/2}^{\ell/2} T(r,\theta,z,t) dz \tag{19}$$

with α as the coefficient of linear thermal expansion and E symbolise Young's Modulus of the material of the plate, respectively.

Eqs. (19) and (20) have to be solved for heated thin simply supported annular sector plate along the edges for which the boundary conditions are

$$w(r,\theta,t)\big|_{r=a} = w(r,\theta,t)_{,r}\big|_{r=a} = 0 \tag{20}$$

$$w(r,\theta,t)\big|_{r=b} = w(r,\theta,t)_{,r}\big|_{r=b} = 0 \tag{21}$$

2.2 Thermal stresses

Now the equations of resultant forces $(N_{ij}, i, j = r, \theta)$, resultant bending moments per unit width (M_{ij}) are defined as:

$$N_r = N_\theta = N_{r\theta} = 0 \tag{22}$$

$$\left.\begin{aligned} M_r &= -D\{w(r,\theta,t)_{,rr} + \upsilon[w(r,\theta,t)_{,r}/r + w(r,\theta,t)_{,\theta\theta/r^2}]\} - M_T/(1-\upsilon) \\ M_\theta &= -D\{\upsilon w(r,\theta,t)_{,rr} + [w(r,\theta,t)_{,r}/r + w(r,\theta,t)_{,\theta\theta/r^2}]\} - M_T/(1-\upsilon) \\ M_{r\theta} &= D(1-\upsilon) w(r,\theta,t)_{,r\theta}/r \end{aligned}\right\} \tag{23}$$

The aforementioned first equation of Eq. (23) must satisfy for simply supported plate

$$M_r\big|_{r=a} = 0, \text{ for all } \theta \text{ in } -\gamma/2 \le \theta \le \gamma/2 \tag{24}$$

The thermal stress components in terms of resultant forces and resultant moments are given as: [24]

$$\left.\begin{aligned}\sigma_{rr} &= \frac{1}{\ell}N_r + \frac{12z}{\ell^3}M_r + \frac{1}{(1-\upsilon)}\left(\frac{1}{\ell}N_T + \frac{12z}{\ell^3}M_T - \alpha ET\right)\\ \sigma_{\theta\theta} &= \frac{1}{\ell}N_\theta + \frac{12z}{\ell^3}M_\theta + \frac{1}{(1-\upsilon)}\left(\frac{1}{\ell}N_T + \frac{12z}{\ell^3}M_T - \alpha ET\right)\\ \sigma_{r\theta} &= \frac{1}{\ell}N_{r\theta} - \frac{12z}{\ell^3}M_{r\theta}\end{aligned}\right\} \tag{25}$$

The Eqs. (1) to (25) constitute the mathematical formulation of the problem under consideration.

3 SOLUTION OF THE PROBLEM

On substituting Eq. (1) in Eq. (18), one obtains the thermal moment as:

$$M_T = \alpha E \sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\{A_q \exp\{[-\kappa(1-\alpha_{mq}^2)m^2/r^2]t\}C_m(\alpha_{mq}r)\cos m\theta \times [\alpha_{mq}\ell(1+\cosh\alpha_{mq}\ell)-2\sinh\alpha_{mq}\ell]/2\alpha_{mq}^2\} \tag{26}$$

Using Eq. (1) and (19), one yield

$$N_T = \alpha E \sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\{A_q \exp\{[-\kappa(1-\alpha_{mq}^2)m^2/r^2]t\}C_m(\alpha_{mq}r)\cos m\theta \times (-1+\cosh\alpha_{mq}\ell)/\alpha_{mq}\} \tag{27}$$

Now substituting Eq. (26) in (16), one obtains the equation of equilibrium as:

$$\begin{aligned}&\nabla^2(\nabla^2-\beta_1^2)w(r,\theta,t) = -\nabla^2\frac{\alpha E}{D(1-\upsilon)}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\{A_q \exp\{[-\kappa(1-\alpha_{mq}^2)m^2/r^2]t\}\times C_m(\alpha_{mq}r)\cos m\theta\\ &[\alpha_{mq}\ell(1+\cosh\alpha_{mq}\ell)-2\sinh\alpha_{mq}\ell]/2\alpha_{mq}^2\}\end{aligned} \tag{28}$$

To solve the completeness of conjugate harmonic axisymmetric equations, that is, Laplacian and Helmholtz type operator, Wang [25] have generalized the Almansi's theorem concerning the representation of the general solution of the equation $\nabla^{2n}=0.$ If $w_1(r,\theta,t)$ and $w_2(r,\theta,t)$ are harmonic functions, then it must satisfy the homogeneous equations

$$\nabla^2 w_1(r,\theta,t) = 0 \tag{29}$$

$$(\nabla^2-\beta_1^2)w_2(r,\theta,t) = 0 \tag{30}$$

The periodic solution of Eqs. (29) and (30) which are axisymmetric can be obtained as:

$$w_1(r,\theta,t) = \sum_{p=1}^{\infty}\sum_{q=1}^{\infty}[B_{pq}J_m(\alpha_{mq}r)+C_{pq}Y_m(\alpha_{mq}r)]\cos m\theta \tag{31}$$

$$w_2(r,\theta,t) = \sum_{p=1}^{\infty}\sum_{q=1}^{\infty}D_{pq}C_m(\alpha_{mq}r)\cos m\theta \tag{32}$$

In which, B_{pq}, C_{pq} and D_{pq} are the arbitrary constants.

Therefore, the conjugate complementary function for Eq. (30), which is finite at the origin, is given by

$$w(r,\theta,t)=\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}[B_{pq}J_m(\alpha_{mq}r)+C_{pq}Y_m(\alpha_{mq}r)+D_{pq}C_m(\alpha_{mq}r)]\cos m\theta \tag{33}$$

The particular solution of the Eq. (28) is given as:

$$w(r,\theta,t)=Z_{pq}\cos m\theta \tag{34}$$

In which

$$Z_{pq}=-\frac{\alpha E}{2D(1-\upsilon)}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\{A_q\exp\{[-\kappa(1-\alpha_{mq}^2)m^2/r^2]t\}\times C_m(\alpha_{mq}r)[\alpha_{mq}\ell(1+\cosh\alpha_{mq}\ell)-2\sinh\alpha_{mq}\ell]/(\alpha_{mq}^2+\beta_1^2)\} \tag{35}$$

Now using boundary conditions Eq. (20) and the first equation of Eq. (21), one obtains the constants B_{pq}, C_{pq} and D_{pq} as:

$$\begin{aligned}B_{pq}&=Z_{pq}(a)[Y_m(b)C'_m(\alpha_{mq}a)-C_m(\alpha_{mq}b)Y'_m(a)]-Y_m(a)[Z_{pq}(b)C'_m(\alpha_{mq}a)-C_m(\alpha_{mq}b)Z'_{pq}(a)]\\&\quad+C_m(\alpha_{mq}a)[Z_{pq}(b)Y'_m(a)-Y_m(b)Z'_{pq}(a)]/Q_{pq}\end{aligned} \tag{36}$$

$$\begin{aligned}C_{pq}&=J_m(a)[Z_{pq}(b)C'_m(\alpha_{mq}a)-C_m(\alpha_{mq}b)Z'_{pq}(a)]-Z_{pq}(a)[J_m(b)C'_m(\alpha_{mq}a)-C_m(\alpha_{mq}b)J'_m(a)]\\&\quad+C_m(\alpha_{mq}a)[J_m(b)Z'_{pq}(a)-Z_{pq}(b)J'_m(a)]/Q_{pq}\end{aligned} \tag{37}$$

$$\begin{aligned}D_{pq}&=J_m(a)[Y_m(b)Z'_{pq}(a)-Z_{pq}(b)Y'_m(a)]-Y_m(a)[J_m(b)Z'_{pq}(a)-Z_{pq}(b)J'_m(a)]\\&\quad+Z_{pq}(a)[J_m(b)Y'_m(a)-Y_m(b)J'_m(a)]/Q_{pq}\end{aligned} \tag{38}$$

In which

$$\begin{aligned}Q_{pq}&=J_m(a)[Y_m(b)C'_m(\alpha_{mq}a)-C_m(\alpha_{mq}b)Y'_m(a)]-Y_m(a)[J_m(a)C'_m(\alpha_{mq}a)-C_m(\alpha_{mq}b)J'_m(a)]\\&\quad+C_m(\alpha_{mq}a)[J_m(b)Y'_m(a)-Y_m(b)J'_m(a)]\end{aligned}$$

Finally by substituting the values of B_{pq}, C_{pq} and D_{pq} in Eq. (33) results in the required expression for the desired thermal deflection. Using the equation of thermal deflection in Eq. (23), one obtains the equations for the bending moments as:

$$\begin{aligned}M_r&=\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\Big\{-\frac{D}{2}\cos m\theta\Big[\alpha E A_q\exp(-km^2(1-\alpha_{mq}^2)t/r^2)\times C_m(\alpha_{mq}r)[\alpha_{mq}\ell(1+\cosh(\alpha_{mq}\ell))\\&-2\sinh(\alpha_{mq}\ell)]/\alpha_{mq}^2(-1+\upsilon)+2\upsilon[-m^2(D_{pq}C_m(\alpha_{mq}r)+B_{pq}J_m(\alpha_{mq}r)+C_{pq}Y_m(\alpha_{mq}r))+\alpha_{mq}r(D_{pq}C'_m(\alpha_{mq}r)\\&+B_{pq}J'_m(\alpha_{mq}r)+C_{pq}Y'_m(\alpha_{mq}r))]/r^2+2\alpha_{mq}^2(D_{pq}C''_m(\alpha_{mq}r)+B_{pq}J''_m(\alpha_{mq}r)+C_{pq}Y''_m(\alpha_{mq}r))\Big]\Big\}\end{aligned} \tag{39}$$

$$\begin{aligned}M_\theta&=\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\Big\{-\frac{D}{2}\cos m\theta\Big[\alpha E A_q\exp(-km^2(1-\alpha_{mq}^2)t/r^2)\times C_m(\alpha_{mq}r)[\alpha_{mq}\ell(1+\cosh(\alpha_{mq}\ell))\\&-2\sinh(\alpha_{mq}\ell)]/\alpha_{mq}^2(-1+\upsilon)-2m^2(D_{pq}C_m(\alpha_{mq}r)+B_{pq}J_m(\alpha_{mq}r)+C_{pq}Y_m(\alpha_{mq}r))/r^2+2\alpha_{mq}r(D_{pq}C'_m(\alpha_{mq}r)\\&+B_{pq}J'_m(\alpha_{mq}r)+C_{pq}Y'_m(\alpha_{mq}r))/r+2\alpha_{mq}^2\upsilon(D_{pq}C''_m(\alpha_{mq}r)+B_{pq}J''_m(\alpha_{mq}r)+C_{pq}Y''_m(\alpha_{mq}r))\Big]\Big\}\end{aligned} \tag{40}$$

$$M_{r\theta}=\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\Big\{D\,m\,\alpha_{mq}(-1+\upsilon)\sin m\theta\,(D_{pq}C'_m(\alpha_{mq}r)+B_{pq}J'_m(\alpha_{mq}r)+C_{pq}Y'_m(\alpha_{mq}r))/r\Big\} \tag{41}$$

3.1 Solution for the associated thermal stresses

The resulting equations of stresses are obtained by substituting the Eqs. (1), (22), (26), (27) and (39)-(40) in Eq. (25) as:

$$\sigma_{rr}=\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\Big\{\frac{1}{r^2\ell^3}\cos m\theta\Big[C_m(\alpha_{mq}r)\Big(12D_{pq}D\,m^2z\,\alpha_{mq}^2(-1+\upsilon)\upsilon+\alpha E\,r^2A_q\exp(-k\,m^2(1-\alpha_{mq}^2)t/r^2)$$
$$[-\ell(\ell+6(1+D)z)\times\alpha_{mq}\cosh(\alpha_{mq}r)+12(1+D)z)\alpha_{mq}\sinh(\alpha_{mq}r)+\alpha_{mq}\ell\times(\ell-6(1+D)z+\alpha_{mq}\ell^2$$
$$\sinh(\alpha_{mq}(z+\ell/2)))]\Big)/\alpha_{mq}^2(-1+\upsilon)-12D\,z[-B_{pq}m^2\upsilon J_m(\alpha_{mq}r)-C_{pq}m^2\upsilon Y_m(\alpha_{mq}r)+\alpha_{mq}r(D_{pq}\upsilon C'_m(\alpha_{mq}r) \tag{42}$$
$$+B_{pq}\upsilon J'_m(\alpha_{mq}r)+C_{pq}\upsilon Y'_m(\alpha_{mq}r)+D_{pq}\alpha_{mq}rC''_m(\alpha_{mq}r)+B_{pq}\alpha_{mq}rJ''_m(\alpha_{mq}r)+C_{pq}\alpha_{mq}rY''_m(\alpha_{mq}r))]\Big]\Big\}$$

$$\sigma_{\theta\theta}=\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\Big\{\frac{1}{r^2\ell^3}\cos m\theta\Big[C_m(\alpha_{mq}r)\Big(12D_{pq}D\,m^2z\,\alpha_{mq}^2\times(-1+\upsilon)+\alpha E\,r^2A_q\exp(-k\,m^2(1-\alpha_{mq}^2)t/r^2)$$
$$\times[-\ell(\ell+6(1+D)z)\alpha_{mq}\cosh(\alpha_{mq}r)+12(1+D)z\sinh(\alpha_{mq}r)+\alpha_{mq}\ell(\ell-6(1+D)z+\sinh(\alpha_{mq}(z+\ell/2))$$
$$\times\alpha_{mq}\ell^2/\alpha_{mq}^2(-1+\upsilon))]\Big)-12D\,z[-B_{pq}m^2J_m(\alpha_{mq}r)-C_{pq}m^2Y_m(\alpha_{mq}r)+\alpha_{mq}r(D_{pq}C_m{}'(\alpha_{mq}r) \tag{43}$$
$$+B_{pq}J'_m(\alpha_{mq}r)+C_{pq}Y'_m(\alpha_{mq}r)+\upsilon\alpha_{mq}r(D_{pq}C''_m(\alpha_{mq}r)+B_{pq}J''_m(\alpha_{mq}r)+C_{pq}Y''_m(\alpha_{mq}r)))]\Big]\Big\}$$

$$\sigma_{r\theta}=\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\Big\{12D\,m\,z\,\alpha_{mq}(-1+\upsilon)\sin m\theta[D_{pq}C'_m(\alpha_{mq}r)+B_{pq}J'_m(\alpha_{mq}r)+C_{pq}Y'_m(\alpha_{mq}r)]/r\,\ell^3\Big\} \tag{44}$$

To find the constant β_1^2 we assume the in-plane displacements [7, 8],

$$u=\sum_{m=1}^{\infty}u(r)\cos m\theta \tag{45}$$

$$v=\sum_{m=1}^{\infty}v(r)\sin m\theta \tag{46}$$

Subjected to the boundary conditions

$$u(a)=v(a)=0 \tag{47}$$

The above form of u and v satisfy the condition that $u=0$ perpendicular to the bounding diameter and $v=0$ along the bounding diameter. These in-plane displacements have been eliminated by integrating Eq. (21) over the area of the plate. Integrating Eq. (21) with respect to r from 0 to a and θ from 0 to π, one gets

$$\beta_1^2=\frac{12}{\ell^2a^2\pi}\left\{\int_0^a\int_0^\pi\Big[[w(r,\theta,t),_r]^2+[w(r,\theta,t),_\theta]^2\Big]dr\,d\theta-a^2\pi(1+\upsilon)\,\alpha N_T\right\} \tag{48}$$

Substituting the value of ω and N_T, Eq. (48) after integration, becomes an Eq. for determining β_1^2.

4 NUMERICAL RESULTS, DISCUSSION AND REMAKS

For the sake of simplicity of calculation, we introduce the dimensionless values as:

$$\left.\begin{array}{l}\bar{r}=r/b,\ \bar{z}=[z-(-\ell/2)]/b,\ \tau=\kappa t/b^2,\\ \bar{T}=T/T_0, \bar{w}=\bar{w}/\alpha T_0 b, \bar{M}_{ij}=M_{ij}/Eb^3,\\ \bar{\sigma}_{ij}=\sigma_{ij}/E\alpha T_0\,(i,j=r,\theta)\end{array}\right\} \quad (49)$$

Substituting the value of Eq. (49) in Eqs. (1), (33) and (42)-(44), we obtained the expressions for the temperature, deflection and thermal stresses respectively for the numerical discussion. The numerical computations have been carried out for Aluminum annular sector plate with physical parameter as $a=0.3m$, $b=1m$, $\ell=0.08m$, $\beta_1^2=0.020$ and reference temperature as 150 ^0C. Applying prescribed surface temperature $g(r,\theta)=(2/\gamma)^2\,[(\gamma/2)^2-\theta^2]$ along $a\le r\le b$ and follows quadratic parabola along $-\gamma/2\le\theta\le\gamma/2$, $\gamma=\pi/2$. The thermo-mechanical properties are considered as modulus of elasticity E = 70 *GPa*, Poisson's ratio υ = 0.35, thermal expansion coefficient α = 23×10^{-6}/^0C, thermal diffusivity κ = 84.18×10-6 m^2s^{-1} and thermal conductivity λ = 204.2 $Wm^{-1}K^{-1}$. The α_{mq} =0.529, 0.679, 0.826, 0.977, 1.129, 1.282, 1.436, 1.599, 1.746, 1.902 are the positive and real roots of the transcendental equation $J_m(\alpha_{mq}b)Y_m(\alpha_{mq}a)-J_m(\alpha_{mq}a)Y_m(\alpha_{mq}b)=0$. To examine the influence of heating on the plate, the numerical calculations were performed for all the variables, and numerical calculations are depicted in the following figures with the help of MATHEMATICA software.

Fig. 2(a) shows that the temperature distribution along the radial direction for various thickness. Temperature distribution approaches to zero at both extreme ends, i.e. at $\bar{r}=a$ and $\bar{r}=b$ due to the more compressive force acting along the edges, whereas temperature gradually increases and attains the maximum value due to a tensile force. Fig. 2(b) as expected due to less thickness, the temperature along $\bar{z}$ the axis for different values of $\bar{r}$ varies linearly from zero temperature to the highest due to the availability of sectional heat supply at $\bar{z}=\ell/2$. Fig. 2(c) shows the temperature distribution along the angular direction for the different radius. It is noted from Fig. that structural size increases the temperature along angular direction attains a maximum at mid-core due to tensile force for different values of $\bar{r}$. Fig. 3(a) illustrates thermal deflection that is sinusoidal in nature and it attains zero at inner and outer boundary surfaces along the radial direction for different angles. From Fig. 3(b), it was observed that the thermal deflection along time τ for different values of $\bar{r}$ increases with increase in radius towards the outer edge due to the accumulation of the heat source.

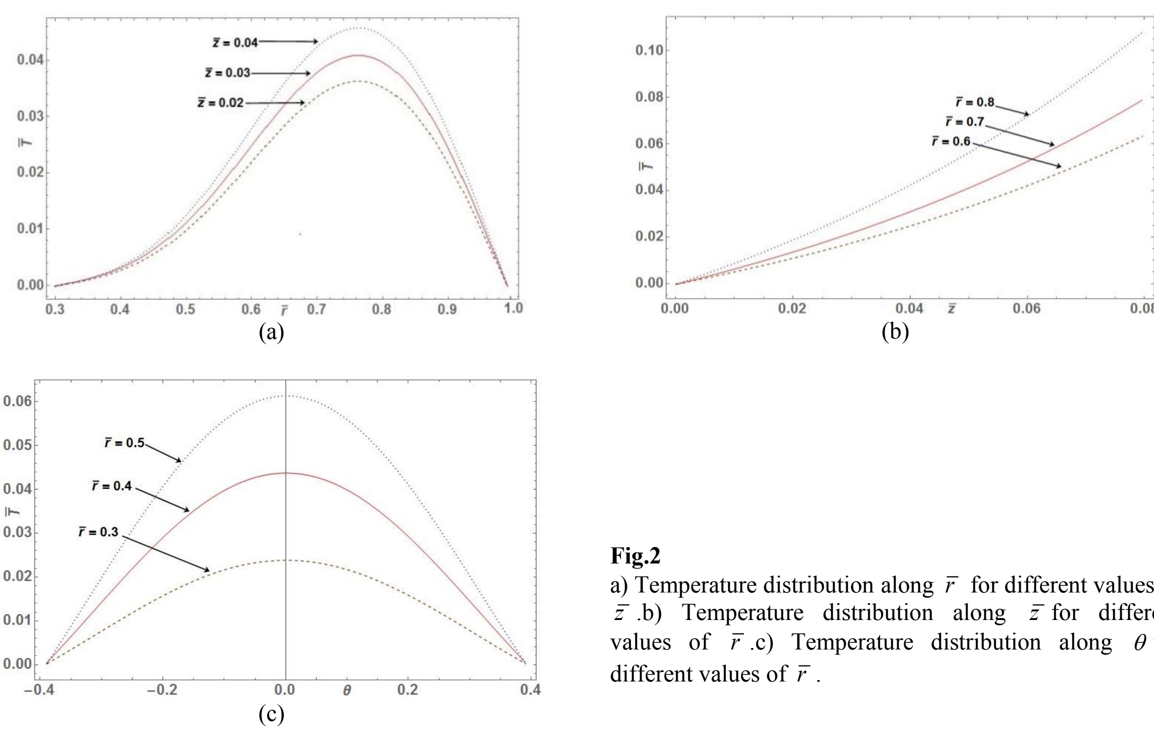
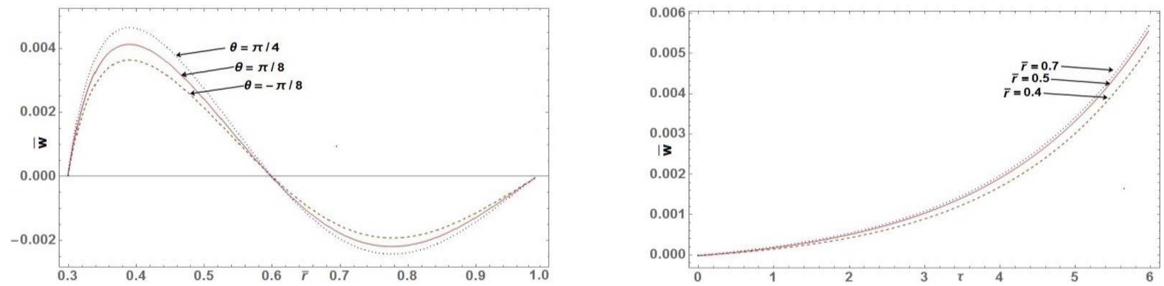


Fig.2
a) Temperature distribution along $\bar{r}$ for different values of $\bar{z}$ .b) Temperature distribution along $\bar{z}$ for different values of $\bar{r}$ .c) Temperature distribution along θ for different values of $\bar{r}$.



(a) (b)

Fig.3
a) Thermal deflection along $\bar{r}$ for different values of θ .b) Thermal deflection along time τ for different values of $\bar{r}$.

It was observed in Fig. 4 that the thermal bending moments ($\bar{M}_r$ and $\bar{M}_\theta$) attains zero at $\bar{r} = \bar{a}$ satisfying Eq. (24) and increases gradually along the radial direction. The moment $\bar{M}_r$ attains maximum magnitude at the mid-core then starts decreasing towards the outer edge, whereas the moment $\bar{M}_r$ increases towards the outer edge. The nature of thermal bending moment $\bar{M}_{r\theta}$ is behaving its effects along the radial direction with an opposite character from negative to positive magnitude value.

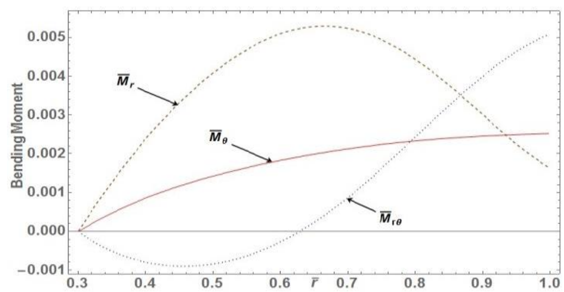
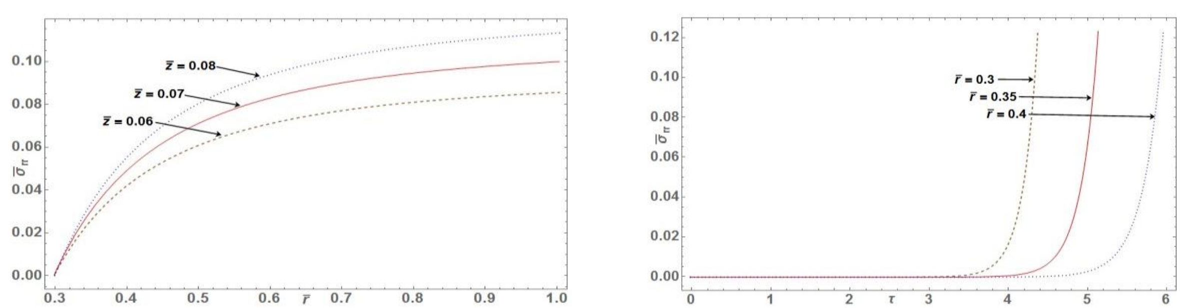


Fig.4
Thermal bending moments along $\bar{r}$ for a different time τ .

Fig. 5(a) illustrates the variation of the radial stress distribution of the plate along the radial direction for various values of $\bar{z}$. The stress profile gradually increases and then it becomes constant. The maximum value of radial stress magnitude occurs at the outer edge due to the accumulation of energy from the available sectional heat supply. Figs. 5(b) and 5(d) indicates the radial and tangential stress along time parameter for different values of $\bar{r}$ and $\bar{z}$ respectively. It is clear from the figures that at the early stage, stress attains zero and suddenly increases attaining maximum stress at the upper limit of the core.

Fig. 5(c), shows that the tangential stress along the radial direction for different time parameters, initially tangential stress is on negative side due to compressive stress occurring at the inner region whereas goes on increasing towards outer radius due to maximum tensile stress. Fig. 5(e), depicts shear stress along the radial direction for different values of $\bar{z}$, initially the shear stress attains maximum expansion due to the accumulation of thermal energy dissipated by sectional heat supply, which is further decreases of the outer surface. In Fig. 5(f), the shear stress along the angular direction for different values of $\bar{r}$, follows the nearly sinusoidal curve.



(a) (b)

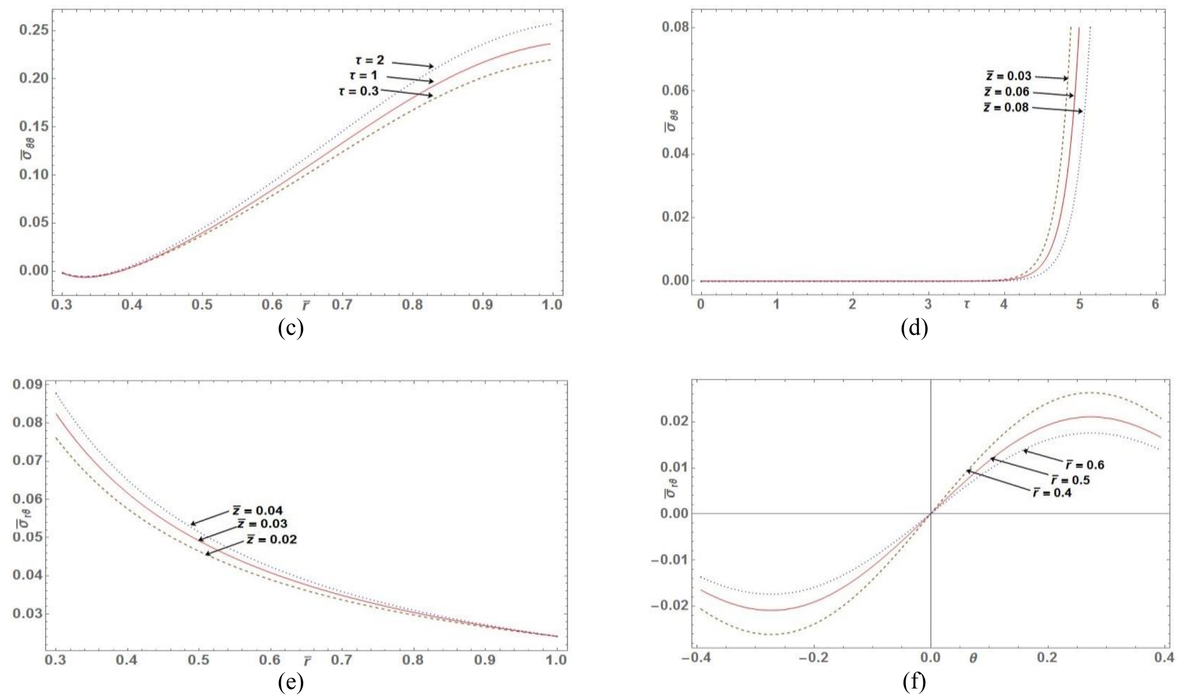


Fig.5
a) Thermal stress $\bar{\sigma}_{rr}$ along $\bar{r}$ for different values of $\bar{z}$ .b) Thermal stress $\bar{\sigma}_{rr}$ along time τ for different values of $\bar{r}$.
c) Thermal stress $\bar{\sigma}_{\theta\theta}$ along $\bar{r}$ for different values of τ .d) Thermal stress $\bar{\sigma}_{\theta\theta}$ along time τ for different values of $\bar{z}$.
e) Thermal stress $\bar{\sigma}_{r\theta}$ along radius $\bar{r}$ for different values of $\bar{z}$ .f) Thermal stress $\bar{\sigma}_{r\theta}$ along θ for different values of $\bar{r}$.

5 CONCLUSIONS

In this article, we have described the theoretical treatment of thermal bending stress analysis for the simply supported annular sector plate with prescribed surface temperature on the top. The temperature distribution and the thermal deflection are used to determine the thermal stresses by classical methods. The expressions for the temperature distribution, thermal deflection and thermal bending stresses are obtained in a series form involving Bessel's functions. The analytical technique proposed here is relatively simple and widely applicable compared with the methods proposed by other researchers. The mentioned results obtained while carrying our research are described as follows:

1. The advantage of this approach is its generality and its mathematical power to handle different types of mechanical and thermal boundary conditions during large deflection under thermal loading.
2. The value of $\bar{\sigma}_{rr}$ at the top face reaches a maximum and then it becomes constant itself at the boundary of the prescribed surface temperature. The value of $\bar{\sigma}_{\theta\theta}$ at the top face reached a maximum at the outer edge due to the accumulation of energy from the available sectional heat supply. The value of $\bar{\sigma}_{r\theta}$ at the top face decreases of the outer surface.
3. The maximum tensile stress shifting from central core to outer region may be due to heat, stress, concentration or available heat sources under considered temperature field.
4. The aforementioned large deflection concept can be very beneficial in the field of micro-devices or micro-system applications, planar continuum robots, predicting the elastoplastic bending and so forth.

REFERENCES

[1] Hetnarski R. B., 2012, *Encyclopedia of Thermal Stresses*, Springer.
[2] Nowacki W., 1962, *Thermoelasticity*, Addison Wesley, NY.

[3] Chia C. Y., 1978, *Nonlinear Analysis of Plates*, McGraw Hill.
[4] Berger H.M.,1955, A new approach to an analysis of large deflection of plates, *Journal of Applied Mechanics* **22**: 465-472.
[5] Tauchert T. R., 2014, *Large Plate Deflections, Berger's Approximation, Encyclopedia of Thermal Stresses*, Springer, Dordrecht.
[6] Basuli S., 1968, Large deflection of plate problems subjected to normal pressure and heating, *Indian Journal of Mechanics and Mathematics* **6**(1): 1-14.
[7] Biswas P., 1976, Large deflection of a heated semi-circular plate under stationary temperature distribution, *Proceedings Mathematical Sciences* **83**(5): 167-174.
[8] Datta S., 1976, Large deflection of a semi-circular plate on elastic foundation under a uniform load, *Proceedings of the Indian Academy of Sciences - Section A* **83**(1): 21-32.
[9] Nowinski J.L., Ohnabe H., 1972, On certain inconsistencies in Berger equations for large deflections of elastic plates, *International Journal of Mechanical Sciences* **14**: 165-170.
[10] Iwinski T., Nowinski J. L., 1957, The problem of large deflections of orthotropic plates, *Archiwum Mechaniki Stosowanej* **9**: 593-603.
[11] Nowinski J. L., 1958, *MRC Technical Summary Report No.34*, *Mathematics Research Center*, US Army University of Wisconsin.
[12] Nowinski J. L., 1958, *MRC Technical Summary Report No.*42, *Mathematics Research Center*, US Army University of Wisconsin.
[13] Nowinski J. L., 1958, *MRC Technical Summary Report No.*67, *Mathematics Research Center*, US Army University of Wisconsin.
[14] Okumura I.A., Honda Y., Yoshimura J., 1989, An analysis for thermal-bending stresses in an annular sector by the theory of moderately thick plates, *Structural Engineering* **6**(2): 347-356.
[15] Wang C.M., Lim G.T., 2000, Bending solutions of sectorial Mindlin plates from Kirchhoff plates, *Journal of Engineering Mechanics* **126**(4): 367-372.
[16] Golmakani M.E., Kadkhodayan M., 2013, Large deflection thermoelastic analysis of functionally graded stiffened annular sector plates, *International Journal of Mechanical Sciences* **69**(1): 94–106.
[17] Eren I., 2013, Analyses of large deflections of simply supported nonlinear beams, for various arc length functions, *Arabian Journal for Science and Engineering* **38**(4): 947-952.
[18] Sitar M., Kosel F., Brojan M., 2014, A simple method for determining large deflection states of arbitrarily curved planar elastica, *Archive of Applied Mechanics* **84**(2): 263-275.
[19] Bakker M.C.M., Rosmanit M., Hofmeyer H., 2008, Approximate large-deflection analysis of simply supported rectangular plates under transverse loading using plate post-buckling solutions, *Thin-Walled Structures* **46**(11): 1224-1235.
[20] Jang T.S., 2014, A general method for analysing moderately large deflections of a non-uniform beam: an infinite Bernoulli-Euler-von Kármán beam on a nonlinear elastic foundation, *Acta Mechanica* **225**(7): 1967-1984.
[21] Choi I.H., 2017, Low-velocity impact response analysis of composite pressure vessel considering stiffness change due to cylinder stress, *Composite Structures* **160**(15): 491-502.
[22] Bhad P., Varghese V., Khalsa L., 2017, A modified approach for the thermoelastic large deflection in the elliptical plate, *Archive of Applied Mechanics* **87**(4): 767-781.
[23] Boley B. A., Weiner J.H., 1960, *Theory of Thermal Stresses*, John Wiley and Sons, New York.
[24] Ventsel E., Krauthammer T., 2001, *Thin Plates and Shells-Theory Analysis, and Applications* Marcel Dekker, New York.
[25] Wang M. Z., Xu X.S.,1990, A generalization of Almansi's theorem and its application, *Applied Mathematical Modelling* **14**: 275-279.