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Modelling Mechanical Properties of AISI 439-430Ti Ferritic Stainless Steel Sheet

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ABSTRACT

The comprehension of the anisotropy impacts on mechanical properties of the rolled steel sheets was investigated using a non-quadratic anisotropic yield function. In this study, experimental and modelling determination regarding the behaviour of an industrial rolled sheet for a ferritic stainless low-carbon steel were carried out. The parameters of the associated yield equation, derived from the three orthotropic yield functions proposed by Hill48, Yld96 and Yld2000-2d, were determined. Predictions and the evolution of normalized yield stress and normalized Lankford parameters (plastic strain ratio) obtained by the presented investigative are considered. The forecasts given by the YLD2000-2d criterion are consistent with that of the experience. In order to describe the path of strain behavior, the isotropic hardening function is described using the following four empirical standard formulae based on: Hollomon, Ludwick, Swift and Voce law. More accurately, the anisotropy coefficients of three yield functions are represented as a function of the longitudinal equivalent plastic strain.

Keywords : Constitutive model; Sheet metal forming; Anisotropy evolution; Orthotropic yield criterion; Isotropic hardening function.

1 INTRODUCTION

THE use of thin sheet metal is very widely adopted in modern industry. Nowadays, with increasing demands for safety, lower weight or reduced fabrication costs, process control and innovative forming processes emerge. The rolling sheets are characterized by the presence of three mutually orthogonal planes of orthotropic symmetry. It is notable that the rolling process promotes the existence of induced anisotropy, which greatly influences the Drawability properties and formability of sheet metals to the desired dimensions and shape. This is especially true for aluminum alloys and ferritic steels, which exhibit strong crystallographic textures after hot rolling [1-3]. To describe and identify the anisotropic mechanical behavior of materials, several such functions (quadratic and non-quadratic) have been proposed. First, Hill [4], which is a simple generalization of the isotropic von Mises plasticity

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to give a quadratic yield criterion mainly associated with the flow rule based on Drucker's postulate. Another isotropic approach of yield functions introduced by Hosford [5] and extended to a planar anisotropic model by Barlat et al. [6-11] and the recent yield function development by Aretz [12] to probe the anisotropic properties of metals. In addition, a quadratic function of Hill48 and as for the constitutive law, a more flexible and adaptable model is the non-quadratic anisotropic yield functions, Yld96 [9] and Yld2000-2d [10], were used to describe the initial anisotropic yield surface with the isotropic hardening law for the yield surface evolution. These are having 7 and 8 material parameters which can be identified respectively by the yield stresses and strain ratios in three uniaxial cases, RD (0°), DD (45°) and TD (90°) and one equibiaxial tension case with a varying cross-section. Yield functions can involve a several anisotropy coefficients for a material. The identification of these parameters requires usually high number mechanical tests in different directions following loading paths. To ensure a certain precision of parameters, the number of experimental data should not be lower than the number of material parameters considered in the identification operation [13]. The description of initial anisotropy of the yield function coupled with optimal hardening evolution, can lead to a good representation of the mechanical behavior [14]. In order to describe the path of strain behavior, the isotropic hardening function is described using the following various empirical standard formulae based on: Hollomon, Ludwick, Swift and Voce model. For more accuracy, Wang et al. [15] proposed an equivalent strain-dependent identification method by taking into account the evolution of anisotropic parameters at different plastic strain levels, which these coefficients of Hill48, Yld96 and Yld2000-2d yield functions are represented as a function of the longitudinal equivalent plastic strain ($\bar{\varepsilon}$).

The remainder of this investigative study is structured as follows. Section 1, a basic mathematical description of the anisotropic behavior of a homogeneous rolled sheet, expressed in accordance with the Hill's formalism and improving Barlat's Yld96 and Yld2000-2d yield function of generalized materials, is presented. In Section 2 describes experiments that were carried out to determine the mechanical characteristics of the material according to the needs of the analysis, since mechanical experiments are used to provide comparison with theoretical results. Then, in Section 3, describes the isotropic hardening for a metal sheet with four classical laws using curve fitting for Stress-Strain experimental data. The yield surface expands and contracts homotheticly in stress space during strain-hardening and strain-softening, respectively. For convenience, the three groups of independent anisotropy coefficients corresponding to the three yield functions, are represented as a function of the longitudinal equivalent plastic strain ($\bar{\varepsilon}$) is employed in Section 4.

2 PHENOMENOLOGICAL MODELS

Considering (RD: the rolling direction, TD: the transverse direction and ND: the normal direction) the three orthotropic directions) wherein the only nonzero component of the stress tensor is $\sigma_{xx} = \sigma$ in the sample frame (*x*, *y*, *z*) (Fig.1). In this section, three yield criteria were used to predict the anisotropic plastic behaviour of the AISI 439-430Ti - Ferritic stainless sheet steels (FSS).

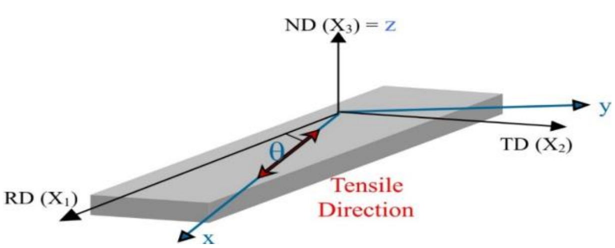


Fig.1
Schematic illustration of the tensile test used in the sheet plane (Symmetric Rolling).

2.1 Hill's 48 yield criteria

The first plastic yield criterion describing the anisotropic behavior of rolled sheets, was the orthotropic and quadratic criterion proposed by Hill [4], insensitive to Bauschinger effect and to Spherical stress characterized by six coefficients independent parameters. In the system of orthotropic coordinates, the Hill's 48 yield criteria are formulated as follows:

$$f(\sigma_{ij}) = Y = F(\sigma_{22}-\sigma_{33})^2 + G(\sigma_{33}-\sigma_{11})^2 + H(\sigma_{11}-\sigma_{22})^2 + 2L\sigma_{23}^2 + 2M\sigma_{13}^2 + 2N_{12}^2 = 2\sigma_0^2 \quad (1)$$

F, *G*, *H*, *L*, *M* and *N* anisotropy coefficients can be derived from texture components (see also [2]) or usually determined from mechanical tests (i.e.; usual uniaxial tensile as well as simple shearing), as hereafter:
whenever, in plane stress state (i.e.; $\sigma_{33} = \sigma_{13} = \sigma_{23} = 0$, and $\sigma_{11}, \sigma_{22}, \sigma_{12} \neq 0$), Eq. (1) can be reduced to

$$F\sigma_{22}^2 + G\sigma_{11}^2 + H(\sigma_{11} - \sigma_{22})^2 + 2N\sigma_{12} = 2\sigma_0^2 \quad (2)$$

where $L = M = N = 3G = 3F = 3H = \frac{3}{2}$, Hill48 yield function reduces to Mises yield function.

If σ_0^1, σ_0^2 and σ_0^3 are the yield stresses in uniaxial tension along the axes RD, TD, and ND, respectively:

$$G + H = 2\left(\sigma_0/\sigma_0^1\right)^2 \qquad F + H = 2\left(\sigma_0/\sigma_0^2\right)^2 \qquad F + G = 2\left(\sigma_0/\sigma^3\right)^2 \quad (3)$$

If σ_0^{23}, σ_0^{13} and σ_0^{12} are the simple shear stresses along the anisotropy axes:

$$L = \left(\sigma_0/\sigma_0^{23}\right)^2 \qquad M = \left(\sigma_0/\sigma_0^{13}\right)^2 \qquad N = \left(\sigma_0/\sigma_0^{12}\right)^2$$

The most common method to obtain *F, G, H* and *N* material parameters of the Hill'48 model, can be related to experimental yield stresses as follows.

For the stress-based method, the Input data is $\sigma_0, \sigma_{45}, \sigma_{90}, \sigma_b$ [16].

$$\begin{aligned} F &= \frac{1}{2}\left[1 - \frac{\sigma_0^2}{\sigma_{90}^2} + \frac{\sigma_0^2}{\sigma_b^2}\right] & G &= \frac{1}{2}\left[\frac{\sigma_0^2}{\sigma_{90}^2} + \frac{\sigma_0^2}{\sigma_b^2} - 1\right] \\ H &= \frac{1}{2}\left[1 + \frac{\sigma_0^2}{\sigma_{90}^2} - \frac{\sigma_0^2}{\sigma_b^2}\right] & N &= \frac{1}{2}\left[\frac{4\sigma_0^2}{\sigma_{45}^2} - \frac{\sigma_0^2}{\sigma_b^2}\right] \end{aligned} \quad (4)$$

Input data is r_0, r_{45}, r_{90} [17].

$$\begin{aligned} F &= \frac{r_0}{r_{90}(1 + r_0)} & G &= \frac{1}{(1 + r_0)} \\ H &= \frac{r_0}{(1 + r_0)} & N &= \frac{(1 + 2r_{45})(r_0 + r_{90})}{2r_{90}(1 + r_0)} \end{aligned} \quad (5)$$

Input data is $\sigma_0, \sigma_{45}, \sigma_{90}, r_{90}$ [18].

$$\begin{aligned} F &= \frac{2\sigma_0^2}{\sigma_{90}^2(1 + r_{90})} & G &= 2 - \frac{2\sigma_0^2 r_{90}}{\sigma_{90}^2(1 + r_{90})} \\ H &= \frac{2\sigma_0^2 r_{90}}{\sigma_{90}^2(1 + r_{90})} & N &= \frac{4\sigma_0^2}{\sigma_{45}^2} - 1 + \frac{\sigma_0^2(r_{90} - 1)}{\sigma_{90}^2(1 + r_{90})} \end{aligned} \quad (6)$$

Note that all these variants imply $\sigma_{ref} = \sigma_0$. Where $\sigma_0, \sigma_{45}, \sigma_{90}$ are unidirectional yield stresses of 0°, 45° and 90° according to the rolling direction (RD). σ_b is the biaxial yield stress determined by a biaxial tensile test experiment. Noting that, all these variants imply $\sigma_{ref} = \sigma_0$. Since formability of any sheet is characterized by $\sigma(\theta)$ mechanical parameter that is primarily related to the size and shape of grains, drawability is usually related to the $r(\theta)$-value, (Lankford's Parameter) which is defined as the ratio of the true strains in the width and in the thickness directions, respectively. Based on the Hill48 quadratic criterion as well as on the associated flow rule according to normality principle, relationships determining mechanical and anisotropic parameters are:

$$\sigma(\theta)=\frac{\sigma_0}{\left(F\sin^4\theta+G\cos^4\theta+H\cos^2 2\theta+2N\sin^2\theta\cos^2\theta\right)^{1/2}} \tag{7a}$$

$$r(\theta)=\frac{H\cos^2 2\theta-(F+G-2N)\cos^2\theta\sin^2\theta}{F\sin^2\theta+G\cos^2\theta} \tag{7b}$$

The anisotropic coefficients *F, G, H, and N* of Hill-48 were obtained from Eqs. (3) and (4) to computed *r*-values.

$$r_0=\frac{H}{G}\ ,\qquad r_{45}=\frac{2N-(F+G)}{2(F+G)}\ ,\qquad r_{90}=\frac{H}{F} \tag{8}$$

2.2 Yld96 yield criteria

The yield criterion Yld96 proposed by Barlat et al. [9] one of the most accurate anisotropic yield functions for rolled sheets presents the following form:

$$f(\sigma_{ij})=\psi=\alpha_1\left|S_2-S_3\right|^a+\alpha_2\left|S_3-S_1\right|^a+\alpha_3\left|S_1-S_2\right|^a=2\sigma_0^a \tag{9}$$

The non-quadratic exponent ‘‘*a*’’ is a material's constant strongly associated with the crystal structure. For an FCC material, the values of the constant *a*=8 are mainly recommended and *a*=6 concerning BCC materials. In Eq. (7), the eigenvalues of the isotropic plasticity equivalent stress space is given by:

$$S=L\sigma$$

where *L* is the fourth order symmetric and deviatoric tensor that represents a linear transformation of the Cauchy stresses. For orthotropic materials, in a 6 by 6 notation, *L* reduces to this following form with C_k are material constants:

$$L_{ij}=\begin{bmatrix} \frac{c_2+c_3}{3} & \frac{-c_3}{3} & -\frac{c_2}{3} & 0 & 0 & 0 \\ \frac{-c_3}{3} & \frac{c_1+c_3}{3} & -\frac{c_1}{3} & 0 & 0 & 0 \\ \frac{-c_2}{3} & -\frac{c_1}{3} & \frac{c_1+c_2}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_6 \end{bmatrix} \tag{10}$$

For plane stress condition in which $(\sigma_{zz}=\sigma_{xz}=\sigma_{yz}=0)$ and the *z*-direction is the third principal direction,

$$S_{ij}=\begin{bmatrix} S_x \\ S_y \\ S_z \\ S_{xy} \end{bmatrix}=\begin{bmatrix} \frac{c_2+c_3}{3} & \frac{-c_3}{3} & \frac{-c_2}{3} & 0 \\ \frac{-c_3}{3} & \frac{c_1+c_3}{3} & \frac{-c_1}{3} & 0 \\ \frac{-c_2}{3} & \frac{-c_1}{3} & \frac{c_1+c_2}{3} & 0 \\ 0 & 0 & 0 & C_6 \end{bmatrix}\begin{bmatrix} \sigma_x \\ \sigma_y \\ 0 \\ \sigma_{xy} \end{bmatrix} \tag{11}$$

and

$$S_{ij}=\begin{bmatrix} S_x & S_{xy} & 0 \\ S_{xy} & S_{yy} & 0 \\ 0 & 0 & S_z \end{bmatrix}=\begin{bmatrix} \frac{c_3(\sigma_x-\sigma_y)-c_2(\sigma_z-\sigma_x)}{3} & c_6\sigma_{xy} & 0 \\ c_6\sigma_{xy} & \frac{c_1\sigma_y-c_3(\sigma_x-\sigma_y)}{3} & 0 \\ 0 & 0 & -S_x-S_y \end{bmatrix} \tag{12}$$

The principal deviatoric stresses S_{ij} in plane sheet can be formulated as:

$$S_{1,2}=\frac{S_x+S_y}{2}\pm\sqrt{\left(\frac{S_x-S_y}{2}\right)^2+S_{xy}{}^2}\ , \quad S_3=-(S_1+S_2) \tag{13}$$

The coefficients $\alpha_{i=1,2,3}=\alpha_1,\alpha_2,\alpha_3$ are computed using the transformation:

$$\alpha_i=\alpha_x P_{1i}^2+\alpha_y .P_{2i}^2+\alpha_z .P_{3i}^2$$

P is the transformation matrix between the principal direction of *s* and the principal axes of anisotropy

$$P=\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The relations between $\alpha_{i=1,2,3}=\alpha_1,\alpha_2,\alpha_3$ and θ are given by

$$\begin{aligned} \alpha_1&=\alpha_x .\cos^2\theta+\alpha_y .\sin^2\theta \\ \alpha_2&=\alpha_x .\sin^2\theta+\alpha_y .\cos^2\theta \\ \alpha_3&=\alpha_{z0} .\cos^2 2\theta+\alpha_{z1}\sin^2 2\theta \end{aligned} \tag{14}$$

In the above equations, $c_1,c_2,c_3,c_6,\alpha_x,\alpha_y,\alpha_{z0},\alpha_{z1}$ are eight coefficients that describe the anisotropy of the material. The value of α_{z0} is usually set to 1. Seven parameters obtained by a numerical identification based on the experimental data $\sigma_0,\sigma_{45},\sigma_{90},r_0,r_{45},r_{90}$ and σ_b . A nonlinear system of seven equations of the seven unknowns is recommended to be solved using the Newton-Raphson method. It should be noted that the function of Yld96 reduces to the simple isotropic case when all the seven anisotropy parameters $c_1,c_2,c_3,c_6,\alpha_1,\alpha_2,\alpha_3$ in the *L* tensor are equal to 1 and finally by setting the exponent *a* = 2 and all the anisotropic parameters equal to unit, the standard von Mises isotropic criteria is restored.

2.2.1 Calculation r_b-value from Yld96

The mechanical parameter $r_b=\dot{\varepsilon}_{yy}/\dot{\varepsilon}_{xx}$ (which is defined as the ratio of the true strains in the width and in the longitudinal directions, respectively) is analogous to the *r*-value, and it characterizes the slope of the yield stress of the balanced biaxial loading stress state $\sigma_b(\sigma_{xx}=\sigma_{yy})$. If the r_b coefficient is equal to 1, the isotropic case will be recovered, else, it is anisotropic behaviour. This coefficient can be evaluated using three different approaches: the first approach concerns experimental way by performing compression tests. The second approach regards the computation of this coefficient from a polycrystal model based on the micro-texture of the material. The third approach to theoretically use Yld96 yield function. In this work, since it was not possible to perform compression tests. The coefficient r_b can be computed by

$$r_b = [-\alpha_x (c_3 + 2c_1)(2c_1 + c_2)\left|2c_1 + c_2\right|^{a-2} + \alpha_y (c_3 - c_1)(c_1 + 2c_2)\left|c_1 + 2c_2\right|^{a-2} - (2c_3 + c_1)(c_1 - c_2) \left|c_1 - c_2\right|^{a-2}] / [-\alpha_x (c_2 - c_3)(2c_1 + c_2)\left|2c_1 + c_2\right|^{a-2} - \alpha_y (2c_2 + c_3)(c_1 + 2c_2)\left|c_1 + 2c_2\right|^{a-2} + (c_2 + 2c_3)(c_1 - c_2)\left|c_1 - c_2\right|^{a-2}] \tag{15}$$

The generalized form of anisotropy equation is described in following equation:

$$r(\theta) = \frac{\dot{\varepsilon}_{yy}}{\dot{\varepsilon}_{zz}} = -\frac{(\frac{\partial \psi}{\partial \sigma_{xx}}).\sin^2\theta - (\frac{\partial \psi}{\partial \sigma_{xy}}).\sin 2\theta + (\frac{\partial \psi}{\partial \sigma_{yy}}).\cos^2\theta}{\frac{\partial \psi}{\partial \sigma_{xx}} + \frac{\partial \psi}{\partial \sigma_{yy}}} \tag{16}$$

Here, θ is the orientation measured anticlockwise form reference x-axis. The rotation rules of stress components from the specimen reference to the sheet axes in uniaxial test are given as:

$$\sigma(\theta) = \frac{\sigma_0}{\left(\alpha_1\left[l - \frac{1}{6}\chi\right]^a + \alpha_2\left[-l - \frac{1}{6}\chi\right]^a + \alpha_3\left[\frac{1}{3}\chi\right]^a\right)^{1/a}} \tag{17}$$

with

$$l = \frac{1}{2}\left[c_2\cos^2\theta + c_1\sin^2\theta\right] \qquad \chi = \sqrt{\beta_1\cos^4\theta + \beta_2\sin^4\theta + \beta_3\sin^2\theta\cos^2\theta}$$

$$\begin{cases} \beta_1 = c_2^2 + 4c_2c_3 + 4c_3^2 \\ \beta_2 = c_1^2 + 4c_1c_3 + 4c_3^2 \\ \beta_3 = 36c_6^2 - 4c_3^2 - 4c_2c_3 - 4c_1c_3 - 2c_1c_2 \end{cases} \qquad \begin{cases} \alpha_1 = \alpha_x.\cos^2\theta + \alpha_y.\sin^2\theta \\ \alpha_2 = \alpha_x.\sin^2\theta + \alpha_y.\cos^2\theta \\ \alpha_3 = \alpha_{z0}.\cos^2 2\theta + \alpha_{z1}\sin^2 2\theta \end{cases} \quad \text{and} \quad \alpha_{z0} = 1$$

where $f(\sigma_{ij}) = \left(\frac{\psi}{2}\right)^{1/a}$

2.3 Yld2000-2d yield criteria

Barlat et al [10] proposed a non-quadratic anisotropic plastic potential model for metals, which is also very successful for steel. Eq. (18) expresses the Yld2000-2d yield criteria in terms of the principal stress deviators tensor:

$$f(\sigma_{ij}) = \Phi = \left|S_1' - S_2'\right|^k + \left|2S_2'' + S_1''\right|^k + \left|2S_1'' + S_2''\right|^k = 2\sigma_0^{\ k} \tag{18}$$

Idem in the case of Yld96, the non-quadratic exponent ‘‘k’’ is a material's constant strongly associated with the crystal structure. For an FCC material, the values of the constant k=8 are mainly recommended and k=6 for BCC materials. For the anisotropic case, the linear transformations reduce to:

$$\begin{bmatrix} S'_{11} \\ S'_{22} \\ S'_{12} \end{bmatrix} = \begin{bmatrix} C'_{11} & C'_{12} & 0 \\ C'_{21} & C'_{22} & 0 \\ 0 & 0 & C'_{66} \end{bmatrix} \begin{bmatrix} s_{11} \\ s_{22} \\ s_{12} \end{bmatrix} , \quad \begin{bmatrix} S''_{11} \\ S''_{22} \\ S''_{12} \end{bmatrix} = \begin{bmatrix} C''_{11} & C''_{12} & 0 \\ C''_{21} & C''_{22} & 0 \\ 0 & 0 & C''_{66} \end{bmatrix} \begin{bmatrix} s_{11} \\ s_{22} \\ s_{12} \end{bmatrix} \tag{19}$$

Or, the transformation can also be applied to the Cauchy stress tensor σ as:

$$\begin{cases} S' = C'S = C'T\sigma = L'\sigma \\ S'' = C''S = C''T\sigma = L''\sigma \end{cases} \tag{20}$$

In this case the first and the second modified principal deviatoric stresses $S'_{1,2}$, $S''_{1,2}$ in plane sheet can be shown as:

$$\begin{aligned} S'_{1,2} &= \frac{1}{2}(S'_{11}+S'_{22}) \pm \frac{1}{2}\sqrt{(S'_{11}-S'_{22})^2+4S'^2_{12}} \\ S''_{1,2} &= \frac{1}{2}(S''_{11}+S''_{22}) \pm \frac{1}{2}\sqrt{(S''_{11}-S''_{22})^2+4S''^2_{12}} \end{aligned} \tag{21}$$

In the previous equations, *C'* and *C''* are linear transformation matrices. Where the transformation matrix, *T*, is

$$T = \frac{1}{3}\begin{bmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix} \overset{\text{Plane}}{=} \begin{bmatrix} 2/3 & -1/3 & 0 \\ -1/3 & 2/3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{22}$$

The tensors *L'* and *L''* representing linear transformations of the stress tensor are.

$$L_{ij} = \begin{bmatrix} L_{11} & L_{12} & L_{13} & 0 & 0 & 0 \\ L_{21} & L_{22} & L_{23} & 0 & 0 & 0 \\ L_{31} & L_{32} & L_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & L_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & L_{66} \end{bmatrix} \overset{\text{Plane}}{=} \begin{bmatrix} L_{11} & L_{12} & 0 \\ L_{21} & L_{22} & 0 \\ 0 & 0 & L_{66} \end{bmatrix} \tag{23}$$

Therefor $L_{1m} + L_{2m} + L_{3m} = 0$ for m = 1, 2, 3. For convenience in the calculation of the anisotropy parameters, the coefficients of *L'* and *L''* can be expressed as follows:

$$L' = \begin{bmatrix} L'_{11} \\ L'_{12} \\ L'_{21} \\ L'_{22} \\ L'_{66} \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & 0 & 0 \\ \frac{-1}{3} & 0 & 0 \\ 0 & \frac{-1}{3} & 0 \\ 0 & \frac{2}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_7 \end{bmatrix}, \quad L'' = \begin{bmatrix} L''_{11} \\ L''_{12} \\ L''_{21} \\ L''_{22} \\ L''_{66} \end{bmatrix} = \frac{1}{9}\begin{bmatrix} -2 & 2 & 8 & -2 & 0 \\ 1 & -4 & -4 & 4 & 0 \\ 4 & -4 & -4 & 1 & 0 \\ -2 & 8 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 9 \end{bmatrix} \begin{bmatrix} \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_8 \end{bmatrix} \tag{24}$$

Finally, the anisotropic parameters can be expressed in the following manner:

$$\begin{cases} \alpha_1 = C'_{11} \\ \alpha_2 = C'_{22} \\ \alpha_3 = 2C''_{21} + C''_{11} \\ 2\alpha_4 = 2C''_{22} + C''_{12} \end{cases} \quad \begin{cases} 2\alpha_5 = 2C''_{11} + C''_{21} \\ \alpha_6 = 2C''_{12} + C''_{22} \\ \alpha_7 = C'_{66} \\ \alpha_8 = C''_{66} \end{cases} \tag{25}$$

The sheet rolled is strongly anisotropic, to describe this property, it is necessary to identify eight independent coefficients of anisotropy such as: $\alpha_1 ... \alpha_7$ and α_8, where they reduce to unite in the simple isotropic case. The eight unknown anisotropy coefficients of the Yld2000-2d yield function were obtained using the eight experimental material data ($\sigma_0, \sigma_{45}, \sigma_{90}, \sigma_b, r_0, r_{45}, r_{90}, r_b$). A nonlinear system of seven equations of the seven unknowns is recommended to be solved using the Newton-Raphson method to determine the α_i parameters.

The reference yield stress is $\sigma_{ref} = \sigma_0$. The rotation rules of stress components from the specimen reference to the sheet axes in uniaxial test are given as:

$$\begin{cases} \sigma_{11} = \sigma(\theta)\cos^2\theta \\ \sigma_{22} = \sigma(\theta)\cos^2\theta \\ \sigma_{12} = \sigma(\theta)\sin\theta\cos\theta \end{cases} \tag{26}$$

$\sigma(\theta)$: is the yield stress of specimen under uniaxial tensile test.

$$\sigma(\theta = \frac{2^{\frac{1}{k}}\sigma_0^2}{\left[\sqrt[k]{X_1} + \left[X_2 - \frac{\sqrt{Y}}{2}\right]^k + \left[X_2 + \frac{\sqrt{Y}}{2}\right]^k\right]^{\frac{1}{k}}} \tag{27}$$

with

$$a_1 = 2\alpha_5 - \frac{1}{2}\alpha_3 \ , \quad a_2 = \alpha_6 - \alpha_4 \ , \quad a_3 = \alpha_3 - \alpha_5, \quad a_4 = 2\alpha_4 - \frac{1}{2}\alpha_6$$

$$a_5 = \frac{4}{3}\alpha_5 - \frac{1}{3}\alpha_3 \ , \ a_6 = \frac{2}{3}a_2 \ , \ a_7 = \frac{4}{3}\alpha_4 - \frac{1}{3}\alpha_6 \ , \ a_8 = \frac{2}{3}a_3$$

$$A = \frac{1}{3}(2\cos^2\theta - \sin^2\theta) \ , \ B = \frac{1}{3}(2\sin^2\theta - \cos^2\theta) \ , \ C = \cos^2\theta.\sin^2\theta$$

$$X_1 = \alpha_1^2 A^2 + \alpha_2^2 B^2 - 2\alpha_1\alpha_2 AB + 4\alpha_7^2 C \ , \ X_2 = (a_1 + a_3)A + (a_2 + a_4)B$$

$$Y = \left(a_5 A + \frac{2a_2 B}{3}\right)^2 - 2\left[\left(a_5 A + \frac{2a_2 B}{3}\right)\left(\frac{2a_3 A}{3} + a_7 B\right)\right] + \left(\frac{2a_3 A}{3} + a_7 B\right)^2 + 4a_8^2 C$$

where

$$f(\sigma_{ij}) = \left(\frac{\Phi}{2}\right)^{1/K} = \left(\frac{\Phi' + \Phi''}{2}\right)^{1/K} \tag{28}$$

A directional *r*-value $r(\theta)$, associated with the orientation angle θ in plane sheet can be calculated from

$$r(\theta) = \frac{\dot{\varepsilon}_{yy}}{\dot{\varepsilon}_{zz}} = -\frac{(\frac{\partial\phi}{\partial\sigma_{xx}}).\sin^2\theta - (\frac{\partial\phi}{\partial\sigma_{xy}}).\sin 2\theta + (\frac{\partial\phi}{\partial\sigma_{yy}}).\cos^2\theta}{\frac{\partial\phi}{\partial\sigma_{xx}} + \frac{\partial\phi}{\partial\sigma_{yy}}} \tag{29}$$

3 UNIAXIAL TENSILE TEST

Uniaxial tensile tests were conducted to determine two essential parameters, the yield stress $\sigma(\theta)$ and the $r(\theta)$ of the overall sheet. The sheet samples prepared along three orientations 0°=RD, 45°=DD and 90°=TD from the rolling direction (RD) were investigated. The anisotropic coefficients of this rolled sheet were also calculated for the 18% pre-strain level and the results were tabulated in Table 1.

Table 1
Material mechanical property for FS steel in three directions.

Direction	v	σ_e (0.2% offset) *(MPa)*	r
0°		278	0.7
45°	0.3	283	1. 41
90°		271	0.82

Fig. 2 shows the uniaxial hardening curves for specimens extracted at three different orientations (RD, DD and TD). The yield curves in TD and RD are even crossing each other. The experimental data were taken from Chahaoui et al. [3].

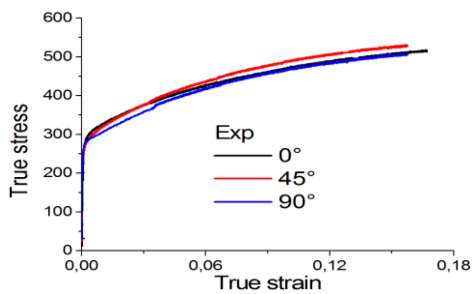


Fig.2
Hardening curves for FS steel.

The normalized flow stresses (yield stress) and *r*-value for different directions are presented in Table 2. Yield stresses for each direction were then normalized by the mono-directional yield stress along the rolling direction.

Table 2
The normalized tensile yield stress by the rolling direction uniaxial yield stress.

Yield stress	σ_0/σ_u	σ_{45}/σ_u	σ_{90}/σ_u	σ_b/σ_u
	1	1.021	0.97	1
r-value	r_0/r_0	r_{45}/r_0	r_{90}/r_0	r_b
	1	2.01	1.17	0.8612

Note that in this work, the equibiaxial yield stress σ_b was assumed $\sigma_b = 1$ and the r_b-value was computed from Yld96.

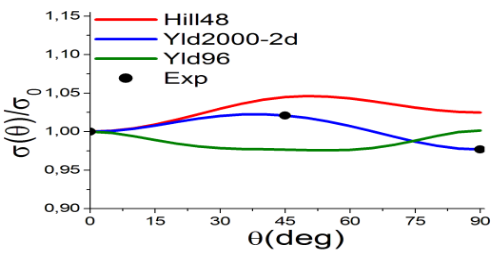
4 DETERMINATION OF ANISOTROPY COEFFICIENTS FOR DIFFERENT YIELD FUNCTIONS

The resulting anisotropic coefficients of Hill's 1948 (*F, G, H* and *N*) and all independent coefficients of Yld96 and Yld2d-2000 for as-received material. The Values are summarized in Table 3.

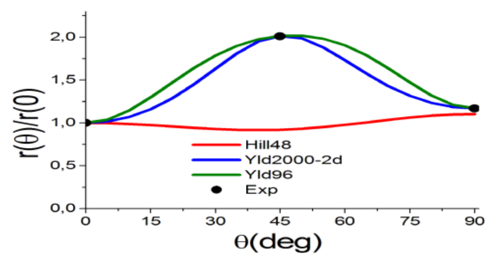
Fig. 3 reflects the variations of the mechanical parameters for three yield criteria (Hill 1948, Yld96 and Yld2000-2d). All of them predict the normalized anisotropic coefficient $\sigma(\theta)$ and $r(\theta)$ evolved with θ in Fig. 3(a-b). The flow stresses $\sigma(\theta)$ and the Lankford *r*-values $r(\theta)$ deducted from Yld2000-2d also agree with experimental data very well. Furthermore, the corresponding yield Loci of the materials that are obtained for the studied yield criteria were plotted in Fig. 3(c).

Table 3
Calculated anisotropy parameters for the FS steel identified from conventional tests.

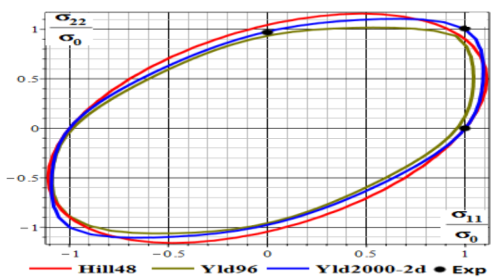
		F		G		H		N	
FS steel	Hill48	0.48		0.53		0.51		1.4	
	YLD96 a=6	c_1	c_2	c_3	c_6	α_x	α_y	α_{z1}	/
		1.023	0.976	1.023	0.945	1.227	1.612	2.490	/
	YLD2000-2d k=6	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
		0.915	1.041	0.914	1.012	1.026	1.001	1.012	0.879



(a)



(b)



(c)

Fig.3
Distribution of the uniaxial mechanical parameters for FS steel. (a) Normalized yield stress.(b) Plastic strain ratio and in. (c) Comparison of three yield contours predicted with Hill48,Yld96, Yld-2000-2d and Experiment behaviour.

5 CURVE FITTING FOR STRESS- STRAIN EXPERIMENTAL DATA WITH ISOTROPIC HARDENING MODEL

The flow stress equation is related with stress and strain data in order to describe the path of strain behavior of FS steel. The isotropic hardening function is described using the following various empirical standard formulae based on: Hollomon, Ludwick, Swift and Voce model in the rolling direction.

- The Hollomon hardening law is: $\sigma_t = K_h \ \varepsilon_t^{\ nh}$
- The Ludwick hardening law is:

$$\sigma_t = \sigma_0 + K_l \ \varepsilon_t^{\ nl} \tag{30}$$

- The Swift hardening law is: $\sigma_t = K_s \ (\varepsilon_0 + \varepsilon_t)^{ns}$ and $\varepsilon_0 = \sigma_0 / E$
- The Voce hardening law is: $\sigma_t = \sigma_{Sat} - (\sigma_{Sat} - \sigma_0)\exp(-nv \ \varepsilon_t)$

In order to quantitatively evaluate the theoretical yield stress-strain relations, we calculated the average error estimation of each equation as:

$$\delta = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{\sigma^{i}_{\ Theoritical} - \sigma^{i}_{\ Experimental}}{\sigma^{i}_{\ Experimental}}\right)^2} \tag{31}$$

where n is the number of the experimental points; $\sigma^i{}_{Theoritical}$ and $\sigma^i{}_{Experimental}$ are the experimental stress and the corresponding theoretical stress, respectively. K_h, K_l, K_s, σ_{sat} , n_h, n_l, n_s, and n_v were obtained by fitting data (σ_t , ε_t) in the uniaxial tensile test along the rolling direction (0°). The results are presented in Table 4.

Table 4
Fitting data of the four hardening laws.

Hollomon		Ludwick		Swift		Voce	
K_h *(MPa)*	600	*Kl (MPa)*	806	*Ks (MPa)*	685.6	σ_{sat} *(MPa)*	541
nh	0.12	*nl*	0.65	*ns*	0.166	*nv*	12.85

In order to describe the plastic property of the material, an isotropic hardening model was fitted to the experimental uniaxial tensile data. It was observed from the previous literatures [19-21]. In Fig. 4, the experimentally measured stress-strain curve and the fit by the four laws, which were utilized for FS steel. It is shown that a good fit is achieved in first by the Voce and it was a better choice of stress-strain evolution of pre-strained material. Therefore, the Voce law is adopted in this work. The Voce hardening model is used to compute the yield stress in terms of the total equivalent plastic strain $\bar{\varepsilon} = 100\%$.

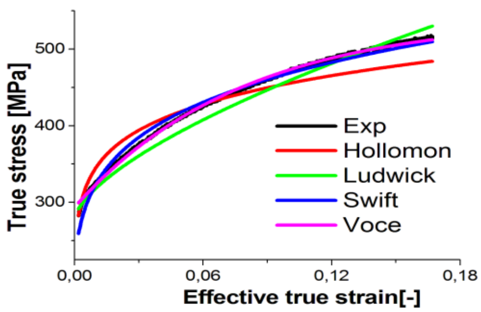


Fig.4
Fitting of experimental hardening curve with different hardening models.

6 EVOLUTION OF YIELD STRESSES NORMALIZED WITH YIELD STRESS FOR THE ROLLING DIRECTION

Directional uniaxial tensile test σ_0/σ_0 ,σ_{45}/σ_0 ,σ_{90}/σ_0 and the assuming of equibiaxial yield stress σ_b/σ_0 and r_b were conducted in order to derive the independent anisotropic coefficients of Hill-48, Yld96 and Yld2000-2d yield functions respectively in sheet plane at five $\bar{\varepsilon}$ levels. The hardening model of voce (Fig.5) was then used as input data to calculate these parameters. Uniaxial stresses were normalized by the uniaxial stresses in the RD for different $\bar{\varepsilon}$ levels, are listed in Table 5. Moreover, the values of anisotropic parameters of Hill-48 and corresponding *r*-values are tabulated in Table 6. The values of anisotropic parameters of Yld96 and Yld2000-2d model are given in Tables 7 and 9 respectively, knowing that at each plastic strain-level the parameters were calculated using the Newton-Raphson iteration method.

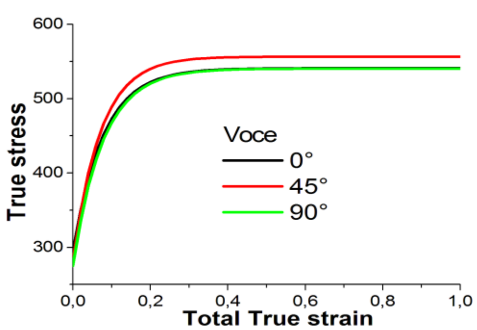


Fig.5
The hardening curves from different testing in terms of 100% of plastic strain.

Table 5
The normalized uniaxial stresses in terms of the plastic work per unit volume w^p.

$\bar{\varepsilon}(-)$	σ_0	σ_{45}	σ_{90}	σ_0/σ_0	σ_{45}/σ_0	σ_{90}/σ_0	σ_b/σ_0
0.001	293	280	275	1	0.955	0.938	0.969
0.16	509	527	506	1	1.035	0.994	1.014
0.30	536	552	535	1	1.030	0.998	1.014
0.40	539	555	539	1	1.030	1.000	1.015
0.56	541	556	540	1	1.027	0.998	1.012

Note that the equibiaxial yield stress σ_b was assumed in this work. Thus, $\sigma_b = (\sigma_0 + \sigma_{90})/2$ at each strain-level $\bar{\varepsilon}$.

Table 6
The values of anisotropic parameters of Hill-48 and corresponding *r*-values.

$\bar{\varepsilon}(-)$	*F*	*G*	*H*	*N*	r_0	r_{45}	r_{90}	r_b
0.001	0.464	0.60	0.535	1.66	0.89	1.060	1.154	0.7670
0.16	0.480	0.49	0.52	1.38	1.055	0.920	1.082	0.9734
0.30	0.484	0.488	0.515	1.398	1.056	0.938	1.064	0.9920
0.40	0.485	0.485	0.514	1.40	1.060	0.940	1.060	1.0000
0.56	0.486	0.490	0.510	1.410	1.040	0.940	0.056	0.9842

Note that the equibiaxial r_b-value was calculated from Yld96 (Barlat et al., [10]).

Table 7
The values of anisotropic parameters of Yld96 function (exponent *a* = 6).

$\bar{\varepsilon}(-)$	c_1	c_2	c_3	c_6	α_x	α_y	α_{z1}
0.001	1.0970	0.9646	1.0347	1.1167	0.7311	1.3396	0.8013
0.16	0.9923	0.9801	1.0197	0.9148	0.9930	1.0534	1.2772
0.30	0.9882	0.9842	0.9273	0.9882	1.0108	1.0298	1.2279
0.40	0.9852	0.9852	1.0147	0.9277	1.0203	1.0203	1.2233
0.56	0.9902	0.9861	1.0138	0.9341	1.0080	1.0352	1.2057

Table 8
The values of anisotropic parameters of Yld2000-2d function (exponent *k* = 6).

$\bar{\varepsilon}(-)$	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
0.001	0.8760	1.1964	0.9348	1.0134	1.0423	1.1216	1.0495	1.0579
0.16	0.9912	1.0214	0.9644	0.9915	0.9943	0.9810	0.9622	0.9541
0.30	0.9990	1.0087	0.9717	0.9917	0.9925	0.9770	0.9677	0.9617
0.40	1.0029	1.0029	0.9734	0.9911	0.9911	0.9734	0.9678	0.9627
0.56	0.9971	1.0092	0.9736	0.9925	0.9945	0.9812	0.9706	0.9654

The anisotropy coefficients listed in Tables 6, 7 and 8 were used to plot the diagrams in Figs. 6. Subsequently, the evolution of normalized mechanical parameters of flow stresses $\sigma(\theta)$ (yield stresses normalized with yield stress for the rolling direction) and Lankford coefficient $r(\theta)$ for seven orientations (0°, 15°, 30°, 45°, 60°, 75° and 90°) is plotted at three levels of effective plastic strain $\bar{\varepsilon}$ at the start of plastic deformation $\bar{\varepsilon} = 0.001$). Thus, it is graphically demonstrated that the evolution of these parameters based on both yield functions (Hill48, Yld96 and Yld2000-2d) change with level of $\bar{\varepsilon}$ at different orientations and the mechanical response is appreciably significant.

The according parameters are given in Tables 6, 7 and 8; the values of anisotropic parameters of Hill-48, Yld96 and Yld2000-2d of FS steel were calculated using stress ratio and *r*-values in each equivalent plastic strain as shown in Fig.7(a, b and c). The values the anisotropy coefficients are varied in relation to the increase of the equivalent plastic strain until 30% of equivalent plastic strain for Hill48 and Yld2000-2d functions but about 40% forYld96. Beyond these values, no significant variation in coefficient evolutions.

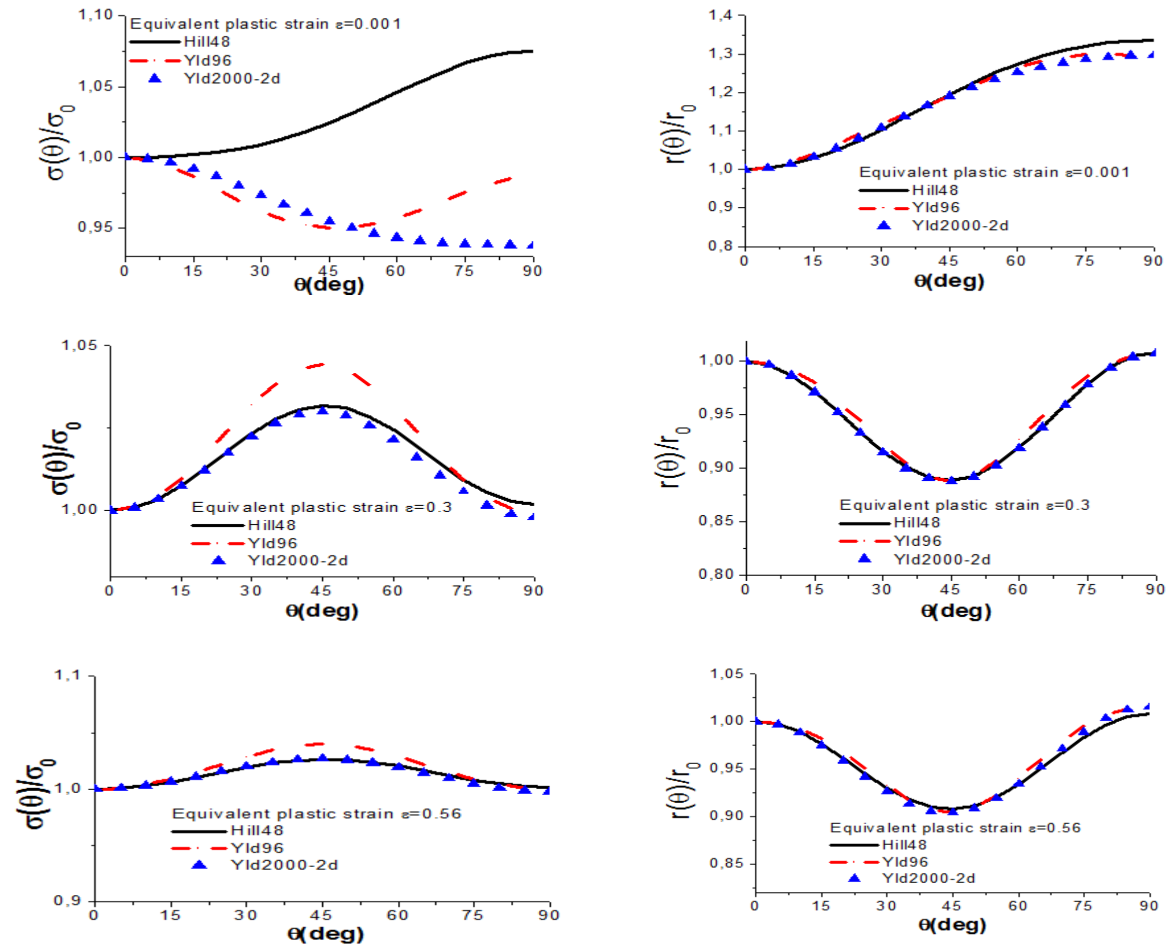
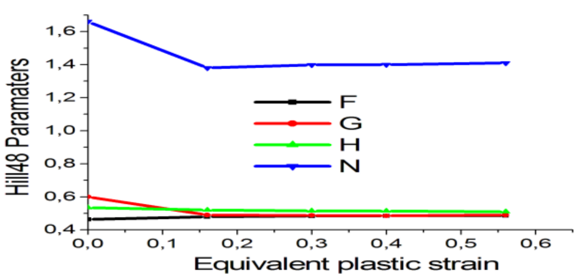
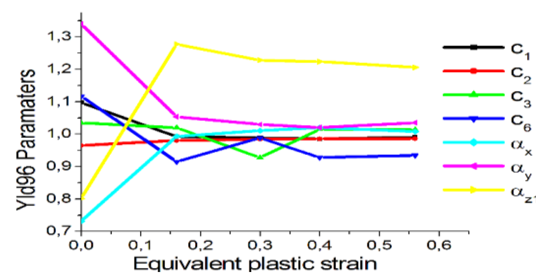


Fig.6
Normalized flow stresses $\sigma(\theta)$ and Lankford $r(\theta)$ for seven orientations at three levels of effective plastic strain based on both yield functions (Hill48, Yld96 and Yld2000-2d).



(a)



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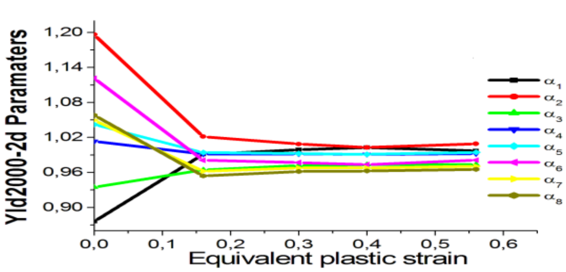

(c)

Fig.7
Anisotropic coefficients were calculated using values of Tables (5- 8).(a) Hill48 Parameters.(b) Yld96 Parameters. c) Yld-2d-2000 Parameters.

For convenience, the three groups of independent anisotropy coefficients corresponding to the three yield functions, are represented as a function of the longitudinal equivalent plastic strain ($\bar{\varepsilon}$).The independent material coefficients calculated at each level of equivalent plastic strains have been optimized using fourth-order polynomial equations functions.

$$d_i = b_0 + b_1 \bar{\varepsilon}^4 + b_2 \bar{\varepsilon}^3 - b_3 \bar{\varepsilon}^2 + b_4 \bar{\varepsilon} \tag{32}$$

These anisotropy coefficients functions are given in Tables 9, 10, and 11 for FS steel witch the varying parameters as they were obtained from the fitting procedure.

Table 9
The anisotropy coefficients of Hill48 yield functions as a function of the equivalent plastic strain ($\bar{\varepsilon}$).

Hill48 coefficients	Fourth-order fit
F	$-0.4364\,\bar{\varepsilon}^4 + 0.7771\,\bar{\varepsilon}^3 - 0.5242\,\bar{\varepsilon}^2 + 0.1658\,\bar{\varepsilon} + 0.464$
G	$11.654\bar{\varepsilon}^4 - 15.797\bar{\varepsilon}^3 + 7.6038\,\bar{\varepsilon}^2 - 1.5475\,\bar{\varepsilon} + 0.6$
H	$-0.3506\,\bar{\varepsilon}^4 - 0.0857\bar{\varepsilon}^3 + 0.2114\bar{\varepsilon}^2 - 0.1283\,\bar{\varepsilon} + 0.535$
N	$32.724\,\bar{\varepsilon}^4 - 44.928\,\bar{\varepsilon}^3 + 21.575\bar{\varepsilon}^2 - 4.1859\,\bar{\varepsilon} + 1.66$

Table 10
The anisotropy coefficients of Yld96 yield functions as a function of the equivalent plastic strain ($\bar{\varepsilon}$).

Yld96 coefficients	Fourth -order fit
c_1	$10.38\bar{\varepsilon}^4 - 14.143\,\bar{\varepsilon}^3 + 6.8914\,\bar{\varepsilon}^2 - 1.4374\,\bar{\varepsilon} + 1.097$
c_2	$-0.363\bar{\varepsilon}^4 + 0.6749\,\bar{\varepsilon}^3 - 0.4763\,\bar{\varepsilon}^2 + 0.1573\,\bar{\varepsilon} + 0.9646$
c_3	$-80.598\bar{\varepsilon}^4 + 90.012\,\bar{\varepsilon}^3 - 30.107\,\bar{\varepsilon}^2 + 2.7492\bar{\varepsilon} + 1.0347$
c_6	$79.667\bar{\varepsilon}^4 - 95.161\,\bar{\varepsilon}^3 + 36.695\,\bar{\varepsilon}^2 - 5.0232\,\bar{\varepsilon} + 1.1167$
α_x	$-24.222\bar{\varepsilon}^4 + 33.077\,\bar{\varepsilon}^3 - 16.285\,\bar{\varepsilon}^2 + 3.4949\,\bar{\varepsilon} + 0.7311$
α_y	$24.603\bar{\varepsilon}^4 - 33.894\,\bar{\varepsilon}^3 + 16.967\,\bar{\varepsilon}^2 - 3.7365\,\bar{\varepsilon} + 1.3396$
α_{z1}	$-61.99\bar{\varepsilon}^4 + 84.221\,\bar{\varepsilon}^3 - 39.689\,\bar{\varepsilon}^2 + 7.4224\,\bar{\varepsilon} + 0.8013$

Table 11
The anisotropy coefficients of Yld2000-2d yield functions as a function of the equivalent plastic strain ($\bar{\varepsilon}$).

Yld2000 coefficients	Fourth -order fit
α_1	$-10.555\bar{\varepsilon}^4 + 14.439\,\bar{\varepsilon}^3 - 7.1295\,\bar{\varepsilon}^2 + 1.5343\bar{\varepsilon} + 0.876$
α_2	$15.381\bar{\varepsilon}^4 - 21.246\,\bar{\varepsilon}^3 + 10.6\,\bar{\varepsilon}^2 - 2.3089\,\bar{\varepsilon} + 1.1964$
α_3	$-0.9401\bar{\varepsilon}^4 + 1.5495\bar{\varepsilon}^3 - 1.0018\,\bar{\varepsilon}^2 + 0.3095\bar{\varepsilon} + 0.9348$
α_4	$2.5877\bar{\varepsilon}^4 - 3.4553\,\bar{\varepsilon}^3 + 1.6271\bar{\varepsilon}^2 - 0.3194\,\bar{\varepsilon} + 1.0134$
α_5	$4.9207\bar{\varepsilon}^4 - 6.6366\,\bar{\varepsilon}^3 + 3.2049\bar{\varepsilon}^2 - 0.663\,\bar{\varepsilon} + 1.0423$
α_6	$14.383\bar{\varepsilon}^4 - 19.532\,\bar{\varepsilon}^3 + 9.4654\bar{\varepsilon}^2 - 1.9521\,\bar{\varepsilon} + 1.1216$
α_7	$10.412\bar{\varepsilon}^4 - 14.227\,\bar{\varepsilon}^3 + 6.7908\bar{\varepsilon}^2 - 1.31061\,\bar{\varepsilon} + 1.0495$
α_8	$12.227\bar{\varepsilon}^4 - 16.836\,\bar{\varepsilon}^3 + 8.0874\bar{\varepsilon}^2 - 1.5618\,\bar{\varepsilon} + 1.0579$

7 CONCLUSIONS

In the present paper, constitutive formulations based on the three orthotropic yield functions of Hill48, Yld96 and Yld2000-2d are presented and analysed to predict the anisotropic plastic behaviour of the AISI 439-430Ti - Ferritic stainless sheet steels (FSS). At several points of experience, Yld2000 criterion gives better agreement of flow stresses predictions and *r*-value anisotropies with experimental data in comparison with Hill'48 and Yld96 functions. For the isotropic hardening, it is shown that, the Voce hardening law a very good compromise with the experimental results for sheet and it is adopted to govern the evolution of the true stress as a function of the equivalent plastic strain. This fact makes the possibility to predict the mechanical behavior of the material beyond the homogeneous zone of deformation. The three groups of independent anisotropy coefficients corresponding to the three yield functions (Hill48; Yld96 and Yld2000-2d yield functions) are represented as a function of the equivalent plastic strain ($\bar{\varepsilon}$) at each level in order to describe the material behavior more accurately.

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