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Fitness for Service Approach (FFS) in Fatigue Life Prediction for a Spherical Pressure Vessel Containing Cracks

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ABSTRACT

During the pressure vessels' operating life, several flaws are likely to grow in long-term operations under cyclic loading. It is therefore essential to take practical and predictive measures to prevent catastrophic events to take place. Fitness for service (FFS) is one safety procedure that is used to deal with maintenance of components in the petroleum industry. In this method, proposed in Codes of practices such as API 579 and BSI 7910, in certain cases, an overly conservative safety prediction is obtained when applied to the operation of pressure vessel containing surface fatigue crack growth. By using improved analytical techniques as well as nonlinear finite element methods critical cracks lengths may be derived more accurately thus reducing conservatism. In this paper, a specific pressure vessel analyzed for fitness for service, which sees fatigue crack growth rate, is assessed using analytical and numerical stress intensity factors. The estimated fatigue life is compared with both methods. It is found that both approaches give similar predictions within a range of scatter assuming that the fatigue properties used are the same in both cases. However, it can be said that the numerical approach gave the more conservative predictions suggesting a detailed analysis is always preferable in FFS examinations.

Keywords: Pressure vessel; Fatigue life assessment; Fitness for service.

1 INTRODUCTION

CYLINDRICAL or spherical pressure vessels are commonly used in industry to carry both liquid s and gases under pressure. According to the ASME Boiler and Pressure Vessel Code (BPVC), Code Section VIII, pressure vessels are containers for the containment of pressure, either internal or external. This pressure may be obtained from an external source or by the application of heat from a direct or indirect source because of a process, or any combination thereof. Theoretically, a spherical pressure vessel twice the strength of a cylindrical pressure vessel

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with the same wall thickness and is the ideal shape to hold internal pressure [1]. However, a spherical shape is difficult to manufacture, and therefore more expensive. Therefore, this type of vessel is preferred for storage of high-pressure fluids. Another advantage of spherical storage vessels is, that they have a smaller surface area per unit volume than any other shape of vessel. This means, that the quantity of heat transferred from warmer surroundings to the liquid in the sphere, will be less than that for cylindrical or rectangular storage vessels [2]. Like all of the industrial structures, the vessels may be damaged and need to be assessed to ensure continued safe operation. Design methods and code structure generally have specific damage tolerances and their application for damage assessment during the operation life is likely to produce improperly conservative assessments. Therefore, design codes do not provide rules to evaluate equipment that degrades during service, and deficiencies due to degradation or original fabrication that may be found during subsequent inspections [3]. Crack-like flaws is one of the major failure modes of pressure vessels. Cracks usually initiate at points of stress concentration. A crack, once initiated, becomes an intense stress concentrator itself. The crack propagates when stress reaches a critical value (fracture toughness). One of the first works on cracked spherical shells was investigated by Folias. In 1970, Folias [4] obtained the critical crack length for a spherical shell using the Griffith energy theory based on the critical crack length in a flat plate. In 1973, Folias [5] published an article about the works done in calculating the stress intensity factor of crack tip in a spherical shell. In addition, Erdogan and Kibler [6] calculated the amount of stress intensity factor in spherical and cylindrical shells containing large cracks. Wang and Hu [7] used a flat spring method to obtain the amount of displacement and stress distribution around a crack tip in a hollow spherical shell and calculated the amount of stress intensity factor. Then, they compared the obtained stress intensity factor with stress intensity factor of flat plate, which is converted by the geometric correction coefficient to the stress intensity factor of a cracked spherical shell. Sun and Ning [8] investigated the elastic plastic-crack fracture in spherical shells. They considered nonlinear spring method based on inelastic shell equations for centered crack. Choa and Chen [9] investigated the stress intensity factor of spherical shells with internal and external cracks. They used finite element and weighted multiplier method for calculating the stress intensity factors. The applied loading in this research is internal pressure. In another method, Brighenti [10] calculated the stress intensity factors using different circumferential stresses around the crack tip. Initial work done to calculate the stress intensity factor in cylindrical and spherical reservoirs is based on the theory of thin walled shells. The values of the stress intensity factor for a broader geometric and loading range are calculated using finite element methods, which includes the Green and Knowles [11], France [12], Zang [13], Anderson [14] works. Fitness-for-service (FFS) assessment is a multi-disciplinary approach to determine whether a structural component is fit for continued service. In 2000, the American Petroleum Institute (API) published API 579, a Recommended Practice for FFS assessment. Although this document was intended primarily for refining and petrochemical assets, it has seen widespread use in a wide range of industries that utilize pressure vessels, piping, and storage tanks. In 2007, API joined forces with the American Society for Mechanical Engineers (ASME) to produce an updated document with the designation API 579-1/ASME FFS-1. This document, which is a Standard rather than a Recommended Practice, contains numerous improvements and explicitly addresses industries outside of refining and petrochemical [15].

In this paper, a specific pressure vessel analyzed for fitness for service. Fatigue crack growth rate is estimated using analytical stress intensity factors. Finally, the estimated fatigue life is compared with a numerical analysis software to derive stress intensity factors. Both methods give similar predictions as the K solutions were similar. However, the latter gave the more conservative predictions suggesting a detailed analysis is preferable for FFS examinations.

2 METHODOLOGY AND FORMULATION

In order to calculate the stress intensity factor, two different formulations, i.e. Barsom method and Folias method are used and the obtained results are compared with the FFS results.

2.1 Stress intensity factor calculation for the elliptical surface crack tip using Barsom formulation[16]

In order to calculate the stress intensity factor (K_{IC}) at the tip of the surrounded crack on the vessel surface, the following equations are developed by Barsom [16]:

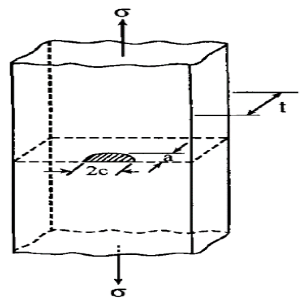


Fig.1
Membrane stress of a spherical pressure vessel with an elliptical crack.

$$\Phi = 1 + 1.464\left(\frac{a}{c}\right)^{1.65}$$
$$M = 1 + 1.2\left[\left(\frac{a}{t}\right) - 0.5\right] \qquad (1)$$
$$K_{IC} = 1.12\sigma \frac{\sqrt[2]{\pi a}}{\Phi} M$$

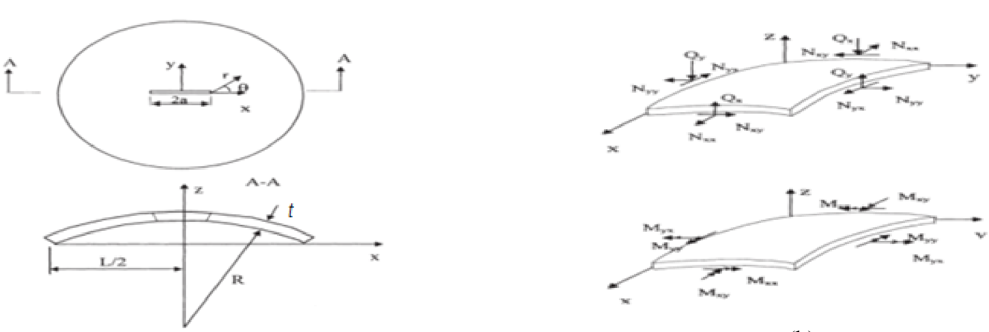
In these equations (according to Fig.1) a is the vertical crack radius (depth of crack), $2c$ is the horizontal diameter (length of crack), M & Φ are the functions of the vessel geometry and crack dimensions, P is the internal pressure, t is the wall thickness and σ is the stress at the crack tip.

2.2 The stress intensity factor calculation by Folias method[5]

Different analytical approaches could give different stress intensity factor (K_{IC}) values for the same conditions. This can be calculated applying the following relations:

$$\lambda = \frac{a}{\sqrt[2]{Rt}}$$
$$M = \left(1 + 0.122\lambda + 0.963\lambda^2 - 0.378\lambda^3 + 0.0423\lambda^4\right)^{0.5} \qquad (2)$$
$$\sigma = \frac{pR}{2t}$$
$$K_{IC} = M\,\sigma\sqrt{\pi c}$$

According to Fig.2, t is the vessel wall thickness and R is the internal radius. M and λ are the functions of geometry. The length of crack is defined as ($2c$):



(a) (b)

Fig.2
a) A curved shell containing a finite line crack. b) The crack tip element loading.

2.3 The stress intensity factor calculation based on API579 [15]

In this paper, the specific pressure vessel with 0.556 *m* inside diameter, 1.71*MPa* working pressure, 2.59*MPa* nominal pressure, 32.2^0C working temperature and 37.8 ^0C design temperature is analyzed for fitness for service. Based on the API597 formulations, the stress intensity factors at internal and external surfaces, K_A and K_B respectively, are computed, as follows:

$$K_A = \sigma G_3 \sqrt{\pi c}$$
$$K_B = \sigma G_4 \sqrt{\pi c} \quad (3)$$

where

$$G_3 = 1 - 0.26066\rho + 0.88766\rho^2 + 0.015826\rho^3 - 0.025266\rho^4 +$$
$$\frac{\left(2.99573 - In\left(\frac{R_m}{t}\right)\right)}{1.38629}\left(0.26785\rho - 0.39378\rho^2 + 0.383574\rho^3 - 0.095384\rho^4\right)$$
$$G_4 = 1 + 0.41551\rho + 0.82404\rho^2 - 0.45458\rho^3 + 0.076714\rho^4 +$$
$$\frac{\left(2.99573 - In\left(\frac{R_m}{t}\right)\right)}{1.38629}\left(-0.05409\rho - 0.24698\rho^2 + 0.35622\rho^3 - 0.099022\rho^4\right)$$
$$\rho = \frac{c}{\sqrt[2]{tR_m}} \quad ; \quad R_m = R + \frac{t}{2}$$

In these equations, *t* is the wall thickness, R_m is the neutral radius and ρ is a geometrical function of crack length (2*c*) and vessel dimensions. It is worth mention that, there are no bending moment in the spherical vessel.

2.4 Fatigue life assessment

We consider the linear elastic fracture mechanic concept and to calculate the fatigue life from crack initiation to failure, Using the Paris law defined by:

$$da/dN = C\Delta K^m \quad (4)$$

where *C* and *m* are the functions of geometry and material and ΔK is stress intensity factor variation at the crack tip. According to the API 579 standard, the parameters *c* and *m* for the spherical vessel made of SA516GR70 are as follows:

$$da/dN = 1.65e^{-8}\Delta K^3 \quad (5)$$

In the first step, one must establish the maximum crack length growth, which will lead to an SIF equal to failure SIF and the number of cycles required for such growth. This will establish the safe life of pressure vessel from the time of initiation of the through crack until the time of failure and fracture. To this end, one needs to establish the failure SIF for SA516GR70 material through the CT tests shown in (Fig. 3), using the following equations:

$$K_{IC} = \frac{P}{B\sqrt{W}} f(\alpha) \qquad \alpha = \frac{a}{W} \qquad 0 \prec a \prec 0.6$$
$$f(\alpha) = \frac{(2+\alpha)\left(0.886 + 4.64\alpha - 13.32\alpha^2 + 14.72\alpha^3 - 5.6\alpha^4\right)}{(1-\alpha)^{3/2}} \quad (6)$$

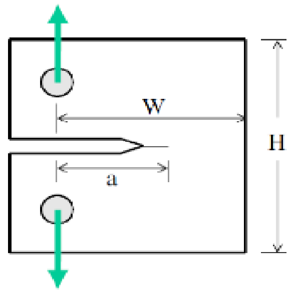


Fig.3
The CT specimen under tensile loading.

where, $f\,(\alpha)$ is a function of geometry of specimen and crack, P is the force applied to specimen, a is the crack length, B is the width of the specimen and W is the total width of the crack ligament. Putting the parameters for the considered vessel expressed in Table 1 in Eq. (6), K_{Ic} =129.37 $MPa(m)^{1/2}$ is calculated. Putting the K_{Ic} in Eq. (2) will result as follows:

$$K_{IC} = M\,\sigma\sqrt{\pi c} = 129.37 Mpa\sqrt[2]{m} \;\therefore 2c = 0.86605 \tag{7}$$

Assuming a through crack with properties in (8) is observed on the vessel, the number of cycles till $2c$ = 0.866 is calculated:

$$\begin{aligned} t &= 0.014m \qquad 2c = 0.014m \\ R &= 0.556m \qquad a = 0.014m \end{aligned} \tag{8}$$

Replacing the parameters in (8) in the Paris Eq. (5), the number of cycles for step increase of 1 *mm* of crack length is calculated, until the crack length of $2c$ = 0.866 is reached. In other words, to grow the crack length from 14 *mm* to 866 *mm*, 1*mm* growth steps will be considered during which the SIF is a constant. The total number of cycles will indicate the number of cycles to failure. The procedure is outlined in Eq. (9)

$$\begin{aligned} &\frac{\Delta a}{\Delta N} = C\,\Delta K^{m} = 1.65e-8\Delta K^{3} \\ &\Delta N = \Delta a / \left[(1.65e-8)\left(M\,\Delta\sigma\sqrt{\pi a}\right)^{3} \right] \\ &\Delta a = crack\;\; length\;\; growth\;\; rate = 0.001m \\ &\Delta N = 0.001 / \left[(1.65e-8)\left(M\,\Delta\sigma\sqrt{\pi a}\right)^{3} \right] \end{aligned} \tag{9}$$

Using a Matlab code, the number of cycles to failure is calculated as 1080 and hence the pressure vessel will have about 1000 safe working cycles from the initiation of through crack till its failure. In order to verify the results which is based on the API579 standard, the same problem is modelled and analyzed in Abaqus.

3 THE FINITE ELEMENT MODEL OF SPHERICAL PRESSURE VESSEL

To obtain the stress intensity factor and the fatigue life of a pressure vessel with surface cracks, the vessel is modeled in ABAQUS.

In addition, using the membrane stress, the stress intensity factor at the crack tip is calculated and applying the Paris law yields the fatigue life. Then, the calculated values are compared with the results of ABAQUS model. The geometrical parameters and mechanical properties of the vessel are given in Table 1.

Table 1
Mechanical characteristics of the pressure vessel.

Max tensile strength, *MPa*	Poisson's ratio	Yield strength *MPa*	Young Modulus *GPa*	Thickness *mm*	Inside diameter *m*
485	0.29	260	200	14	0.556
Fracture toughness $MPa\sqrt{m}$ *	Nominal pressure MPa	Working Pressure MPa	Working temperature ^oC	Density *g/cc*	Material type
129.37	2.59	1.72	32.2	7.8	SA516 GR70

*The fracture toughness is taken from reference [17].

The pressure vessel with several crack depths, i.e. 5.6, 8.4, 11.2 and 14 *mm* is modeled in ABAQUS (See Fig.4) using a semi-sphere to model the whole geometry. At first the pressure vessel is modeled by considering the uncracked conditions. Then, using the extended finite element method, the above-mentioned cracks are created. The crack growth rate is calculated at each loading cycle and the number of cycles up to critical crack length is estimated using Eqs. (12). The comparison of the obtained results from ABAQUS model with the linear elastic fracture mechanics method (API) are shown in Fig. 5 and Tables 2-5 and Fig. 6 to Fig.11. Tables 2-5 highlight the differences in deriving stress intensity factors from the different codes at the different crack depth-to-length ratios, the ratio of the crack depth to the crack length (*a/c*), and crack length. Clearly, the difference in these calculations could be termed a source of error. However, there is generally good agreement between the different results. The ideal is to use the highest value of Stress intensity factor for further analysis to give conservative results.

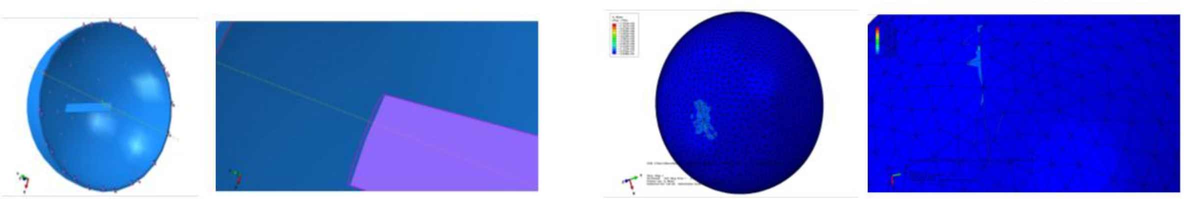


Fig.4
The finite element model showing the meshed sphere.

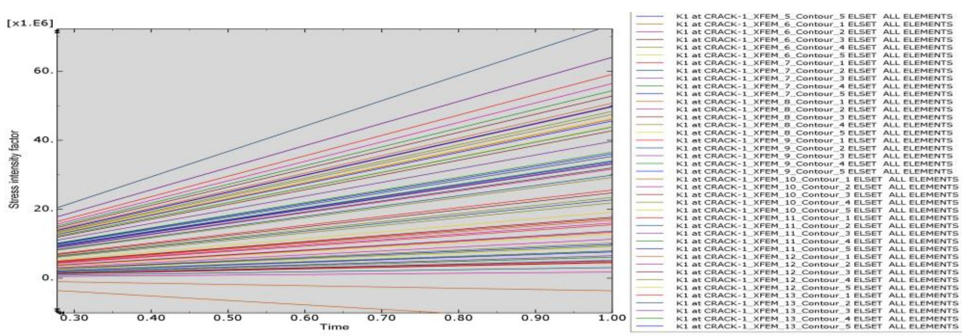


Fig.5
Stress intensity factor vs. time curves from Abaqus.

Table 2
Comparison of Stress intensity factor for a crack with depth of 5.6 *mm*.

Stress intensity factor Pa.m^0.5			crack depth-to-length ratio	crack length *mm*
ABAQUS	API	Folias' relations		
7.04e8	6.59e8	1.90e7	0.0129	433.44
2.98e8	2.57e8	1.73e7	0.02	280
2.34e8	2.11e8	3.93e7	0.04	140
1.63e8	1.37e8	4.61e7	0.08	70
1.18e8	1.23e8	5.94e7	0.16	35
6.41e7	6.33e7	3.58e7	0.24	23.33
7.60e7	6.72e7	4.19e7	0.32	17.5
1.28e8	1.79e8	1.15e8	0.4	14
4.59e7	3.34e7	1.96e7	0.8	7

Table 3
Stress intensity factor for a crack with depth of 8.4 *mm*.

Stress intensity factor Pa.m^0.5			crack depth-to-length ratio	crack length *mm*
ABAQUS	API	Folias' relations		
2.40e8	2.45e8	2.50e8	0.0194	433.44
1.20e9	1.22e9	1.47E8	0.03	280
2.57e8	2.42E8	7.83E7	0.06	140
2.32E8	1.95E8	7.51E7	0.12	70
2.12E8	1.31E8	7.43E7	0.24	35
1.85E8	1.16E8	7.36E7	0.36	23.33
1.30E8	8.39E7	5.50E7	0.48	17.5
1.58E8	1.36E8	8.73E7	0.6	14
5.71E7	3.93E7	1.81E7	1.2	7

Table 4
Stress intensity factor for a crack with depth of 11.2 *mm*.

Stress intensity factor Pa.m^0.5			crack depth-to-length ratio	crack length *mm*
ABAQUS	API	Folias' relations		
2.93E8	2.91E8	2.11E7	0.0258	433.44
3.47E8	3.54E8	5.91E7	0.04	280
2.05E8	2.10E8	9.29E7	0.08	140
2.81E8	2.59E8	1.82E8	0.16	70
1.47E8	1.34E8	9.99E7	0.32	35
1.21E8	1.17E8	7.71E7	0.48	23.33
9.04E7	9.09E7	5.81E7	0.64	17.5
5.56E7	5.80E7	3.47E7	0.8	14
3.36E7	2.52E7	1.13E7	1.6	7

Table 5
Stress intensity factor for a crack with depth of 14 *mm.*

Stress intensity factor Pa.m^0.5				crack depth-to-length ratio	crack length *mm*
Barson's relations	ABAQUS	API	Folias' relations		
3.60E8	7.04E8	6.59E8	4.93E7	0.0129	433.44
3.54E8	3.84E8	3.97E8	8.64E7	0.02	280
5.75E8	5.27E8	5.21E8	2.94E8	0.04	140
3.42E8	2.93E8	3.06E8	2.62E8	0.08	70
2.67E8	2.50E8	2.49E8	2.06E8	0.16	35
3.84E8	3.80E8	3.66E8	2.73E8	0.24	23.33
2.40E8	2.22E+08	2.31E8	1.63E8	0.32	17.5
4.82E7	5.24E7	4.75E7	2.24E7	0.4	14
7.53E7	7.33E7	7.31E7	4.8E7	0.8	7

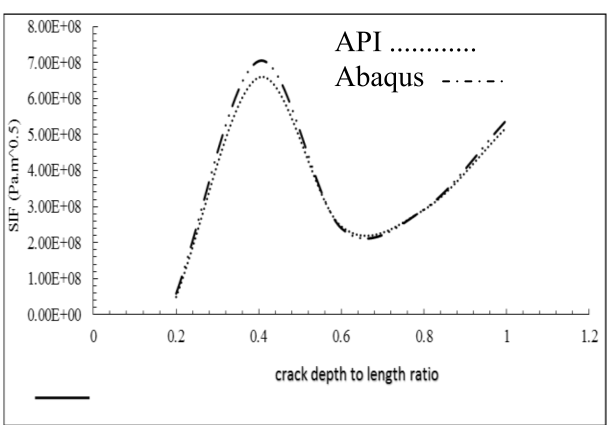


Fig.6
The comparison of the ABAQUS model results with the linear elastic fracture mechanics method (API) results for crack length of 433.44*mm*.

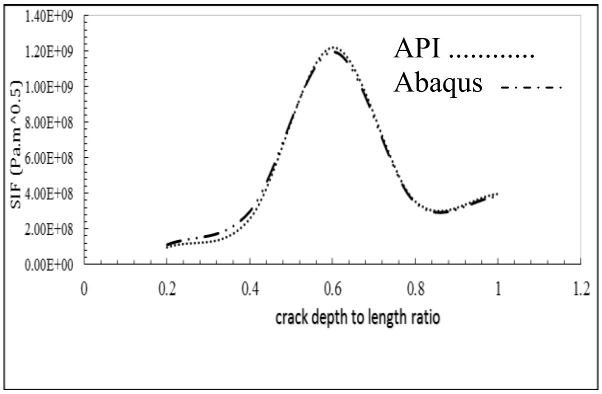


Fig.7
The comparison of the ABAQUS model results with the linear elastic fracture mechanics method (API) results for crack length of 280 *mm*.

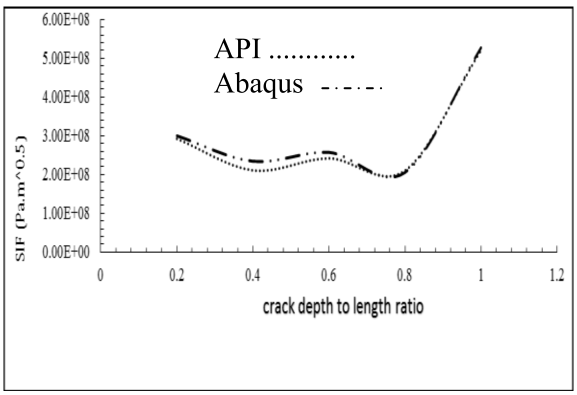


Fig.8
The comparison of the ABAQUS model results with the linear elastic fracture mechanics method (API) results for crack length of 140 *mm*.

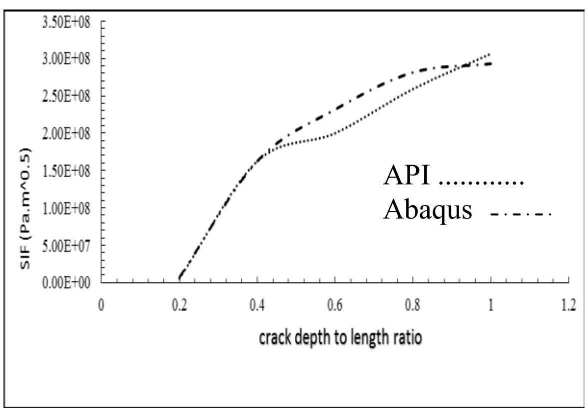


Fig.9
The comparison of the ABAQUS model results with the linear elastic fracture mechanics method (API) results for crack length of 70 *mm*.

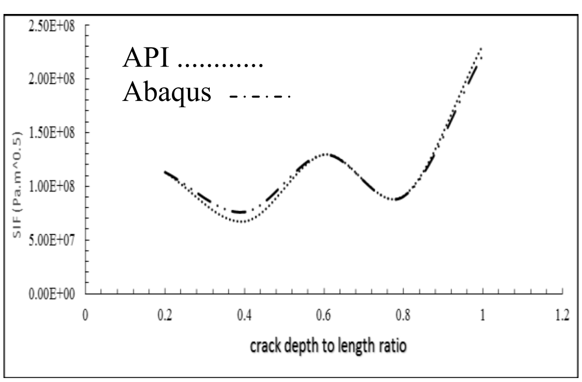


Fig.10
The comparison of the ABAQUS model results with the linear elastic fracture mechanics method (API) results for crack length of 17.5 *mm.*

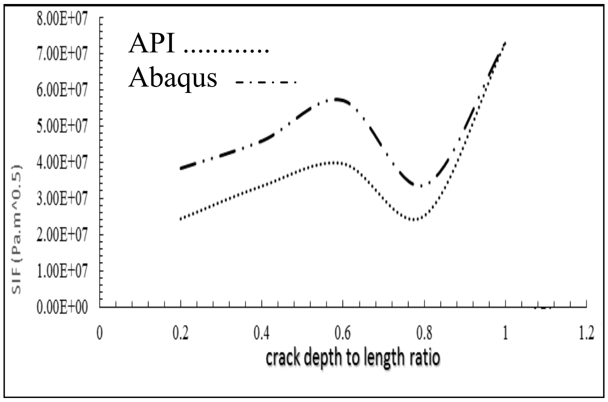


Fig.11
The comparison of the ABAQUS model results with the linear elastic fracture mechanics method (API) results for crack length of 7 *mm*.

3.1 Fatigue crack growth analysis

As already established, the results of API579 and Abaqus correlate well and hence, Abaqus is used to calculate the crack growth analysis. The results are demonstrated in Figs. 12 and 13. The number of allowable cycles is plotted with the calculated SIF in Fig. 13 for the optimized condition of the vessel.

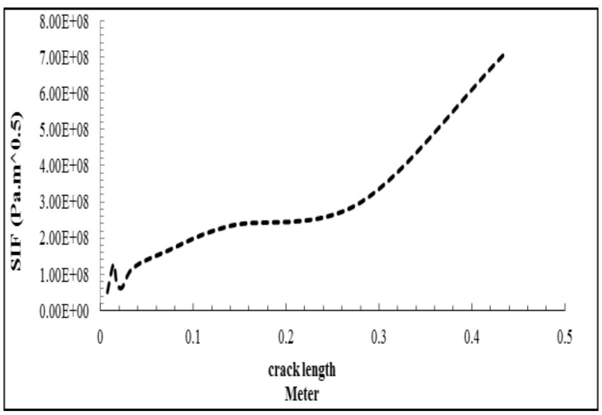


Fig.12
The stress intensity factor versus the surface crack length_ ABAQUS model.

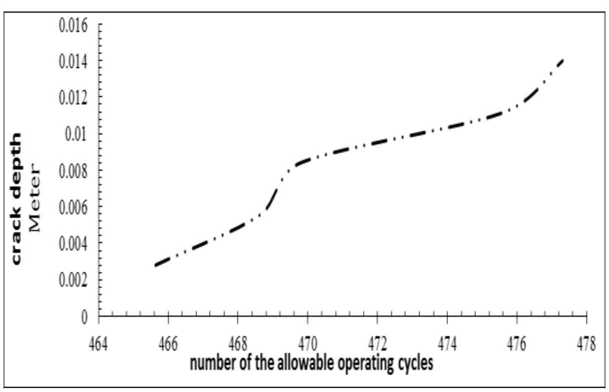


Fig.13
The number of the allowable operating cycles versus the crack depth_ ABAQUS model.

4 CONCLUSIONS

The importance of establishing the correct crack -t tip parameter is paramount to improved failure assessment. In FEM analysis, it is possible to identify difference in the effects of boundary conditions so that appropriate stress intensification is used. The main findings of the paper are thus as follows.

1. The main cause of the crack existence and growth in the vessel is the membrane stress due to cyclic filling and emptying operations.
2. The cracks in the internal surfaces of the vessel (under cyclic loads) are found to be generally elliptical.
3. Comparing with the elastic fracture mechanic method, the FFS technique results are used using the ABAQUS model. Therefore, the predictive maintenance using an accurate numerical technique is an adequate life assessment procedure giving the conservative predictions for a range of crack depths.
4. The rate of stress intensity factor grows by increasing the length of the crack and accordingly crack growth in terms of fatigue cycles can be therefore estimated in an optimum fashion. For purpose of design, using either the numerical estimation or the analytical results will both give similar design or life estimation. However, for deeper crack length the numerical method is more conservative and should be adopted for a more conservative life estimation.
5. To produce the best failure assessment predictions reliable numerical analysis of the stress intensity factor is needed. The variation of crack length and/or decreasing aspect ratio varies the stress intensity factor in both methods of analysis.
6. There are only small differences in the analytical and numerical calculations for SIF leading to differences in the predicted failure times or cycle for the short crack length of a semi-elliptical surface crack. However, increase in crack length increases the value of K derived numerically compared to the analytical solution'
7. Graphs indicate that increasing the ratio of the crack depth to the wall thickness, causes the relative rise in the stress intensity factor.
8. The present study suggest that the analytical method is like the numerical methods for short crack lengths. However, the divergence increases with the increase in crack depth. Further work needs to perform on different components and loading conditions to derive definitive answers for different crack/geometry cases.

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