

Study of Love Waves in a Clamped Viscoelastic Medium with Irregular Boundaries

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ABSTRACT

A mathematical model is presented to investigate the effects of sandiness, irregular boundary interfaces, heterogeneity and viscoelasticity on the phase velocity of Love waves. Geometry of the problem is consisting of an initially stressed viscoelastic layer with corrugated irregular boundaries, which is sandwiched between heterogeneous orthotropic semi-infinite half-space with initial stress and pre-stressed dry sandy half-space. Heterogeneity arises in the upper half-space is due to trigonometric variation in elastic parameters of the orthotropic medium. Inclusion of the concept of corrugated irregular viscoelastic layer clamped between two dissimilar half-spaces under different physical circumstances such as initial stress and heterogeneity brings a novelty to the existing literature related to the study of Love wave. Dispersion equation for Love wave is obtained in closed form. The obtained dispersion relation is found to be in well agreement with classical Love wave equation. Numerical example and graphical illustrations are made to demonstrate notable effect of initial stress, internal friction, wave number and amplitude of corrugations on the phase velocity of Love waves. © 2019 IAU, Arak Branch. All rights reserved.

Keywords: Corrugation; Orthotropic; Heterogeneity; Phase velocity; Initial stress.

1 INTRODUCTION

IN the recent years, a number of authors investigated seismic waves phenomenon at regular interfaces of elastic/viscoelastic/poroelastic media [1, 2, 3]. However, the interface between any two adjacent layers of the Earth is very complicated and irregular in nature. These irregular interfaces may be in the shape of corrugation (cyclic), rectangular, parabolic or much complicated. The studies of Love waves in corrugated boundary surface of the Earth layer is highly interested to seismologist and earthquake engineers due to its relevance in many scientific and engineering fields. In addition to this the physical problem of Love wave propagations in irregular boundary surface, which is assumed here, is utilization of continued interest in earth crust and ground response. Many authors

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have given some valuable information through their studies about the boundary surface of the Earth layer by considering different type of irregularity such as parabolic, hyperbolic and corrugated boundary surfaces. Behavior of Seismic waves produced through a layered half-space having geometrically corrugated is discussed by several authors. Tomar and Saini [4] have studied the behavior of reflection and refraction of SH-waves at a corrugated interface between two-dimensional transversely isotropic half spaces. Selim [5] discussed the static deformation of an irregular initially stressed medium. Singh and Tomar [6] studied the qP-wave at a corrugated interface between two dissimilar pre-stressed elastic half-space. SH-waves propagation through the irregular boundary layer is studied by Alam et al. [7].

Stresses, which exist in a body without action of some external forces is termed as initial stress. Due to atmospheric pressure, gravity, creep, difference in temperature, large amount of initial stresses may exist inside the earth. It should be noted that initial stresses are present in structural elements during their manufacture and assembly, in the Earth's crust under the action of geostatic and geodynamic forces, in rocks, in composite materials, etc. The problems related to the dynamics of the initially stressed deformable bodies have a great practical importance due to its varied and wide applications in many engineering sciences such as those in the mechanics of composites and earthquake engineering. Effect of initial stress on the propagation behavior of Love waves in a layered piezoelectric structure was presented by Liu [8] et al. Qian et al [9] have discussed propagation of Love waves in a piezoelectric layered structure with initial stresses. Biot [10] has delineated the influence of initial stress on elastic waves. Influence of gravity and initial stress on the Love waves in a transversely isotropic medium was studied by Dey and Chakraborty [11]. Akbarov and Ilhan [12] investigated the dynamic response to a moving load of a system comprising an initially stressed covering orthotropic layer and initially stressed orthotropic half-plane.

Heterogeneity is a trivial feature inside the earth or in a geo-structure and it makes a strong basis for the consideration in the study of geo-mechanical problems. Heterogeneity may arise in Earth's medium in the form of linear, quadratic, trigonometric, harmonic or exponential variations of depth. These variations in material properties works as a catalyst in affecting the propagation behavior of waves through which they travel. Researchers and seismologists mostly favor the heterogeneous coupled field structures to analyze the underground response of seismic surface waves. Kundu et al. [13] and Alam et al. [14] investigated the SH-wave and torsional wave propagations in heterogeneous viscoelastic medium respectively. Gupta and Bhengra [15] examined the dispersion characteristic of torsional waves through heterogeneous anisotropic medium.

Materials, in which mechanical and thermal properties are unique and independent in three mutually orthogonal twofold axes of rotational symmetry, are known as orthotropic materials. Hence, these planes of symmetries reduce the number of independent elastic constants to 9 in orthotropic materials. They are a subset of anisotropic materials, as their properties change when measured from different directions. Materials, such as ceramics, cold-rolled steel, wood, bone and many fiber-reinforced composites exemplify the orthotropic materials. The space of orthotropic materials in engineering applications is very vast than other anisotropic materials. Some notable references related to the study of wave phenomena in orthotropic media are Abd-Alla and Ahmed [16], Gupta and Ahmed [17] and Singhal and sahu [18], etc.

Imperfect elastic bodies can be considered to have properties intermediate between those of elastic and viscous bodies, and they are called viscoelastic bodies. Sediments, coal tar, as well as salt exemplify viscoelastic materials. The rocks in the lithosphere and asthenosphere behave like viscoelastic materials and most of the earthquakes occurred in these zones in the last few decades. Therefore, it is enthralling the researchers to consider the viscoelastic materials for the seismic wave studies. The reflection and transmission of plane wave from a plane surface separating a micro polar viscoelastic solid (MVES) half-space and a fluid saturated (FS) incompressible porous solid half-space is investigated by Barak and Kaliraman [19]. Kielczynski et al. [20] have discussed the effect of viscous liquid on Love wave propagation. Kumari et al. [21] have investigated the possibility of propagation of torsional waves in a viscoelastic layer over an inhomogeneous half space. Kumari [22] studied reflection and refraction of longitudinal wave through the plane surface separating a micro polar viscoelastic solid half-space and a fluid saturated incompressible half-space.

The real Earth materials such as soil or sand are found at each level and made of loosely connected mesoparticle grains or platelets. A dry sandy medium is nothing but a medium that consists of sandy particles retaining no moistures or water vapors. Rayleigh wave dispersion in an irregular sandy Earth's crust over orthotropic mantle is analyzed by Vishwakarma and Xu [23] et al. Dey and Chandra [24] considered surface waves in a dry sandy medium under gravity. Alam et al. [25] developed torsional wave propagations in medium sandwiched between similar kinds of two heterogeneous dry sandy medium. Vishwakarma and Xu [26] considered a substratum over a dry sandy Gibson half-space to study the torsional surface waves.

The objective of the present study is to investigate the behavior of Love waves in a corrugated viscoelastic layer sandwiched between a heterogeneous orthotropic half-space under initial stress and an initially stressed dry sandy

half-space. Using the mathematical methods dispersion relation has been obtained in closed form, which specifies the dependence of phase velocity over the wave number. Dispersion curves are plotted to highlight the effects of heterogeneity of upper half-space, sandiness of lower half-space, initial stresses, wave number of corrugation, internal friction parameter of viscoelastic layer, amplitude of upper corrugations on the propagation of Love wave. Obtained result has been matched with classical Love wave equation as a particular case of the considered problem. The effect of internal friction arises in the layer due to imperfect elasticity is also considered in this study.

2 GEOMETRY OF THE PROBLEM

To analyze the present problem geometrically we have taken Cartesian coordinate system in such a way that origin, O is at the common interface of the layer and the lower half-space, x -axis is along the direction of propagation of Love wave and z -axis is pointing vertically downward. A viscoelastic layer [$M_2 : g_1(x) - H \leq z \leq g_2(x)$] of thickness H is clamped between a heterogeneous orthotropic half-space [$M_1 : -\infty \leq z \leq g_1(x) - H$] and a dry sandy half-space [$M_3 : g_2(x) \leq z \leq \infty$]. The common interfaces are considered as the corrugated irregular and all the media are initially stressed as shown in Fig. 1.

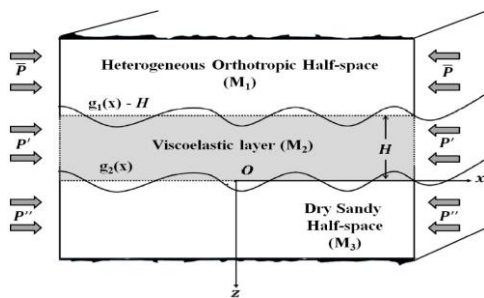


Fig.1 Geometry of modeled structure.

3 GOVERNING EQUATIONS

The corrugation of the upper boundary surface of the half-space is $z = g_1(x) - H$ and the Corrugation of the common interface between layer and the half-space is defined by $z = g_2(x)$ where $g_1(x)$ and $g_2(x)$ are continuous periodic functions and independent of vertical direction of xz -plan. Taking a suitable origin of coordinates the Trigonometric Fourier series of $g_1(x)$ and $g_2(x)$ can be represented as follows (Asano [22])

$$g_l = \sum_{n=1}^{\infty} (g_n^{(l)} e^{in\lambda x} + g_{-n}^{(l)} e^{-in\lambda x}), \quad l = 1, 2 \tag{1}$$

where $g_n^{(l)}$ and $g_{-n}^{(l)}$ are Fourier series expansion coefficients and n is the series expansion order such that

$$g_{\pm 1}^{(l)} = \frac{a}{2}, \quad g_{\pm 2}^{(l)} = \frac{b}{2}, \quad g_{\pm n}^{(l)} = \frac{A_n^{(l)} \mp B_n^{(l)}}{2}, \quad l = 1, 2, \dots \quad \text{and} \quad n = 2, 3, \dots \tag{2}$$

Here $A_n^{(l)}$, $B_n^{(l)}$ are the cosine and sine coefficients of Fourier series expansion. In view of above expressions of $g_n^{(l)}$ and $g_{-n}^{(l)}$, Eq. (2) gives (Alam et al. [7])

$$g_1 = a \cos \gamma x + A_2^{(1)} \cos 2\gamma x + B_2^{(1)} \sin 2\gamma x + A_3^{(1)} \cos 3\gamma x + B_3^{(1)} \sin 3\gamma x + \dots A_n^{(1)} \cos n\gamma x + B_n^{(1)} \sin n\gamma x + \dots, \tag{3}$$

$$g_2 = b \cos \gamma x + A_2^{(2)} \cos 2\gamma x + B_2^{(2)} \sin 2\gamma x + A_3^{(2)} \cos 3\gamma x + B_3^{(2)} \sin 3\gamma x + \dots A_n^{(2)} \cos n\gamma x + B_n^{(2)} \sin n\gamma x + \dots, \tag{4}$$

Here, γ the wave number of corrugation so that wavelength of corrugation is $2\pi/\gamma$. The corrugated interfaces of the concerned problem can be expressed by only one cosine term $g_1(x) = a \cos(\gamma x)$ and $g_2(x) = b \cos(\gamma x)$, where a and b are amplitudes of respected corrugations.

Let us assume (u_1, v_1, w_1) , (u_2, v_2, w_2) and (u_3, v_3, w_3) are the displacement components for upper initially stressed heterogeneous orthotropic layer, intermediate viscoelastic layer and lower dry sandy half-space. Then by the characteristic of Love wave propagation along the x -direction

$$u_i = 0, \quad v_i = v_i(x, z, t), \quad w_i = 0 \quad \text{for } i = 1, 2, 3 \tag{5}$$

4 DYNAMICS OF THE UPPER HETEROGENEOUS ORTHOTROPIC HALF-SPACE

The stress–strain relation for the upper heterogeneous orthotropic half-space is given by

$$\begin{bmatrix} \eta_{11} \\ \eta_{22} \\ \eta_{33} \\ \eta_{23} \\ \eta_{13} \\ \eta_{12} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & 0 & 0 & 0 \\ A_{12} & A_{22} & A_{23} & 0 & 0 & 0 \\ A_{13} & A_{23} & A_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2M & 0 & 0 \\ 0 & 0 & 0 & 0 & 2L & 0 \\ 0 & 0 & 0 & 0 & 0 & 2N \end{bmatrix} \begin{bmatrix} \delta_{11} \\ \delta_{22} \\ \delta_{33} \\ \delta_{23} \\ \delta_{13} \\ \delta_{12} \end{bmatrix} \tag{6}$$

where η_{ij} are the stress components, A_{ij} ($i, j = 1, 2, 3$) and L, M, N are elastic constant and δ_{ij} are strain components. Using Eq. (5), the strain–displacement relation for the upper half-space are obtained as:

$$\delta_{11} = 0, \delta_{22} = 0, \delta_{33} = 0, \delta_{23} = \frac{1}{2} \frac{\partial v_1}{\partial z}, \delta_{13} = 0, \delta_{12} = \frac{1}{2} \frac{\partial v_1}{\partial x} \tag{7}$$

Using Eq. (7), in Eq. (6) we have

$$\eta_{11} = \eta_{22} = \eta_{33}, \eta_{13} = 0, \eta_{23} = M \frac{\partial v_1}{\partial z}, \eta_{21} = N \frac{\partial v_1}{\partial x} \tag{8}$$

In view of Eq. (8), the only non-vanishing equation is given by

$$\frac{\partial \eta_{21}}{\partial x} + \frac{\partial \eta_{23}}{\partial z} - \frac{P}{2} \frac{\partial^2 v_1}{\partial x^2} = \rho_1 \frac{\partial^2 v_1}{\partial t^2} \tag{9}$$

Now the heterogeneity in the upper orthotropic half-space is taken in the following form

$$M = M'(1 + \sin \alpha z), \quad N = N'(1 + \sin \alpha z), \quad \rho_1 = \rho_0(1 + \sin \alpha z), \quad \bar{P} = P(1 + \sin \alpha z) \tag{10}$$

where α is the heterogeneity parameter. Therefore, the equation of motion for the propagation of Love wave in upper orthotropic half-space with initial stress is given by

$$\frac{\partial^2 v_1}{\partial z^2} + \alpha \frac{\partial v_1}{\partial z} + \left(\frac{N'}{M'} - \frac{P}{2} \right) \frac{\partial^2 v_1}{\partial x^2} = \frac{1}{\beta_1^2} \frac{\partial^2 v_1}{\partial t^2} \tag{11}$$

where $\beta_1 = \sqrt{\frac{M'}{\rho_0}}$ and P is the initial stress of the upper half-space.

Let us assume $v_1(x, z, t) = V_1(z)e^{ik(x-ct)}$ be the solution of Eq. (11) then which takes the form

$$\frac{d^2V_1}{dz^2} + \frac{\alpha \cos(\alpha z)}{1 + \sin \alpha z} \frac{dV_1}{dz} + k^2 \left(\frac{c^2}{\beta_1^2} - \frac{N'}{M'} + \frac{P}{2} \right) V_1 = 0 \quad (12)$$

Taking $V_1(z) = \frac{\phi(z)}{\sqrt{1 + \sin \alpha z}}$ in Eq. (12) to eliminate the $\frac{dV_1}{dz}$ term we get

$$\frac{d^2\phi}{dz^2} + \lambda^2\phi = 0 \quad (13)$$

where

$$\lambda^2 = k^2 \left(\frac{\alpha^2}{4k^2} \frac{c^2}{\beta_2^2} - \frac{N'}{M'} + \frac{P}{2} \right) \quad (14)$$

The solution of Eq. (13) is given by

$$\phi(z) = A_1 e^{i\lambda z} + B_1 e^{-i\lambda z} \quad (15)$$

where A_1 and B_1 are arbitrary constant. Hence, we have

$$V_1(z) = \frac{(A_1 e^{i\lambda z} + B_1 e^{-i\lambda z})}{\sqrt{1 + \sin \alpha z}} \quad (16)$$

The appropriate solution of Eq. (16) in view of condition $v_1(z) \rightarrow 0$ when $z \rightarrow -\infty$ is given by

$$V_1(z) = \frac{A_1 e^{i\lambda z}}{\sqrt{1 + \sin \alpha z}} \quad (17)$$

Therefore, the expression of displacement in upper-half space is obtained as:

$$v_1(x, z, t) = \frac{A_1 e^{i\lambda z} e^{ik(x-ct)}}{\sqrt{1 + \sin \alpha z}} \quad (18)$$

5 DYNAMICS OF THE INITIALLY STRESSED VISCOELASTIC LAYER

With condition (5), the stress-displacement relations for non-vanishing stresses in viscoelastic medium are as:

$$S_{23} = (\mu_2 + i\omega\mu'_2) \left(\frac{\partial v_2}{\partial z} \right) \quad (19)$$

$$S_{21} = (\mu_2 + i\omega\mu'_2) \left(\frac{\partial v_2}{\partial x} \right) \quad (20)$$

In view of Eqs. (19) and (20), the only non-vanishing equation of motion for propagation of Love wave in initially stressed viscoelastic layer is given by

$$\left(\mu_2 + \mu_2' \frac{\partial}{\partial t} \right) \left(\frac{\partial^2 v_2}{\partial x^2} + \frac{\partial^2 v_2}{\partial z^2} - \frac{P'}{2} \frac{\partial^2 v_2}{\partial x^2} \right) = \rho_2 \frac{\partial^2 v_2}{\partial t^2} \tag{21}$$

where μ_2 is the rigidity modulus, μ_2' is a parameter representing internal friction due to viscoelasticity, ρ_2 is the density of medium and P' is the initial stress acting along x -direction in the medium.

Let us assume $v_2(x, z, t) = V_2(z)e^{ik(x-ct)}$ be the solution of Eq. (21) then it takes the form

$$V_2''(z) + DV_2(z) = 0 \tag{22}$$

where

$$D = -k \sqrt{\left(1 - \frac{P'}{2} \right) + \frac{\rho_2 c^2}{\mu_2 + \mu_2' i \omega}} \quad \text{and} \quad \beta_2 = \sqrt{\frac{\mu_2}{\rho_2}}; \quad \text{where } \omega = kc. \tag{23}$$

Hence, solution of above Eq. (22) is given by

$$V_2(z) = (A_2 \cos Dz + B_2 \sin Dz) \tag{24}$$

The displacement in sandwiched viscoelastic layer with initial stress is given by

$$v_2(z) = (A_2 \cos Dz + B_2 \sin Dz) e^{ik(x-ct)} \tag{25}$$

6 DYNAMICS OF THE PRE-STRESSED DRY SANDY HALF-SPACE

In view of condition (5), the stress-displacement relations for non-vanishing stresses in dry sandy medium are given by:

$$\kappa_{23} = A \left(\frac{\partial v_3}{\partial z} \right) \tag{26}$$

$$\kappa_{23} = A \left(\frac{\partial v_3}{\partial x} \right) \tag{27}$$

where, $A = \mu_3 \eta$; μ_3 is the rigidity modulus, η is the sandy parameter.

Therefore, the only non-vanishing equation of motion for propagation of Love type wave in dry sandy half-space under initial stress is given by

$$A \left[\left(1 - \frac{P''}{2A} \right) \frac{\partial^2 v_3}{\partial x^2} + \frac{\partial^2 v_3}{\partial z^2} \right] = \rho_3 \frac{\partial^2 v_3}{\partial t^2} \tag{28}$$

where ρ_3 is the density of medium and P'' is the initial stress acting along x -direction in the medium.

Let us assume $v_3(x, z, t) = V_3(z)e^{ik(x-ct)}$ be the solution for Eq. (28) is as follows.

Then the generated ordinary differential is obtained as:

$$V_3''(z) - y^2 V_3(z) = 0 \tag{29}$$

where, $y = \sqrt{1 - \frac{P''}{2A} - \frac{\rho_3 c^2}{A}}$. Then solution of Eq. (29) is given as:

$$V_3 = A_3 e^{-yz} + B_3 e^{yz} \quad (30)$$

In view of Eqs. (28) and (30) we have

$$v_3(x, z, t) = (A_3 e^{-yz} + B_3 e^{yz}) e^{ik(x-ct)} \quad (31)$$

where A_3 and B_3 are arbitrary constants. The appropriate solution of above equation in view of condition $V_3(z) \rightarrow 0$ as $z \rightarrow \infty$.

Therefore, the solution of sandy half-space in presence of hydrostatic stress is given by

$$v_3(x, z, t) = A_3 e^{-yz} e^{ik(x-ct)} \quad (32)$$

7 BOUNDARY CONDITIONS AND DISPERSION RELATION

Stresses and displacements are continuous at $z = g_1(x) - H$, which leads the following relations

$$v_1 = v_2 \quad (33)$$

$$\eta_{23} - g'_1(x) \eta_{21} = S_{23} - g'_1(x) S_{21} \quad (34)$$

Stresses and displacements are continuous at $z = g_2(x)$, which leads the following relations

$$v_2 = v_3 \quad (35)$$

$$S_{23} - g'_2(x) S_{21} = \kappa_{23} - g'_2(x) \kappa_{12} \quad (36)$$

Using the above boundary conditions and eliminating all the constants, we get dispersion equation for Love wave as:

$$\begin{aligned} & \frac{e^{i\lambda(g_1-H)}}{\sqrt{1+\sin\alpha(g_1-H)}} \{[(\mu_2 + i\omega\mu'_2)\{D \cos Dg_2 - ikg'_2 \sin Dg_2\} + A \sin Dg_2(y + g'_2 ik)] [(\mu_2 + i\omega\mu'_2) \\ & [-D \sin D(g_1-H) - ikg'_1 \cos D(g_1-H)] - [(\mu_2 + i\omega\mu'_2)\{D \sin Dg_2 - ikg'_2 \cos Dg_2\} \\ & + A \cos Dg_2(y + g'_2 ik)] [(\mu_2 + i\omega\mu'_2)(D \cos D(g_1-H) - ikg'_1 \sin D(g_1-H))] \} - \\ & (M'(1+\sin\alpha(g_1-H))) \frac{i\lambda e^{i\lambda(g_1-H)}}{\sqrt{1+\sin\alpha(g_1-H)}} - \frac{\alpha e^{i\lambda(g_1-H)} \cos\alpha(g_1-H)}{2[1+\sin\alpha(g_1-H)]^{3/2}} - \\ & g'_1 N'(1+\sin\alpha(g_1-H)) ik \frac{e^{i\lambda(g_1-H)}}{\sqrt{1+\sin\alpha(g_1-H)}} \{[(\mu_2 + i\omega\mu'_2)\{D \cos Dg_2 - ikg'_2 \sin Dg_2\} \\ & + A \sin Dg_2(y + g'_2 ik)] [-\cos D(g_1-H)] + \sin D(g_1-H) [(\mu_2 + i\omega\mu'_2)\{D \sin Dg_2 - ikg'_2 \cos Dg_2\} \\ & + A \cos Dg_2(y + g'_2 ik)] \} = 0 \end{aligned} \quad (37)$$

8 PARTICULAR CASE OF THE PROBLEM

If the uppermost half-space is omitted, and both the intermediate layer and half-space are considered as stress free and homogeneous isotropic elastic with planer boundary surfaces, then the dispersion Eq. (37) reduces to the classical equation of Love wave [27]

$$\tan kH \sqrt{\frac{c^2}{\beta_2^2} - 1} = \frac{\mu_3 \sqrt{1 - \frac{c^2}{\beta_3^2}}}{\mu_2 \sqrt{\frac{c^2}{\beta_2^2} - 1}} \tag{38}$$

where, $\beta_2 = \sqrt{\frac{\mu_3}{\rho_3}}$.

9 NUMERICAL CALCULATIONS AND DISCUSSION

In order to study the influence of various affecting parameters on the phase velocity of Love wave propagating in the said model, we have considered following data for computation purpose:

- (a) For the upper corrugated heterogeneous orthotropic half-space under initial stress (M_1) (Prosser and Green [28]):

$$M = 2.64 \times 10^9 \text{ N / m}^2, N = 1.87 \times 10^9 \text{ N / m}^2 \text{ and } \rho_1 = 1442 \text{ kg / m}^3$$

- (b) For intermediate viscoelastic layer with initial stress (M_2) (Gubbins [29])

$$\mu_2 = 203.9 \times 10^9 \text{ N / m}^2 \text{ and } \rho_2 = 4744 \text{ kg / m}^3$$

- (c) For the lower corrugated dry sandy half-space under initial stress (M_3) (Gubbins [29])

$$\mu_3 = 6.54 \times 10^9 \text{ N / m}^2 \text{ and } \rho_3 = 3404 \text{ kg / m}^3.$$

The variation of dimensionless phase velocity (c / β_2) of Love wave against dimensionless wave number (kH) for different value of various affecting parameters such as heterogeneity (α), sandy parameter (η), initial stress acting in upper half-space (P), sandwiched layer (P') and lower half-space (P''), wave number of corrugation (γ), internal friction parameter ($\omega \mu_2 / \mu_2$) of viscoelastic layer, amplitude of upper corrugation (a) and for lower half-space (b) have been shown in following Figs. 2 to 10.

Figs. 2 and 3 represent the effect of amplitude parameters a for upper corrugation and b for lower corrugation boundary surfaces on the phase velocity of Love wave. The minute observations of both figures conclude that phase velocity of the wave decreases with an increment in the amplitude a associated with upper corrugation, whereas the phase velocity of increases with an increment in the amplitude b associated with the lower half-space. Moreover, we can see that the effect of upper corrugation amplitude a on the phase velocity is almost negligible at the lower frequency region compared to the higher frequency region. It can also be established through the figures that the phase velocity of the wave can be enhanced by taking flatter (regular) upper interface, whereas the flatter (regular) lower interface diminishes the velocity.

Fig.4 describes the effect of wave number of corrugation (γ) on the phase velocity against wave number of the wave. The figure clearly indicates that the wave number of corrugation retard down the phase velocity of the wave. More expressly, the longer wave number of corrugation can be used to diminish the phase velocity of Love waves and smaller wave number of corrugation can be used to enhance the phase velocity of Love waves. It can be

observed from the figure that, phase velocity curves become closer at higher magnitude of wave number of corrugation. It means that, the higher wave number of corrugation affects the phase velocity of the wave negligibly.

Fig.5 describes the variation of phase velocity against wave number for different value of heterogeneity parameter (α). The meticulous observation of the figure irradiates that phase velocity of the wave increases with an increment in the heterogeneity parameter. It means that the heterogeneity present in upper orthotropic half-space enhancing the phase velocity of wave. Moreover, we can see that the effect of heterogeneity associated with the bonded layer on the phase velocity is almost negligible at the higher frequency region compared to the lower frequency region.

Fig.6 manifest the variation of phase velocity against wave number for different value of dry sandy parameter (η) associated with lower half-space. It can be observed by the figure that sandiness present in lower half-space has admiration effect on the wave. Here the phase velocity of the wave got enhanced by the sandiness present in the lower half-space medium. It means that Love waves travels slowly when the lower half-space is an elastic solid, whereas it travels fast when the lower half-space is dry sandy. It is also clear from the concerned figure that the phase velocity curves accumulated in the higher frequency region, which shows that dray sandiness has almost negligible effect on the phase velocity of the wave.

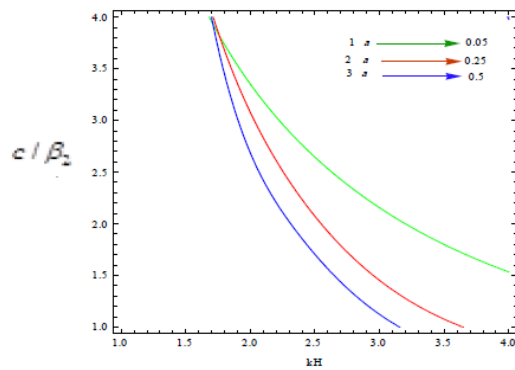


Fig.2
Variation of dimensionless phase velocity (c/β_2) against dimensionless wave number (kH) for different values of upper corrugation amplitude parameter (a) when $b = 0.45$.

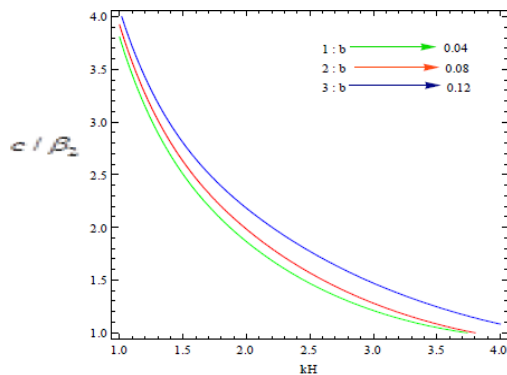


Fig.3
Variation of dimensionless phase velocity (c/β_2) against dimensionless wave number (kH) for different values of lower corrugation amplitude parameter (b) when $a = 0.18$.

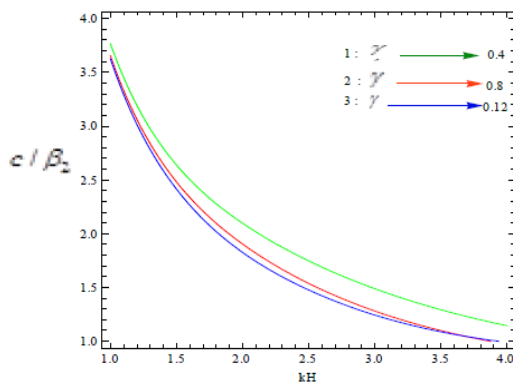


Fig.4
Variation of dimensionless phase velocity (c/β_2) against dimensionless wave number (kH) for different values of wave number of corrugation (γ) when $b = 0.1$ and $a = 0.12$.

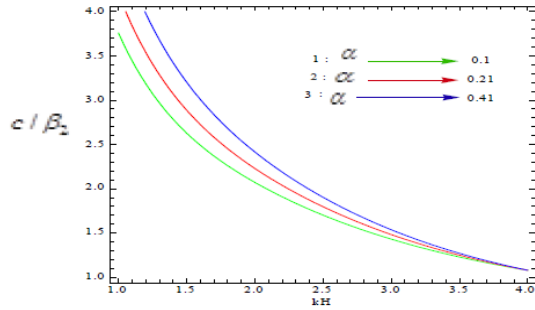


Fig.5
Variation of dimensionless phase velocity (c/β_2) against dimensionless wave number (kH) for different values of intermediate heterogeneity parameter (α) when $\eta = 0.4$.

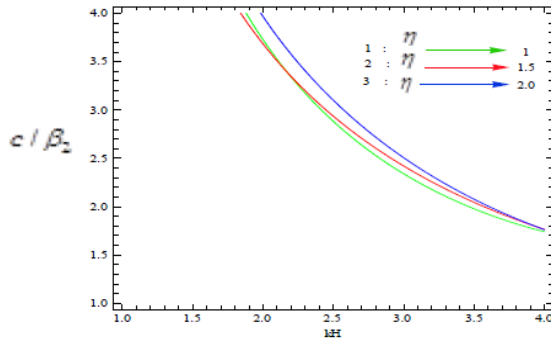


Fig.6
Variation of dimensionless phase velocity (c/β_2) against dimensionless wave number (kH) for different values of dry sandy parameter (η) associated with lower dry sandy half-space when $\alpha = 0.4$.

Figs. 7, 8 and 9 represent the variation of phase velocity against with respect to the wave number due to the effect of initial stresses present in the said model. Fig. 7 shows the effect of initial stress present in the upper half-space (M_1), Fig. 8 reveals the effect of initial stress present in the sandwiched layer (M_2) and Fig. 8 shows the effect of initial stress present in the lower half-space (M_3). The Figs. 7 and 9 clearly irradiate that the initial stresses associated with the upper and lower half-spaces have inverse impact on the phase velocity of wave, while from Fig. 8 it is found that the initial stresses associated with the layer has proportional impact phase velocity of wave.

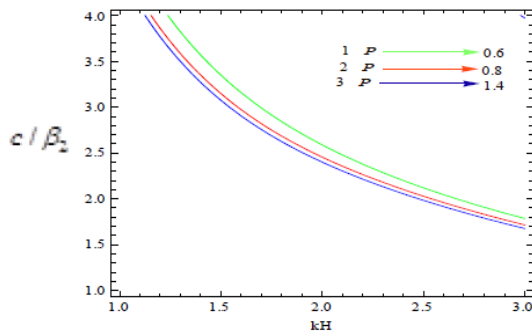


Fig.7
Variation of dimensionless phase velocity (c/β_2) against dimensionless wave number (kH) for different values of initial stress (P) associated with uppermost heterogeneous orthotropic half-space when $P' = 0.11$ and $P'' = 0.61$.

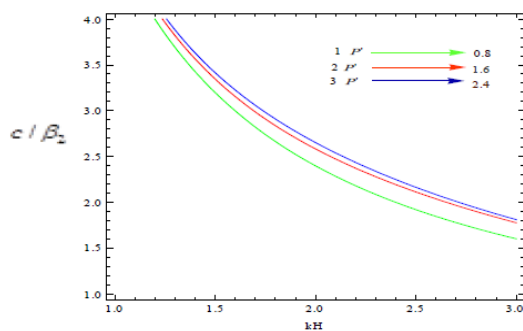
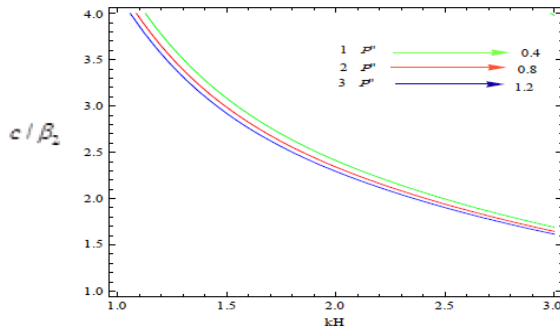
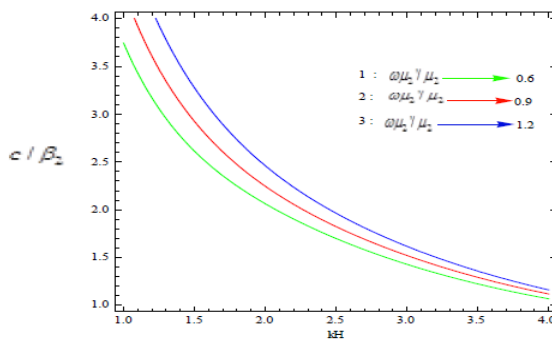


Fig.8
Variation of dimensionless phase velocity (c/β_2) against dimensionless wave number (kH) for different values of initial stress (P') acting in intermediate viscoelastic layer when $P' = 0.4$ and $P'' = 0.2$.

**Fig.9**

Variation of dimensionless phase velocity (c/β_2) against dimensionless wave number (kH) for different values of initial stress (P'') acting in lowermost dry sandy half-space when $P' = 0.11$ and $P = 0.61$.

The effect of internal friction arises due to viscoelasticity of the layer is discussed in the Fig. 10. The curves in this figure have been plotted between the phase velocity and the wave number for different values of internal friction parameter. It can be noticed from the figure that the internal friction arises due to viscoelasticity of the layer enhances the phase velocity of the wave. It can also be seen from the figure that the effect of internal friction on the phase velocity is significant in the lower frequency region of the wave number.

**Fig.10**

Variation of dimensionless phase velocity (c/β_2) against dimensionless wave number (kH) for different values of internal friction parameter ($\omega\mu_2'/\mu_2$) acting in intermediate viscoelastic layer.

10 CONCLUSIONS

The earth is a combination of layers, which have different material properties. The interface between any two adjacent layers is very complicated and irregular in nature and work as a catalyst in affecting the propagation behavior of Love waves. It may not be enough to catch all the engineering problems by the assumption that material mediums are perfect elastic, isotropic, homogeneous, without initial stress and have a planar boundary surface, as the concept cannot indulge many features of the continuum response, which are of great significance. This motivates us to study the Love waves propagation in an irregular corrugated initially stressed viscoelastic layer clamped between a initially stressed heterogeneous orthotropic half-space and a initially stressed dry sandy half-space. We have investigated the various affective parameters, which influences the propagation of Love waves in the assumed model. Major observations of the study has pointed out as follows:

- Regular interface in upper half-space, heterogeneity present in the upper half-space, sandiness nature of lower half-space, initial stress acting in the intermediate layer and the internal friction of the intermediate viscoelastic layer are enhancing the phase velocity of the Love wave.
- Regular interface in lower half-space, wave number of corrugation and initial stress acting in upper and lower half-spaces are diminishing the phase velocity of Love wave.

It is very clear from the study that the corrugated irregular interfaces, heterogeneity, dry sandiness, viscoelasticity and initial stress present in the Earth model have prominent impact on the propagation of Love waves. Today, Earthquake is considered as unrestrained problem in the path of development, and hence various studies in the earthquake-related domain are being carried out to seek to minimize the impact of earthquakes hitting vulnerable areas across the globe. The results obtained in this paper give some essential information to seismologists about the velocity of Love wave propagation in an Earth's layer which have properties intermediate between those of elastic and viscous (Viscoelastic) with irregular interfaces, and clamped between two dissimilar kind of half-spaces. These results may play a vital role to understand well and predict the seismic wave behavior at continental

margins, mountain roots, etc. In the present study, the considered irregularity is cyclic, so that it can be extended by considering the common interfaces as rectangular or parabolic.

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