

On Plane Waves for Mode-I Crack Problem in Generalized Thermoelasticity

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ABSTRACT

A general model of the equations of generalized thermoelasticity for an infinite space weakened by a finite linear opening Mode-I crack is solving. The material is homogeneous and has isotropic properties of elastic half space. The crack is subjected to prescribed temperature and stress distribution. The formulation is applied to generalized thermoelasticity theories, the Lord-Shulman and Green-Lindsay theories, as well as the classical dynamical-coupled theory. The normal mode analysis is used to obtain the exact expressions for the displacement components, force stresses, temperature, couple stresses and micro-stress distribution. The variations of the considered variables through the horizontal distance are illustrated graphically. Comparisons are made with the results between the three theories.

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Keywords : Mode-I crack; (L-S) theory; (G-L) theory; Thermoelasticity.

1 INTRODUCTION

DURING the second half of the twentieth century, non-isothermal problems in the theory of elasticity have become increasingly important due to their many applications in widely diverse fields. The high velocities of modern aircraft give rise to aerodynamic heating, which produces intense thermal stresses, reducing the strength of aircraft structure. In the technology of modern propulsive system, such as jet and rocket engines, the high temperatures associated with combustion processes are the origins of severe thermal stresses. Similar phenomena are encountered in the technologies of space vehicles and missiles in mechanics of large steam turbines and even in shipbuilding, where, strangely enough, ship factories are often attributed to thermal stresses of moderate intensities [1], [2]. The classical uncoupled theory of thermoelasticity predicts two phenomena not compatible with physical observation. First, the equation of heat conduction of this theory does not contain any elastic terms contrary to the fact that elastic changes produce heat effects. Secondly, the heat equation is parabolic type predicting infinite speeds of propagation for heat waves. Biot [3] introduced the theory of coupled thermoelasticity to overcome the first shortcoming. The governing equations for this theory are coupled, eliminating the first paradox of the classical theory. However, both theories share the second shortcoming since heat equation for the coupled theory is also parabolic. Two generalizations to the coupled theory were introduced. The first is due to Lord and Shulman [4], who obtained a wave-type heat equation by postulating a new law of heat conduction to replace the classical Fourier's

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law. This new law contains the heat flux vector as well as its time derivative. It also contains a new constant that acts as a relaxation time. Since the heat equation of this theory is of the wave type, it automatically ensures finite speeds of propagation for heat and elastic waves. The remaining governing equations for this theory, namely, the equations of motions and constitutive relations, remain the same as those for the coupled and the uncoupled theories. This theory was extended by Dhaliwal and Sherief [5] to generalize an isotropic media in the presence of heat sources. Sherief and Dhaliwal [6] solved a thermal shock problem. These problems are valid for short times. Othman [7] studied the dependence of the modulus of elasticity on the reference temperature in a two dimensional generalized thermo-elasticity with one relaxation time. The second generalization to the coupled theory of thermo-elasticity is what is known as the theory of thermo-elasticity with two relaxation times or the theory of temperature-rate-dependent thermo-elasticity. Müller [8], in review of the thermo-dynamics of thermoelastic solids, proposed an entropy production inequality, with the help of which he considered restrictions on a class of constitutive equations. A generalization of this inequality was proposed by Green and Laws [9]. The theory of couple thermo-elasticity was extended by Lord and Shulman [10] and Green and Lindsay [11] by including the thermal relaxation time in constitutive relations. These theories eliminate the paradox of infinite velocity of heat propagation and are termed generalized theories of thermoelasticity. This exist in the following differences between the two theories.

1. The Lord-Şulman theory (L-S) involves one relaxation time of thermoelastic process (τ_0). The Green and Lindsay (G-L) involves two relaxation times (τ_0, ν_0).
2. The (L-S) energy equation involves first and second time derivatives of strain, whereas the corresponding equation in (G-L) theory needs only the first time derivative of strain.
3. In the linearized case according to the approach of (G-L) theory the heat cannot propagate with finite speed unless the stresses depend on the temperature velocity, whereas according to (L-S) theory the heat can propagate with finite speed even though the stresses there are independent of the temperature velocity.
4. The Lord-Şulman theory (L-S) can not be obtained from Green and Lindsay (G-L) theory.

These equations were also obtained independently by Şuhubi [12]. The theory contains two constants that act as relaxation times and modifies all the equations of the coupled theory, not only the heat equation. The classical Fourier's law of heat conduction is not violated if the medium under consideration has a center of symmetry. Ignaczak [13] and [14] studied a strong discontinuity wave and obtained a decomposition theorem of this theory. Dhaliwal and Rokne [15] solved a thermal shock problem in generalized thermo-elasticity. Lotfy et al. [16] studied two-dimensional problem of generalized magneto-thermoelasticity under the effect of temperature dependent properties. Othman et al. [17] studied transient disturbance in a half-space under generalized magneto-thermoelasticity with moving internal heat source. Lotfy [18] studied the plane waves in generalized thermoelasticity elastic half-space by using a general model of the equations of generalized Photothermal theory for a homogeneous isotropic elastic half space. Lotfy et al. [19] studied the generalized thermo-microstretch elastic medium with temperature dependent properties for different theories.

In the recent years, considerable efforts have been devoted the study of failure and cracks in solids. This is due to the application of the latter generally in industry and particularly in the fabrication of electronic components. Most of the studies of dynamical crack problem are done using the equations of coupled or even uncoupled theories of thermoelasticity [20-26]. This is suitable for most situations where long time effects are sought. However, when short time are important, as in many practical situations, the full system of generalized thermoelastic equations must be used [4].

In this paper, we shall use the normal mode analysis to the problem of generalized thermoelasticity for an infinite space weakened by a finite linear opening Mode-I crack is solving based on the dynamical coupling theory, the Lord-Shulman theory and the Green- Lindsay theory. In addition, the effects of different times are discussed numerically and illustrated graphically.

2 FORMULATION OF THE PROBLEM

Consider a linear, infinite homogeneous and isotropic thermoelastic continuum occupying the region G given by $G = \{ (x, y, z) \mid, -\infty < x < \infty, -\infty < y < \infty \}$, with a crack on the x -axis, $|x| \leq a$, $y=0$ is considered. The crack surface is subjected to a known temperature and normal stresses distributions. There are many types of crack and this study will be devoted to Mode-I, shown in Fig. 1.

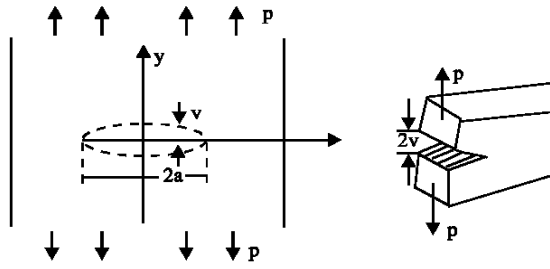


Fig.1 Displacement of an external Mode-I crack.

when all body forces are neglected the governing equations are
Strain-displacement relations

$$e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}), \quad i,j = 1,2 \tag{1}$$

Equation of motion

$$\rho \frac{\partial^2 \bar{u}}{\partial t^2} = (\lambda + \mu) \nabla (\nabla \cdot \bar{u}) + \mu \nabla^2 \bar{u} - \hat{\gamma} (1 + \nu_0 \frac{\partial}{\partial t}) \nabla T \tag{2}$$

Heat conduction equation

$$K \nabla^2 T = \rho C_E (n_1 + \tau_0 \frac{\partial}{\partial t}) \dot{T} + \hat{\gamma} T_0 (n_1 + n_0 \tau_0 \frac{\partial}{\partial t}) \dot{e} \tag{3}$$

Stress displacement relation

$$\sigma_{ij} = 2 \mu e_{ij} + \lambda e \delta_{ij} - \hat{\gamma} (1 + \nu_0 \frac{\partial}{\partial t}) T \delta_{ij} \tag{4}$$

The state of plane strain parallel to the xy -plane is defined by

$$u_2 = u(x, y, t), \quad u_3 = v(x, y, t), \quad u_1 = 0 \tag{5}$$

The partial derivative with the field Eqs. (1)-(4) reduce to

$$(\lambda + \mu) (\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y}) + (\mu) (\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) - \hat{\gamma} (1 + \nu_0 \frac{\partial}{\partial t}) \frac{\partial T}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2}, \tag{6}$$

$$(\lambda + \mu) (\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2}) + (\mu) (\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}) - \hat{\gamma} (1 + \nu_0 \frac{\partial}{\partial t}) \frac{\partial T}{\partial y} = \rho \frac{\partial^2 v}{\partial t^2}, \tag{7}$$

$$K (\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}) = \rho C_E (n_1 + \tau_0 \frac{\partial}{\partial t}) \frac{\partial T}{\partial t} + \hat{\gamma} T_0 (n_1 + n_0 \tau_0 \frac{\partial}{\partial t}) \frac{\partial e}{\partial t} \tag{8}$$

where

$$\hat{\gamma} = (3\lambda + 2\mu) \alpha_i, \quad \text{and} \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{9}$$

The constants $\hat{\gamma}$ depend on mechanical as well as the thermal properties of the body and the dot denotes the partial derivative with respect to time. Eqs. (6)-(8) are the field equations of the generalized linear thermoelasticity, applicable to the coupled theory and two generalizations, as follows:

The equations of the coupled thermoelasticity CD theory, when

$$v_0 = \tau_0 = 0, \quad n_0 = 0, \quad n_i = 1. \quad (10)$$

Eqs. (6)-(8) has the form

$$(\lambda + \mu) \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} \right) + (\mu) \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \hat{\gamma} \frac{\partial T}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2}, \quad (11)$$

$$(\lambda + \mu) \left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2} \right) + (\mu) \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \hat{\gamma} \frac{\partial T}{\partial y} = \rho \frac{\partial^2 v}{\partial t^2}, \quad (12)$$

$$K \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = \rho C_E \frac{\partial T}{\partial t} + \hat{\gamma} T_0 \frac{\partial e}{\partial t} \quad (13)$$

The constitutive relation can be written as:

$$\sigma_{xx} = (2\mu + \lambda) \frac{\partial u}{\partial x} + \lambda \frac{\partial v}{\partial y} - \hat{\gamma} T \quad (14)$$

$$\sigma_{yy} = (\lambda + 2\mu) \frac{\partial v}{\partial y} + \lambda \frac{\partial u}{\partial x} - \hat{\gamma} T \quad (15)$$

$$\sigma_{xy} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \quad (16)$$

$$\sigma_{yx} = \mu \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \quad (17)$$

Lord and Shulman's (L-S) theory, when

$$v_0 = 0, \quad n_0 = n_i = 1, \quad \tau_0 > 0. \quad (18)$$

Eqs. (6) and (7) is the same as Eqs. (11) and (12) and Eq. (8) has the form:

$$K \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = \rho C_E \left(1 + \tau_0 \frac{\partial}{\partial t} \right) \frac{\partial T}{\partial t} + \hat{\gamma} T_0 \left(1 + \tau_0 \frac{\partial}{\partial t} \right) \frac{\partial e}{\partial t} \quad (19)$$

Green and Lindsay's theory (G-L), when

$$n_0 = 0, \quad n_i = 1, \quad v_0 \geq \tau_0 > 0. \quad (20)$$

Eq. (6) and (7) remains without change and Eq. (8) has the form:

$$K \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = \rho C_E \left(1 + \tau_0 \frac{\partial}{\partial t} \right) \frac{\partial T}{\partial t} + \hat{\gamma} T_0 \frac{\partial e}{\partial t} \quad (21)$$

For convenience, the following non-dimensional variables are used:

$$\bar{x}_i = \frac{\omega^*}{C_2} x_i, \quad \bar{u}_i = \frac{\rho C_2 \omega^*}{\hat{\gamma} T_0} u_i, \quad \bar{t} = \omega^* t, \quad \bar{\tau}_0 = \omega^* \tau_0, \quad \bar{v}_0 = \omega^* v_0, \quad \bar{T} = \frac{T}{T_0}, \quad \bar{\sigma}_{ij} = \frac{\sigma_{ij}}{\hat{\gamma} T_0}, \quad \omega^* = \frac{\rho C_E C_2^2}{K}, \quad C_2^2 = \frac{\mu}{\rho}. \quad (22)$$

By using Eqs. (22), Eqs. (6)-(8) become (dropping the dashed for convenience)

$$\frac{\partial^2 u}{\partial t^2} = \frac{\mu}{\rho C_2^2} \nabla^2 u + \frac{(\mu + \lambda)}{\rho C_2^2} \frac{\partial e}{\partial x} - (1 + \nu_0) \frac{\partial}{\partial t} \frac{\partial T}{\partial x}, \quad (23)$$

$$\frac{\partial^2 v}{\partial t^2} = \frac{\mu}{\rho C_2^2} \nabla^2 v + \frac{(\mu + \lambda)}{\rho C_2^2} \frac{\partial e}{\partial y} - (1 + \nu_0) \frac{\partial}{\partial t} \frac{\partial T}{\partial y} \quad (24)$$

$$\nabla^2 T - (n_1 + \tau_0) \frac{\partial}{\partial t} \frac{\partial T}{\partial t} - \frac{\hat{\gamma}^2 T_0}{\rho K \omega^*} (n_1 + n_0 \tau_0) \frac{\partial}{\partial t} \frac{\partial e}{\partial t} = 0 \quad (25)$$

Assuming the scalar potential functions $\varphi(x, y, t)$ and $\psi(x, y, t)$ defined by the relations in the non-dimensional form:

$$u = \frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \varphi}{\partial y} - \frac{\partial \psi}{\partial x} \quad (26)$$

By using (26) in Eqs. (23)- (25), we obtain.

$$\left[\nabla^2 - a_0 \frac{\partial^2}{\partial t^2} \right] \varphi - a_0 \left(1 + \nu_0 \frac{\partial}{\partial t} \right) T = 0 \quad (27)$$

$$\left(\nabla^2 - a_1 \frac{\partial^2}{\partial t^2} \right) \psi = 0 \quad (28)$$

$$\left[\nabla^2 - \left(n_1 \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) \right] T - \varepsilon \left(n_1 \frac{\partial}{\partial t} + n_0 \tau_0 \frac{\partial^2}{\partial t^2} \right) \nabla^2 \varphi = 0 \quad (29)$$

where

$$c_1^2 = \frac{\lambda + 2\mu}{\rho}, \quad a_0 = \frac{C_2^2}{C_1^2}, \quad a_1 = \frac{\rho C_2^2}{\mu + k}, \quad \varepsilon = \frac{\hat{\gamma}^2 T_0}{\rho \omega^* K} \quad (30)$$

The solution of the considered physical variables can be decomposed in terms of normal mode as the following form:

$$[\phi, \psi, T](x, y, t) = [\phi^*, \psi^*, T^*](x) \exp(\omega t + i a y) \quad (31)$$

where $[\phi^*, \psi^*, T^*](x)$ are the amplitude of the functions, ω is a complex and a is the wave number in the y -direction and using Eq. (31), then Eqs. (27) – (29) become respectively

$$(D^2 - A_1)\phi^* - A_2 T^* = 0 \quad (32)$$

$$(D^2 - A_3)\psi^* = 0 \quad (33)$$

$$\{(D^2 - A_4)T^* - A_6(D^2 - a^2)\phi^* = 0 \quad (34)$$

where $D = \frac{d}{dx}$,

$$A_1 = a^2 + a_0\omega^2 \quad (35)$$

$$A_2 = a_0(1 + \nu_0\omega) \quad (36)$$

$$A_3 = a^2 + a_1\omega^2 \quad (37)$$

$$A_4 = a^2 - A_5 \quad (38)$$

$$A_5 = \omega(n_1 + \tau_0\omega) \quad (39)$$

$$A_6 = \varepsilon\omega(n_1 + n_0\tau_0\omega) \quad (40)$$

Eliminating T^* , ϕ^* between Eqs. (32) and (34), we get the following fourth order ordinary differential equation satisfied by T^* and ϕ^* (coupled)

$$(D^4 - AD^2 + B)\{\phi^*, T^*\} = 0 \quad (41)$$

We obtain also the uncoupled equation

$$(D^2 - A_3)\psi^* = 0 \quad (42)$$

where

$$A = A_1 + A_4 + A_2A_6 \quad (43)$$

$$B = A_1A_4 + a^2A_2A_6 \quad (44)$$

The solution of Eqs. (41) and (42), has the form

$$\phi^* = \sum_{j=1}^2 M_j(a, \omega) e^{-k_j x} \quad (45)$$

$$T^* = \sum_{n=1}^2 M'_n(a, \omega) e^{-k_n x} \quad (46)$$

$$\psi^* = M_3(a, \omega) e^{-k_3 x} \quad (47)$$

where $M_j(a, \omega)$, $M'_j(a, \omega)$ and $M_3(a, \omega)$ are some parameters depending on a and ω . k_j , ($j=1,2$) are the roots of the characteristic equation of Eq. (41) and k_3 , is the root of the characteristic equation of Eq. (42). Using Eqs. (45) and (46) into Eq. (32) we get the following relations

$$T^* = \sum_{n=1}^2 a_j^* M_j(a, \omega) e^{-k_j x} \tag{48}$$

where,

$$a_j^* = \frac{(k_j^2 - A_1)}{A_2}, \quad j = 1, 2 \tag{49}$$

3 APPLICATION

3.1 Instantaneous mechanical source acting on the surface

The plane boundary is subjected to an instantaneous normal point force and the boundary surface is heat conduction problem, the boundary conditions at the vertical plan $y = 0$ and in the beginning of the crack at $x = 0$ are

$$\sigma_{yy} = -p(x), \quad |x| < a \tag{50}$$

$$T = f(x), \quad |x| < a \quad \text{and} \quad \frac{\partial T}{\partial y} = 0 \quad |x| > a \tag{51}$$

$$\sigma_{xy} = 0, \quad -\infty < x < \infty \tag{52}$$

Using (22), (26) and (27)-(29) with the non-dimensional boundary conditions and using (45), (46) and (48), we obtain the expressions of displacement components, force stress, coupled stress and temperature distribution for the medium as follows:

$$\bar{u}(x) = -k_1 M_1 e^{-k_1 x} - k_2 M_2 e^{-k_2 x} + ia M_3 e^{-k_3 x} \tag{53}$$

$$\bar{v}(x) = ia M_1 e^{-k_1 x} + ia M_2 e^{-k_2 x} - k_3 M_3 e^{-k_3 x} \tag{54}$$

$$\bar{\sigma}_{yy}(x) = s_1 M_1 e^{-k_1 x} + s_2 M_2 e^{-k_2 x} + s_3 M_3 e^{-k_3 x} \tag{55}$$

$$\bar{\sigma}_{xy}(x) = r_1 M_1 e^{-k_1 x} + r_2 M_2 e^{-k_2 x} + r_3 M_3 e^{-k_3 x} \tag{56}$$

$$\bar{T}(x) = a_1^* M_1 e^{-k_1 x} + a_2^* M_2 e^{-k_2 x} \tag{57}$$

where

$$s_1 = -a^2 f_1 + f_2 k_1^2 - (1 + \nu_0 \omega) a_1^*, \quad s_2 = -a^2 f_1 + f_2 k_2^2 - (1 + \nu_0 \omega) a_2^*, \quad s_3 = -iak_3 (f_1 + f_2), \tag{58}$$

$$r_1 = -2iak_1 f_3, \quad r_2 = -2iak_2 f_3, \quad r_3 = f_3 (-a^2 + k_3^2), \quad f_1 = \frac{\lambda + 2\mu + k}{\rho c_2^2}, \quad f_2 = \frac{\lambda}{\rho c_2^2}, \quad f_3 = \frac{\mu}{\rho c_2^2}$$

Applying the boundary conditions (50-52) at the surface $x = 0$ of the plate, we obtain a system of three equations. After applying the inverse of matrix method, we obtain the values of the three constants $M_j, j = 1, 2, 3$. Hence, we obtain the expressions of displacements, force stress, coupled stress and temperature distribution for generalized thermoelastic medium.

4 NUMERICAL RESULTS AND DISCUSSIONS

In order to illustrate our theoretical results obtained in preceding section and to compare these in the context of various theories of thermoelasticity, we now present some numerical results. In the calculation process, we take the case of copper crystal as material subjected to mechanical and thermal disturbances for numerical calculations consider the material medium as that of copper. Since, ω is the complex constant then we take $\omega = \omega_0 + i\zeta$. The other constants of the problem are taken as $\omega_0 = -2$; $\zeta = 1$; the physical constants used are:

$$\rho = 8954 \text{ kgm}^{-3}, \quad \lambda = 7.76 \times 10^{10} \text{ N/m}^2, \quad T_0 = 293\text{K}, \quad \mu = 3.86 \times 10^{10} \text{ N/m}^2, \quad a = 1,$$

$$\alpha_t = 1.78 \times 10^{-5} \text{ K}^{-1}, \quad K = 0.6 \times 10^{-2} \text{ cal/cm sec } ^\circ\text{C}, \quad C_E = 383.1 \text{ J kg}^{-1}\text{K}^{-1}.$$

The results are shown in Figs. 2-7. The graph shows the three curves predicted by different theories of thermoelasticity. In these figures, the solid lines represent the solution in the Coupled theory, the dotted lines represent the solution in the generalized Lord and Shulman theory and dashed lines represent the solution derived using the Green and Lindsay theory. We notice that the results for the temperature, the displacement and stresses distribution when the relaxation time is including in the heat equation are distinctly different from those when the relaxation time is not mentioned in heat equation, because the thermal waves in the Fourier's theory of heat equation travel with an infinite speed of propagation as opposed to finite speed in the non-Fourier case. This demonstrates clearly the difference between the coupled and the generalized theories of thermoelasticity. For the value of y , namely $y = 0.1$, were substituted in performing the computation. It should be noted (Fig.2) that in this problem, the crack's size, x , is taken to be the length in this problem so that $0 \leq x \leq 2, y = 0$, represents the plane of the crack that is symmetric with respect to the y -plane. It is clear from the graph that T has maximum value at the beginning of the crack ($x = 0$), it begins to fall just near the crack edge ($x = 2$), where it experiences sharp decreases (with maximum negative gradient at the crack's end). The value of temperature quantity converges to zero with increasing the distance x .

Fig. 3, the horizontal displacement, u , begins with a line decrease then a parabolic increases again to reach its maximum magnitude just at the crack end. Beyond it u falls again to try to retain zero at infinity. Fig. 4, the vertical displacement v , we see that the displacement component v always starts from the zero value and terminates at the zero value. Also, at the crack end to reach maximum value, beyond reaching zero at the double of the crack size (state of particles equilibrium). The displacements u and v show different behaviors, because of the elasticity of the solid tends to resist vertical displacements in the problem under investigation. Both of the components show different behaviors, the former tends to increase to maximum just before the end of the crack. Then it falls to a minimum with a highly negative gradient. Afterwards it rises again to a maximum beyond about the crack end.

All horizontal stresses, σ_{xx} reach coincidence with zero values (Fig. 5) and satisfy the boundary condition at $x = 0$ and converges to zero with increasing the distance x . However σ_{yy} (Fig.6) had a same behavior as it retains its maximum strength until reaching the crack end when it falls to a minima then increases again just beyond the crack edge to coincide with other vertical stresses to reach zero after their relaxations at infinity. These trends obey elastic and thermoelastic properties of the solid under investigation. Fig. 7, shows that the stress component σ_{yx} satisfy the boundary condition at $x = 0$. It increases in the start and start decreases (maximum) in the context of the three theories at the end of crack.

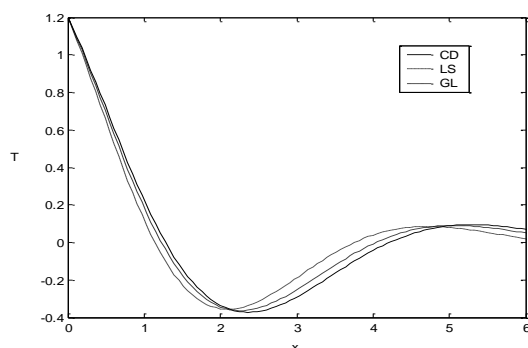


Fig.2
Variation of temperature distribution T with the different theories.

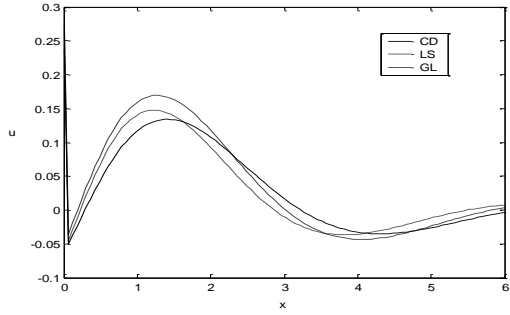


Fig.3
Variation of horizontal displacement distribution u with the different theories.

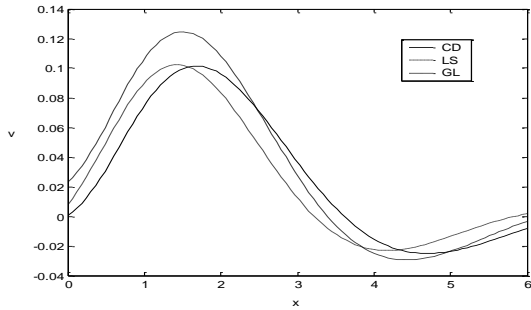


Fig.4
Variation of vertical displacement distribution v with the different theories.

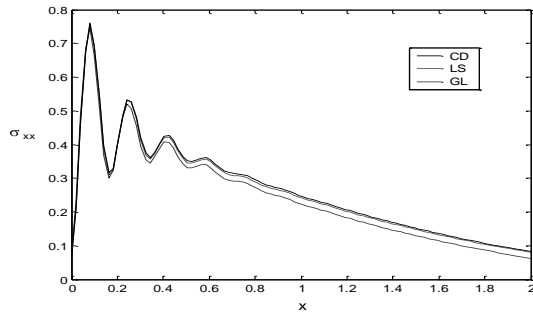


Fig.5
Variation of horizontal stress distribution σ_{xx} with the different theories.

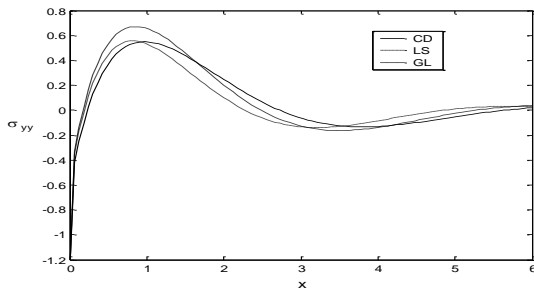


Fig.6
Variation of vertical stress distribution σ_{yy} with the different theories.

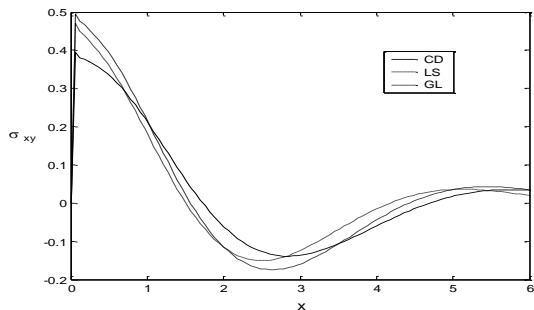


Fig.7
Variation of stress component distribution σ_{xy} with the different theories.

Figs. 8-11 show the comparison between the temperature T , displacement components u, v and the force stress component σ_{yy} , in the case of different three values of y , (namely $y = 0.1, y = 0.2$ and $y = 0.3$) under GL theory.

It should be noted (Fig. 8) that in this problem. It is clear from the graph that zero has maximum value at the beginning of the crack ($x = 0$), it begins to fall just near the crack edge ($x = 2$), where it experiences sharp decreases (with maximum negative gradient at the crack's end). Graph lines for both values of y show different slopes at crack ends according to y -values. In other words, the temperature line for $y = 0.1$ has the highest gradient when compared with that of $y = 0.2$ and $y = 0.3$ at the edge of the crack. In addition, all lines begin to coincide when the horizontal distance x is beyond the double of the crack size to reach the reference temperature of the solid. These results obey physical reality for the behavior of copper as a polycrystalline solid.

Fig. 9, the horizontal displacement u , despite the peaks (for different vertical distances $y = 0.1, y = 0.2$ and $y = 0.3$) occur at equal value of x , the magnitude of the maximum displacement peak strongly depends on the vertical distance y . it is also clear that the rate of change of u increases with increasing y as we go farther apart from the crack. On the other hand, Fig. 10 shows atonable increase of the vertical displacement, v , near the crack end to reach maximum value beyond $x = a = 2$ reaching zero at the double of the crack size (state of particles equilibrium).

Fig.11, the vertical stresses σ_{yy} graph lines for both values of y show different slopes at crack ends according to y -values. In other words, the σ_{yy} component line for $y = 0.1$ has the highest gradient when compared with that of $y = 0.2$ and $y = 0.3$ at the edge of the crack. In addition, all lines begin to coincide when the horizontal distance x is beyond the double of the crack size to reach zero after their relaxations at infinity. Variation of y has a serious effect on both magnitudes of mechanical stresses (Fig.11). These trends obey elastic and thermoelastic properties of the solid under investigation.

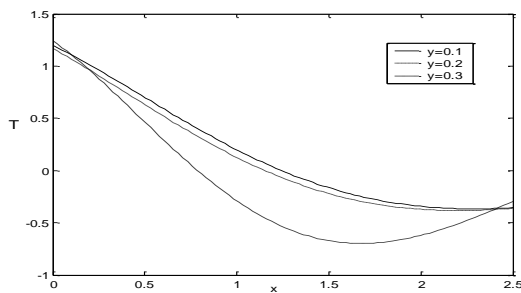


Fig.8
Variation of temperature distribution T with the different depths under G-L theory.

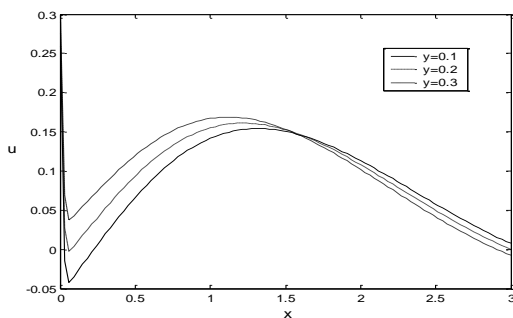


Fig.9
Variation of horizontal displacement distribution u with the different depths under G-L theory.

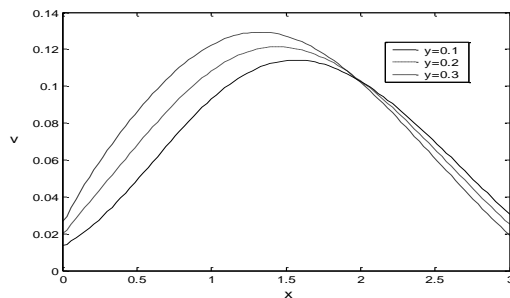
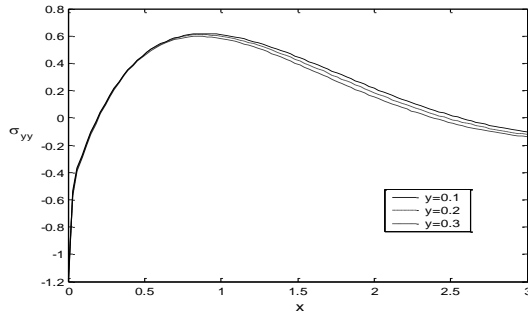


Fig.10
Variation of vertical displacement distribution v with the different depths under G-L theory.

**Fig.11**

Variation of vertical stress distribution σ_{yy} with the different depths under G-L theory.

5 CONCLUSIONS

1. The curves in the context of the (CD), (L-S) and (G-L) theories decrease exponentially with increasing x , this indicate that the thermoelastic waves are attenuated and non-dispersive, where purely thermoelastic waves undergo both attenuation and dispersion.
2. The curves of the physical quantities with (L-S) theory in most of figures are lower in comparison with those under (G-L) theory, due to the relaxation times.
3. Analytical solutions based upon normal mode analysis for thermoelastic problem in solids have been developed and utilized.
4. A linear opening mode-I crack has been investigated and studied for copper solid.
5. Temperature, radial and axial distributions were estimated at different distances from the crack edge.
6. Horizontal and vertical stress distributions were evaluated as functions of the distance from the crack edge.
7. Crack dimensions are significant to elucidate the mechanical structure of the solid.
8. Cracks are stationary and external stress is demanded to propagate such cracks.
9. It can be concluded that a change of volume is attended by a change of the temperature while the effect of the deformation upon the temperature distribution is the subject of the theory of thermoelasticity.
10. The value of all the physical quantities converges to zero with an increase in distance y and all functions are continuous.

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