Performace Evaluation of Maximum A-Posteriori Estimator in the Nakagami-m Fading MIMO Channels for m<1

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ABSTRACT:
The Nakagami-m model has garnered significant attention in the literature as a versatile channel model suitable for describing channels experiencing varying degrees of fading, from severe to moderate. This model aligns closely with the characteristics observed in the majority of measured fading radio channels. The estimation of Nakagami-m fading in a multiple-input multiple-output (MIMO) channel poses a crucial challenge. Deriving the probability density function (pdf) for a Nakagami distribution random vector with correlated entries and developing closed-form classical and/or Bayesian estimators for a linear MIMO channel proves to be impractical. In this study, we simplify the analysis by assuming that the entries of the Nakagami fading random vector are uncorrelated. Consequently, the joint distribution of the channel vector entries is computed by multiplying the pdfs of the individual entries. Subsequently, the maximum a-posteriori (MAP) estimation of the channel entries is determined. Under the assumption of orthogonal training symbols, the obtained results lead to second-order nonlinear complex equations. To evaluate the performance of the MAP channel estimator in MIMO Nakagami-m frequency-flat fading channels, these nonlinear complex equations are solved numerically. The findings indicate that one-sided Gaussian fading represents the worst-case scenario, yet the channel estimation results surpass those of the least squares (LS) estimation. Additionally, fewer errors are observed in Rayleigh fading channel estimation. Furthermore, it is demonstrated that the performance of the MAP estimator improves with an increase in the Nakagami shape parameter. The numerical results affirm that the proposed estimator serves as a suitable method for estimating Nakagami-m fading in uncorrelated MIMO channels with m<1.

KEYWORDS: Nakagami-m Fading, One-sided Gaussian Fading, Rayleigh Fading, Maximum A-Posteriori Estimator.

1. INTRODUCTION

The focus on multiple input multiple output (MIMO) systems in wireless communications stems from their notable attributes of high capacity and diversity gain. Extensive research has demonstrated that in scenarios where the fades between pairs of transmit and receive antenna elements are independent and identically distributed (i.i.d.), the capacity of a Rayleigh distributed flat fading channel exhibits almost linear growth with the minimum number of transmitter and receiver antennas [1]-[3]. The study in [3] additionally highlights that Rician fading can enhance the capacity of a multiple antenna system, particularly when the transmitter possesses knowledge of the Rice factor. Furthermore, findings in [4], [5] reveal that in Nakagami-m fading, the MIMO channel capacity experiences an increase as the fading parameters are elevated.

To harness the benefits of MIMO systems, it is imperative that the receiver and/or transmitter possess access to channel state information (CSI). One prevalent method for determining MIMO CSI is through training-based channel

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estimation (TBCE) [6], [7]. The selection of optimal training signals typically involves exploring the minimization of the mean square error (MSE) of the linear MIMO channel estimator. Existing literature suggests that the optimal design of training sequences for MIMO channel estimation is intricately linked to the statistical characteristics of the channel, such as the fading model and the channel noise model.

While the Rayleigh model is commonly assumed for fading in many wireless communication systems, it is often conjectured that the MIMO channel fading follows a Rayleigh distribution. However, the Nakagami-m model proves to be a more suitable fit for the fading channel distribution. The Nakagami-m distribution fading model [8] stands out as one of the most versatile, demonstrating greater flexibility and accuracy in aligning with experimental data compared to Rayleigh, log-normal, or Rician distributions. This model is characterized by two parameters: the scale parameter and the shape parameter, denoted as 'm'. It incorporates the Rayleigh distribution when 'm' equals 1 and the one-sided Gaussian distribution when 'm' equals 1/2. Considered a versatile statistical distribution, Nakagami-m accurately models a variety of fading environments.

Various studies, including [4], [5], and [9], have delved into the modeling of Nakagami fading in MIMO channels. The challenge lies in determining the parameter 'm' during the estimation of the Nakagami probability density function (pdf). To effectively utilize the Nakagami-m distribution for modeling a given set of empirical data, it becomes necessary to ascertain or estimate the shape parameter from the data. The receiver, for optimal signal reception in Nakagami fading, also requires knowledge of this shape parameter. Different methods can be employed to estimate the required knowledge of channel statistics.

For example, in [10], the problem of estimating the Nakagami 'm' parameter is addressed using maximum likelihood (ML) estimation. In [11], a maximum a-posteriori (MAP) estimator is introduced for Nakagami-m fading parameter estimation. The derivation of the covariance matrix for correlated Nakagami-m fading channels is presented in [12]. Additionally, [13] introduces a copula-based method for estimating the Nakagami fading parameter in the received signal, which is subject to fading and contaminated by dependent noise.

In [7], [14], the authors proposed shifted scaled least squares (SSLS) and minimum mean square error (MMSE) estimators for the estimation of Rician fading in MIMO channels. Subsequently, in [15], the correlation between the channel Rice factor and the Nakagami shape parameter is utilized to formulate the MIMO channel covariance matrix. As a result, the SSLS and MMSE estimators can leverage the knowledge of Nakagami channel statistics, leading to an improvement in their performance. Numerical findings affirm the suitability of both estimators for Nakagami MIMO channel estimation, with the MMSE channel estimator demonstrating superior performance compared to the SSLS and least squares (LS) estimators. However, it is noted that the SSLS and MMSE estimators in [15] are specifically effective for scenarios where 'm' is greater than 1.

In this study, the MAP estimator is employed to estimate the Nakagami fading MIMO channel when 'm' is less than 1. The joint pdf of the channel vector entries is derived by multiplying the pdfs of the entries, assuming the vector entries are uncorrelated. Although the MAP estimation result does not yield a closed-form estimator, it leads to a set of nonlinear second-order complex equations. An algorithm is utilized to solve these equations and estimate the channel. Numerical results indicate an enhancement in the performance of the MAP estimator with an increase in the Nakagami shape parameter. Even in the most challenging scenario, where 'm' equals 0.5, the results are superior to those of the LS estimator.

The remainder of this paper is structured as follows: The next section introduces the system model of interest and outlines some assumptions regarding the fading process. Section 3 delves into the study of the MAP estimator. Numerical examples are presented in Section 4, while Section 5 serves as the conclusion for this paper.

2. THE SYSYEM MODEL

We consider a MIMO system with n_t transmitter and n_r receiver antennas. The frequency-flat block fading model is assumed for the MIMO channel. It means that the channel response is fixed within one block and can vary from one block to another one randomly. Each transmitted block contains training and data symbols. The frame structure is the same for all Tx antennas. Training and data symbols are located at the beginning and the end of the blocks, respectively. In practice, the channel is estimated using training symbols in the training phase, which will be used for data detection. To estimate the MIMO channel in each block, it is required that $n_p \geq n_t$ training signals are transmitted by each transmitter antenna. The $n_r \times n_p$ complex received signal matrix can be expressed as

$$\mathbf{Y} = \mathbf{HX} + \mathbf{V}, \qquad (1)$$

where $\mathbf{X}$ and $\mathbf{V}$ are the complex n_t -vector of transmitted sequences on the n_t transmit antennas and n_r -vector of additive noise, respectively, and $\mathbf{H}$ is the $n_r \times n_t$ channel matrix. The elements of noise matrix are i.i.d. complex Gaussian random variables with zero mean and the variance σ_n^2 (i.e., $\mathcal{CN}(0, \sigma_n^2)$). The MIMO channel model (1) can be expressed in the following vector form:

$\mathbf{y} = \tilde{\mathbf{X}}\mathbf{h} + \mathbf{v}$ (2)

where $\mathbf{y} = \text{vec}(\mathbf{Y})$, $\mathbf{v} = \text{vec}(\mathbf{V})$, $\tilde{\mathbf{X}} = \mathbf{X}^T \otimes \mathbf{I}_{n_r}$, and $\mathbf{h} = \text{vec}(\mathbf{H})$. The notation $(\cdot)^T$ is reserved for the matrix transpose, $\otimes$ for the Kronecker product. $\mathbf{I}_r$ denotes the $r \times r$ identity matrix. The operand $\text{vec}(\cdot)$ stacks all the columns of the matrix argument into one tall column vector.

The entries of the channel matrix **H** in (1) or the vector **h** in (2) are assumed to be complex random variables with the following general form

$h_{ij} = \mathrm{R}e^{j\Theta}$ (3)

where R is the envelope and Θ is the phase. The Nakagami-m fading envelope R has the following pdf [8]

$$f_{\mathrm{R}}(r) = \frac{2\left(\frac{m}{\Omega}\right)^m r^{2m-1}}{\Gamma(m)} \exp\left(-\frac{m}{\Omega}r^2\right); \quad r \geq 0\,, \; m \geq 0.5 \tag{4}$$

Where $\Omega = \mathrm{E}[\mathrm{R}^2]$ is the expected value of the average power, and $m = \Omega^2/\mathrm{V}[\mathrm{R}^2]$ is the shaping parameter which controls the shape of the distribution, and $\mathrm{E}[\mathrm{R}^2]$ and $\mathrm{V}[\mathrm{R}^2]$ respectively denote the expectation and variance of R^2.

In (4), $\Gamma(\cdot)$ is the gamma function as follows

$$\Gamma(m) = \int_0^{+\infty} x^{m-1} e^{-x}\, dx \tag{5}$$

The mean and variance of R can be written as follows:

$$\mathrm{E}[\mathrm{R}] = \frac{\Gamma(m+1/2)}{\Gamma(m+1)} \sqrt{\frac{\Omega}{m}} \tag{6}$$

$$\mathrm{V}[\mathrm{R}] = \Omega\left(1 - \frac{1}{m}\left(\frac{\Gamma(m+1/2)}{\Gamma(m+1)}\right)^2\right) \tag{7}$$

The k-th moment of the Nakagami-m distribution is given by [10]

$$\mathrm{E}[\mathrm{R}^k] = \frac{\Gamma(m+k/2)}{\Gamma(m)}\left(\frac{\Omega}{m}\right)^{k/2} \tag{8}$$

In (3), the phase Θ is assumed to be uniformly distributed as follows:

$$f_\Theta(\Theta) = \frac{1}{2\pi}\,, \qquad -\pi \leq \Theta \leq \pi \tag{9}$$

The Nakagami-m distribution covers a wide range of fading conditions. For example, when m=0.5, it is reduced to a one-sided Gaussian distribution and when m=1, it is reduced to a Rayleigh distribution. In the limit when $m \rightarrow \infty$, the channel becomes static, and its corresponding pdf becomes an impulsive function located at $\sqrt{\Omega}$. For m < 1, the fading is more severe than the Rayleigh fading, and for values of m > 1, the fading is less severe. For the values of m > 1, the Nakagami-m distribution closely approximates the Rician distribution [15]. In the rest of the paper, we assume that m<1, unless otherwise specified.

3. MAP CHANNEL ESTIMATION

We assume that channels between each pair of transmit and receive antennas, i.e., h_{ij}'s, are independent. Therefore, the joint pdf of the entries of **h** is computed by multiplying the pdfs of the entries using (4) and (9) as

$$\boldsymbol{p}(\mathbf{h}) = \mathrm{C} \prod_{i=1}^{n_r}\prod_{j=1}^{n_t}\left|h_{ij}\right|^{2(m-1)} \exp\left(-\frac{m}{\Omega}\sum_{i=1}^{n_r}\sum_{j=1}^{n_t}\left|h_{ij}\right|^2\right) \tag{10}$$

Where $\mathrm{C} = \left(m^m/\pi\, \Omega^m\, \Gamma(m)\right)^{n_r n_t}$ and $h_{ij}, i = 1, 2, \ldots, n_r; j = 1, 2, \ldots, n_t$ are the elements of the channel matrix **H** in (1). The conditional pdf can be computed as

$$\boldsymbol{p}(\mathbf{y}|\mathbf{h}) = \frac{1}{\pi^{n_r n_t} \det(\mathbf{C_V})}\, exp\left(-\frac{1}{\sigma_n^2}\sum_{i=1}^{n_r}\sum_{j=1}^{n_t}\left|y_{ij} - z_{ij}\right|^2\right) \tag{11}$$

Where σ_n^2 is the variance of the elements of additive receiver noise matrix $\mathbf{V}$ in (1), $\mathbf{C_V}$ is the covariance matrix of the elements of the vector $\mathbf{v}$ in (2), $y_{ij}, i = 1, 2, \dots, n_r;\ j = 1, 2, \dots, n_t$ are the elements of the received signal matrix $\mathbf{Y}$ in (1), $z_{ij} = \sum_{n=1}^{n_t} h_{in}\, x_{nj}\,,\ i = 1, 2, \dots, n_r;\ \ j = 1, 2, \dots, n_t$, and $x_{ij}, i, j = 1, 2, \dots, n_t$ are the elements of the training matrix $\mathbf{X}$ in (1).

In MAP, the channel is estimated in order to maximize $\boldsymbol{p}(\mathbf{h})\, \boldsymbol{p}(\mathbf{y}|\mathbf{h})$ as follows:

$$\hat{\mathbf{h}}_{MAP} = \underset{\mathbf{h}}{\operatorname{argmax}}\ \left(\boldsymbol{p}(\mathbf{h})\, \boldsymbol{p}(\mathbf{y}|\mathbf{h})\right), \qquad (12)$$

or equivalently,

$$\hat{\mathbf{h}}_{MAP} = \underset{\mathbf{h}}{\operatorname{argmax}}\ \left(\ln \boldsymbol{p}(\mathbf{h}) + \ln \boldsymbol{p}(\mathbf{y}|\mathbf{h})\right). \qquad (13)$$

Using (10), (11) and by differentiating $\ln \boldsymbol{p}(\mathbf{h}) + \ln \boldsymbol{p}(\mathbf{y}|\mathbf{h})$ with respect to $h_{kl},\ k = 1, 2, \dots, n_r;\ l = 1, 2, \dots, n_t$ and setting the results equal to zero, $n_r n_t$ complex second-order equations are obtained as

$$-(m-1) + \frac{m}{\Omega}\, |h_{kl}|^2 - \left(\sum_{j=1}^{n_t} x_{lj} \bar{y}_{kj}\right) h_{kl}$$
$$-\sum_{j=1}^{n_t} \sum_{\substack{n=1 \\ n \neq l}}^{n_t} x_{lj} \bar{x}_{nj}\, \bar{h}_{kn} h_{kl} + \left(\sum_{j=1}^{n_t} |x_{lj}|^2\right) |h_{kl}|^2 = 0$$
$$k = 1, 2, \dots, n_r;\quad l = 1, 2, \dots, n_t \qquad (14)$$

Where $\overline{(.)}$ denotes the complex conjugate. Assuming that a training matrix $\mathbf{X}$ with orthogonal rows is used here, we have

$$\sum_{j=1}^{n_t} x_{lj} \bar{x}_{nj} = 0 \qquad for\ l, n = 1,2, \dots, n_t \qquad (15)$$

Using (15), it is straightforward to show that under orthogonal training for the MAP estimator, (14) reduces to

$$a_l |h_{kl}|^2 - b_{kl}\, h_{kl} - (m-1) = 0$$

$$k = 1, 2, \dots, n_r;\quad l = 1, 2, \dots, n_t \qquad (16)$$

Where,

$$a_l = \frac{m}{\Omega} + \sum_{j=1}^{n_t} |x_{lj}|^2, \qquad (17)$$

$$b_{kl} = \sum_{j=1}^{n_t} x_{lj} \bar{y}_{kj} \qquad (18)$$

Generally, the second-order equations of (16) have two roots for any k, l. The roots that maximize the function $\boldsymbol{p}(\mathbf{h})\, \boldsymbol{p}(\mathbf{y}|\mathbf{h})$ are chosen. Suppose $h_{kl} = h_{kl_R} + j h_{kl_I}$, where h_{kl_R} is the real part of h_{kl} and h_{kl_I} is the imaginary part of h_{kl}. Also, suppose $b_{kl} = b_{kl_R} + j b_{kl_I}$, where b_{kl_R} is the real part of b_{kl} and b_{kl_I} is the imaginary part of b_{kl}. Substituting them in (16) and with some calculations, for $k = 1, 2, \dots, n_r;\ \ l = 1, 2, \dots, n_t$ we will have

$$d_{kl} \left(h_{kl_R}\right)^2 + e_{kl}\, h_{kl_R} - (m-1) = 0, \qquad (19)$$

$$h_{kl_I} = -h_{kl_R} \frac{b_{kl_I}}{b_{kl_R}}\ . \qquad (20)$$

Where,

$$d_{kl} = a_l \left(1 + \left(\frac{b_{kl_I}}{b_{kl_R}}\right)^2\right), e_{kl} = -\left(b_{kl_R} + \frac{(b_{kl_I})^2}{b_{kl_R}}\right). \qquad (21)$$

In order to estimate the channel matrix **H**, We use Algorithm 1, as follows:

Algorithm 1: The MAP estimation's steps.

Step 1: Solve the second order equation (19) for $k = 1$ and $l = 1, 2, \dots, n_t$ (there are two roots for any k, l, generally)
Step 2: Calculate (20) for $k = 1, l = 1, 2, \dots, n_t$ and for both roots of (19)
Step 3: Calculate $h_{kl} = h_{kl_R} + jh_{kl_I}$ for $k = 1$ and $l = 1, 2, \dots, n_t$ and for both roots of (19)
Step 4: Calculate the function
$f(k) = \sum_{j=1}^{n_t} ln|h_{kj}|^{2(m-1)} - \frac{m}{\Omega}\sum_{j=1}^{n_t}|h_{kj}|^2 - \sum_{j=1}^{n_t}|y_{kj-}z_{kj}|^2$
for 2^{n_t} combinations of roots obtained in step 3 and choose a combination of roots that maximizes $f(k)$
Step 5: Repeat steps 1-4 for $k = 2, \dots, n_t$

In Step 4 of the Algorithm 1, we are using the following relation

$$\ln \boldsymbol{p}(\mathbf{h}) + \ln \boldsymbol{p}(\mathbf{y}|\mathbf{h}) = \ln \mathrm{C} + \ln \mathrm{B} + \sum_{k=1}^{n_t} f(k) \qquad (22)$$

Where $\mathrm{B} = 1/\pi^{n_r n_t} \det(\mathbf{C_V})$, and B, C are independent of h_{kl}.

4. SIMULATION RESULTS

In this section, the performance of the MAP estimator is numerically evaluated. To measure the accuracy of the channel estimation, we use the normalized mean square error (NMSE) defined as follows

$$NMSE = \frac{E\{\|\mathbf{h} - \hat{\mathbf{h}}\|_F^2\}}{E\{\|\mathbf{h}\|_F^2\}} \qquad (23)$$

For simulations, we generate samples of MIMO channels using the Nakagami-m distribution. For the training sequences, we use the orthogonal sequences proposed in [6] ad [7]. For each signal-to-noise ratio (SNR), we run 5000 simulations and average find the NMSE using (23).

Fig. 1 shows NMSE of the LS estimator [10] and the MAP channel estimator (algorithm 1) with orthogonal training versus signal to noise ratio (SNR) for various Nakagami shape parameters when n_r= n_t=1. As it is expected, the LS estimator can not exploit the knowledge of m, a phenomenon that is confirmed in [15]. In [6], [7] and [15] it was shown that the LS estimator does not require any knowledge of the channel and that the performance of the LS estimator is independent of the channel shape parameter, m, and the correlation coefficients. Here, we use the LS estimator as a benchmark for comparison. It is observed that one-sided Gaussian distribution (m = 0.5) is the worst case for the MAP estimation, however, the result is still better than LS estimation. Moreover, by increasing m the performance of the MAP estimator improves, especially at low SNRs.

Figs. 2 and 3 show the NMSE of the LS and MAP estimators for higher number of transmitter and receiver antennas. As can be seen, at high SNRs, the performances of the MAP estimator for different values of m are similar particularly for lower number of transmitter and receiver antennas. It is notable that in the special case of m = 1, i.e Rayleigh fading, the MAP estimator is the same as the MMSE estimator of [6] for low correlations.

Figs. 4, 5, and 6 compare the NMSE of the MAP channel estimator versus SNR for various number of transmitter and receiver antennas when m=0.5, m = 0.75, and m=1, respectively. As expected, increasing the number of transmitter and receiver antennas results in higher channel estimation error.

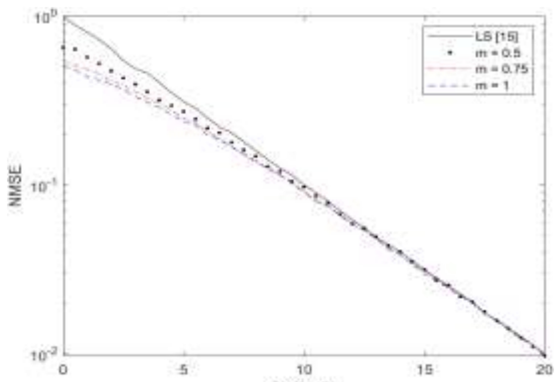

Fig. 1. NMSE of the LS and MAP estimators vs. SNR for various Nakagami shape parameters and $n_r = n_t = 1$.

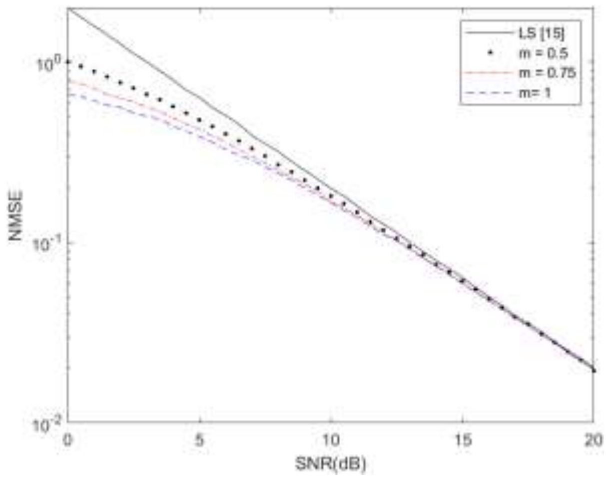

Fig. 2. NMSE of the LS and MAP estimators vs. SNR for various Nakagami shape parameters and $n_r = n_t = 2$.

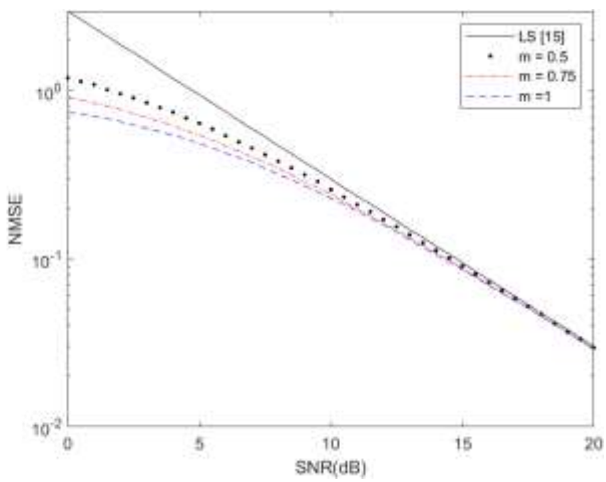

Fig. 3. NMSE of the LS and MAP estimators vs. SNR for various Nakagami shape parameters and $n_r = n_t = 3$.

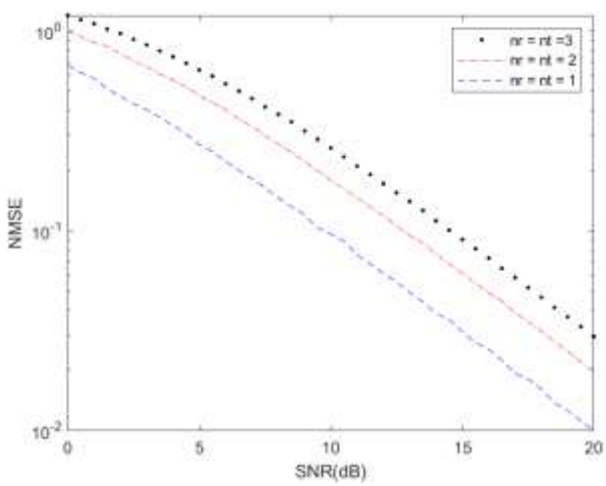

Fig. 4. NMSE of the MAP estimator vs. SNR for various number of antennas $n_r = n_t$ and m = 0.5.

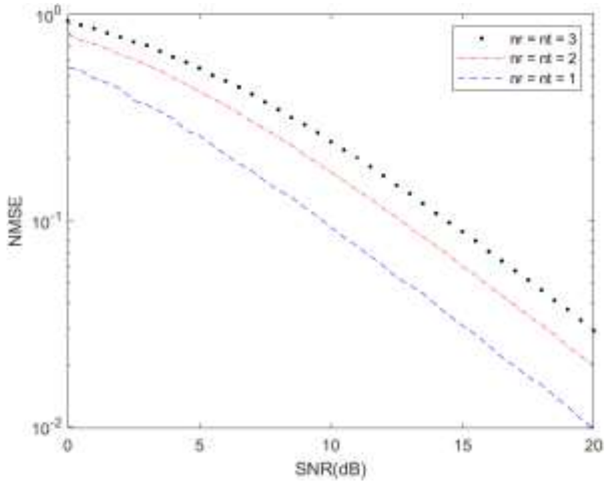

Fig. 5. NMSE of the MAP estimator vs. SNR for various number of antennas $n_r = n_t$ and m = 0.75.

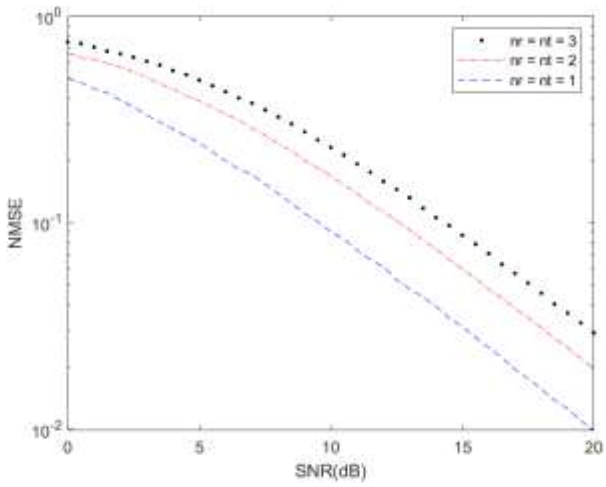

Fig. 6. NMSE of the MAP estimator vs. SNR for various number of antennas $n_r = n_t$ and m = 1.

5. CONCLUSIONS

This paper introduces the MAP estimator for estimating Nakagami fading in uncorrelated MIMO channels with m < 1. The proposed approach yields a set of second-order nonlinear equations characterized by complex coefficients. An algorithm is employed to solve these equations and obtain the channel coefficients. As anticipated, an increase in the Nakagami shape parameter contributes to an enhancement in channel estimation accuracy. Remarkably, the MAP estimation results outperform classical LS estimation, even under severe fading conditions, such as when m=0.5. In the special case where m=1, corresponding to the Rayleigh fading model, the MAP approach presented in this paper aligns with the MMSE technique from [6] for uncorrelated channel scenarios. It is noteworthy that for m > 1, the channel follows a Rician distribution and experiences less severe fading. Previous work in [15] has addressed the estimation of this channel type, and the results align with the findings presented in this article. Nakagami fading is recognized as a suitable model for wireless environments, with the Nakagami-m distribution often providing the best fit for land-mobile and indoor-mobile multipath propagation.

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