

Channel Estimation Techniques for MIMO-OFDM Systems Using Pilot Carriers on Multipath Channels

Navid Daryasafar¹, Omid Borazjani²

1-Department of Electrical Engineering, Dashtestan Branch, Islamic Azad University, Borazjan, Iran
E-mail: navid_daryasafar@yahoo.com (Corresponding author)

2- Department of Electrical Engineering, Dashtestan Branch, Islamic Azad University, Borazjan, Iran.
E-mail: omidborazjani@yahoo.com

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ABSTRACT:

In high capacity systems, as the bit transmission rate increases, Intersymbol Interference (ISI) caused by the multi-path channel reduces the system efficiency. The technique of OFDM acts very well against this phenomenon. Moreover, an accurate estimation of the communication channel coefficients improves the performance of communication systems effectively. In this paper, the estimation algorithms of the MIMO-OFDM channels are investigated to compare them in terms of estimation error and calculation complexity. In the following research, a method is proposed to improve the estimation of the MIMO-OFDM channel. In this method, combining the LS algorithm with adaptive algorithms improves the estimation performance at different Doppler frequencies. We observe in simulations that the use of adaptive algorithms improves the estimation of the channel at Doppler frequencies.

KEYWORDS: Multiple Input Multiple Output Systems (MIMO), LS Algorithm, LMS Algorithm, RLS Algorithm.

1. INTRODUCTION

Propagation environment phenomena such as fading and ISI reduce the performance of mobile wireless networks. Communication systems should be designed in such a way that they dominate these problems reliably. One of the modern methods to overcome the problems and also increase the transmission rate is to use several antennas in the transmitter and receiver or the so called MIMO system [1]. Using spatial-temporal processing, problems of a wireless channel can be overcome in these systems. Since most of MIMO algorithms are used for narrow band channels, the technique of OFDM can be used in the MIMO system to confront with the selective nature of wideband wireless channels frequencies. OFDM converts effectively a wideband channel to a few parallel flat sub-channels. Given that in detecting the data vectors transmitted in MIMO-OFDM systems, it is assumed that the channel coefficients matrix is quite known for the receiver, it is necessary to estimate the channel matrix in all sub carriers and at every time period to correctly detect the transmitted data.

In order to reveal the coherent of received signals, digital communication systems must have an exact estimation of the situation of exchange channel between transmitter and receiver. Since increasing the number of transmitter and receiver antennas causes an increase in the number of unknowns (coefficients of the

channel between both antennas of transmitter and receiver) the estimation of channels in multi-antenna systems is a lot more challenging than in one-antenna ones [2].

Many references have so far been published on channel estimation in OFDM based single antenna systems [3-8]. However, since the signal received by each sub carrier of MIMO-OFDM is sum of the faded signals transmitted by different transmitter antennas plus noise, the channel estimation techniques commonly used in SISO-OFDM systems are not applicable to MIMO-OFDM systems [9].

In recent years, numerous papers dealt with the subject of channel estimation in MIMO-OFDM systems [9-16]. Most of the publish papers discussed the channel estimation assuming perfect frequency synchronization between the transmitter and receiver (lack of any frequency offset between the carrier and the local reference of the receiver). Although the synchronization module is an integral part of all communication receivers, there is always a time varying frequency offset, caused by the phase noise and Doppler frequency, between the transmitter and receiver oscillators.

Adaptive CE algorithms are gaining more attention these days. Least Mean Square (LMS) [17-20] is widely used for its simplicity. If complexity is not an issue then Recursive Least Square (RLS) [17], [18] is a

good choice. Moreover to use the best part of the above given Adaptive Channel Estimation (ACE) algorithms they can be combined to build the hybrid algorithms.

2. SYSTEM DESCRIPTION

The block diagram of a MIMO-OFDM system is shown in Figure 1. Basically, the MIMO-OFDM transmitter has N_t parallel transmission paths which are very similar to the single antenna OFDM system, each branch performing serial-to-parallel conversion, pilot insertion, N -point IFFT and cyclic extension before the final TX signals are up-converted to RF and transmitted. It is worth noting that the channel encoder and the digital modulation, in some spatial multiplexing systems [21], can also be done per branch, not necessarily implemented jointly over all the N_t branches. The receiver first must estimate and correct the possible symbol timing error and frequency offsets, e.g., by using some training symbols in the preamble. Subsequently, the CP is removed and N -point FFT is performed per receiver branch.

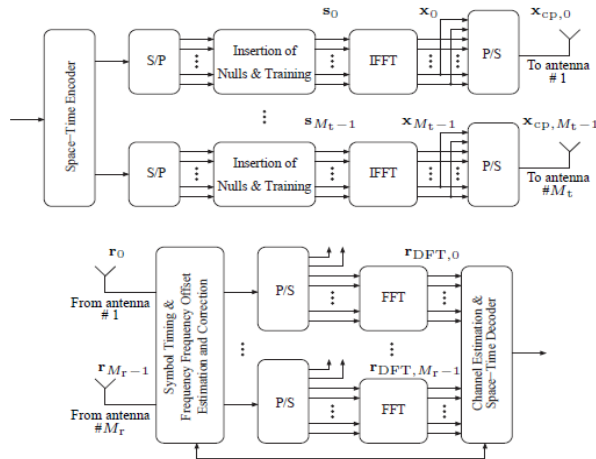


Fig.1. Block diagram of transmitter and receiver in a MIMO-OFDM system

For a 2×2 MIMO-OFDM channel, the impact response under the channel between i_{th} transmitter antenna and j_{th} receiver is represented by $h_{i,j}$ [22]:

$$H(K) = \sum_{K=0}^{L-1} h_L e^{-j\frac{2\pi}{N}KL}; K = 0, \dots, N-1 \quad (1)$$

Signal model is as follows:

$$S^{cp}(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K) e^{j\frac{2\pi}{N}K(n-N_{cp})}; n = 0, \dots, N + N_{cp} - 1 \quad (2)$$

Then vector r rate in the presence of carrier frequency offset is:

$$r^{cp}(n) = e^{j2\pi\Delta f n} \times S^{cp}(n) * h(n) + z(n); n = 0, \dots, N-1 \quad (3)$$

With eliminating cp and doing a series of operations, we have:

$$r(n) = e^{j2\pi\Delta f(n+N_{cp})} \frac{1}{\sqrt{N}} \sum_{K=0}^{N-1} X(K) \sum_{k=0}^{L-1} h_L e^{-j\frac{2\pi}{N}KL} e^{j\frac{2\pi}{N}Kn} + z(n) \quad (4)$$

Where $Z(n)$ is White Gaussian Noise with an average of 0. The output of the receiver is as follows:

$$y(n) = \frac{1}{N} \sum_{n=0}^{N-1} r(n) e^{-j\frac{2\pi}{N}Kn}; k = 0, \dots, N-1 \quad (5)$$

Result

$$y(n) = e^{j2\pi\Delta f N_{cp}} \sum_{i=0}^{N-1} X(i) H(i) \delta_{i,k} + z(k) \quad (6)$$

And

$$\delta_{i,k} = \frac{1}{N} \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}n(CF0+i-k)} = \text{sinc}(CF0+i-k) e^{j\pi(CF0+i-k)}, i, k = 0, \dots, N-1 \quad (7)$$

$$Y_{1 \times N \times 1} = [X_1 X_2] W L h_1 = A h_1 + z_1 \quad (8)$$

$$Y_{2 \times N \times 1} = [X_1 X_2] W L h_2 = A h_2 + z_2 \quad (9)$$

Finally Channel's coefficients estimation in MIMO-OFDM system is as follows:

$$\text{if } : k \in \text{pilot}_{2L} \Rightarrow \hat{h}_1 = A^{-1}(k) \hat{Y}_1(k), \hat{h}_2 = A^{-1}(k) \hat{Y}_2(k) \quad (10)$$

Where

$$h_1 = [h_{11}; h_{21}], h_2 = [h_{12}; h_{22}] \quad (11)$$

And WL is a matrix $N \times L$ consisting of all $e^{-jkL\frac{2\pi}{N}}$

3. CHANNEL ESTIMATION TECHNIQUES

1.3. LS Channel Estimation

The combination of orthogonal frequency division multiplexing (OFDM) with space-time coding has received much attention recently to combat multipath delay spread and increase system capacity. Channel parameters are needed in order to coherently decode the transmitted signal. Least square (LS) channel estimation for MIMO-OFDM systems has been addressed in. But if the multipaths are not sample-spaced, the well known leakage problem for DFT based channel estimation induces an irreducible error floor for estimation error. To reduce this error floor, more taps have to be used, which not only increases computational complexity but also makes estimation problem more ill-conditioned and thus enhances noise. As an alternative, channel estimation algorithm based on parametric model has been proposed in and extended to MIMO-OFDM in.

At the receiver, $n_r \times n_t$ sets of extracted received pilot tones are used for channel estimation, which LS method is chosen due to its simplicity. The standard formula for this approach at m th symbol is computed as:

$$H_{LS}(m) = ((x^p(m))^H x^p(m))^{-1} ((x^p(m))^H Y^p(m)) \quad (12)$$

Where, $X^p(m)$ and $Y^p(m)$ respectively show the transmitted and received pilots.

The LS estimate of the channel can be obtained as

$$H_{LS} = \arg \left\{ \min \left\{ (Y - XH_{LS})^H (Y - XH_{LS}) \right\} \right\} \quad (13)$$

Set

$$\frac{\partial \left\{ (Y - XH_{LS})^H (Y - XH_{LS}) \right\}}{\partial H_{LS}} \quad (14)$$

Consequently

$$\hat{H}_{LS} = \frac{Y}{X} = H + \frac{N}{X} \quad (15)$$

Ignore the impact of noise, then

$$\hat{H}_{LS} = \{H(0)H(1)\dots H(K-1)\}^T \quad (16)$$

Although, LS estimation algorithm is very simple, its performance is sensitive to the noise. The veracity of the estimation is reduced at the low SNR.

2.3. MMSE Channel Estimation

The MMSE estimate of the channel can be obtained as

$$\hat{H}_{MMSE} = \hat{F} h_{MMSE} = FR_{KY}R_{YY}^{-1}Y \quad (17)$$

Where

$$R_{KY} = E[hY^H] = R_{KK}F^H X^H \quad (18)$$

$$R_{YY} = E[YY^H] = XFR_{KK}F^H X^H + \sigma_n^2 I_N \quad (19)$$

Then

$$\hat{H}_{MMSE} = FR_{KK}F^H X^H (XFR_{KK}F^H X^H + \sigma_n^2 I_N)^{-1}Y \quad (20)$$

Where $R_{KY} = E[hY^H]$ is the channel autocorrelation matrix, σ_n^2 is the variance of noise, $F = [W_k^{nk}]$ is the

DFT matrix with $W_k^{nk} = \frac{1}{\sqrt{k}} e^{-j2\pi \frac{nk}{K}}$

The MMSE channel estimation has well performance but higher complexity. It requires the inversion of a $K \times K$ matrix, which implies a high complexity when K is large. Notice that it requires the channel statistical properties including the channel autocorrelation matrix and noise variance which is always unknown in the practice systems.

3.3. LMS Algorithm

LMS algorithm is one of the most used algorithms in weighting arrays. This algorithm is based on weight change with regard to an objective function. Weight update relation is given in Eq. 21.

$$\bar{w}(n+1) = \bar{w}(n) - \mu \bar{g}(\bar{w}(n)) \quad (21)$$

Where, $\bar{w}(n+1)$ is $(n+1)$ th weights and μ is a positive scalar named gradient step which controls algorithm convergence. $\bar{g}(\bar{w}(n))$ is an estimation of mean square error (MSE) gradient given by Eq. 22.

$$MSE(\bar{w}(n)) = E[|r(n+1)|^2] + \bar{w}^\dagger(n)R\bar{w}(n) - 2\bar{w}^\dagger(n)\bar{z} \quad (22)$$

Where, R is the correlation matrix of array signals, \bar{z} is the correlation vector of reference signal with arrived signal to the array and $r(n+1)$ is the reference signal. MSE gradient in n th iteration is expressed by Eq. 23.

$$\nabla_{\bar{w}} MSE(\bar{w}) \Big|_{\bar{w}=\bar{w}(n)} = 2R\bar{w}(n) - 2\bar{z} \quad (23)$$

It should be noted that n th weights are used for calculation in $(n+1)$ th iteration. Output of the array is given by Eq. 24.

$$y(n) = \bar{w}(n)\bar{x}(n+1) \quad (24)$$

Where, $\bar{x}(n+1)$ is the vector of array signals. The estimation of MSE gradient is performed by replacing R and \bar{z} with their estimations. This in $(n+1)$ th iteration is expressed by Eq. 25.

$$g(\bar{w}(n)) = 2\bar{x}(n+1)\bar{x}^\dagger(n+1)\bar{w}(n) - 2\bar{x}(n+1)r(n+1) \\ = 2\bar{x}(n+1)\varepsilon^*(\bar{w}(n)) \quad (25)$$

Where, $\varepsilon(\bar{w}(n))$ is the error between array output and reference signal expressed by Eq. 26.

$$\varepsilon(\bar{w}(n)) = \bar{w}^\dagger(n)\bar{x}(n+1) - r(n+1) \quad (26)$$

As expressed by Eq. 25, estimation of MSE gradient is the array output multiplied by array signal in $(n+1)$ th iteration. An important issue in LMS algorithm is the algorithm convergence. If λ_{\max} is the highest

eigenvalue of R and $\mu < \frac{1}{\lambda_{\max}}$, LMS algorithm will converge. Mean value of weights also converge toward optimum weights.

Convergence speed of the algorithm is expressed by the time weights reach optimum weights. An important parameter which can be used to measure LMS algorithm convergence is time constant of the eigenvalues given by Eq. 27.

$$\tau_l = \frac{1}{2\pi\lambda_l} \quad (27)$$

λ_l is the lth eigenvalue of matrix R. As seen, these time constants are related to the amount of eigenvalues. The more the eigenvalues, the less the time constants are. High eigenvalues are usually related to sources with high power, while low eigenvalues correspond to the small received power or environment noise. Thus, when eigenvalue of the matrix R is highly scattered, the algorithm convergence time increases. Convergence is a vital issue in the application of an algorithm in system. For example, due to its convergence time, LMS algorithm is not recommended to be used in mobile communication network since. Moreover, convergence speed should be considered due to the used signal states such as each user presence time, each time interval in TDMA system.

4.3. RLS Algorithm

LMS algorithm convergence is related to eigenvalues of correlation matrix. LMS algorithm convergence is less in environments where eigenvalues of correlation matrix is high. This problem is solved in RLS algorithm where inverse of correlation matrix is used instead of step size, μ . Thus, update relation of weights is demonstrated by Eq. 29.

$$\bar{w}(n) = \bar{w}(n-1) - R^{-1}(n)\bar{x}(n)\varepsilon^*(\bar{w}(n-1)) \quad (28)$$

Where, R(n) is given by:

$$R(n) = \delta_0 R(n-1) + \bar{x}(n)\bar{x}^\dagger(n) = \sum_{k=0}^n \delta_0^{n-k} \bar{x}(k)\bar{x}^\dagger(k) \quad (29)$$

Where, δ_0 is a positive scalar less than unity and very close to it. RLS algorithm needs correlation matrix inverse, and the recursive relation for calculation of this matrix is given by:

$$R^{-1}(n) = \frac{1}{\delta_0} \left[R^{-1}(n-1) - \frac{R^{-1}(n-1)\bar{x}(n)\bar{x}^\dagger(n)R^{-1}(n-1)}{\delta_0 + \bar{x}^\dagger(n)R^{-1}(n-1)\bar{x}(n)} \right] \quad (30)$$

Initial value of correlation matrix inversion is

$$R^{-1}(0) = \frac{1}{\varepsilon_0} I \text{ where } \varepsilon_0 > 0.$$

RLS algorithm minimizes mean square error (MSE).

MSE is expressed by $J(n) = \sum_{k=0}^n \delta_0^{n-k} |\varepsilon(k)|^2$. The

main advantage of RLS algorithm is that its convergence is independent on eigenvalues of correlation matrix dissipation. RLS and LMS algorithms, among the algorithms work based on gradient, have the highest and lowest convergence speed, respectively.

5.3. Proposed Algorithm

The main idea of this paper is to use the coefficients of the estimated channel by the LS estimator and its use in recursive equations of the LMS and RLS algorithms. In this way, the channel coefficients are first estimated using the LS estimator. Using the coefficients, new coefficients of the channel are calculated in recursive algorithms of LMS and RLS. It will be seen that the BER system will be improved to some extent.

In the LMS method, the coefficients of the \hat{H}_n vector are calculated as follows, using the recursive equations of the LMS method:

$$H_n = \hat{H}_{n-1} - \mu \times e \times X^* \quad (31)$$

n, e and μ is respectively the number of iterations, the error signal and the LMS coefficient between 0 and 1.

Equation (32) can be rewritten as the following form:

$$\hat{H}_{LMS} = \hat{H}_{LS} - \mu \times e \times X^* \quad (32)$$

4. ANALYSING PROPOSED METHODS

Simulation for MIMO-OFDM system with 64 sub carriers and different number of transmitter and receiver antennas is repeated in switch frequency fading channels. The bandwidth has been assumed 1 MHz and its modulation QPSK.

In the following figures, mean square error is shown according to signal to noise ratio. The carrier frequency offset is assumed a stable amount in simulations ($\Delta f=0.1$).

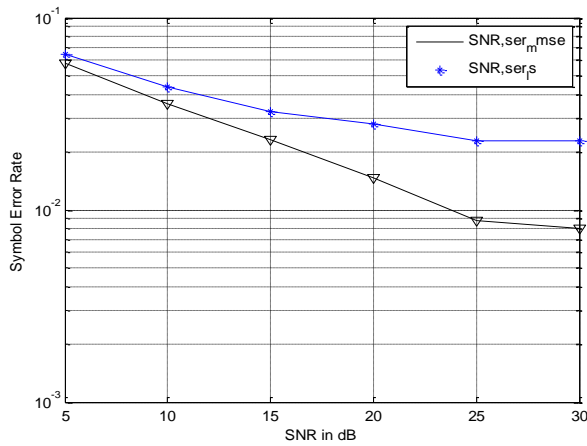


Fig.2. Plot of SNR V/S SER for an OFDM system with MMSE/LS estimator based receivers

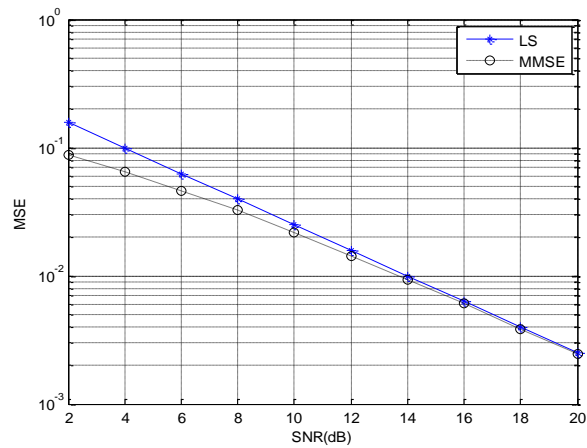


Fig. 3. Channel estimation in 2*2 MIMO-OFDM systems L=4 with LS and MMSE methods

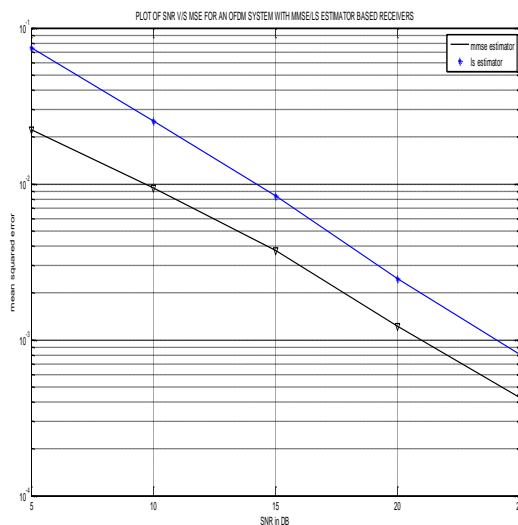


Fig. 4. Comparison of the performances of the LS and the MMSE channel estimators for a 64 sub carrier OFDM system based on the parameter of Mean square error

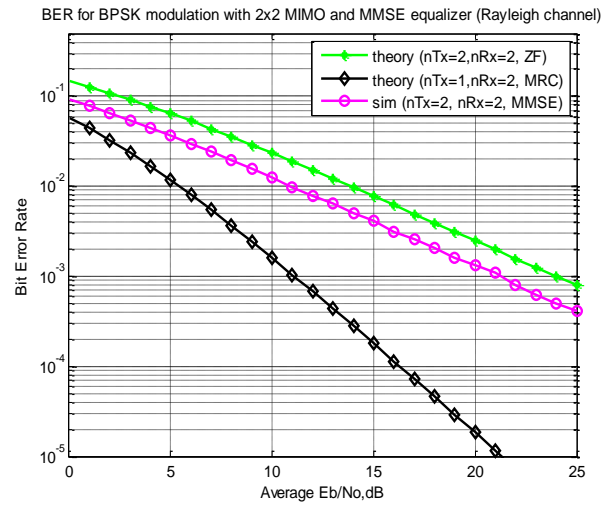


Fig. 5. BER plot for 2x2 MIMO with MMSE equalization for BPSK in Rayleigh channel

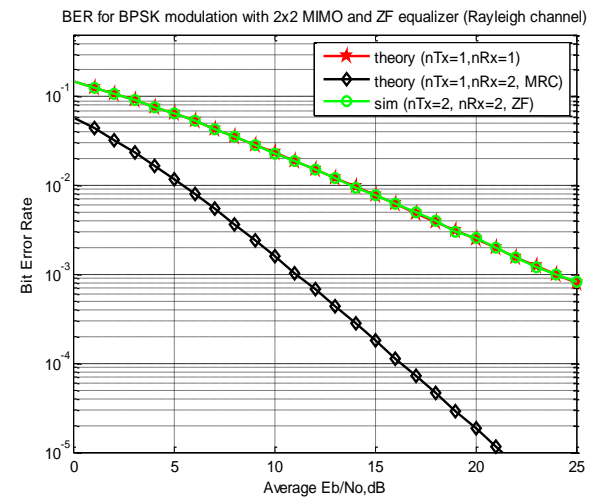


Fig. 6. BER plot for 2x2 MIMO channel with ZF equalizer (BPSK modulation in Rayleigh channel)

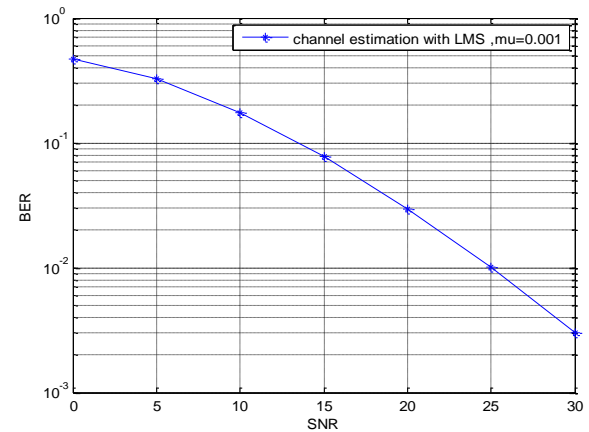


Fig. 7. Channel estimation in 2*2 MIMO-OFDM systems L=4 with synchronization and LMS algorithm $\mu=0.001$

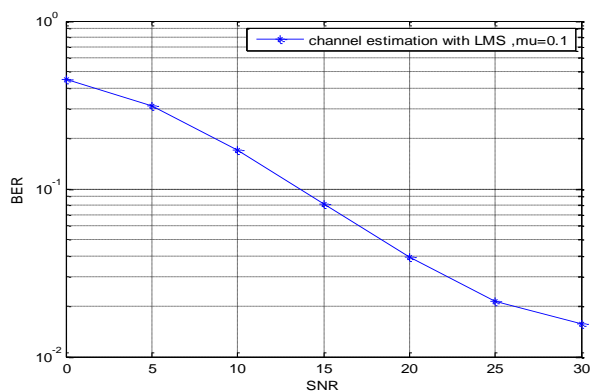


Fig. 8. Channel estimation in 2*2 MIMO-OFDM systems $L=4$ with synchronization and LMS algorithm $\mu=0.1$

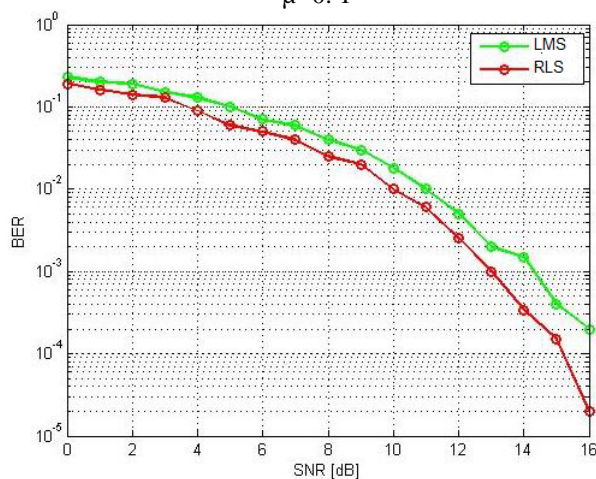


Fig. 9. BER vs SNR for QPSK MIMO-OFDM system using LMS and RLS

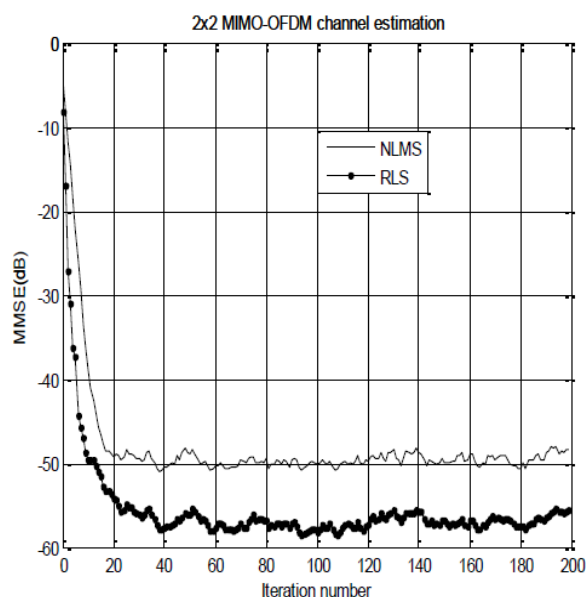


Fig. 10. Channel estimation in 2*2 MIMO-OFDM systems with LMS and RLS algorithms.

As you can see in figures 2 to 4, the MMSE algorithm has a better performance in the estimation of the channel coefficients than the LS algorithm. However, in comparison to the LS estimator, the algorithm has more calculation complexity and also requires knowledge of the channel situation including the channel auto correlation matrix.

In figures 9 and 10, LMS and RLS adaptive channel estimator are shown for MIMO-OFDM systems. The simulation results showed that the RLS CE algorithm is better to use for MIMO OFDM systems.

5. CONCLUSION

This paper used the LS method for the primary channel estimation. In order to improve the accuracy of the channel estimation, the LMS and RLS algorithms, including an output feedback, were added to the receiver, which improved the performance of the BER system.

Accurately selecting μ , the channel estimation with the help of the LMS algorithm can get closer to the LS method. As we see in the conducted simulations, the accurate selection of μ causes the channel estimation to be quite similar to the estimation done by the LS method. The LMS method is highly dependent on the μ parameter. Using simple recursive equations, the method provides an appropriate estimate of a channel. The RLS algorithm exhibits better performance in the channel estimation than the LMS algorithm, but it is more complicated than the LMS algorithm in terms of calculations.

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