Spectrum sensing of multiband signals using periodic non-uniform sampling

Javad Shadbahr ,M.Farzan Sabahi
1- Islamic Azad university of Ahar, department of electric Engineering, Ahar, Iran
Email: javadshadbahr@gmail.com
2- Isfahan University, department of electric Engineering, Isfahan, Iran
Email: sabahi@eng.ui.ac.ir

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Abstract
In this essay, we propose a Multiband Spectrum Sensing (MSS) approach that utilizes randomized sampling and its spectral analysis to accomplish the sensing task using considerably low sampling rates. Since the spectrum sensing procedure does not necessitate signal reconstructing, it is shown that the sampling rates can be arbitrarily low for some randomized sampling schemes. Most importantly, general guidelines are provided to ensure that the developed MSS satisfies certain detection probabilities set by the user. Spectrum sensing involves scanning predefined part(s) of the radio spectrum in search of meaningful activity such as an ongoing transmission or the occurrence of an event. Standard communications laboratory test equipment, e.g. spectrum analyzers, can perform similar functionality. They conventionally sweep the spectrum using a narrowband tunable bandpass filter(s) and determine the energy within each of the scanned spectral bands.

Keywords: Spectrum sensing, Non-uniform Sampling, Gaussian White Noise

1.INTRODUCTION

The spectral support of the present signal is displayed based on whether the attained energy in a given frequency band exceeds a certain noise floor. Such instruments are usually bulky and expensive; consequently their solutions are inadequate for the investigated wireless communication systems whose resources (e.g. cost, weight, size and power) are limited.

Spectrum sensing has a wide diversity of application areas including astronomy and seismology. The majority of these methods, which deploy digital signal processing, are uniform sampling based.

2.WIDEBAND SPECTRUM SENSING AND DETECTION PROBABILITIES

We consider that the studied systems operate over L non-overlapping contiguous spectral subbands/channels occupying the frequency range $B = [fmin, fmin + B]$. Each of the channels is of width B_c and $B_c = LB$ is the total processed bandwidth. The incoming

multiband signal consists of an unknown M number of concurrently active transmissions

and is given by:

$$x(t) = \sum_{m=1}^{M} x_m(t) = \sum_{m=1}^{M} x_{T,m}(t) * h_m(t) \quad (1)$$

Where $*$ denotes the convolution operation.

The maximum expected number of simultaneously active subbands at any time is $L_A \geq M$. Whereas, $x_m(t)$ and $h_m(t)$are the incoming signal corresponding to the m-th subband and the impulse response of its propagation channel respectively. The collected samples of the received signal $y(t)$ are contaminated with Additive White Gaussian Noise (AWGN) with variance p_N :

$$y(t_n) = x(t_n) + n(t_n) \quad (2)$$

The objective is to devise a method that is capable of scanning the overseen frequency range B and reliably identifying the active subbands. It should operate at rates significantly lower than those requested by uniform-sampling-based approaches where

$f_{us} \geq 2B$; $2B$ is referred to as Nyquist rate thereafter.

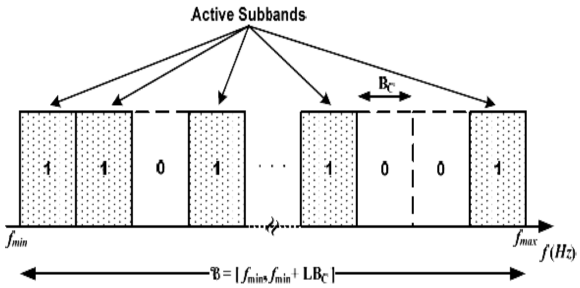

Figure 1: Illustration of the occupancy of the system subbands

Evidently, the spectrum sensing problem boils down to choosing between the hypothesis ("0"), i.e. $H0, k$ which represents the absence of any activity within the *k*-th subband, and the alternative hypothesis ("1"), i.e. $H1, k$ which infers the presence of a transmission. An example where some of the subbands are occupied is exhibited in Figure 1.

The performance of a spectrum sensing technique is commonly measured by the Receiver Operating Characteristics (ROC) that captures the relation between the probability of false alarm $p_{f,k} = P_r\{H_{1,k}|H_{0,k}\}$ and the probability of detection $p_{d,k} = P_r\{H_{1,k}|H_{1,k}\}$ in the *k*-th system subband [1, 2, 3, 4].

These probabilities are typically interrelated via the detection threshold γ_k whose value trades $p_{d,k}$ and $p_{f,k}$. Accordingly, the reliability and robustness of the proposed spectrum sensing approach is reflected by its ability to fulfill a set of sought detection probabilities, i.e. $p_{d,k}$ and $p_{f,k}$ for one or more of the monitored system channels. Here, guidelines are derived to guarantee meeting such demands.

3.PROPOSED SENSING APPROACH

The introduced sensing procedure comprises three steps:

1) Randomly sampling the incoming signal at substantially low rates

2) Estimating the magnitude of the signal spectrum at selected frequency point(s)

3) Comparing the magnitude(s) with pre calculated threshold(s)

Having a spectrograph that is relatively smooth would permit assessing fewer frequency points per system subband to determine its status. We seek to inspect one frequency point per channel. Consequently, the spectrum sensing structure can be described as follow:

$$\begin{cases} H_{0,k} : \hat{X}_e(f_k) < \gamma_k \\ H_{1,k} : \hat{X}_e(f_k) \geq \gamma_k \end{cases} \quad k = 1,2, \dots L \tag{3}$$

where $\hat{X}_e(f_k)$ is the estimated magnitude spectrum and γ_k is the threshold. The frequency points $\{f_k\}_{k=1}^{L}$ are placed at the centre of the system subbands. To control such uncertainties, we average a *K* number of estimates in (3) which are calculated from *K* signals with T_0 width, $\tau_r = [t_r, t_r + T_0]$:

$$\hat{X}_e(f_k) = \frac{1}{k} \sum_{r=1}^{k} X_e(t_r, f) \tag{4}$$

The width of the time analysis T_0 , the average sampling rate $\alpha = \frac{N}{T_0}$ and the number of estimate averages *K* . The latter two parameters are the available means to curb any perturbations or anomalies in the estimation process, i.e. guarantee a certain level of accuracy.

Any recommendations on the values of α and K should take into account severe system conditions to safeguard the response of the MSS method, e.g. the maximum spectrum occupancy and the highest harmful effect of the smeared-aliasing phenomenon. This approach attempts to present Non-uniform sampling scheme to develop analytical expression of spectrum sensing.

4.PERIODIC NON-UNIFORM SAMPLING

There are quiet lots of techniques which are used in Non-uniform sampling, but in this essay we present Periodic Non-uniform Sampling to achieve our purposes. In periodic sampling, we first pick a suitable sampling period *T* (such that uniform sampling at rate *1/T* causes no aliasing), and a suitable integer *L > 0*, and then sample the input signal *x*(*t*) non-uniformly at the instants $t_i(n) = (nL + c_i)T$ for$1 \leq i \leq p$ and $n\epsilon Z$. The set $\{c_i\}$ contains *p* distinct integers chosen from set $L = \{0,1, \dots, L-1\}$. [6][7]

$$0 \leq c_1 \leq c_2 \leq \dots \leq c_p \leq L-1$$

5.POWER SPECTRUM ANALYSIS

Radio signals are not necessarily time limited and so do not have direct Fourier transform to provide a frequency domain representation. This Hypothesis play important roles to simplify signal analysis. An autocorrelation function $R_x(\tau)$ which shows the time delay between the tow samples of the process:

$$R_x(\tau) = E[x(t)x(t+\tau)]$$

$$R_x(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)x(t+\tau)dt \tag{5}$$

The power spectral density (PSD) function $W_x(f)$ is time invariant and is defined by the Weiner Kintchine theorem:

$$W_x(f) = F\{R_x(\tau)\}$$
$$W_x(f) = \int_{-\infty}^{\infty} R_x(\tau) e^{-j2\pi f t}\, dt \qquad (6)$$

If we consider $x_T(t)$ as a time limited version of $x(t)$ such that $x_T(t) = x(t)$ for $t \in [-T, T]$ then the Fourier transform $X_T(f)$ exist and Parseval's theorem tells us that square of its magnitude is the energy density function. For deterministic signals the PSD function is found by dividing the energy density function by time duration 2T, and for random signals the expected value of this is taken:

$$W_x(f) = \lim_{T\to\infty} \frac{1}{2T} E\left[\left|\int_{-T}^{T} x_T(t) e^{-j2\pi f t}\, dt\right|^2\right] \qquad (7)$$

It is well known that the spectrum of a finite number of non-uniform samples can be represented by:

$$X_s(f) = \sum_{n=1}^{N} x(t_n) e^{-j2\pi f\, t_n} \qquad (8)$$

Which is in the form of a non-uniform discrete Fourier Transform (DFT). An approximation to the sampled signals PSD function for a finite number of samples is found by taking the magnitude squared value of the DFT and dividing by the number of samples:

$$W_{xs}(f) \approx \frac{1}{N} |X_s(f)| \qquad (9)$$

6. SPECTRUM SENSING USING NON-UNIFORM SAMPLING

In Table 1, we list a number of popular spectrum sensing techniques; review papers such as [1,2,4] compare these methods stating their advantages and disadvantages.

We divided them into two categories: parametric and nonparametric. The parametric ones rely on previous knowledge of the incoming signal and its characteristics.

On the other hand, nonparametric techniques are not specific to a particular type of signals and thus branded as generic. They are deemed to be more appropriate for the pursued multiband spectrum sensing where limited information about the incoming signal is presumed. Some spectrum sensing methods are intrinsically geared to deal with one frequency subband at a time i.e. narrowband spectrum sensing. To apply these methods to the multiband scenario, the incoming multiband signal should be filtered using a bank of fine-tuned narrowband bandpass filters to separate the concurrently active transmissions into their corresponding spectral subbands. Alternatively, wideband spectrum sensing approaches permit the simultaneous sensing of the system channels; they usually involve estimating the spectrum of the received signal. A major implementation challenge lies in the prohibitively high sampling rates required by conventional spectral analysis tools; they are uniform sampling based and have to operate at or above the Nyquist rate. Whereas, we usher in a new wideband spectrum sensing approach that overcomes the sampling rate limitation by using the randomized sampling and processing methodology.

Table 1: Spectrum Sensing approaches

	Approach	Minimum Sampling rate	Multiband	Computational Complexity**
Nonparametric	Energy detector	Nyquist	✓	Low
	Multitaper spectrum estimation	Nyquist	✓	High
	Wavelet-based detection	Nyquist	✓	Moderate
	Compressive-sensing based	Sub-Nyquist	✓	High
Parametric	Matched-filtering detection	Nyquist	✗	Low
	Cyclostationary Detector	Nyquist	✗	Moderate
	Covariance-based detector	Nyquist	✗	Moderate

Now we introduce Spectrum Sensing based on Non-uniform sampling practically. We are seeking for signal at the distinct rang [$f_0 - \Delta f, f_0 + \Delta f$]. In this regard we should take non-uniform sampling and try to evaluate spectrum of the signal in a specified frequency and then compare with threshold we have chosen, then plot Receiver Operating Characteristics (ROC) curves. To achieve the best rang of threshold first we add a White Gaussian Noise to signal and then by calculating the spectrum of the Noise in subband of interest we consider this value as threshold

$$y(t) = x(t) + n(t).$$

In the figures below we have simulated Spectrum Sensing with both uniform an non-uniform sampling to compare their performance. In these diagrams f_s, f_c, γ and n are sampling frequency, career frequency, threshold and number of samples respectively.

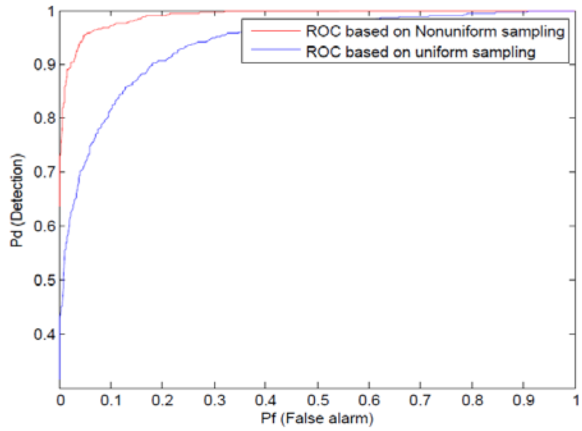

Figure 2: Detection probabilities of the targeted subbands $f_{c1} = 10Hz$, $f_{c2} = 20Hz$ $f_s = 100Hz$ and $SNR = 10dB$

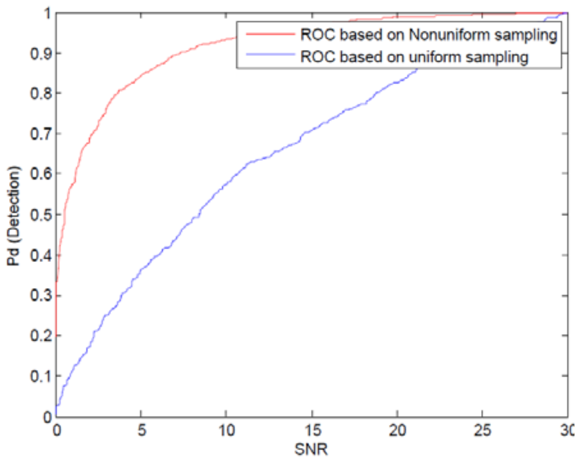

Figure 3: Detection probabilities of the targeted subbands $f_{c1} = 10Hz$, $f_{c2} = 20Hz$ $f_s = 100Hz$ and $\gamma = 0.2$

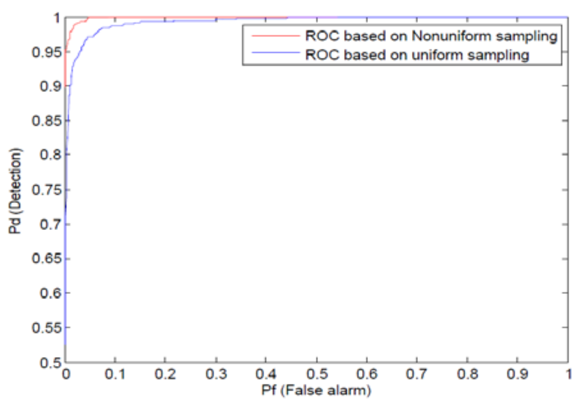

Figure 4: Detection probabilities of the targeted subbands $f_{c1} = 10Hz$, $f_{c2} = 20Hz$ $f_s = 250Hz$ and $SNR = 10dB$

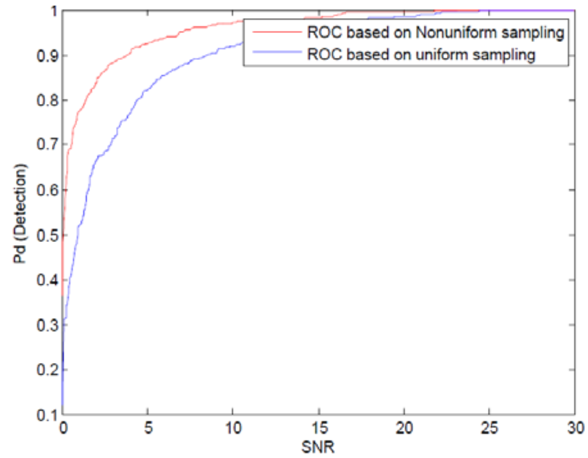

Figure 5: Detection probabilities of the targeted subbands $f_{c1} = 10Hz$, $f_{c2} = 20Hz$ $f_s = 250Hz$ and $\gamma = 0.2$

In figure 6 and 7 we reduced sampling rate, as we all know in this condition aliasing will happen in terms of taking uniform sampling. It can be seen that, in the same sampling rate, results of detection probability of non-uniform sampling scheme are more likely than uniform sampling.

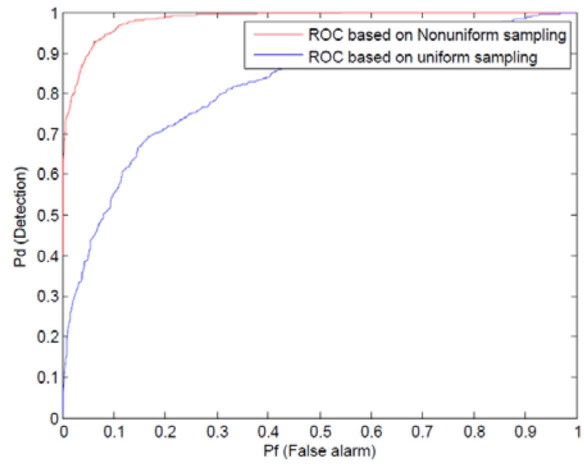

Figure 6: Detection probabilities of the targeted subbands $f_{c1} = 10Hz$, $f_{c2} = 20Hz$ $f_s = 20Hz$ and $SNR = 10dB$

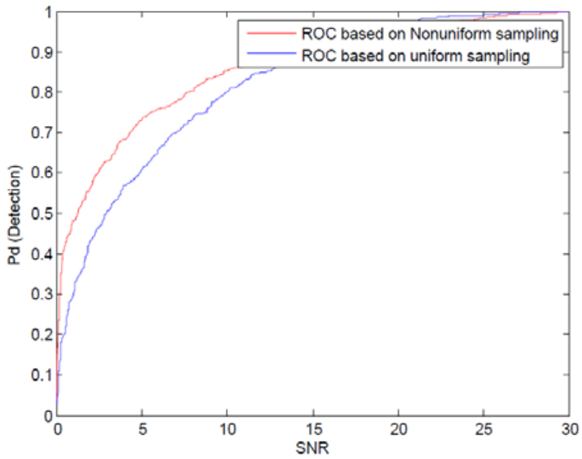

Figure 7: Detection probabilities of the targeted subbands $f_{c1} = 10Hz$, $f_{c2} = 20Hz$ $f_s = 20Hz$ and $\gamma = 0.2$

7. CONCLUSION

A wideband randomized-sampling-based spectrum sensing approach was introduced in this essay where simplicity and low computational complexity are among its key merits. This paper investigates the spectrum sensing using periodic non-uniform sampling is discussed. By comparing this method with spectrum sensing using uniform sampling we found that in the method assigned the restrictions of uniform sampling (Aliasing) does not exist, and considering that in the spectrum sensing no need to reconstruct the signal, so we able to detect the existence of signal with lower sampling rate.

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