

# Simulation and Comparison Possibility of Cavity Quantum Systems at Different Levels in Various Regimes, Coupled with the Fock and Coherent Initial Conditions

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## ABSTRACT:

In this article, we have given the size of the coupling constant, a system of quantum electro-dynamics in weak coupling regime, strong and very strong case our analysis and simulation. Also by choosing the type of quantum system and the electrical field of the electromagnetic wave, the size of Rabi frequency to the extent that we are less than excited states and photons in the cavity decay rate is. In this case, the interaction between the quantum system and electromagnetic field coupling regime is weak. It is one of the properties of quantum light emitting modes of the cavity modes can be in resonance with each other, resulting in a sudden increase in the spontaneous emission rate, causing a sharp peak in the density of the spontaneous emission spectrum. The nature of this regime leads to appropriate in light production applications, such as it is possible to increase efficiency, reduce the threshold lasers emitting light in alignment with the vertical cavity structure light emitting diodes noted. Also property is listed under the regime of entangled photons is used in the production of instant. While this research, the rotating wave approximation is used in this context, survival and annihilation operators of photons Sinusoidal behavior.

**KEYWORDS:** Quantum Electro-Dynamic, Weak Coupling Regime, Strong Coupling Regime, Ultra-strong Coupling Regime, Electromagnetic Wave, Rabi Frequency, Photon, Decay Rate, Spontaneous, Vertical Cavity Surface Emitting Laser, Light Emitting Diode.

## 1. INTRODUCTION

One of the very important issues and applications in devices small cavity quantum electro-dynamics, quantum dot emitting interval control frequency spectrum transition between the cavity mode. The ability to control the gap in the construction of single photons with high efficiency, nano-cavity lasers with high bandwidth and low threshold and even single point optical components used. The various techniques such as adjusting the temperature of the network, compress inert gases at low temperatures, electric control is proposed. Third coupled regime, coupling is very strong where Rabi frequency is greater than the rate of spontaneous radiation [3]. In fact, this type of coupling, the interaction energy and coupling the product of Planck's constant multiples Rabi frequency has the same order of magnitude of the energy transition, the transition frequency energy to turn the product of the Planck's constant is obtained. Medical progress in this regime has been in solid state systems,

in this system, solid state, micro-cavities located on the doped potential wells, active electrical transition between the lower bands well. The system includes superconducting quantum circuits in the transmission line resonator is observed [6, 7]. In this type of system, the atoms align with the atoms has two artificial energy levels obtained from Josephson junctions, is coupled. In a two-dimensional electron gas with high mobility that other material well terahertz resonance coupling, the mode coupling of photons to transfer the two-dimensional electron gas magnetic cyclotron magnetic field applied perpendicular to the plane of the quantum wells is obtained. It is shown that cyclotron frequency proportional to the applied magnetic field and the system will be very manageable. For excitation of the material is adjustable by changing the applied magnetic. a key aspect of the dependence of the transition dipole moment of a cyclotron cyclotron-long campaign, and we can see that the system can be very high amount of dipole moment coupling constant in

magnitude and frequency of its transition. The system frequency cyclotron Rabi frequency was about 58%. The use of metal - dielectric - metal micro-cavity with cavity polariton quantum states of an ideal system for generating terahertz range [4]. Metal cavity, enclosed radiation modes known that the transition following the intensification of the middle band are potential wells, The coupling system is very strong view and indicated that the frequency of the transition Rabi frequency is more than 0.5 [2, 5].

Integration of operations acquired in describing the cavity resonator with nonlinear atomic oscillator if the field is weak. In the most general possible cavity quantum electro-dynamics system is considered and it is assumed that the number of arbitrary quantum system with light-emitting quantum coupling constant interaction with the cavity mode are defined by an arbitrary number. The most general Hamiltonian appropriate for the operating system obtained, and then the image is used Heisenberg interaction Hamiltonian image is obtained. The Hamiltonian obtained in a time-dependent Schrödinger equation is replaced and generalized linear differential equation that Rabi frequency's famous equation is obtained [1], [8]. Finally, the equation obtained in the previous step, change the system time is obtained. The remarkable thing is that the work done and the work that has been done in this rotating wave approximation are used in the space of survival and annihilation operators of photons sinusoidal behavior [8]. In order to complete the profile of the system, one of the cases, the initial state of the system is determined by many studies on different types of early states, and all these studies suggest that cavity electro-dynamic system behaves differently in different initial conditions. Initial conditions can be determined on a case by Fock state be determined where exactly the quantum system in which the energy levels, and exactly how many photons in the cavity mode is located. In the other type where the exact number of photons in each cavity mode Poisson distribution is unknown and has been known to the coherent state [6], [9].

In the second part of the article analyzes the events in the system and pay calculations. The third and fourth parts of the article analyzes the Fock and Coherent initial conditions and initial conditions we will how to produce initial conditions, the subject of the article will be the fifth and in the end we will have a conclusion.

## 2. ANALYSIS OF THE INCIDENCE OF CAVITIES AND COMPUTING SYSTEM

The potential well in this article, is provided  $\text{In}_{0.52}(\text{Al}_x\text{Ga}_{1-x})_{0.48}\text{As}/\text{In}_{0.53}\text{Ga}_{0.47}\text{As}/\text{In}_{0.52}(\text{Al}_x\text{Ga}_{1-x})_{0.48}\text{As}$  [4], [7]. Assuming strip electrons in the conduction band and also for the presence of heavy and light holes

in the valence band two bars, as an acceptable approximation is used. The design of this potential well by choosing  $x = 0.9$  and  $Z0 = 0.9$  nm (the wells) high electron-hole transition energy is obtained 799 me V and the energy difference between heavy and light holes around 30 me V is obtained.

According to the results of three levels model quantum system, the energy levels of the model, respectively, 0, 30 and 829 me V consider [7-9].

The following equations can be written in the form of a coat Hamiltonian system:

$$|\varphi(t)\rangle = \sum_{A_i=lh, hh, e} \sum_{f=1}^8 \phi(A_i, f) |A_i\rangle |f\rangle \quad (1)$$

$$H = +\hat{H}_0 + \hat{H}_{r,r} + \hat{H}_{r,E} \quad (2)$$

$$H_0 = \sum_{i,j} E_i^1 \hat{\sigma}_i^1 + \hbar\Omega \hat{a}^\dagger a = E_{lh}^1 \hat{\sigma}_{lh}^1 + E_{hh}^1 \hat{\sigma}_{hh}^1 + E_e^1 \hat{\sigma}_e^1 + \hbar\Omega \hat{a}^\dagger a \quad (3)$$

Hamiltonian  $\hat{H}_{r,r}$  is zero because only values of the coefficient matrix and the Hamiltonian represents a quantum system we three levels interaction between the two, mood here is not [5, 14]. Hamiltonian of Jaynes-Cummings-Paul Model is.  $\hat{\sigma}_{i,j}^l$  is quantum dot operator atomic transition means transition from level L 'th to level i,  $\hat{a}^\dagger$  The number of photons in a quantum system that increases the number of photons in (3) equation [3].

In this section, the system calculates the probability of the initial state seal or as Fock or coherent as we consider the impact of the coupling coefficient varies with time consider the possibility of the presence system, and then the effect of this increase and the change of regime, coupled to the other coupling regime in the use of rotating wave approximation in a quantum system three levels examine [4-9].

In this way the system has been used to coat coefficients were resolved manually. Schrödinger equation is solved when the suit states are a function of time, and the impact of the sentence  $(\hat{\sigma}_{i,j}^l)^\dagger \hat{a}^\dagger = \hat{\sigma}_{j,i}^l \hat{a}^\dagger$  cannot be overlooked [7].

Jackets states of the entire system are as follows:

$$|\varphi(t)\rangle = \sum_{A,F} \phi(A, F) |A\rangle |F\rangle \quad (4)$$

$$|A\rangle = \bigotimes_{n=1}^k \begin{pmatrix} n \\ r_n \end{pmatrix} = \begin{pmatrix} 1 \\ r_1 \end{pmatrix} \begin{pmatrix} 2 \\ r_2 \end{pmatrix} \dots \begin{pmatrix} k \\ r_k \end{pmatrix}$$

$$|F\rangle = \bigotimes_{v=1}^w |f_v\rangle = |f_1\rangle |f_2\rangle \dots |f_v\rangle$$

That  $|A\rangle$  and  $|F\rangle$  the quantum dots and cavity modes which have swept the N number.

$$N = (P+1)^m \times (E_j)^n \quad (5)$$

The maximum number of photons in each cavity mode P and m is the number of cavity modes and  $E_j$  is number of energy levels and n is the number of quantum dots are [9].

Each row shows that each quantum dot is what energy balance, as well as the number of photons in each cavity mode. As a result, according to the state energy levels of quantum dots and cavity modes in each row is obvious that the Hamiltonian what values on the coat of the effect will be. The jackets are available depending on the type of operator in Hamilton and their impact on changing the system of Katy to coat the resulting equation will be left. Thus the equation will be as follows.

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$$\frac{d}{dt}[\phi(t)]_{N \times 1} = [M]_{N \times N} [\phi(t)]_{N \times 1} \quad (6)$$

According to equation (6) the system will be calculated as follows specifies that the initial conditions of the system.

$$\{\phi(t)\} = e^{[M]t} \{\phi(0)\} \quad (7)$$

One of the things that set the initial conditions to solve the various systems according to the number of quantum dots and the number of energy levels of each of them, as well as the number of cavity modes and the maximum number of photons used in them will be different [7]. According to the algorithm used in equation (4) and the production of matrix [M], to solve the equation (7) matrix [M] diagonal matrix into eigenvalues and the matrix [R] -1 and [R] matrix R put together the eigenvalues of matrix [M] as its pillars is obtained [13].

$$\begin{aligned} [R]^{-1}[M][R] &= [D] = [d_i \sigma_{ij}] \\ \Rightarrow \{\phi(t)\} &= e^{[M]t} \{\phi(0)\} = [R]e^{[D]t}[R]^{-1}\{\phi(0)\} \\ &= [R][e^{(d_i t)} \sigma_{ij}][R]^{-1}\{\phi(0)\} \end{aligned} \quad (8)$$

$\{\phi(0)\}$  is matrix and size of matrix is  $N \times 1$  will be the initial condition for each of its lines of each of the states will be swept in two forms, Fock state and coherence state will be determined. the written program that you want to calculate the coefficient matrix inputs coat conditions and initial conditions required to receive the seal as well as a Fock or coherent produce. Then, the algorithm written in the program is explained.

### 3. ANALYSIS OF THE LIKELIHOOD OF SEAL SYSTEM WITH FOCK INITIAL CONDITIONS

Due to the expansion of equation (3),  $\hat{\sigma}_{j,i}^{\dagger} a^{\dagger}$  including an increase in the number of photons and the transition

to a higher energy level is a quantum system. First, as mentioned in effect when  $|lh\rangle$  the transition from one energy level to  $|e\rangle$  energy level occurs, if we consider that by this sentence from another case the Hamiltonian to be, and in the case of the initial conditions, the primary mode is intended to take zero value is taken into account and the rest of the states. Finally, the possibility to examine the case. Moment of the electron-hole transition levels of heavy holes and light holes is zero [4-9].

As a result, the effect of applying the first coat  $\hat{\sigma}_{j,i}^{\dagger} a^{\dagger}$  and the rest  $|1, lh\rangle = 1$  of the states  $|1, lh\rangle, |2, e\rangle$  in the form of zero initial conditions, we obtain the probability that the results in the following figures are shown [14].

It is also evident that the probability of the system, while increasing the coupling constant, growing and very strong coupling regime much more likely and therefore no longer rotating wave approximation. Because of the three levels system and the transition between levels of electrons and heavy holes are also in the process of this transition, we try. As a result,  $\hat{\sigma}_{j,i}^{\dagger} a^{\dagger}$  the effect of applying the  $|1, hh\rangle$  first coat and the other state with zero initial conditions,  $|2, e\rangle$  we obtain the probability [5-8].

In developing the equation (3) and in the case of weak coupling  $|2, e\rangle$  a very low chance of the match, but in the strong coupling regime, and the regime is much more powerful, much more is, so as not credit the rotating wave approximation. In addition, as there is the possibility  $|1, hh\rangle$  of the system in the form of Sinusoidal out and volatility is very high. In general, total square is equal to the size of the various states. In setting the Fock initial conditions as a seal of this size alone are the certain conditions quantum dots and cavity modes. As mentioned above in the form of a matrix  $N \times 1$  are the initial states of different energy levels of quantum dots where N states and cavity modes are [7, 8]:

$|\Phi(t)\rangle = |\text{Different modes in the cavity in different states in } t_0\rangle * |\text{Different levels of energy states in } t_0\rangle$

For example, on a system with a cavity mode and a maximum of 3 photons and a quantum dot with two energy levels of state may be written as follows (number of 3 photons are intended to reduce the number of sentences).

$\Phi(g)$ : The energy levels of the quantum point or base.

$\Phi(e)$ : The second energy level or excited quantum dot.

Different states of the quantum dot is:

$$\phi(t_0)|g\rangle + \phi(t_0)|e\rangle \tag{9}$$

Because of different states have only one cavity mode cavity mode:

$$|\phi(t_0)\rangle = \sum_{n=0}^3 |n\rangle \times \phi(t_0)|g\rangle + \phi(t_0)|e\rangle \tag{10}$$

In determining the initial conditions, is considered as a Fock state exactly how many photons in each cavity mode in which there is energy balance [9]. For example, if a system is the primary form of photons in the cavity mode and quantum dot based in state. According to the above equation must consider the third mode and zero the rest of our states. As can be seen by increasing the cavity modes and maximum exposure and the number of photons of energy levels of quantum dots and significantly increases the number of possible states. That actually needed to determine the initial state in the analysis of different systems manually complex and likely will be a lot of mistakes. However, although the initial conditions as Fock virtually no physical value and the initial conditions are generally determined as a coherent state.

The result is clear and stable with increasing electric field coupling, the oscillation probability has increased. With the arrival of strong coupling regime probability function varies with time, but retains its sinusoidal mode by entering the coupling is not very strong.

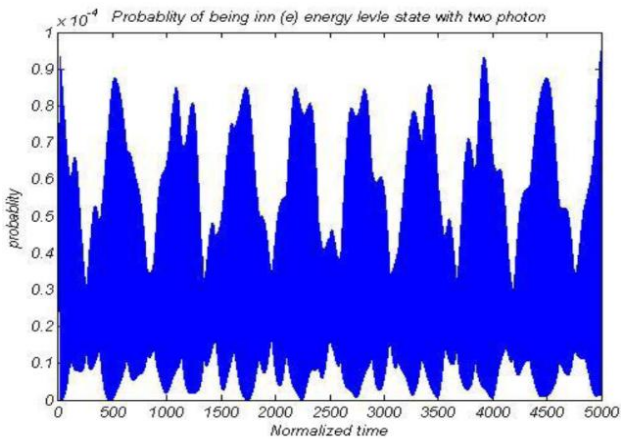


Fig. 1. The probability of the system under various conditions, weak coupling.

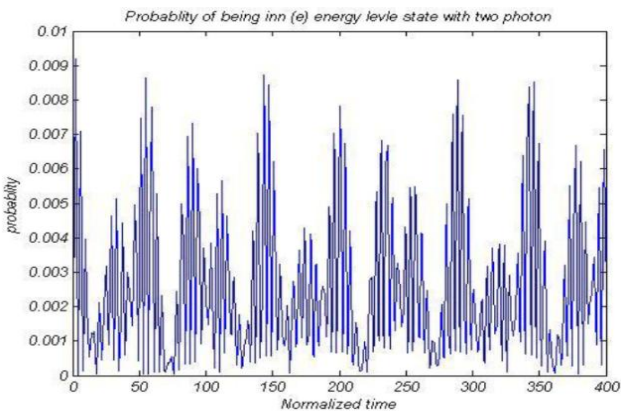


Fig. 2. The probability of the system under various conditions, strong coupling.

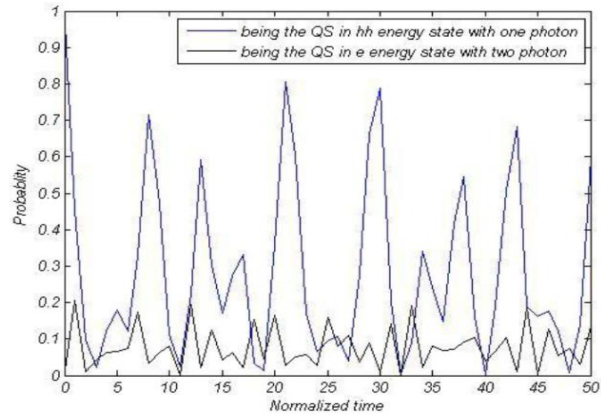


Fig. 3. The probability of the system under various conditions, very strong coupling

#### 4. ANALYSIS OF THE LIKELIHOOD OF SEAL SYSTEM WITH COHERENT INITIAL CONDITIONS

As mentioned above, assuming that the number of well-defined in the Fock state of the photons in each cavity modes are, The initial conditions were determined that this assumption leads to the creation of microscopic electric fields that do not have a physical value. To resolve this issue of initial conditions used in the coherent state [1-4].

As defined, coherent states, especially states are annihilation operator.

$$|\phi(t_0)\rangle = \frac{1}{\sqrt{\sum_{n=0}^m \left| \frac{\lambda^n}{n!} e^{-\lambda} \right|^2}} \sum_{n=0}^m \sqrt{\frac{\lambda^n}{n!}} e^{-\lambda} |n\rangle$$

\* |Different levels of energy states in  $t_0$ >

m is the maximum number of photons can be placed in each cavity mode.

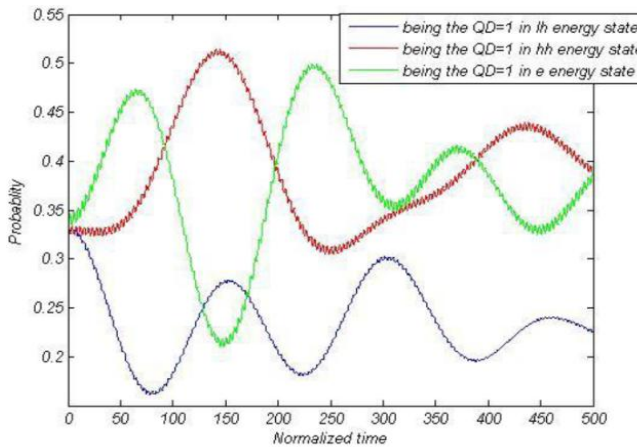


Fig. 4. The probability of the system under various conditions, weak coupling

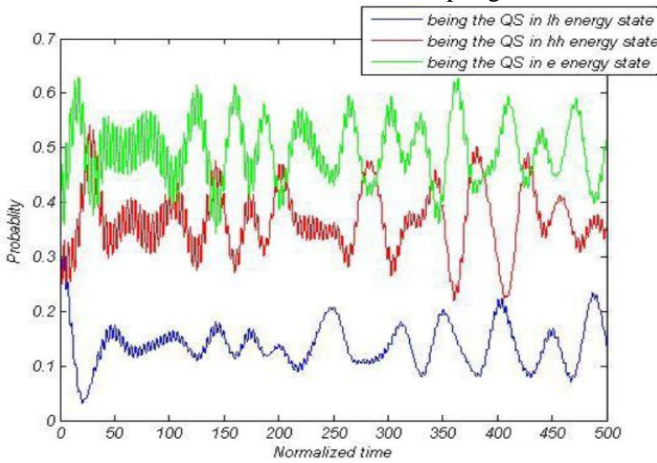


Fig. 5. The probability of the system under various conditions, strong coupling

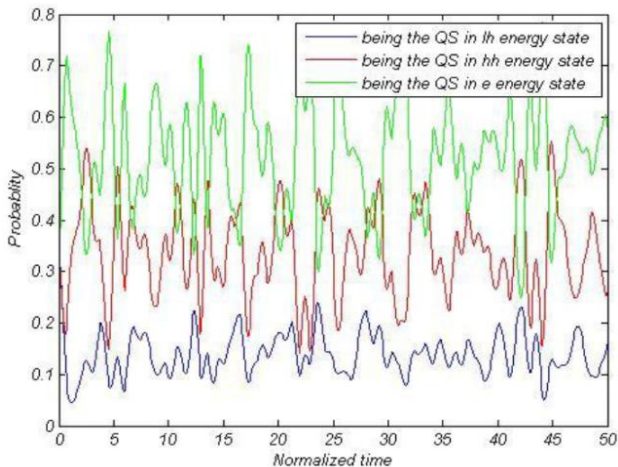


Fig. 6. The probability of the system under various conditions, very strong coupling

**5. HOW TO GENERATE INITIAL CONDITIONS**

The algorithm used in the written application for evaluation of different combinations of quantum dots and cavity modes as follows if the system is state coat

coefficients for each combination of compounds scenarios coat, cavity modes possible combinations to suit the energy levels of quantum dots assume states energy levels of quantum dots would be swept, in first cavity number of  $m$  (the last mode cavity) without photons, and while all the quantum dots in their first energy level in the first row of the quantum number  $n$ -th point (the last point quantum) will be the first energy level, and in the next row remains unchanged. But quantum dots will be  $n$ -th in a higher energy state.

This situation is repeated until all the energy levels of the quantum dot to be swept  $n$ -th [12-14]. Then, in the next row of dots and  $n-1$ -th have a higher energy level and all the previous steps repeated. The cycle continues and with the energy levels of the quantum dot to last your energy levels of next quantum dot will increase energy levels. Finally, get all your energy levels of quantum dots to the latest mode of the cavity photons  $m$ -th have an increased number and cycle to be repeated in this case all the photons in the cavity so that replication continues to be my number of  $m$ . The number of photons  $m-1$ -th mode cavity numbers have increased a unit to repeat the entire process. Finally, this process continues until all the photons in the cavity modes are [11].

First, determine the energy levels of quantum dots, which are placed in the system and then determine that the cavity mode in the initial state is located exactly how many photons. Finally, according to the state obtained from the combination of quantum dots and cavity modes matrix elements basic scenarios that cover both defined. The state program, a consideration, others are zero and thus the Fock initial state is produced.

The other and more important in determining the initial conditions consistent if necessary, according to the above relationships, the initial conditions of the two stages will be carried out coherently. in the first stage in any combination of modes of energy levels of quantum dots, each quantum dot state probability calculated the energy levels and in the second stage probabilities associated with each of the states of the photons in each cavity modes in any combination of modes of the cavity is calculated. Finally, the initial state of each of the compounds, in the case of quantum dots and cavity modes of the probability of all cases relating to the composition will be calculated. In addition, given that there is a need to analyze the system in cases between states energy levels in the quantum dot phase difference there, the need to apply the conditions of the original agreement, to produce the initial conditions in terms of coherent states of the phase difference between the energy levels, in the case of each of the state's energy levels of phase, they will be asked The phase of each of the levels to be determined. In other words, if the phase difference

between two energy levels or be addressed by the zero phase one and phase to the other, enters the desired phase difference. First, the possibility of any state energy levels of the quantum dot is calculated with respect to its phase. And results in a matrix of the quantum dot is likely the first level to the last level of the quantum dot and the likely mode are [9]. After calculating the coefficient matrix of system states due to the possibility of the system in different states of quantum dots and cavity modes Fock and coherent with the initial conditions, will be calculated.

## 6. CONCLUSION

Quantum optical systems with any number of cavity modes to investigate and obtain the necessary relations programs for analysis and production of initial conditions may be written in the most general case. A real system in which a potential well is a quantum equivalent of a three level system. Finally, a multi-part system consisting of six interacting quantum dot in a cavity mode in the weak coupling regime, strong and very strong evaluated. All levels of probability in various positions in the system of initial conditions Fock state and coherent state were calculated. The probability of the system in different scenarios as Sinusoidal or riding on a frequency modulated signal no change scenarios fall and revival as much increase. As the diagrams related to the Fock initial conditions specified:

By increasing the coupling constant electric field and increase the possibilities of increased volatility.

With the arrival of strong coupling regime probability function retains its time-varying sinusoidal form, but with the arrival of the coupling is not very strong.

The most important conclusion is clearly seen that the system is likely to increase the coupling constant rising and the very strong coupling regime much more likely and therefore no longer rotating wave approximation. The existing solution, the most complete solution possible in the strong coupling regime and more importantly, very strong.

What is the probability plots comparing the different energy levels of the quantum system is in the initial conditions specified coherent expressed as follows.

The probability of weak and strong coupling regime system as Sinusoidal and without distortion is changing on a modulated signal.

By increasing the applied electric field and thus increase the coupling constant frequency observed change is likely to increase in the coupling system is very strong.

The very strong coupling regime likely not on a

modulated signal as shown in a smaller time frame does not change as a Sinusoidal and regular.

In general, by increasing the applied electric field while the transition moment is constant and steady increase in the energy levels of the quantum system coupled probability of behavior is erratic.

## REFERENCES

- [1] W. P. Schleich, "Quantum Optics in Phase Space," 1st ed. Berlin: Wiley-VCH, 2001.
- [2] D. Press et al., "Photon anti bunching from a single quantum-dot-micro cavity system in the strong coupling regime," *Phys. Rev. Lett.*, Vol. 98, pp. 117402-117405, 2007.
- [3] J. P. Reithmaier et al., "Strong coupling in a single quantum dot-semiconductor micro cavity system," *Nature*, Vol. 432, pp.197-200, 2004.
- [4] D. Englund, "Controlling the spontaneous emission rate of single quantum dots in a 2D photonic crystal," *Phys. Rev. Lett.* Vol. 95, pp. 013904-013908, 2005.
- [5] A. Anappara et al., "Signatures of the ultra strong light-matter coupling regime," *Phys. Rev. B*, Vol. 79, pp. 201303-201306, 2009.
- [6] T. Niemczyk et al., "Circuit quantum electrodynamics in the ultra strong-coupling regime," *Nat. Phys.* Vol. 6, pp. 772, 2010.
- [7] A. H. Sadeghi et al., "Interaction of Quantum Dot Molecules with Multi-mode Radiation Fields," *Scientia Iranica*, Vol. 17, pp. 59-70, 2010.
- [8] Y. Todorov et al., "Ultra strong Light-Matter Coupling Regime with Polariton Dots," *Phys. Rev. Lett.* Vol. 105, pp. 196402-196405, 2010.
- [9] M. Geiser et al., "Ultra strong Coupling Regime and Plasmon Polaritons in Parabolic Semiconductor Quantum Wells," *Phys. Rev. Lett.* vol. 108, pp. 106402-1066406, 2012.
- [10] G. Scalari et al., "Ultra strong Coupling of the Cyclotron Transition of a 2D Electron Gas to a THz Met material," *Science*, Vol. 335, No. 6074, pp. 1323-1326, 2012.
- [11] S. D. Liberato "Quantum Vacuum Radiation Spectra from a Semiconductor Micro cavity with a Time-Modulated Vacuum Rabi Frequency," *Phys. Rev. Lett.* Vol. 98, pp.103602-103605, 2007.
- [12] G. Günter et al., "Sub-cycle switch-on of ultra strong light-matter interaction," *Nature*, Vol. 458, pp. 178-181, 2009.
- [13] A. Laucht et al., "Electrical control of spontaneous emission and strong coupling for a single quantum dot," *New J. Phys.*, Vol. 11, pp. 023034, 2009.
- [14] M. Kaniber et al., "Highly efficient single-photon emission from single quantum dots within a two dimensional, photonic band-gap," *Phys. Rev. B*, Vol. 77, 073312-073315, 2008.