Optimal Power Allocation in MIMO Channel

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ABSTRACT:
In this paper, we introduce a spatial MIMO channel for modern communication systems. Parameters required in the spatial MIMO channel modeling are studied and the channel is simulated for two types of modeling named "un-polarized" and "cross-polarized" antennas. Then, the channel capacity is computed at different signal to noise ratio. Besides, the channel capacity is analyzed for two different scenarios including high SNR and low SNR. To reach the maximum capacity, the optimal power allocation on antennas is also investigated. The simulation results show that, at low SNR, the power allocation to the sub-channel corresponding to maximum singular value is optimal and equal power allocation to the sub-channels is optimal at high SNR.

KEYWORDS: MIMO channel- Spatial Channel Model- Diversity- Degree of freedom- Optimal power- Capacity

1. INTRODUCTION

The MIMO system was introduced by *Telatar* in 1995. He showed that MIMO system made different paths between transmitter and receiver and caused various sub-channels. If we use one path to send a signal, there will be a considerable probability that this channel is in a deep fade. Hence, by transmitting signals that carry similar information via different paths, numerous independently faded copies of data symbols are taken at the receiver and more trustworthy detection can be achieved. As a result, signal reception from different sub-channels increases the channel capacity [1].

Early studies about these channels were related to decreasing noise effects and increasing channel capacity. Afterward, *Alamuti* proposed the use of orthogonal codes or low correlation codes in these channels. These codes were introduced in order to make independent paths between transmitter and receiver.

These systems are known as 802.11n and used in modern communication systems. The important characteristics of these systems are increasing the capacity, degree of freedom, system flexibility, diversity gain, and received power by transmit beamforming (BF).

There are three accepted techniques for MIMO channel modeling including ray tracing model, correlation model, and scattering model. The first technique models the exact positions of the scatterers. This technique is too complex for most outdoor environments and illogical for system simulations. In the second technique, temporal variations of the channel are modeled without taking the spatial correlation of parameters. Then, there may be no relation between temporal and spatial domains of the channel. The third technique, known as Spatial Channel Model (SCM), presumes a specified distribution of scatterers, and the channel is considered based on them. In this model, wireless propagation medium is not clearly modeled. Hence, its complexity is less than two other models. On the other hand, some disadvantages of this model are complexity in parameters, scattering distributions assembly for different channel environments, and inclusion of high number of parameters [2].

Using cross-polarized antennas in developed cellular systems with numerous transmitter and receiver antennas has attracted so much attention. Therefore, we should consider this feature as well as other parameters of the MIMO channel. In two-dimensional (2-D) spatial channel modeling, disregarding elevation spectrum, we can indicate numerous effects of electromagnetic propagation in outdoor environments.

In some propagation environments which focus on angular spectrum, the assumption of 2-D propagating waves is not efficient. In these cases, erroneous results are obtained by just considering azimuth spectrum. So in order to reach desired results, the 2-D model is corrected and elevated to the next three dimensional (3-D) model. In this article, we have only presented 2-D channel modeling [3], [4].

2. MIMO CHANNEL MODELING

The aim of this section is the recognition of necessary parameters for spatial and temporal channel, in which a single base station transmits the information to a single mobile station. Furthermore, two cases of un-polarized and cross-polarized models are introduced in this section.

The overall procedure for generating the channel matrix consists of three basic steps. The first one is specifying an environment such as suburban macro, urban macro or urban micro, since every environment has its own features that differ from other environments. In the second step, the parameters associated with the environment, which are used in simulations, are obtained. In the last step, channel coefficients are generated on the basis of required parameters.

The received signal at the mobile station (MS) is composed of exact copies of N time-delayed multipath, associated with the transmitted signal. These N paths have been defined by different powers and delays, and chosen randomly according to the channel generation procedure. Additionally, each path consists of M sub-paths [5].

Angular parameters, used in channel modeling have been shown in fig. 1.

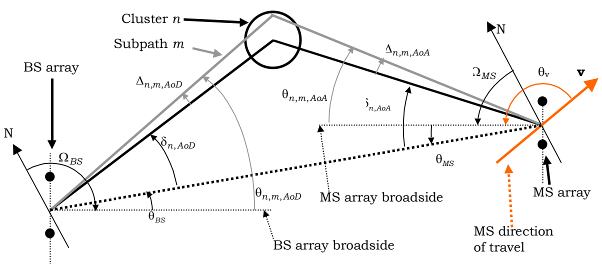

Fig. 1. Angle parameters used for modeling [2]

These parameters are defined in the following [2]:

Ω_{BS} : Base station (BS) antenna array direction.

θ_{BS} : Angle between the BS-MS Line Of Sight (LOS) and the BS broadside.

$\delta_{n,\mathrm{AoD}}$: Angle of Departure (AoD) for the nth (n=1, …, N) path.

$\Delta_{n,m,\mathrm{AoD}}$: Offset for m^{th} (m=1,…,M) sub-path of the n^{th} path.

$\theta_{n,m,\mathrm{AoD}}$: AoD for m^{th} (m=1,…,M) sub-path of the n^{th} path.

Ω_{MS} : Mobile Station (MS) antenna array direction.

θ_{MS} : Angle between the BS-MS Line Of Sight (LOS) and the MS broadside.

$\delta_{n,\mathrm{AoA}}$: Angle of Arrival (AoA) for the n^{th} (n=1,…,N) path.

$\Delta_{n,m,\mathrm{AoA}}$: Offset for m^{th} (m=1,…,M) sub-path of the n^{th} path.

$\theta_{n,m,\mathrm{AoA}}$: AoA for m^{th} (m=1,…,M) sub-path of the n^{th} path.

θ_v : Angle of the velocity vector.

Channel coefficients are generated using defined parameters. The numbers of MS and BS antennas are S and U, respectively. So, the matrix of the channel coefficients for each of the N paths is given by means of a $U \times S$ matrix shown with $\mathbf{H}_n(t)$ [2], [6]. The $(u,s)^{\mathrm{th}}$ component of this matrix is expressed as:

$$h_{u,s,n}(t) = \sqrt{\frac{P_n \sigma_{\mathrm{SF}}}{M}} \times \sum_{m=1}^{M} \left(\begin{array}{l} \sqrt{G_{\mathrm{BS}}\left(\theta_{n,m,\mathrm{AoD}}\right)} \\ \times \exp\left(j \left(\begin{array}{l} 2\pi d_s \sin\left(\theta_{n,m,\mathrm{AoD}}\right) / \lambda \\ + \Phi_{n,m} \end{array} \right) \right) \\ \times \sqrt{G_{\mathrm{MS}}\left(\theta_{n,m,\mathrm{AoA}}\right)} \\ \times \exp\left(j 2\pi d_u \sin\left(\theta_{n,m,\mathrm{AoA}}\right) / \lambda \right) \\ \times \exp\left(\dfrac{j 2\pi |v| \cos\left(\theta_{n,m,\mathrm{AoA}} - \theta_v\right) t}{\lambda} \right) \end{array} \right) \quad (1)$$

The parameters used in (1) are defined in the following:

P_n : Power of the n^{th} path.

σ_{SF} : Log-normal SF.

M: Number of sub-paths per-path.

$G_{\mathrm{BS}}(\theta_{n,m,\mathrm{AoD}})$: BS antenna gain.

$G_{\mathrm{MS}}(\theta_{n,m,\mathrm{AoA}})$: MS antenna gain.

λ : Wavelength in meters.

d_s : Distance from s^{th} element of BS antenna to the reference antenna in meters.

d_u : Distance from u^{th} element of MS antenna to the reference antenna in meters.

$\Phi_{n,m}$: Phase of the m^{th} sub-path of the n^{th} path.

$|v|$: Magnitude of the MS velocity vector.

The $(u,s)^{\mathrm{th}}$ component of the channel matrix indicates path gain from s^{th} element of the BS antenna to u^{th} element of the MS antenna and is calculated as:

$$h_{u,s}(t) = \sum_{n=1}^{N} h_{u,s,n}(t) \quad (2)$$

So the channel matrix from transmitter to receiver is expressed according to:

$$\mathbf{H}(t)=\begin{bmatrix} h_{1,1}(t) & \cdots & h_{1,S}(t) \\ \vdots & \ddots & \vdots \\ h_{U,1}(t) & \cdots & h_{U,S}(t) \end{bmatrix}_{U\times S} \tag{3}$$

In the next step, we consider introducing parameters in MIMO channel modeling using cross-polarized antennas. In the SCM model, mentioned in this section, the vertical spectrum is not considered, and the 2-D spatial channel modeling is introduced.

In this model, the polarization is decomposed into vertical and horizontal directions. The components with analogous polarity are mixed with each other, but others have less mixing. So, four channels are considered between the MS and the BS antennas. These channels show the relationship between components with horizontal and vertical polarity in BS array, and components with horizontal and vertical polarity in MS array. Therefore, the antenna patterns in the MS and the BS are considered into the vertical and the horizontal directions. The presented model in this section has original step similar to the un-polarized model, however, the polarity components are added [3].

The $P2$ power of each path in the horizontal direction depends on the $P1$ power in the vertical direction that is defined as cross-polarization discrimination by $XPD=P2/P1$. The whole M sub-paths of the n^{th} path (n=1, 2,…,6) have same XPD, but each path has independent XPD [3].

For this section, ideal tilted dipole antennas are assumed. Mixing of the horizontal and the vertical components arises from path effects, and antenna polarization outflow effects are neglected.

The ideal dipole antenna with polarization vector $\mathbf{p}$ and α angle from $\mathbf{z}$ axis has the vertical and the horizontal components corresponding to:

$$\boldsymbol{\chi}(\mathbf{k})=\begin{bmatrix} \chi^{\mathrm{v}}(\mathbf{k}) \\ \chi^{\mathrm{h}}(\mathbf{k}) \end{bmatrix}\exp(j\mathbf{kr})= \begin{bmatrix} \cos\alpha \\ \sin\alpha\cos\varphi \end{bmatrix}\exp(j\mathbf{kr}) \tag{4}$$

Vector $\mathbf{r}$ is the distance of the antenna from the center of the antenna array. Vector $\mathbf{k}$ is 2-D wave vector with the carrier wavelength λ and azimuth angle φ. This vector is defined as:

$$\mathbf{k}=\frac{2\pi}{\lambda}\begin{bmatrix}\cos\varphi & \sin\varphi\end{bmatrix} \tag{5}$$

According to (4), complex response of the BS antenna for the horizontal and the vertical components is obtained in the following [3]:

$$\boldsymbol{\chi}_{\mathrm{BS}}(\mathbf{k})=\begin{bmatrix} \chi_{\mathrm{BS}}^{(\mathrm{v})}(\theta_{n,m,\mathrm{AoD}}) \\ \chi_{\mathrm{BS}}^{(\mathrm{h})}(\theta_{n,m,\mathrm{AoD}}) \end{bmatrix}= \begin{bmatrix} \cos\alpha \\ \sin\alpha\cos(\theta_{n,m,\mathrm{AoD}}) \end{bmatrix}\exp(j\mathbf{kr}) \tag{6}$$

Where, the first element is the complex response of the BS antenna for the vertical component, and the second element is the complex response of the BS antenna for the horizontal component.

Using (4), complex response of the MS antenna for a wave component with the vertical and the horizontal polarity is obtained as [3]:

$$\boldsymbol{\chi}_{\mathrm{MS}}(\mathbf{k})=\begin{bmatrix} \chi_{\mathrm{MS}}^{(\mathrm{v})}(\theta_{n,m,\mathrm{AoA}}) \\ \chi_{\mathrm{MS}}^{(\mathrm{h})}(\theta_{n,m,\mathrm{AoA}}) \end{bmatrix}= \begin{bmatrix} \cos\alpha \\ \sin\alpha\cos(\theta_{n,m,\mathrm{AoA}}) \end{bmatrix}\exp(j\mathbf{kr}) \tag{7}$$

In which, the first element is the complex response of the MS antenna for a component with the vertical polarity, and the second element is the complex response of the MS antenna for a component with the horizontal polarity.

The random wave components departing from the BS antenna (with either V or H polarizations) and arriving at the MS antenna (with either V or H polarizations) are expressed as:

$$\mathbf{G}_i^{\mathrm{2D}}=\begin{bmatrix} z_i^{\mathrm{vv}} & \sqrt{r_{n1}}\,z_i^{\mathrm{vh}} \\ \sqrt{r_{n2}}\,z_i^{\mathrm{hv}} & z_i^{\mathrm{hh}} \end{bmatrix}= \begin{bmatrix} \exp\left(j\Phi_{n,m}^{(\mathrm{v,v})}\right) & \sqrt{r_{n1}}\exp\left(j\Phi_{n,m}^{(\mathrm{h,v})}\right) \\ \sqrt{r_{n2}}\exp\left(j\Phi_{n,m}^{(\mathrm{v,h})}\right) & \exp\left(j\Phi_{n,m}^{(\mathrm{h,h})}\right) \end{bmatrix} \tag{8}$$

where, the z terms (i=1,…, M) in matrix $\mathbf{G}_i^{\mathrm{2D}}$ are defined as independent identical distributions (i.i.d.) and are complex exponential variables that z_i is i^{th} wave component for each of the polarization channels, namely, VV, VH, HV, and VV. Variables $\Phi_{n,m}^{(x,y)}$ represent phase offset between wave component departing with x polarization from the BS and arriving with y polarization at the MS for m^{th} (m=1,…,M) sub-path of n^{th} (n=1,…,N) path [3]. Also, the variables r_{n1} and r_{n2} are i.i.d. and they are defined as:

$$r_{n1}=\frac{P_{\mathrm{vh}}}{P_{\mathrm{vv}}},\quad r_{n2}=\frac{P_{\mathrm{hv}}}{P_{\mathrm{hh}}} \tag{9}$$

Where:

P_{vh}: power of wave leaving the BS in the vertical direction and arriving at the MS in the horizontal direction.

P_{hv}: power of wave leaving the BS in the horizontal direction and arriving at the MS in the vertical direction.
P_{vv}: power of wave leaving the BS in the vertical direction and arriving at the MS in the vertical direction.
P_{hh}: power of wave leaving the BS in the horizontal direction and arriving at the MS in the horizontal direction.
The channel coefficients are calculated by using the parameters, introduced in this section. The $(u,s)^{\text{th}}$ component of the channel coefficients matrix for each of the N paths is evaluated as [3]:

$$h_{u,s,n}^{2D}(t)=\sqrt{\frac{P_n\sigma_{SF}}{M}}\times \sum_{m=1}^{M}\left(\begin{array}{l}\begin{bmatrix}\cos\alpha & \sin\alpha\cos\left(\theta_{n,m,\text{AoD}}\right)\end{bmatrix}\\ \times\begin{bmatrix}\exp\left(j\Phi_{n,m}^{(\text{v,v})}\right) & \sqrt{r_{n1}}\exp\left(j\Phi_{n,m}^{(\text{h,v})}\right)\\ \sqrt{r_{n2}}\exp\left(j\Phi_{n,m}^{(\text{v,h})}\right) & \exp\left(j\Phi_{n,m}^{(\text{h,h})}\right)\end{bmatrix}\\ \times\begin{bmatrix}\cos\alpha\\ \sin\alpha\cos\left(\theta_{n,m,\text{AoA}}\right)\end{bmatrix}\\ \times\exp\left(j2\pi d_s\sin\left(\theta_{n,m,\text{AoD}}\right)/\lambda\right)\\ \times\exp\left(j2\pi d_u\sin\left(\theta_{n,m,\text{AoA}}\right)/\lambda\right)\\ \times\exp\left(j2\pi|v|\cos\left(\theta_{n,m,\text{AoA}}-\theta_v\right)t/\lambda\right)\end{array}\right) \quad (10)$$

Where, parameters used in (10) have been described in previous paragraphs. The superscript 2D indicates wave propagation in two dimensions. Therefore, the $(u,s)^{\text{th}}$ component of the channel matrix is channel gain from the s^{th} element of the BS antenna to the u^{th} element of the MS antenna and is computed as:

$$h^{2D}{}_{u,s}(t)=\sum_{n=1}^{N}h^{2D}{}_{u,s,n}(t) \quad (11)$$

So, the channel coefficients matrix from the BS antenna to the MS antenna is shown by $\mathbf{H}^{2D}(t)$ and is indicated according to:

$$\mathbf{H}^{2D}(t)=\begin{bmatrix}h^{2D}{}_{1,1}(t) & \cdots & h^{2D}{}_{1,S}(t)\\ \vdots & \ddots & \vdots\\ h^{2D}{}_{U,1}(t) & \cdots & h^{2D}{}_{U,S}(t)\end{bmatrix}_{U\times S} \quad (12)$$

3. MIMO CHANNEL CAPACITY

The performance of the various digital modulations at fading channels is not so good, because the error probabilities of all of them decline very slowly versus increasing the SNR. The cause of this bad performance is that trusty communication depends on the strength of one signal path. Then, there is a high probability that this signal path will be in a deep fade, and the receiver could not detect the main signal. One solution to modify this bad performance is passing the information symbols through multiple signal paths that each of them fades independently. There is a high probability that one of these paths is strong enough. This technique is named diversity, and it improves the performance over fading channels.
There are three ways to get diversity, called, diversity over time, diversity over frequency, and diversity over space. The main diversity in the MIMO channel can be obtained over space.
Antenna diversity, or spatial diversity, can be gotten by locating multiple antennas at the transmitter and/or the receiver. If the antennas are located sufficiently far apart, the channel gains between different antenna pairs fade separately from together, and they cause independent signal paths. The necessary antenna separation depends on the local scattering environment and the carrier frequency [1], [7], [8].
The channel capacity is one of the cases that is more important in the MIMO system. The diversity cause increasing the channel capacity. In the next, we are analyzing the MIMO channel and introducing the capacity.
If the received signal and the transmitted signal are shown by y and x, respectively; then, time-invariant channel is described as:

$$\mathbf{y}=\mathbf{Hx}+\mathbf{w} \quad (13)$$

For simplicity, time domain channel variations have been ignored. Dimensions of the vectors **y** and **x** are $(n_r\times 1)$ and $(n_t\times 1)$, respectively. In addition, **w** is a zero-mean Gaussian random noise with variance N_0.
The capacity is determined by decomposing the channel matrix into a group of parallel and independent scalar Gaussian sub-channels. Using singular value decomposition (SVD), the channel matrix is written as [1]:

$$\mathbf{H}=\mathbf{U\Lambda V}^{*} \quad (14)$$

Where, **U** and **V** are unitary matrices and their dimensions are $n_r\times n_r$ and $n_t\times n_t$, respectively. Also, $\mathbf{\Lambda}$ is a rectangular matrix with $n_r\times n_t$ dimensions that its off-diagonal elements are zero and other elements are real and positive singular values of the matrix **H**, arranged as:

$$\lambda_1\geq\lambda_2\geq\cdots\geq\lambda_{n_{\min}} \quad ,n_{\min}:=\min\left(n_t,n_r\right) \quad (15)$$

By defining new parameters according to (16), the channel equation is obtained as (17) [1].

$$\tilde{\mathbf{x}}:=\mathbf{V}^{*}\mathbf{x}, \quad ,\tilde{\mathbf{y}}:=\mathbf{U}^{*}\mathbf{y}, \quad \tilde{\mathbf{w}}:=\mathbf{U}^{*}\mathbf{w} \quad (16)$$

$$\tilde{\mathbf{y}}=\mathbf{\Lambda}\tilde{\mathbf{x}}+\tilde{\mathbf{w}} \quad (17)$$

Where, $\tilde{\mathbf{w}}$ is Gaussian random noise and has the same distribution as **w**. The representation of the channel is defined as parallel channels in accordance with (17).

Consequently, the total capacity is sum of the capacity of all of the sub-channels. So, the capacity of the MIMO channel can be computed by using the SVD decomposition and it may be expressed as:

$$C=\sum_{i=1}^{n_{\min}}\log\left(1+\frac{P_i^*\lambda_i^2}{N_0}\right) \quad \text{bits/s/Hz} \tag{18}$$

Where, $P_1^*,\dots,P_{n_{\min}}^*$ are located powers to each of these sub-channels.

The optimal power allocation to each of these sub-channels is one of the ways that causes increasing the channel capacity. The *waterfilling* algorithm is one of the optimal power allocation strategies for MIMO channel. In this algorithm, the transmitter allocates more power to the stronger sub-channels, taking benefit of the well again channel conditions, and less or no power to the weaker ones. These powers are selected in a way to achieve maximum capacity as:

$$P_i^*=\left(\mu-\frac{N_0}{\lambda_i^2}\right)^+ \tag{19}$$

$$\sum_i P_i^*=P$$

In which, μ is used to satisfy the total power.

The spreading out the singular values and the SNR are the key parameters that determine the performance of the MIMO channel. We want to analyze the performance for both of high and low SNR.

At high SNR, equal power allocation between all the transmit antennas is optimal and the channel capacity is depend on the rank of the channel matrix that its maximum is $n_{\min}$ (fig. 2). Then, the channel capacity is simplified as:

$$C\approx\sum_{i=1}^{b}\log\left(1+\frac{P\lambda_i^2}{kN_0}\right) \tag{20}$$

Where, b is the number of non zero singular values that is equal to the rank of the channel matrix, and SNR=P/N_0. Also, b is degree of freedom. The degree of freedom is equal to the number of detection symbols per second.

This result says that among the channels with the same total power, the one with the less spreading out the singular values has the highest capacity.

At low SNR, the optimal power allocation strategy is to allocate power only to the strongest singular value (fig. 3). In this case, the channel capacity is simplified as:

$$C\approx\log\left(1+\frac{P}{N_0}\left(\max_i\lambda_i^2\right)\right) \tag{21}$$

In this strategy, the rank of the channel matrix is not important but the value of the maximum singular value is significant.

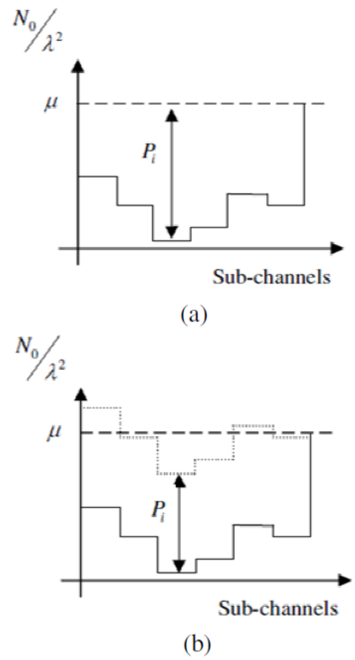

Fig. 2. Optimal power allocation at high SNR
(a) Power allocation using *waterfilling* algorithm.
(b) Equal power allocation between antennas.

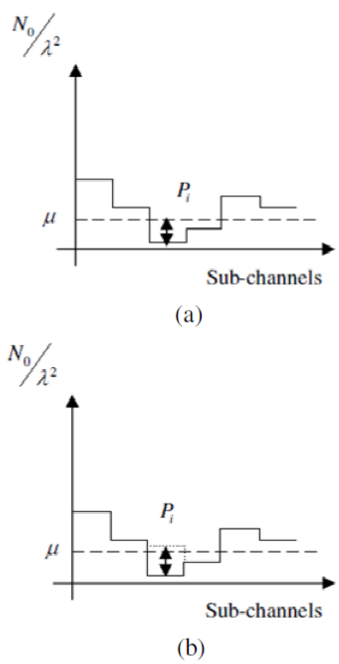

Fig. 3. Optimal power allocation at low SNR
(a) Power allocation using *waterfilling* algorithm.
(b) Power allocation only to strongest singular.

4. SIMULATION OF THE MIMO CHANNEL

In this section, first, by using previously presented parameters, we have simulated introduced models by MATLAB software. Then, we have computed the channel capacity using the *waterfilling* algorithm. After that, the channel capacity has been computed for high and low SNR. Finally, we have compared them together. Since in each simulation run some of the channel parameters like cluster statistics are varied, we have compared the outputs in many simulation runs.

The *suburban macro* has been used in simulation. The parameters values, used for simulation are given in table 1.

Table 1. The parameters values used for simulation

parameter	value	parameter	value
BS antenna pattern	3 sectors in each cell	environment	Suburban macrocell
MS antenna pattern	Omni directional	f_c	1900×10^6 Hz
AS at BS	2 degrees	PAS	Laplacian
AS at MS	35 degrees	Ω_{MS}	U(0,360)
MS antenna gain	0 dB	Ω_{BS}	U(0,360)
BS antenna gain	[2]	$\Phi_{n,m}$	U(0,360)
Pathloss model	31.5+35log10 (d) [2]	r_u, r_s	0.5 λ

In figures 4 to 9, the MIMO channel capacity at high SNR for MIMO 2×2, 4×4, and 8×8 (for both of models, un-polarized and cross-polarized antennas) have been plotted, respectively. The channel capacity by using equation 19 and 20 has been computed and compared with together. These figures demonstrate that equal power distribution between antennas is optimal at high SNR, nearly.

In figures 10 to 15, also, the MIMO channel capacity at low SNR for MIMO 2×2, 4×4, and 8×8 (for both of models, un-polarized and cross-polarized antennas) have been illustrated and compared with together, respectively. These figures show that allocation power to the sub-channel equivalent to the strongest singular value is optimal at low SNR, approximately.

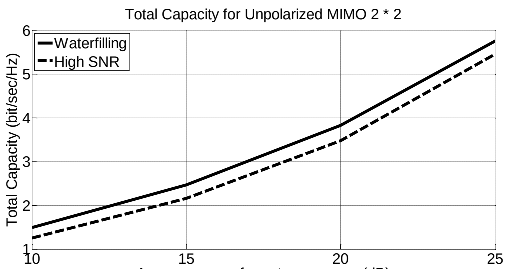

Fig. 4. The un-polarized MIMO 2×2 capacity using *waterfilling* algorithm and high SNR method

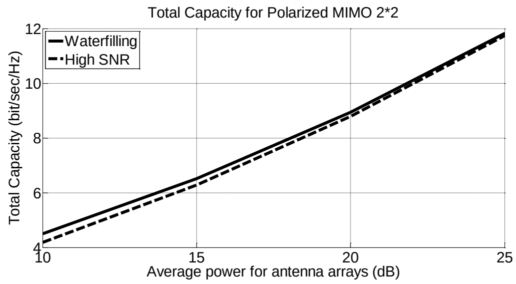

Fig. 5. The cross-polarized MIMO 2×2 capacity using *waterfilling* algorithm and high SNR method

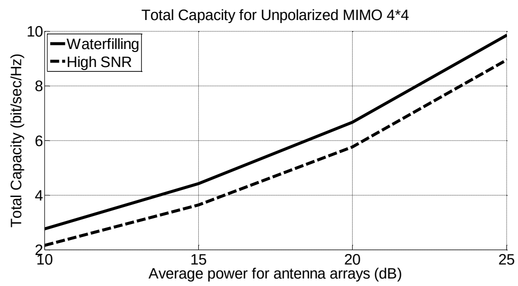

Fig. 6. The un-polarized MIMO 4×4 capacity using *waterfilling* algorithm and high SNR method

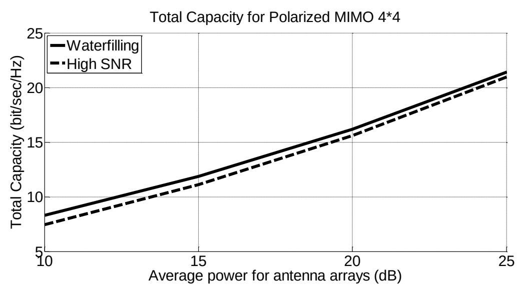

Fig. 7. The cross-polarized MIMO 4×4 capacity using *waterfilling* algorithm and high SNR method

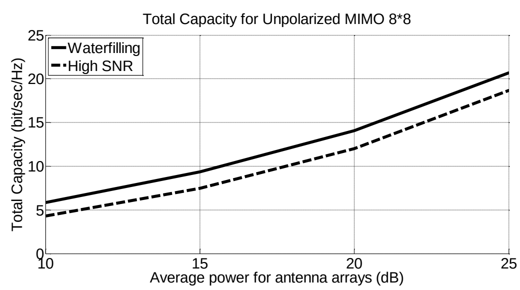

Fig. 8. The un-polarized MIMO 8×8 capacity using *waterfilling* algorithm and high SNR method

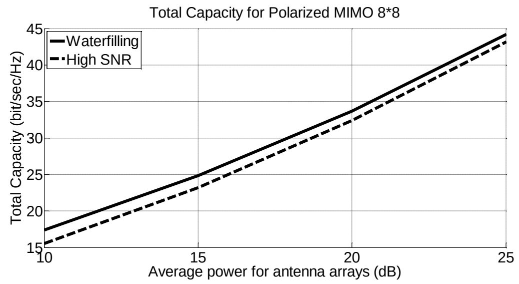

Fig. 9. The cross-polarized MIMO 8×8 capacity using *waterfilling* algorithm and high SNR method

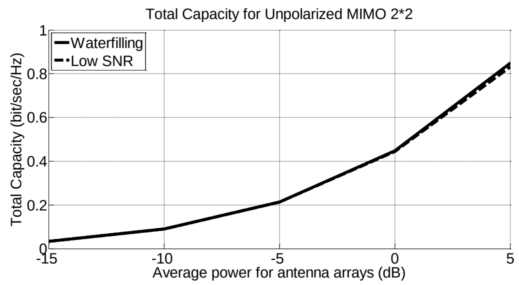

Fig. 10. The un-polarized MIMO 2×2 capacity using *waterfilling* algorithm and low SNR method

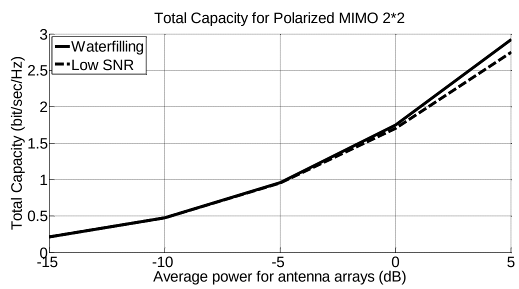

Fig. 11. The cross-polarized MIMO 2×2 capacity using *waterfilling* algorithm and low SNR method

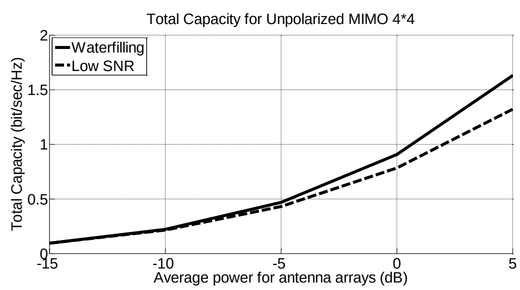

Fig. 12. The un-polarized MIMO 4×4 capacity using *waterfilling* algorithm and low SNR method.

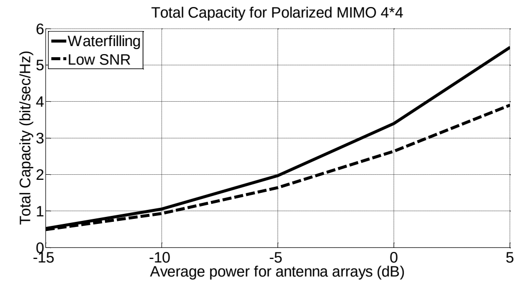

Fig. 13. The cross-polarized MIMO 4×4 capacity using *waterfilling* algorithm and low SNR method

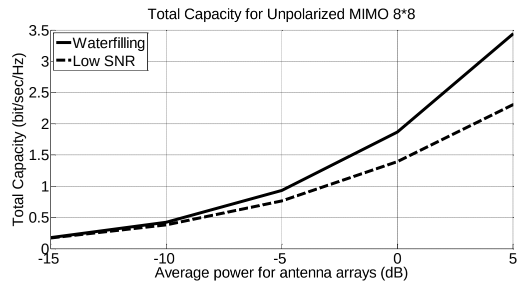

Fig. 14. The un-polarized MIMO 8×8 capacity using *waterfilling* algorithm and low SNR method

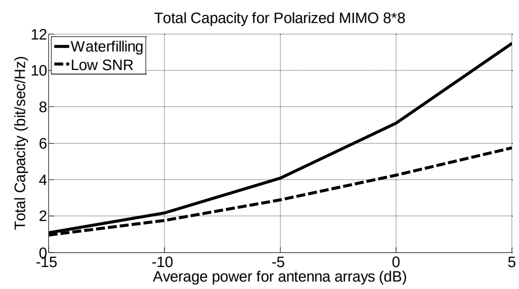

Fig. 15. The cross-polarized MIMO 8×8 capacity using *waterfilling* algorithm and low SNR method

5. CONCLUSION

Simulation results demonstrate that, at high SNR, the equal power allocation between the antennas is optimal while, at low SNR, the power allocated to the sub-channel corresponding to the strongest singular value is optimal.

As well, at low SNR, the channel capacity deviates from the optimal value with increasing the number of antennas if the power is allocated to the sub-channel corresponding to the strongest singular value. The reason is that although the number of independent paths increases by increasing the number of antennas, but we just use one path corresponding to the strongest singular value to transmit a signal, instead of all existing paths.

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