Calculation of Third-Order Harmonic Distortion in an Operational Transconductance Amplifier

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ABSTRACT
Due to its nonlinear behavior, operational transconductance amplifier causes substantial distortion in transfer conductance that weakens the intended signal. Analyzing Taylor series, this paper proposes a method to estimate third-order harmonic distortion. As a result, an equation to calculate output current is proposed. Moreover, third-order harmonic distortion is explained in an equation in terms of different parameters. Behavioral simulation was used to investigate harmonic distortion characteristics for the purpose of investigating the validity of the results. Final results show that analytical relationships and the simulation results are perfectly matched.

KEYWORDS: Operational Transconductance Amplifier, Third-Order Harmonic Distortion, Source Degeneration.

1. INTRODUCTION

OTA is an important part of the analog circuits and systems whose most important utility is in receiver filters [1] – [3]. OTA main properties are lower noise level and power consumption, better linear behavior, and wider dynamic range; and the nonlinear behavior still needs improvement [4]- [5]. Nonlinear conditions cause substantial distortion of transconductance. In this context, some research is performed on promoting good linearization practices for circuits made of nonlinear active elements [1][6]– [10]. Therefore, a more comprehensive analysis is needed. In [1]- [6][10], linearization is analyzed using source degeneration model. In [6][7] [9][11], linearization is provided using the linear area transistor along with variable bias. On the other hand, focus is on the combination of two methods of source degeneration linearization and variable bias in [9]. Linearization through using op-amp and linearization. by fixing drain current and source - drain voltage are offered in [6][7]. Since mathematical analysis of the circuit characteristics is very important among designers, we perform a third-order harmonic distortion analysis using equations dominating transistors.

This paper is organized as follows: In Section II, the OTA architecture discussed in references is briefly reviewed. The third section describes the third-order harmonic distortion. In the fourth section, Taylor series are used for extracting features of the third-order harmonic distortion. Accuracy of the proposed analysis is evaluated in the fifth section.

Finally, conclusions are made in section six.

2. OTA INTRODUCTION

Since the proposed circuit is expressed based on the OTA, an overview of it is given in this section.

OTA is a differential amplifier with input voltage and output current. In OTA, the output is a function of input voltage difference. If OTA is ideal, its input and output impedance will be infinite. By changing the current of input transistors (their Gm) [and] amount of transconductance (Gm), we can control the OTAs. For an ideal OTA, it can be written [12]:

$$i_0 = g_m V_i \tag{1}$$

$$Z_i = \infty \qquad Z_0 = \infty \tag{2}$$

Where i_0 and V_i respectively represent the output current and input voltage. OTA equivalent circuit and symbol are also shown in figure 1.

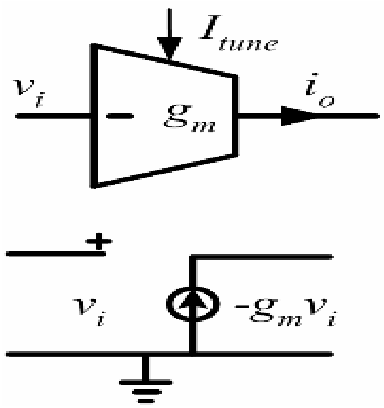

Fig. 1. Simple single-output OTA

OTA transconductance values vary from tens to hundreds of µA/V.

OTAs can also be single, double or multi-output [12]. OTA is one of the active components and is available as an integrated circuit.

Among various OTAs, the simplest and most commonly used method is based on source degeneration. Linearization using source degeneration method is in shown in figure 2.

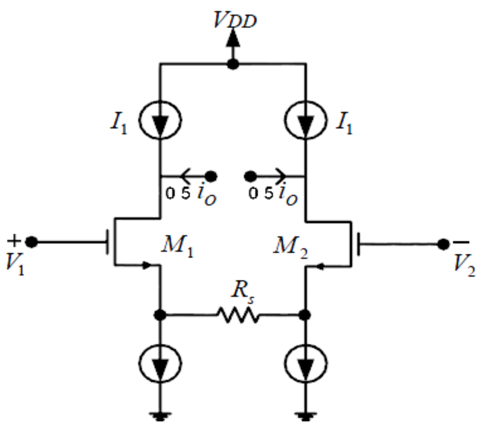

Fig. 2. OTA linearization using source degeneration

Source degeneration causes conflict among linearity, noise, power and efficiency losses. Degeneration reduces the swing between the gate and transistor source and thereby linearizes the input-output characteristic. For the output current and third order harmonic distortion we have [6]:

$$i_o = \left(\frac{g_{m1,2}}{\alpha}\right) V_{id} \sqrt{1 - \frac{V_{id}^2}{(2\alpha V_{eff})^2}} \quad (3)$$

$$HD_3 = \frac{1}{32}\left(\frac{V_P}{\alpha V_{eff1,2}}\right)^2 \quad (4)$$

Where $\alpha = 1 + gm_{1,2}R_s$ It is therefore seen that the nonlinear factor under the radical decreases with the coefficient $1/\alpha^2$; *i.e.* the linear range is increased and this increase in linear range is specified in relation (4). It means that the third-order distortion is decreased with the coefficient $1/\alpha^2$. Using source degeneration method reduces the Gm with the $1/\alpha$ factor. Simulation shows that relation (4) has not sufficient accuracy [9].

3. THIRD ORDER HARMONIC DISTORTION

random, but it mostly appears in it periodically. This means that consecutive cycles are almost identical and may slowly change. This concept describes the harmonic term. Harmonic distortion is caused by nonlinear elements transforming the system into a nonlinear one. A Linear System is the one whose output can be expressed as the linear combination of its response to the individual inputs. Specifically, if we have for the inputs $x_1(t)$ and $x_2(t)$:ss

$x_1(t) \rightarrow y_1(t)$, $x_2(t) \rightarrow y_2(t)$

For all constant values of a and b, we have:

$$ax_1(t) + bx_2(t) \rightarrow ay_1(t) + by_2(t) \quad (5)$$

Any system that does not meet the above condition is nonlinear.

Nonlinear property often leads to important and interesting phenomena. Harmonic distortion is one of them. If the system is introduced with the equation (6):

$$y(t) = a_1x(t) + a_2x^2(t) + a_3x^3(t) \quad (6)$$

We can also determine the nonlinear effect by applying a sinusoidal signal to the input and measurement in the output. In equation (6), if we have $X(t) = A\cos\omega t$ then

$$Y(t) = a_1A\cos\omega + a_2A^2\cos^2\omega t + a_3A^3\cos^3\omega t = \quad (7)$$
$$a_1A\cos\omega t + \frac{a_2A^2}{2}(1+\cos 2\omega t) + \frac{a_3A^3}{4}(3\cos\omega t + \cos 3\cos\omega t) =$$
$$\frac{\alpha_2A^2}{2} + \left[a_1A + \frac{3\alpha_3A^3}{4}\right]\cos\omega t + \cos\omega t + \frac{\alpha_3A^3}{4}\cos 3\omega$$

In equation (7), we see that high-order sentences produce higher harmonics. Phrases with even exponents make even harmonics, and the odd-exponent sentences produce odd harmonics. It is notable that range of the n^{th} harmony is increased almost proportional to the n^{th} exponent of the input. This is called the harmonic distortion effect [6].

HD_3 can be calculated from the expansion of equation (7):

$$HD_3 = \frac{\frac{\alpha_3A^3}{4}}{\alpha_1A} = \frac{1}{4}\frac{\alpha_3}{\alpha_1}A^2 \quad (8)$$

4. THE PROPOSED METHOD FOR CALCULATING HD_3

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In this paper, the OTA in figure 3 is used which is based on the source degeneration.

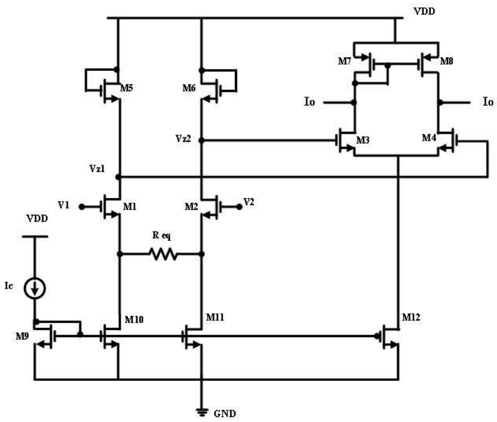

Fig.(3). The Proposed OTA circuit

Regarding figure (3), gate voltage of the transistors M_3 and M_4 are V_{z2} and V_{z1} . V_{z2} and V_{z1} can be obtained [13].

$$V_{z1} - V_{z2} = (V_{DD} - V_{gs5}) - (V_{DD} - V_{gs6}) \tag{9}$$

$V_{z1} - V_{z2} = V_{gs6} - V_{gs5}$, because drain current of M_1, M_5 and drain current of M_6, M_2 are the same.

$$(V_{gs5} - V_T)^2 = (V_{gs1} - V_T)^2 = (V_1 - V_{S1} - V_T)^2 \tag{10}$$

$$(V_{gs6} - V_T)^2 = (V_{gs2} - V_T)^2 = (V_2 - V_{S2} - V_T)^2 \tag{11}$$

$$V_{gs5} = V_{gs1} = V_1 - V_{S1} \tag{12}$$

$$V_{gs6} = V_{gs2} = V_2 - V_{S2} \tag{13}$$

In equations (12) - (13), supposing $I_{D5} > I_{D6}$ and current through R_{eq}, ΔI is:

$$V_{z1} - V_{z2} = (V_2 - V_{S2}) - (V_1 - V_{S1}) \tag{14}$$

$$= (V_2 - V_1) - (V_{S2} - V_{S1}) = (V_2 - V_1) + R_{eq}\Delta I$$

$$(V_1 - V_2) = (V_{z2} - V_{z1}) + R_{eq}\Delta I \tag{15}$$

For the current through M5 and M6 transistors, we may have:

$$I_{D_5} = I_{C_1} + \Delta I = \frac{K}{2}(V_{gs_1} - V_T)^2 \tag{16}$$

$$I_{D_6} = I_{C_1} + \Delta I = \frac{K}{2}(V_{gs_2} - V_T)^2 \tag{17}$$

Where $\mu_n C_{ox}(w/l)$ and L, W are respectively the width and length; C_{ox} is the oxide capacitance per channel and μ_n is the mobility. Therefore:

From equations (12) and (13), we can conclude:

$$V_{gs_1} = \sqrt{\frac{2(I_{C_1} + \Delta I)}{K}} + V_T \tag{18}$$

$$V_{gs2} = \sqrt{\frac{2(I_{C1} - \Delta I)}{K}} + V_T \tag{19}$$

$$V_{Z_2} - V_{Z_1} = V_{gs_2} - V_{gs_1} = \tag{20}$$

$$\sqrt{\frac{2(I_{C_1} + \Delta I)}{K}} + V_T - \sqrt{\frac{2(I_{C_2} + \Delta I)}{K}} + V_T =$$

$$\sqrt{\frac{2}{K}}\left(\sqrt{I_{C_2} + \Delta I} - \sqrt{I_{C_1} + \Delta I}\right) =$$

$$\sqrt{\frac{2I_{C_1}}{K}}\left(\sqrt{1 + \frac{\Delta I}{I_{C_1}}} - \sqrt{1 - \frac{\Delta I}{I_{C_1}}}\right)$$

Supposing that $\Delta I / I_{C1} << 1$ And $\Delta I << I_{C1}$;

$$V_{Z_2} - V_{Z_1} = \sqrt{\frac{2I_{C_1}}{K}}\left(\sqrt{1 + \frac{\Delta I}{2I_{C_1}}} - \sqrt{1 - \frac{\Delta I}{2I_{C_1}}}\right) = \sqrt{\frac{2}{K\, I_{C_1}}}\Delta I \tag{21}$$

By placement of (21) in (15), we have:

$$V_1 - V_2 = \Delta I + R_{eq}\Delta I \sqrt{\frac{2}{K\, I_{C_1}}} \tag{22}$$

Hence:

$$\Delta I = \frac{1}{\sqrt{\frac{2}{K I_{C_1}}} + R_{eq}} \tag{23}$$

$$V_{Z_2} - V_{Z_1} = \frac{\sqrt{\frac{2}{K I_{C_1}}}}{\sqrt{\frac{2}{K I_{C_1}}} + R_{eq}} \times V_1 - V_2 \tag{24}$$

For obtaining the output differential current, *i.e.* $I_o = I_{D3} - I_{D4}$, we also know that $I_{D4} + I_{D3} = I_{C2}$ and by writing KVL, we have:

$$I_0 = I_{D_3} - I_{D_4} = \frac{K}{2}\Delta V_Z \sqrt{\frac{4I_{C2}}{K} - \Delta V_Z^2} \tag{25}$$

$$\Delta V_Z \leq \sqrt{\frac{4IC_2}{K}}$$

$$\longrightarrow \quad = \frac{K}{2}\Delta V_Z \sqrt{\frac{4IC_2}{K}} I_0 = I_{D_3} - I_{D_4}$$

By placing ΔV_Z in the output current equation, we obtain the following:

$$I_0 = \frac{IC_2}{\sqrt{\frac{IC_2}{K}} + \frac{I_{C_1} R_{eq}}{2}\sqrt{\frac{2IC_2}{I_{C_1}}}} \times (V_1 - V_2)$$

The proposed HD_3 ،OTA can be roughly obtained by analyzing the Tailor series:

$$HD_3 = \frac{1}{16}\frac{1}{\left[R_{eq} + \sqrt{\frac{2}{K I_{C_1}}}\right]^2 I_{C_1} I_{C_2}}(V_1 - V_2)^2 \tag{26}$$

If we consider the channel length modulation, the harmonic distortion is obtained as follows:

$$HD_3 = \frac{1}{16}\frac{1}{\left[R_{eq}+\sqrt{\frac{2}{K' I_{C_1}(1+\lambda V_{DS})}}\right]^2 I_{C_1} I_{C_2}}(V_1 - V_2)^2 \qquad (27)$$

4. SIMULATION RESULTS

The proposed OTA in figure 3 is simulated using HSPICE software.

In Table 1, the values of OTA parameters designed for simulation are given. For obtaining the third-order harmonic distortion through simulation, different domains and resistances of 1K to 5K are applied, and the harmonic distortion is measured in each resistor. In figures (4) - (6), the theorized and simulated harmonic distortion for designed OTA is depicted. Comparison between simulation results and the theory shows the appropriate accuracy of the proposed analysis. As the analysis predicts, the third-order harmonic distortion decreases with increased resistance. Harmonic distortion values for various domains and resistances are given in table (2) – (4).

Table 1. Proposed OTA parameters

Unit	Simulation	Parameters
V	1.8	Voltage source
µ	0.18	Technology
mA	300	IC
mV	120	V_{eff}
µM	65/0.5	$(W/L)_{1,2}$

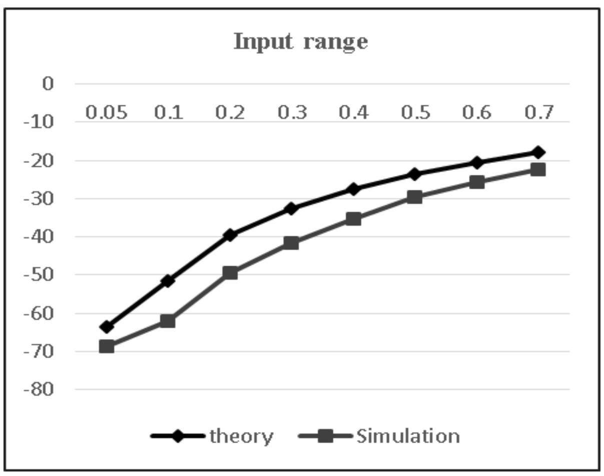

Fig. 4. simulation and theory HD_3 for 1k resistance and different domains

Table 2. Parameter values for R = 1K and $G_m = 3\times 10^{-4}$

V_i	a_1	a_3	HD_3	
			Theory	Simulation
0.1	32.09u	24.08n	-51.96	-62.49
0.3	94.31u	764.40n	-32.86	-41.82
0.5	148.55u	4.67u	-23.98	-30.05
0.7	187.37u	13.78u	-18.14	-22.66

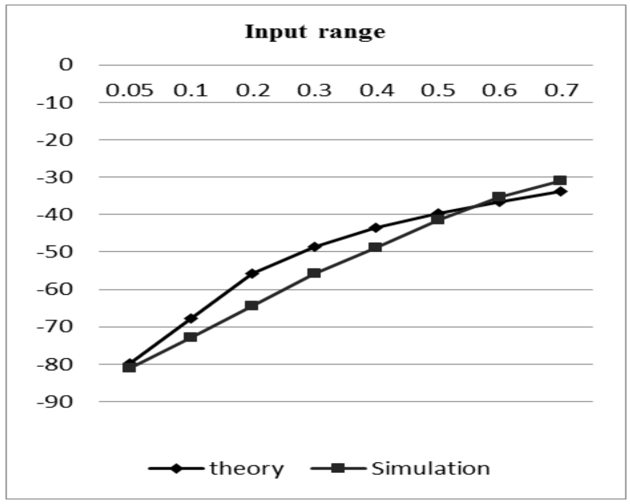

Fig. 5. simulation and theory HD_3 for 3k resistance and different domains

Table 3. Parameter values for R = 3K and $G_m = 1.3\times 10^{-4}$

V_i	a_1	a_3	HD_3	
			Theory	Simulation
0.1	13.58u	3.07n	-67.70	-72.91
0.3	40.60u	66.73n	-48.62	-55.68
0.5	66.71u	558.16n	-39.74	-41.54
0.7	89.56u	2.52u	-33.90	-31.01

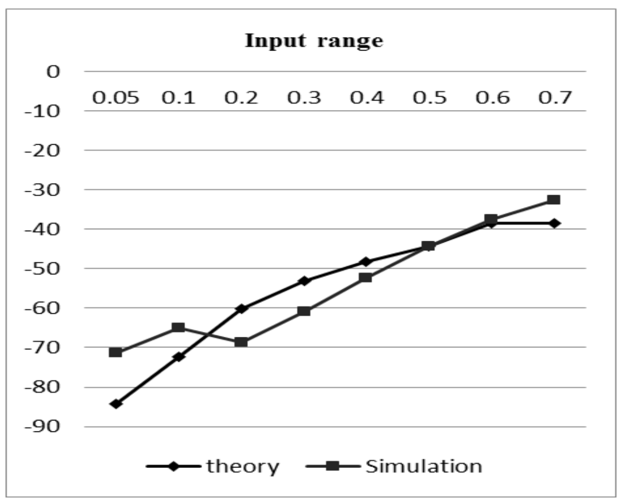

Fig. 6. simulation and theory HD_3 for 5k resistance and different domains

Table 4. Parameter values for R = 5K and G_m =9× 10^{-5}

V_i	a_1	a_3	HD_3	
			Theory	Simulation
0.1	8.81u	4.96n	-75.83	-64.98
0.3	26.41u	19.35n	-56.74	-62.70
0.5	43.67u	209.70n	-47.87	-46.37
0.7	59.33u	1.19u	-42.03	-33.95

5. CONCLUSION

One of the simplest methods of linearization is to use source degeneration resistance. In this paper, the third order harmonic distortion is given using analysis of the Taylor series to give a precise analysis relation for it. Simulation is done using different values of the circuit parameters and HSPICE software and is given in various curves. Effort is made to provide a proper mathematical relation for calculating HD_3 such that designers can use it for achieving an optimum characteristic.

REFERENCES

[1] Farzan Rezaei and Seyed Javad Azhari, ″**A High Linear Operational Transconductance Amplifier (OTA) with High Common Mode Rejection Ratio**″, International Conference on Signal Acquisition and Processing _ IEEE Computer Society ,2010, 18-22.

[2] R. L. Geiger and E. Sanchez-Sinencio, ″**Active filter design using operational transconductance amplifiers: A tutorial** ″, IEEE Circuit and Device Magazine, vol1, pp. 20-32, Mar. 1985.

[3] X.Zhu and Y.Sun, ″ **Low-Distortion Low-Voltage Operational Transconductance Amplifier** ″, Electronics Leters ,No 25,2008.

[4] Arun Ramachandran, ″ **Nonlinearity and Noise Modeling of Operational Transconductance Amplifiers for Continuous Time Analoge Filtar**″**,** A thesis submitted to the Office of Graduate Studies of Texas A&M University, May 2005.

[5] Achim Gartez, ″**Operational Transconduction Amplifiers** ″**,**1-5, july 6,2008.

[6] B. Razavi, ″ **Design of Integrated Circuit for Optical Communications**″**,** First Ed., New York: McGraw Hill Co., 200.3.

[7] D. Johns and K. W. Martin,″**Analog Integrated Circuit Design** ″**,** John Wiley and Sons Canada, Limited, 1996.

[8] Pietro Monsurro and Salvatore Pennisi, ″**Linearization Technique for Source Degenerated CMOS Diffrential Transconductors**, ″ IEEE Transactions on Circuits and Systems, No10, 848-852,2007 **.**

[9] Ko- Chi Kuo and Adrian Leuciuc, ″ **A linear MOS Transconductor Using source degeneration and Adaptive Biasing**″ ,IEEE Transactions on Circuits and Systems, No.10 , 937-943, 2001.

[10] Artur Lewinski and Jose Silva- Martinez**,** ″ **A High Frequency Transconductor Using a Robust Nonlinearity Cancellation**″, IEEE Transactions on Circuits and Systems, No9 ,896-900,2006.

[11] Tuan Vu Cao, Dag T. Wisland, Tor Sverre Lande and Farshad Moradi,″ **Low-Power, Enhanced-Gain Adaptive-Biasing-based Operational Transconductance Amplifiers** ″**,** IEEE Nanoelectronics group, Department of Informatics University of Oslo ,2009.

[12] YouZheng, ″**Operational Transconductance Amplifier for Gigahertz Application**″**,** A thesis submitted to the Department of Electrical and Computer Engineering**,** Queen's University, September, 2008.

[13] Tien.Yulo and Chung Hung, ″**A Wide Tuning Range Gm-C Continuous-Time Analog Filter**″, IEEE Transactions on Circuits and Systems, No9 ,896-900, 2006.