Evaluation of Critical Buckling Load in FG Plate using Analytical and Finite Elements Methods

Hossein Ahmadi Rashid, Amir Ghiasvand *
Department of Mechanical Engineering,
University of Tabriz, Tabriz, Iran
E-mail: Hossein.ahmadi@Tabrizu.ac.ir, Amir.Ghiasvand@Tabrizu.ac.ir
*Corresponding author

Maziar Mahdipour Jalilian
Department of Mechanical Engineering,
Kermanshah Branch, Islamic Azad University, Kermanshah, Iran
E-mail: Maziar.1986.2000@Gmail.com

Mahdi Kazemi
Assistant Professor of Mechanical Engineering,
Malayer University, Malayer, Iran
E-mail: Kazemii.m@Gmail.com

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Abstract: In this paper, analytical and finite element solutions of mechanical buckling of a thick Functionally Graded (FG) plate have been investigated. Boundary conditions have been assumed as simply supported at all edges and three different loadings have been applied. In analytical section the procedure of developing the critical buckling force by third order shear theory has been presented and then the stability Equations have been reduced from 5 to 2. In continue, the problem has been solved using numerical simulation by ABAQUS. To validate the FEM, results have been compared and validated with analytical solution. The results show that the bi-axial compression loading case with the loading ratio of R to one and R to zero are the most possible and most unlikely case in buckling occurrence, respectively.

Keywords: Buckling, Finite Elements Analysis, Functionally Graded Materials (FGM), Third Order Shear Theory

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Biographical notes: Hossein Ahmadi Rashid received his MSc in Mechanical Engineering at University of Tabriz in 2012. **Amir Ghiasvand** received his PhD in Mechanical Engineering from University of Tabriz in 2020. His current research interest includes Welding, Fatigue and Fracture mechanics. **Maziar Mahdipour Jalilian** received his PhD in Mechanical Engineering from Razi University in 2018. He is Assistance Professor of Mechanical Engineering at the Kermanshah Branch, Islamic Azad University, Iran. He published various papers in referred journals. His area of research is Welding and Optimization. **Mahdi Kazemi** received his PhD in Mechanical Engineering from Buali Sina University in 2017.

1 INTRODUCTION

Functionally Graded Materials (FGM), due to their great properties are used in advanced engineering applications. High strength, corrosion, thermal resistance, hardness and low weight are some of their properties [1]. These inhomogeneous materials of made of isotropic matters and their properties are changed in one or more directions which cause unique and interstitial properties. Ceramic-Metal FG materials have both corrosion and abrasion resistance properties like ceramics and ductility like metals [2]. FG plates can be used in aerospace applications. Nowadays the demand for using FGMs are extremely increased in various industries, e.g. the structures facing buckling and post-buckling [3].

FGM can be represented as a non-homogenous material which its mechanical properties vary continuously along the thickness direction from top one surface to the bottom surface. This is achieved by varying the volume fraction of the constituents. FGM are typically designed for a specific function or application. Most of the times they are manufactured to achieve good strength to weight ratios and good thermal or electrical conductivity [4].

The buckling behaviour of rectangular FGM plates subjected to compressive loads has attracted the attention of many researchers working on structural analysis and design. Many studies have been performed on buckling and its critical force under different mechanical and thermal loadings. Hashemi et al. has studied an exact solution for the buckling of isotropic rectangular Mindlin plates. They considered a combination of six different boundary conditions in which two opposite edges are simply supported. Monoaxial in-plane compressive loads on both directions were considered as well as equal biaxial compressive loads. They presented the non-dimensional critical buckling loads and mode shapes for the six cases analysed [5]. Featherston and Watson investigated the behaviour of a number of optimised fibre composite plates of differing geometry, simply supported along two edges and built in along the other two. In their analysis, they analysed a varying combination of shear and in-plane bending, for which no theoretical solution exists, and assessed the suitability of analytical techniques and finite element analysis to predict this behaviour [6].

Piscopo investigated the Shimpi theory for buckling analysis of thick rectangular plates and taking into account the shear deformations. The finite element method has long been recognized as one of the most effective numerical methods for analysing the buckling load of thin plate like structures under arbitrary loading and boundary conditions. Chin et al. presented a finite element method using thin-plate elements. This method was capable of predicting the buckling capacity of arbitrarily shaped thin-walled structural members under any general load and boundary conditions [7]. Mohammadi, et al. obtained an exact solution for the buckling analysis of thin functionally graded rectangular plates. Their work is based on the classical plate theory and using the principle of minimum total potential energy, the equilibrium Equations are obtained [8]. Javaheri and Eslami studied the thermal buckling of FG plates using shell and plates theory and energy principals [9]. Applying high order displacement field and five stability Equations, they found an acceptable solution. Najafizadeh et al. investigated the buckling of an almost thick FG plate using first order shear deformation theory [10]. They derived a stability Equation and found that by solving this Equation, one can estimate the experimental critical buckling force. Ma and Wang used the third order shear deformation theory to solve the axisymmetric bending problem of a circular FG plate [11]. Kim and Na used Finite Element Method (FEM) to investigate the buckling response of FG plate [12]. Han et al studied the mechanical and thermal buckling of a FGM cylindrical shell analytically and numerically [13]. In this paper, analytical and finite element solutions of mechanical buckling of a thick FG plate have been investigated. The novelty of the present work is to obtain closed-form solutions for the buckling loads based on the third order shear deformation theory and FEM approach. First, the procedure of developing the critical buckling force by third order shear theory has been presented and then the stability Equations have been reduced from 5 to 2. In continue, the problem has been solved using numerical simulation by ABAQUS. At the end, to validate the FEM, results have been compared and validated with analytical solution.

2 GENERAL EQUATIONS DEVELOPMENT

Consider a FG plate with the planar dimension of ‘a’ and ‘b’, and ‘h’ in thickness which the properties vary through the thickness direction (-h/2, h/2) due to following relations:

$$P = P_m - P_{cm}\left(\frac{2z+h}{2h}\right) \quad (1)$$

$$P_{cm} = P_c - P_m \quad (2)$$

In “Eq. (1)” P denotes the properties like elastic modulus, density and so on. The subscripts m and c mention the metal and ceramic, respectively. The Poisson’s ratio is also assumed to be constant through the thickness. Third order shear deformation theory is used as:

$$u(x,y,z) = u_0 + zu_1 - \frac{4z^3}{3h^2}\left(u_1 + w_{0,x}\right) \quad (3)$$

$$v(x,y,z)=v_0+zv_1-\frac{4z^3}{3h^2}\left(v_1+w_{0,y}\right) \quad (4)$$

$$w(x,y,z)=w_0 \quad (5)$$

Where u, v and w are displacements along x, y and z directions, respectively. Also u_0, v_0 and w_0 are the mid-plane displacements and, u_1 and v_1 are the rotations about x and y axes, respectively. The fundamental relations of a FG plate due to third order shear deformation theory are as follow:

$$\left(N_x,M_x,P_x\right)=\frac{1}{1-\upsilon^2}[\left(E_1,E_2,E_4\right)\left(\varepsilon_x^0+\upsilon\varepsilon_y^0\right) +\left(E_2,E_3,E_5\right)\left(k_x^0+\upsilon k_y^0\right)+\left(E_4,E_5,E_7\right)\left(k_x^2+\vartheta k_y^2\right)] \quad (6)$$

$$\left(N_y,M_y,P_y\right)=\frac{1}{1-\upsilon^2}[\left(E_1,E_2,E_4\right)\left(\varepsilon_y^0+\upsilon\varepsilon^0\right) +\left(E_2,E_3,E_5\right)\left(k_y^0+\upsilon k_x^0\right)+\left(E_4,E_5,E_7\right)\left(k_y^2+\vartheta k_x^2\right)] \quad (7)$$

$$\left(N_{xy},M_{xy},P_{xy}\right)=\frac{1}{2(1+\upsilon)}[\left(E_1,E_2,E_4\right)\gamma_{xy}^0 +\left(E_2,E_3,E_5\right)k_{xy}^0+\left(E_4,E_5,E_7\right)k_{xy}^2] \quad (8)$$

$$\left(Q_x,R_x\right)=\frac{1}{2\left(1+\upsilon\right)}\left[\left(E_1,E_3\right)\gamma_{xz}^0+\left(E_3,E_5\right)k_{xz}^1\right] \quad (9)$$

$$\left(Q_y,R_y\right)=\frac{1}{2\left(1+\upsilon\right)}\left[\left(E_1,E_3\right)\gamma_{yz}^0+\left(E_3,E_5\right)k_{yz}^1\right] \quad (10)$$

Where:

$$\begin{pmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{pmatrix}=\begin{pmatrix}u_{0,x}+\frac{1}{2}w_{0,x}^2\\ u_{0,y}+\frac{1}{2}w_{0,y}^2\\ u_{0,y}+v_{0,x}+w_{0,x}w_{0,y}\end{pmatrix} \quad (11)$$

$$\begin{pmatrix}\gamma_{xz}^0\\ \gamma_{yz}^0\end{pmatrix}=\begin{pmatrix}u_1+w_{0,x}\\ v_1+w_{0,y}\end{pmatrix} \quad (12)$$

$$\begin{pmatrix}k_x^0\\ k_y^0\\ k_{xy}^0\end{pmatrix}=\begin{pmatrix}u_{1,x}\\ v_{1,y}\\ u_{1,y}+v_{1,x}\end{pmatrix} \quad (13)$$

$$\begin{pmatrix}k_{xz}^1\\ k_{yz}^1\end{pmatrix}=\begin{pmatrix}\frac{-4}{h^2}\left(u_1+w_{0,x}\right)\\ \frac{-4}{h^2}\left(v_1+w_{0,y}\right)\end{pmatrix} \quad (14)$$

$$\begin{pmatrix}k_x^2\\ k_y^2\\ k_{xy}^2\end{pmatrix}=\begin{pmatrix}\frac{-4}{3h^2}\left(u_{1,x}+w_{0,xx}\right)\\ \frac{-4}{3h^2}\left(v_{1,y}+w_{0,yy}\right)\\ \frac{-4}{3h^2}\left(u_{1,y}+v_{1,x}+2w_{0,xy}\right)\end{pmatrix} \quad (15)$$

And:

$$E_1=E_mh+\frac{E_{cm}h}{2} \quad (16)$$

$$E_2=\frac{E_{cm}h^2}{12} \quad (17)$$

$$E_3=\frac{E_mh^3}{12}+\frac{E_{cm}h^3}{24} \quad (18)$$

$$E_4=\frac{E_{cm}h^4}{80} \quad (19)$$

$$E_5=\frac{E_mh^5}{80}+\frac{E_{cm}h^5}{160} \quad (20)$$

$$E_6=\frac{E_{cm}h^6}{448} \quad (21)$$

$$E_7=\frac{E_mh^7}{448}+\frac{E_{cm}h^7}{896} \quad (22)$$

In "Eq. (96 to 10)", N_i، M_i، P_i، Q_i and R_i are the components of stress and are calculated as:

$$\left(N_i,M_i,P_i\right)=\int_{-\frac{h}{2}}^{\frac{h}{2}}\left(\sigma_i\left(1,z,z^3\right)dz\right),\mathrm{i}=\mathrm{x,y,xy} \quad (23)$$

$$\left(Q_i,R_i\right)=\int_{-\frac{h}{2}}^{\frac{h}{2}}\left(\sigma_{iz}\left(1,z^2\right)dz\right),\mathrm{i}=\mathrm{x,y} \quad (24)$$

In addition, the equilibrium Equation of a FG plate based on the third order shear deformation theory is a system of five Equations as follows:

$$N_{x,x}+N_{xy.y}=0 \tag{25}$$

$$N_{xy,x}+N_{y,y}=0 \tag{26}$$

$$Q_{x,x}+Q_{y,y}-\frac{4}{h^2}\left(R_{x,x}+R_{y,y}\right)+\frac{4}{3h^2}\left(P_{x,xx}+2P_{xy,xy}+P_{y,yy}\right) +N_xw_{,xx}+2N_{xy}w_{,xy}+N_yw_{,yy}+N_yw_{,yy}=0 \tag{27}$$

$$M_{x,x}+M_{xy,y}-Q_x+\frac{4}{h^2}R_x-\frac{4}{3h^2}\left(p_{x,x}+P_{xy,y}\right)=0 \tag{28}$$

$$M_{y,y}+M_{xy,x}-Q_y+\frac{4}{h^2}R_y-\frac{4}{3h^2}\left(p_{y,y}+P_{xy,x}\right)=0 \tag{29}$$

Using fundamental Equations related to third order shear deformation theory, the equilibrium Equation reduces to:

$$D\nabla^2\left(u_{1,x}+v_{1,y}\right)-\frac{4}{3h^2}B\nabla^4w+N_xw_{,xx} +2N_xw_{,xx}+2N_{xy}w_{,xy}+N_yw_{,yy}=0 \tag{30}$$

$$\frac{4}{3h^2}\left[E\nabla^2\left(u_{1,x}+v_{1,y}\right)-\frac{4}{3h^2}c\nabla^4w\right]+F\left(u_{1,x}+v_{1,y}\right) +F\left(w_{,xx}+w_{,yy}\right)+N_xw_{,xx}+2N_{xy}w_{,xy}+N_yw_{,yy}=0 \tag{31}$$

Where:

$$A=\frac{E_1E_3-E_2^2}{\left(1-\upsilon^2\right)E_1} \tag{32}$$

$$\mathrm{B}=\frac{E_1E_5-E_2E_4}{\left(1-\upsilon^2\right)E_1} \tag{33}$$

$$C=\frac{E_1E_7-E_4^2}{\left(1-\upsilon^2\right)E_1} \tag{34}$$

$$D=A-\frac{4}{3h^2}B, E=B-\frac{4}{3h^2}C \tag{35}$$

$$F=\frac{1}{2(1+\upsilon)}\left(E_1-\frac{8}{h^2}E_3+\frac{16}{h^4}E_5\right) \tag{36}$$

The equilibrium Equation can be derived according to Brash et al research [13]. In this research it was assumed that equilibrium of plate can be expressed by u_0, v_0 and w_0. The displacement vector of a point in vicinity of equilibrium state is shown by u^1, v^1 and w^1. So, the total displacement of this point is:

$$N_x=N_x^0+N_x^1 \tag{37}$$

$$N_y=N_y^0+N_y^1 \tag{38}$$

$$N_{xy}=N_{xy}^0+N_{xy}^1 \tag{39}$$

Where, N_x^1, N_y^1 and N_{xy}^1 represent the linear force growth corresponding to u^1, v^1 and w^1, respectively. By substitution of presented relations in equilibrium Equation, the stability Equations are achieved. After that the terms containing superscript 1, are reduced from the Equations due to satisfaction of equilibrium Equation. Also, nonlinear terms containing superscript 1, are reduced from the Equations because of negligibility in value. The final equilibrium Equation is written as:

$$D\nabla^2\left(u_{1,x}^1+v_{1,y}^1\right)-\frac{4}{3h^2}B\nabla^4w^1 +N_x^0w_{,xx}^1+2N_{xy}^0w_{,xy}^1+N_y^0w_{,yy}^1=0 \tag{40}$$

$$\frac{4}{3h^2}\left[E\nabla^2\left(u_{1,x}^1+v_{1,y}^1\right)-\frac{4}{3h^2}c\nabla^4w^1\right]+F\left(u_{1,x}^1+v_{1,y}^1\right) +F\left(w_{,xx}^1+w_{,yy}^1\right)+N_x^0w_{,xx}^1+2N_{xy}^0w_{,xy}^1+N_y^0w_{,yy}^1=0 \tag{41}$$

In these Equations, the superscripts, 1 and 0 denote the stability and equilibrium, respectively.

3 MECHANICAL LOADING CASE

A simply supported plate has been considered which F_x and F_y uniform forces are exerted on x= 0, a and y=0, b, respectively. The resultant pre-buckling components are:

$$N_x^0=-\frac{F_x}{b} \tag{43}$$

$$N_y^0=-\frac{F_y}{a} \tag{44}$$

$$N_{xy}^0=0 \tag{45}$$

Substitution of these components in equilibrium Equation, results in formation of two differential Equation with incremental variables, w^1 and $\left(u_{1,x}^1+v_{1,y}^1\right)$.

To solve this system of Equation, one can use the following approximate solution.

$$u_1^1 = u_{mn} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \quad (46)$$

$$v_1^1 = v_{mn} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \quad (47)$$

$$w_1^1 = w_{mn} sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \quad (48)$$

Finally, the critical buckling load can be calculated:

$$F_c = \frac{bG_{mn}^2 \left[\frac{D}{Q_{mn}} \left(F + \frac{16}{9h^4} CG_{mn} \right) + \frac{4}{3h^2} B \right]}{G_{mn}^R \left(1 + \frac{DG_{mn}}{Q_{mn}} \right)} \quad (49)$$

Where:

$$G_{mn} = \left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 \quad (50)$$

$$G_{mn}^R = \left(\frac{m\pi}{a} \right)^2 + R\left(\frac{n\pi}{b} \right)^2 \quad (51)$$

$$Q_{mn} = F - \frac{4}{3h^2} EG_{mn} \quad (52)$$

$$R = \frac{F_y b}{F_x a} \quad (53)$$

R is a dimensionless parameter which describes the loading situation. In the case of bi-axial compression, x-axis compression and compression-tension bi-axial loading, R is assumed as positive, zero and negative, respectively.

4 NUMERICAL SIMULATION

The ABAQUS software is a powerful tool in solving structural and non-structural problems in buckling and post-buckling cases. Using "*Lanczos*" and "*Subspase*" solvers, ABAQUS can predict mode shapes and critical buckling. In this paper, numerical buckling of FG plate in various thickness to width ratios and loadings has been investigated and the results have been validated with analytical Equations. In this manner, FG plates with planar dimension of a=1 and b=0.5, in twenty different thickness to width ratios (h/b) starting form 0.01 with the step of 0.2, have been considered which are shown in "Fig. 1".

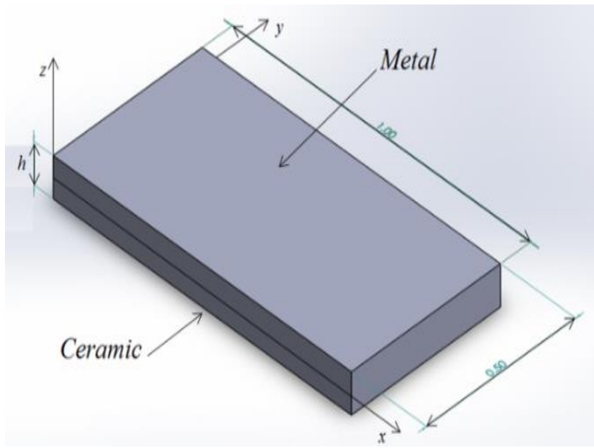

Fig. 1 Considered FG plate.

The FG plate has been considered as a composition of Aluminum and Alumina with linear properties variation through the thickness. The properties of Aluminum and Alumina are illustrated in "Table 1".

Table 1 The properties of Aluminum and Alumina

Material	State	E (GPa)	ϑ
Aluminum	Metal	70	0.3
Alumina	Ceramic	380	0.3

In the properties module of ABAQUS software, one cannot define the FG material. So, the USDFLD subroutine has been applied to define the FG plate properties. In this manner, first the properties were coded in MATLAB software for twenty different thicknesses to width ratios and introduced to ABAQUS as field variables.

In the solution, the "Lanczos" was used and buckling forces and the first three mode shapes were derived from software. The longitudinal boundary condition is assumed to be simply supported. In the first case, a 1 newton compressive uni-axial load (introduced by R=0) was exerted on longitudinally. After validation with analytical relations, R was considered as 1 and -1, and results were obtained. The shell element S4R with seeding size of 2 mm was applied for all models. It should be mentioned that due to 20 thickness ratio and 3 different R, 60 samples were modeled and the results were obtained.

5 RESULTS AND DISCUSSION

To validate the models, numerical solution was compared with the calculated critical buckling load in a FG plate, the results of uni-axial compression loading with R=0 combined with TSDT and CPT were used (Eq.14). The critical buckling load vs. h/b for R=0, is shown in “Fig. 2”.

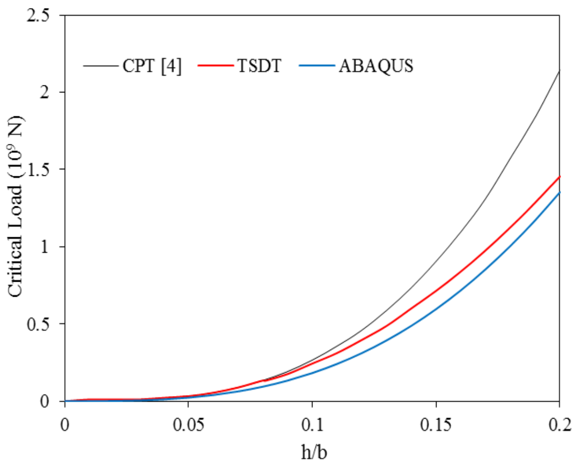

Fig. 2 Critical buckling load obtained from CPT, TSTD and numerical approach using USDFLD in ABAQUS software.

The results obtained from three approaches with R=0, have similar response against h/b. By increasing the R, critical buckling load increases because of increment in thickness of plate and this increment is more intense in higher h/b values. Based on the obtained results and presented by other researchers [9], using CPT results in predicting the higher critical buckling value in FG plates compared with real values. FSDT and TSDT result in more accurate prediction compared with CPT.

Based on “Fig. 2”, the results obtained from numerical solution and analytical solution (TSDT) are in good agreement, and both results are better than CPT [4] results due to realistic conditions, which demonstrate that numerical solution by USDFLD subroutine in ABAQUS software can predict the critical buckling load in FG plates in different boundary conditions accurately.

Three first mode shapes contours of FG plate with h/b=0.05 under buckling are shown in “Fig. 3”.

After validation of the results, critical buckling load and first, second and third mode shapes of plate with different R=0, 1 and -1 have been studied. Critical buckling load curve in first, second and third mode at various R values and h/b are illustrated in “Fig. 4, 5 and 6”, respectively.

According to “Fig. 4 to 6”, by increasing the h/b, increment in critical buckling load in first, second and third modes occurs which is more intense in higher h/b values. In addition, it has been illustrated that critical buckling load increases by increasing the mode numbers. This increment is more intense at R=0 compared with other values of R.

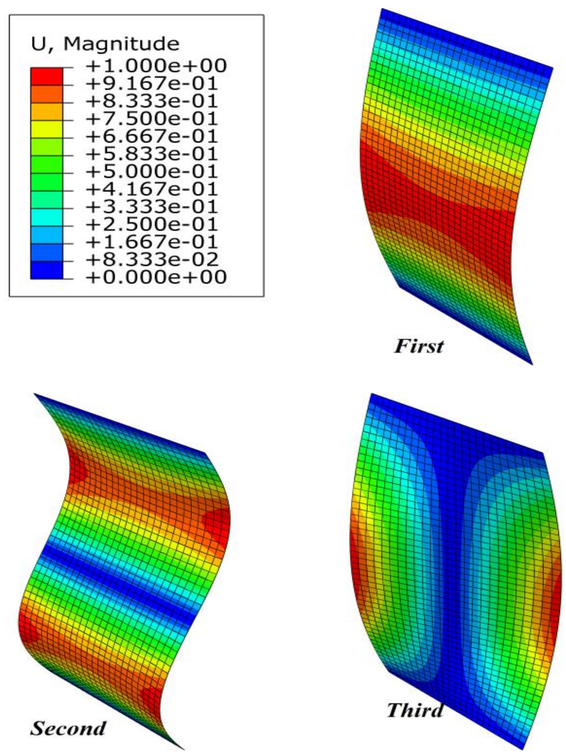

Fig. 3 Three first mode shapes contours of FG plate with h/b=0.05 and R=0.

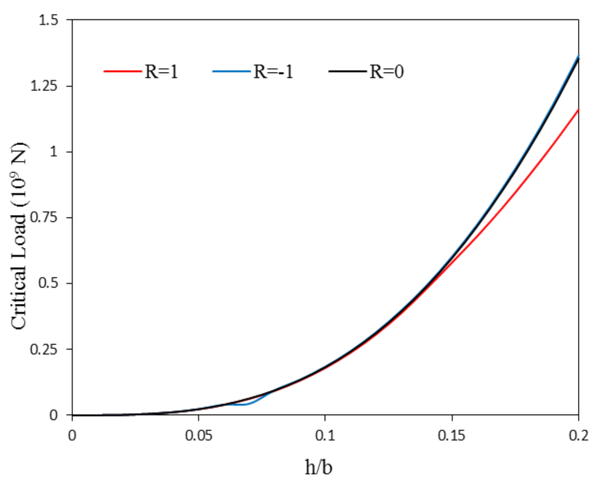

Fig. 4 Critical buckling load in first mode.

According to “Fig. 4”, the conditions R=0 and R= -1 coincide which results are same in calculation of critical buckling force in first mode. This outcome can be seen in Eslami and Javaheri’s paper [9], too. Critical buckling force at higher h/b, in R=1 shows 10% decrement compared with R=0 and R=-1.

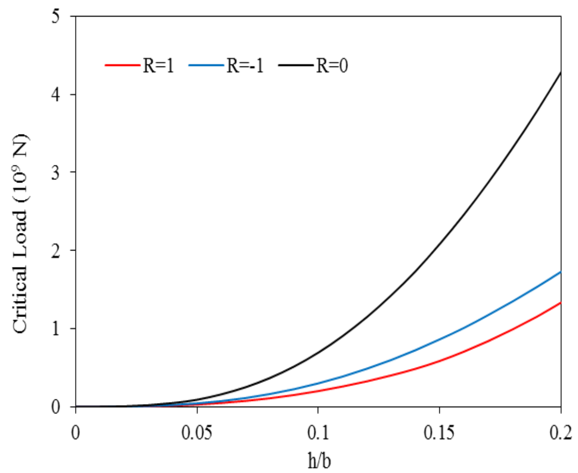

Fig. 5 Critical buckling load in second mode.

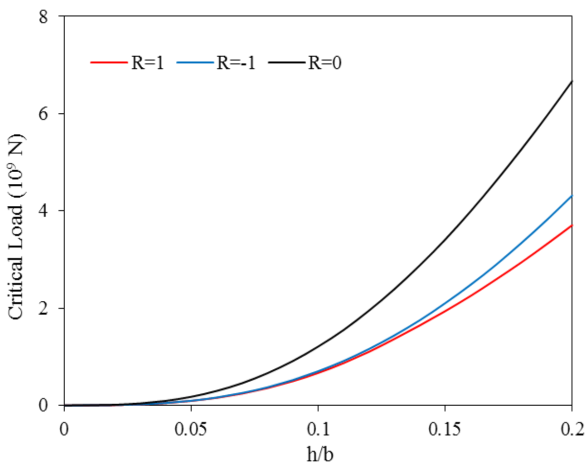

Fig. 6 Critical buckling load in third mode.

According to "Fig. 5 and 6", the critical buckling force curve in second and third modes with R=0, is wider than other values of R and its maximum is 2.5 times higher. Finally based on "Fig. 3 to 5", it was observed that in every buckling mode, the maximum force is related to R=0, R= -1 and R=1, respectively. According to presented Figures, bi-axial loading (R=1) is the most favorable situation for buckling occurrence in FG plate and lower force is required to start buckling. Also, the most resistant situation to start buckling is the situation related to R=0 which buckling occurs at higher force values. Buckling modes displacement contours related to R=1 and R= -1 are shown in "Fig.6 and 7", respectively. According to "Fig. 4, 5 and 8", the shape of first mode of buckling in all loading situation are same and buckling occurs at mid-plane, but second and third modes are different which may be due to bi-axial loading.

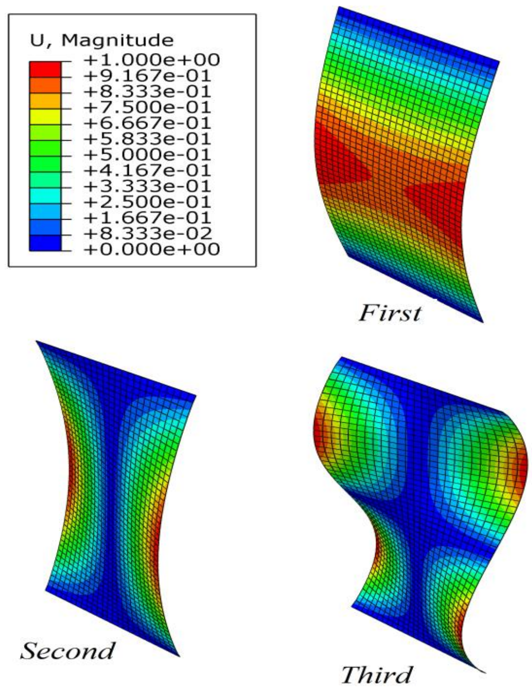

Fig. 7 Displacement contours and three first modes of buckling (R=1).

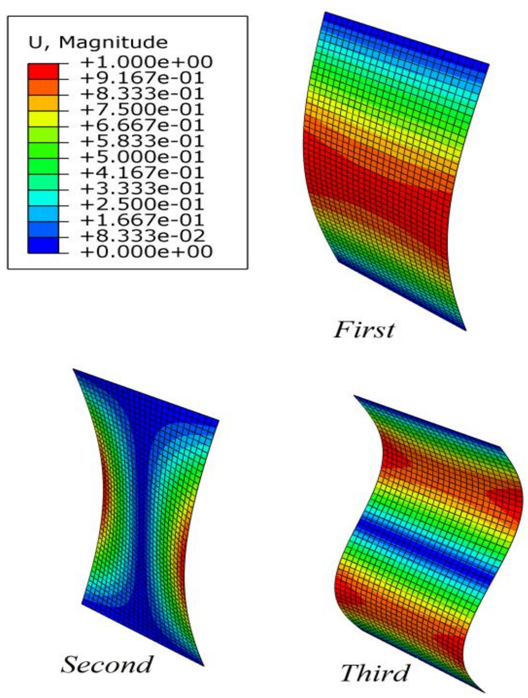

Fig. 8 Displacement contours and three first modes of buckling (R=-1).

6 CONCLUSIONS

In this paper, buckling of FG plate was investigated using analytical and numerical approaches. Analytical relations were used to estimate the critical buckling force. Also, the ABAQUS software was used to estimate the critical buckling force in 60 different positions. Uni-axial compressive (R=0), bi-axial compressive (R=1) and bi-axial compressive-tension loadings (R=-1) were exerted on plate. The width and length of plate were considered as constant value and only thickness of plate was assumed to change in different positions. The effect of thickness change was investigated on critical buckling load and mode shapes and following results were obtained:

- The results obtained from numerical simulation are in good agreement with analytical results using TSDT.
- By increasing the thickness to with ratio (h/b), the critical buckling force increases in all loading cases and this increment is more intense at higher h/b values.
- The highest force growth related to buckling mode change occurs at R=0 and this increment is more intense at higher h/b values.
- The critical buckling force in second and third modes at R=0 situation are significantly higher compared to other loadings and the maximum values are approximately 2.5 times higher.
- Based on force analysis, the highest force value is related to R=0, R=-1 and R=1, for three first modes.
- Based on the results, bi-axial loading (R=1) is the most favorable situation for buckling occurrence in FG plate and lower force is required to start buckling. Also, the most resistant situation to start buckling is the situation related to R=0 which buckling occurs at higher force values.

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