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Function Generation Synthesis of the Four-bar Linkage Based on Four and Five Precision Points using Newton-HCM

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Abstract: The length values selection for a determined type of linkage to achieve the necessary task, dimensional synthesis, is classified into three classes based on the mechanism's task: function generation, path generation, and motion generation. The case considered in this study, Function generation synthesis, aims to create a relation between the angular motions of the input and output links of the mechanism. For this problem, a semi-analytical method called the Newton-HCM is used for numerical solutions, which combines Newton's method with the semi-analytical Homotopy Continuation Method (HCM). Function generation synthesis of a planar four-bar linkage for four and five precision points is the main challenge of the current study, which is highly nonlinear and complicated to solve. Numerical examples of the function generation problem for a four-bar linkage with four and five precision points are presented and authenticate the excellent performance of the proposed algorithm.

Keywords: Function Generation Synthesis, HCM, Newton's Method, Planar Four-Bar Linkage, Precision Points

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Research paper

1 INTRODUCTION

Various analytical and numerical methods are used to solve different engineering problems [1-2]. The solution's convergence and accuracy depend on the problem's type, its degree of non-linearity, and the utilized solution method. Dimensional synthesis of the planar four-bar linkage based on the function generation purpose is the main challenge of the current study. Determining the length values for a determined linkage type to achieve the necessary task is called dimensional synthesis. Dimensional synthesis can be classified into three classes based on the mechanism’s task: function generation, path generation, and motion generation [3]. The purpose of the function generator mechanism is to create a relation between the angular motions of the input and output links. The path generation mechanism moves a particular point of the mechanism along prescribed points. Motion generation is similar to path generation, except that the position and orientation of the rigid body are considered together.

Chebyshev [4] and Freudenstein [5] presented beginning studies on this problem. Chebyshev solved this problem in 1854 for a straight-line path problem with a determined interval for the input link [4], also known as Chebyshev’s fundamental theorem. Moreover, Freudenstein presented a simple position Equation for the four-bar linkage in 1954 [5]. This simplification results in its broad application for finding solutions to several problems, including mechanism analysis and synthesis. Modern approaches in mechanism synthesis problems result from recent improvements in calculations, using powerful calculators, genetic and evolutionary algorithms, and artificial neural networks. Rao [6-7] has optimized a 4bar linkage using Freudenstein’s Equation for three precision points by minimizing the least-squares of error. Sun [8] has used the quadratic interpolation method to decrease the error of the designed four-bar linkage. Chen [9] has performed a study compared to Rao’s [7] by optimizing the function generation four-bar linkage using Mudguardt’s method and improving the error value. Guj et al. [10] proposed an optimization algorithm using the penalty function method to reduce the inertial force in the four-bar mechanism to optimize the high-speed mechanisms. For spatial 4-link mechanisms, Soylemez and Freudenstein [11] have optimized the transmission force for the skew crank-and-rocker linkage and the skew slider-crank mechanism. Gosselin and Angeles [12] proposed an algorithm for minimizing the transmission defect in planar and spherical function generator mechanisms. Angeles et al. [13-14] optimized the coupler curve in the path generator and the motion generator four-bar linkage using the unconstrained nonlinear least square optimization. Shariati and Norouzi [15] used the SQP method to find the optimal mechanism for five precision point syntheses of the four-bar linkages. Moreover, some researchers consider joints clearance in their design process [16-18]. Daniali et al. [16] proposed a new algorithm for simultaneous kinematic and dynamic optimization. Their method reduced the path generation error arising from joints clearance. The possibility, powerfully and simplicity of the evolutionary and genetic algorithms used for solving a wide range of problems, especially mechanism synthesis, are described in publications [18-22]. For instance, Sardashti et al. [18] synthesized the free defect four-bar linkage with clearance joints using the particle swarm optimization method. They considered branch and circuit defects in their design process and designed the mechanism without these defects. Moreover, Penunuri et al. [19] used the differential evaluation method for synthesizing the mechanisms with single and hybrid tasks. Shpli [20] used the GA method for synthesizing the path generator four-bar linkage with maximum mechanical advantage. Bustos et al. [21] used an algorithm with a combination of the finite element method and the genetic algorithm. Cabrera et al. [22] utilized the genetic algorithm for optimal synthesis of the mechanisms, too. Recently, advancements in numerical calculations and the mathematical theory of polynomials led to new solutions called continuation (or homotopy) methods. Wampler [23-25] used this method for kinematic analysis of the mechanisms. He extracted all solutions of a system of algebraic polynomial Equations using numerical continuation. After that, some researchers have used this approach in the mechanism design field [26-29]. Varedi et al. [26] used the homotopy continuation method (HCM) for solving the forward kinematics problem of the 3UPU parallel robot. Tari et al. [27] utilized HCM to exclude the unwanted solutions arising in kinematics problems. Moreover, he used this approach for kinetostatic synthesis of a compliant four-bar mechanism [28]. Furthermore, the HCM method is utilized for the kinematic analysis of the parallel robots, too [29-30].

The current study presents a combination of Newton's method and the HCM algorithm for the numerical solution of the nonlinear Equations arising in function generation synthesis of a planar four-bar linkage based on the four and five precision points. Based on the highly nonlinear nature of these problems, the numerical methods used before have some drawbacks, described before. It is the first time the Newton-HCM algorithm has been utilized for function generation synthesis problems.

2 HOMOTOPY CONTINUATION METHOD (HCM)

Proper initial guesses and convergence possibility are two troublesome points in most numerical methods, including the Newton–Raphson. The Newton-HCM can eliminate these deficiencies [24]; consequently, several researchers have been used this method in the past decades [23-26]. In the Newton-HCM method, firstly, some new functions are written using auxiliary homotopy functions, and then the Newton-Raphson method is used for solving this system of nonlinear Equations.

If a system of nonlinear Equations is considered as:

$$F(X)=0 \quad \text{i.e} \begin{cases} a(x,y,\dots,z)=0, \\ b(x,y,\dots,z)=0, \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{cases} \tag{1}$$

One can solve these Equations using the iterative algorithm of Newton's method as:

$$\begin{bmatrix} \frac{\partial a(x_n,y_n,\dots)}{\partial x} & \frac{\partial a(x_n,y_n,\dots)}{\partial y} & \cdot & \cdot & \cdot \\ \frac{\partial b(x_n,y_n,\dots)}{\partial x} & \frac{\partial b(x_n,y_n,\dots)}{\partial y} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \\ \cdot & \cdot & & \cdot & \\ \cdot & \cdot & & & \cdot \end{bmatrix} \begin{bmatrix} x_{n+1}-x_n \\ y_{n+1}-y_n \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} = \begin{bmatrix} -a(x_n,y_n,\dots) \\ -b(x_n,y_n,\dots) \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \tag{2}$$

HCM approach converts this system of nonlinear Equations to new ones by eliminating some terms and adding auxiliary homotopy functions, which leads to solving these Equations more efficiently. Indeed, Newton's method is used for solving the new system of Equations, which is easier, and its solutions are accessible. The converted system of Equations, called homotopy continuation functions, is as follows [23-25]:

$$H(X,t) \equiv t\,F(X)+(1-t)A(X)=0 \tag{3}$$

Where, $A(X)$ are new simple Equations for initializing the solution process or auxiliary homotopy functions that must be solved easily. Moreover, t is an iteration setting parameter that changes from 0 to 1 and defines two boundary conditions [23-25]:

$$H(X,0)=A(X), \quad H(X,1)=F(X) \tag{4}$$

The above Equation shows that HCM solves $A(X)=0$ at the first iteration (when $t=0$), and solves $F(X)=0$ at the final iteration (when $t=1$). All of these solutions in each iteration are using Newton's method:

$$\begin{bmatrix} \frac{\partial H_1(x_n,y_n,\dots)}{\partial x} & \frac{\partial H_1(x_n,y_n,\dots)}{\partial y} & \cdot & \cdot & \cdot \\ \frac{\partial H_2(x_n,y_n,\dots)}{\partial x} & \frac{\partial H_2(x_n,y_n,\dots)}{\partial y} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \\ \cdot & \cdot & & \cdot & \\ \cdot & \cdot & & & \cdot \end{bmatrix} \begin{bmatrix} x_{n+1}-x_n \\ y_{n+1}-y_n \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} = \begin{bmatrix} -H_1(x_n,y_n,\dots) \\ -H_2(x_n,y_n,\dots) \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \tag{5}$$

Indeed, the HCM iterative algorithm transforms the converted system's solution into the numerical results of the basic system of Equations. The HCM's purpose is to solve the $H(X,t)=0$ instead of $F(X)=0$ by changing the parameter t from 0 to 1. This algorithm leads to avoiding divergence in the numerical solution of the system of nonlinear Equations.

3 PLANAR FOUR-BAR MECHANISM

The simplest closed-loop mechanism is the four-bar linkage, which has four rigid links and four revolute joints. A typical planar four-bar linkage is shown in Fig.1. In this figure, the lengths of the links A_0B_0, A_0A, AB and B_0B are denoted by the angles l_1, l_2, l_3 and l_4, respectively. A_0B_0 is fixed while the two links A_0A and B_0B can only rotate about their respective fixed axes A_0 and B_0. Moreover, their position angles are indicated respectively by ψ and φ. The link connected to the actuator or driving motor is called the input link (A_0A) and B_0B is known as the output link. In "Fig. 1", if the vector x is along the ground, the vectors of A_0A, A_0B_0 and B_0B are:

$$\overrightarrow{A_0A}=\begin{bmatrix} l_2\cos\psi \\ l_2\sin\psi \end{bmatrix}, \quad \overrightarrow{A_0B_0}=\begin{bmatrix} l_1 \\ 0 \end{bmatrix}, \quad \overrightarrow{B_0B}=\begin{bmatrix} l_4\cos\varphi \\ l_4\sin\varphi \end{bmatrix} \tag{6}$$

Therefore, the vector of the coupler link can be obtained as:

$$\overrightarrow{AB} = \overrightarrow{A_0B_0} + \overrightarrow{B_0B} - \overrightarrow{A_0A} = \begin{bmatrix} l_1 + l_4 \cos\varphi - l_2 \cos\psi \\ l_4 \sin\varphi - l_2 \sin\psi \end{bmatrix} \quad (7)$$

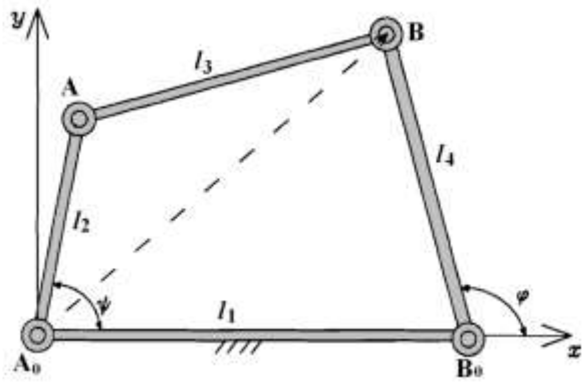


Fig. 1 Planar four-bar mechanism.

The length of the coupler link is designated as l_3. Thus, one can write:

$$l_3^2 = (l_1 + l_4 \cos\varphi - l_2 \cos\psi)^2 + (l_4 \sin\varphi - l_2 \sin\psi)^2 \quad (8)$$

Simplifying "Eq. (8)", Freudenstein obtained a simple scalar Equation [5]:

$$k_1 + k_2 \cos\varphi - k_3 \cos\psi = \cos(\psi - \varphi) \quad (9)$$

Where:

$$k_1 = \frac{l_1^2 + l_2^2 + l_4^2 - l_3^2}{2 l_2 l_4}, \quad k_2 = \frac{l_1}{l_2}, \quad k_3 = \frac{l_1}{l_4} \quad (10)$$

Equation (9) is known as the Freudenstein Equation and is readily applicable to the kinematics analysis of planar four-bar linkage. However, in a planar four-bar mechanism, the functional relationship $\varphi = \varphi(\psi)$ between output and input angles depends on three independent parameters (k_1, k_2, k_3).

If the lengths of the links (l_1, l_2, l_3 and l_4) are known, one can obtain parameters (k_1, k_2, k_3) from "Eq. (11)", and therefore, the output angle φ can be determined in any input angle ψ. This is done utilizing tan-half-angle identities:

$$T = \tan\left(\frac{\varphi}{2}\right), \quad \sin\varphi = \frac{2T}{1+T^2}, \quad \cos\varphi = \frac{1-T^2}{1+T^2} \quad (11)$$

Substituting of the preceding Equation into "Eq. (9)", a quadratic Equation in T is achieved:

$$A(\psi)T^2 + B(\psi)T + C(\psi) = 0 \quad (12)$$

Where:

$$A(\psi) = (k_1 - k_2) + (1 - k_3)\cos\psi, \quad B(\psi) = -2\sin\psi, \quad C(\psi) = (k_1 + k_2) - (1 + k_3)\cos\psi \quad (13)$$

Considering "Eq. (12)", the solution can be found by utilizing the well-known method for the roots of the quadratic Equation.

On the other hand, if the parameters (k_1, k_2, k_3) are known and supposing $l_1 = 1$, one can find the length of the link as:

$$l_2 = \frac{1}{k_2}, \quad l_4 = \frac{1}{k_3}, \quad l_3 = \sqrt{1 + l_2^2 + l_4^2 - \frac{2k_1}{k_2 k_3}} \quad (14)$$

4 FUNCTION GENERATION SYNTHESIS

Function generation synthesis is the problem of obtaining link's lengths, l_1, l_2, l_3 and l_4, for a determined set of input and output angles values, ψ_i and φ_i. For function generation synthesis based on three precision points, "Eq. (9)" can be utilized directly. Given three pairs of the input and output angles (ψ_i, φ_i), i=1,2,3, one can use these angle pairs in "Eq. (9)" to get three linear Equations in k_1, k_2, and k_3:

$$k_1 + k_2 \cos\varphi_i - k_3 \cos\psi_i = \cos(\psi_i - \varphi_i) \quad, i = 1..3 \quad (15)$$

"Eq. (16)" shows this system of linear Equations in matrix form.

$$\begin{bmatrix} 1 & \cos\varphi_1 & -\cos\psi_1 \\ 1 & \cos\varphi_2 & -\cos\psi_2 \\ 1 & \cos\varphi_3 & -\cos\psi_3 \end{bmatrix} \begin{Bmatrix} k_1 \\ k_2 \\ k_3 \end{Bmatrix} = \begin{bmatrix} \cos(\psi_1 - \varphi_1) \\ \cos(\psi_2 - \varphi_2) \\ \cos(\psi_3 - \varphi_3) \end{bmatrix} \quad (16)$$

Once k_1, k_2, and k_3 are resulting from the solution of the linear system of Equations ("Eq. (16)"), the link's lengths can be obtained easily using "Eq. (14)".
It has shown that the function generation synthesis problem for three precision points is easy and straightforward because the system of Equations is linear. However, if precision points are more than three, the system of Equations becomes nonlinear, and therefore, one must use other analytical or numerical solutions for them. For designs with a higher number of precision points, two new variables (ψ_0 and φ_0) indicating the rotation angles from undefined and arbitrary starting points are proposed by Freudenstein.

4.1. Four-Precision Points (4PP)

There are four angle pairs in the 4PP problem, and utilizing "Eq. (9)" for these pairs leads to four Equations. Thus, in this case, ψ_0 is considered as the fourth unknown parameter. Indeed, the input angle ψ, is considered as the sum of two angles, the reference angle ψ_0, and the variable angle $\Delta\psi_i$:

$$\psi_i = \psi_0 + \Delta\psi_i, \quad i = 1..4 \tag{17}$$

Substituting "Eq. (17)" in "Eq. (9)" for four angle pairs are as follows:

$$k_1 + k_2 \cos\varphi_i - k_3 \cos(\psi_i + \psi_0) = \cos(\psi_i + \psi_0 - \varphi_i) \quad , i = 1..4 \tag{18}$$

Therefore, The Equations in "Eq. (18)" are a set of four nonlinear Equations with four unknowns: k_1, k_2, k_3 and ψ_0. This system of nonlinear Equations can be solved using the Newton-HCM.
Based on "Eq. (18)", the homotopy continuation functions can be written as follows:

$$\begin{aligned}
&(k_1 + k_2 \cos\varphi_1 - k_3 \cos(\psi_1 + \psi_0) - \cos(\psi_1 + \psi_0 - \varphi_1)) \times t + (1-t) \times G_1 = 0\\
&(k_1 + k_2 \cos\varphi_2 - k_3 \cos(\psi_2 + \psi_0) - \cos(\psi_2 + \psi_0 - \varphi_2)) \times t + (1-t) \times G_2 = 0\\
&(k_1 + k_2 \cos\varphi_3 - k_3 \cos(\psi_3 + \psi_0) - \cos(\psi_3 + \psi_0 - \varphi_3)) \times t + (1-t) \times G_3 = 0\\
&(k_1 + k_2 \cos\varphi_4 - k_3 \cos(\psi_4 + \psi_0) - \cos(\psi_4 + \psi_0 - \varphi_4)) \times t + (1-t) \times G_4 = 0
\end{aligned} \tag{19}$$

The solution of these Equations can be obtained by changing the auxiliary homotopy functions (G_i) and solved by the Newton-Raphson method.

4.2. Five-Precision Points (5PP)

For the 5PP problem, one can add the other reference angle φ_0 to the Equations ($\varphi_i = \varphi_0 + \Delta\varphi_i$). Therefore, "Eq. (18)" changes to:

$$k_1 + k_2 \cos(\varphi_i + \varphi_0) - k_3 \cos(\psi_i + \psi_0) = \cos(\psi_i + \psi_0 - (\varphi_i + \varphi_0)) \quad , i = 1..5 \tag{20}$$

Following the previous section, we have five unknowns: k_1, k_2, k_3, ψ_0 and φ_0. As a result, "Eq. (20)" is a set of five nonlinear Equations with five unknowns. Therefore, homotopy continuation functions are as follows:

$$\begin{aligned}
&(k_1 + k_2 \cos\varphi_1 - k_3 \cos(\psi_1 + \psi_0) - \cos(\psi_1 + \psi_0 - \varphi_1)) \times t + (1-t) \times G_1 = 0\\
&(k_1 + k_2 \cos\varphi_2 - k_3 \cos(\psi_2 + \psi_0) - \cos(\psi_2 + \psi_0 - \varphi_2)) \times t + (1-t) \times G_2 = 0\\
&(k_1 + k_2 \cos\varphi_3 - k_3 \cos(\psi_3 + \psi_0) - \cos(\psi_3 + \psi_0 - \varphi_3)) \times t + (1-t) \times G_3 = 0\\
&(k_1 + k_2 \cos\varphi_4 - k_3 \cos(\psi_4 + \psi_0) - \cos(\psi_4 + \psi_0 - \varphi_4)) \times t + (1-t) \times G_4 = 0\\
&(k_1 + k_2 \cos\varphi_5 - k_3 \cos(\psi_5 + \psi_0) - \cos(\psi_5 + \psi_0 - \varphi_5)) \times t + (1-t) \times G_5 = 0
\end{aligned} \tag{21}$$

It is worth noting that the solution of the above nonlinear Equations ("Eqs. (19) and (21)") has many answers; however, we choose one of the answers, which satisfies some constraints, including Grashof criteria, free defects, etc.

5 NUMERICAL EXAMPLES

Consider a planar four-bar linkage to produce the precision points of "Table 1". (For the 4PP problem: i=1...4 and for the 5PP problem: i=1...5). Fixed variations for the homotopy parameter t and the initial guesses for the unknown parameters are considered as $\Delta t = 0.0001$ and $[k_{1,0}, k_{2,0}, k_{3,0}, \psi_{0,0}, \varphi_{0,0}] = [1,1,1,1,1]$, respectively. The auxiliary homotopy functions along with the results of the solution of the two problems are given in "Table 2".

Table 1 Desired values for the input and output angles

	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$
ψ_i (deg)	100	123	141	158	188
φ_i (deg)	38.5	61	77	90.5	108

Using “Eq. (14)”, one can obtain lengths of the link as “Table 3”. Moreover, we show the reference angles ψ_0 and φ_0 (in degree) in this table. These values must be added to the values of “Table 1”. The reason is, here, the designed mechanisms show these new values in their input and output angles. These new values are shown in “Table 4”.

Table 2 The auxiliary homotopy functions and their results

Problem	auxiliary homotopy functions (G_i)					Results				
	G_1	G_2	G_3	G_4	G_5	k_1	k_2	k_3	ψ_0	φ_0
4PP	k_1	$-k_2$	$-k_3-1$	$\cos\psi_0$	---	1.0797	−0.5083	−0.4497	5.3558	---
5PP	$-k_1$	$-k_2$	$-k_3$	$-\sin\psi_0$	$-\cos\psi_0$	1.1697	0.9165	0.8222	2.4188	3.3032

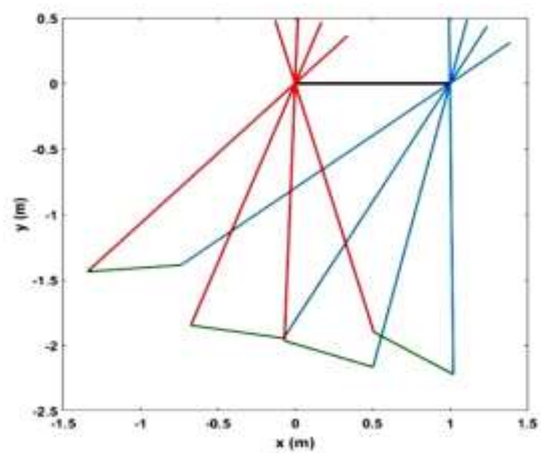
Table 3 Link’s lengths of the designed mechanisms

Problem	l_1	l_2	l_3	l_4	ψ_0 (deg)	φ_0(deg)
4PP	1	−1.9673	0.6072	−2.2236	306.86°	---
5PP	1	1.0911	0.7518	1.2162	138.58°	189.26°

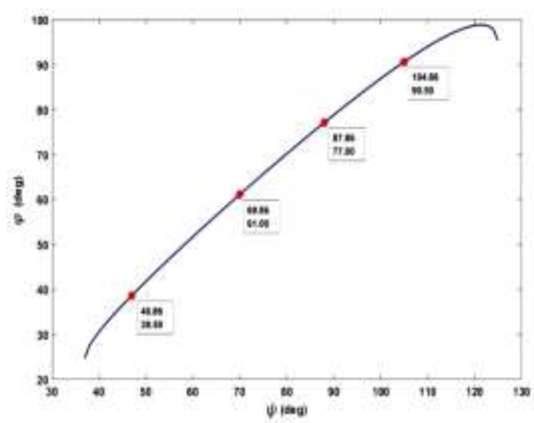
Table 4 New values for input and output angles of the two problems

Problem	Angles	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$
4PP	ψ_i (deg)	46.86	69.86	87.86	104.86	---
	φ_i (deg)	38.50	61.00	77.00	90.50	---
5PP	ψ_i (deg)	238.58	261.58	279.58	296.58	326.58
	φ_i (deg)	227.76	250.26	266.26	279.76	297.26

Figure 2a shows the mechanism designed for the first problem in four positions. Moreover, one can plot the diagram of θ_4 (or φ) with respect to θ_2 (or ψ) for this mechanism (“Fig. 2b”). These figures show that the designed mechanism precisely covers the values of “Table 4”. Furthermore, for a better comparison, “Fig. 3” shows these precision points separately. Similarly, these figures can be plotted for the second problem. Figure 4 shows the mechanism designed for the five-precision point problem. One can find the best matching between “Fig. 4a and Fig. 4b”. Moreover, detailed angles for these five precision points are shown in “Fig. 5”.



(a)



(b)

Fig. 2 The designed mechanism for the 4PP problem.

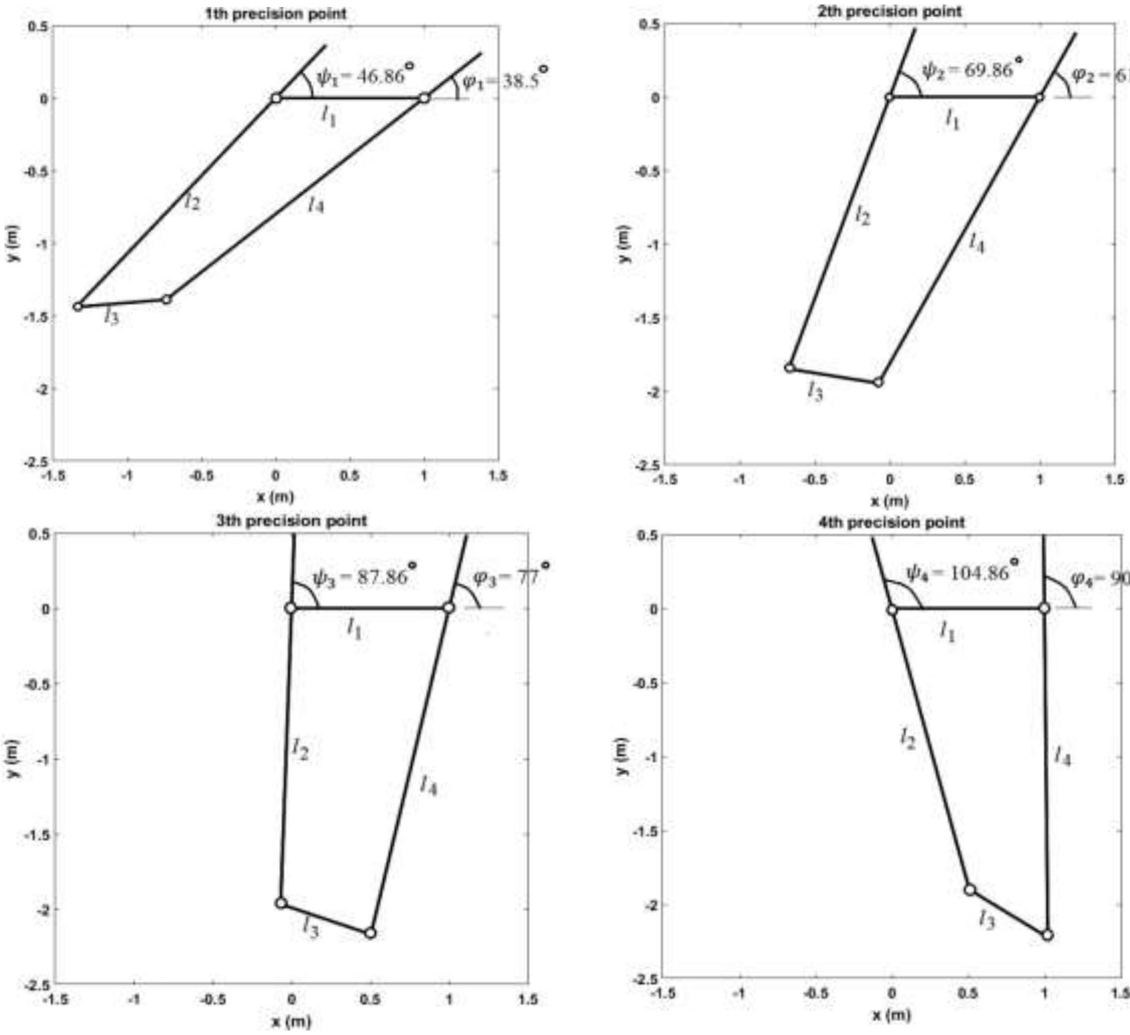
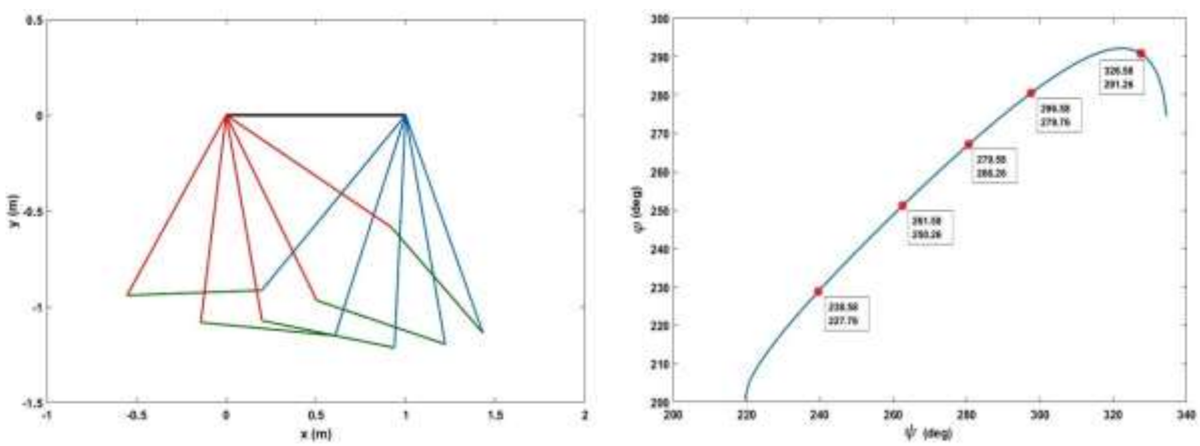


Fig. 3 Four precision points of the designed mechanism for the 4PP problem.



(a) **(b)**

Fig. 4 The designed mechanism for the 5PP problem

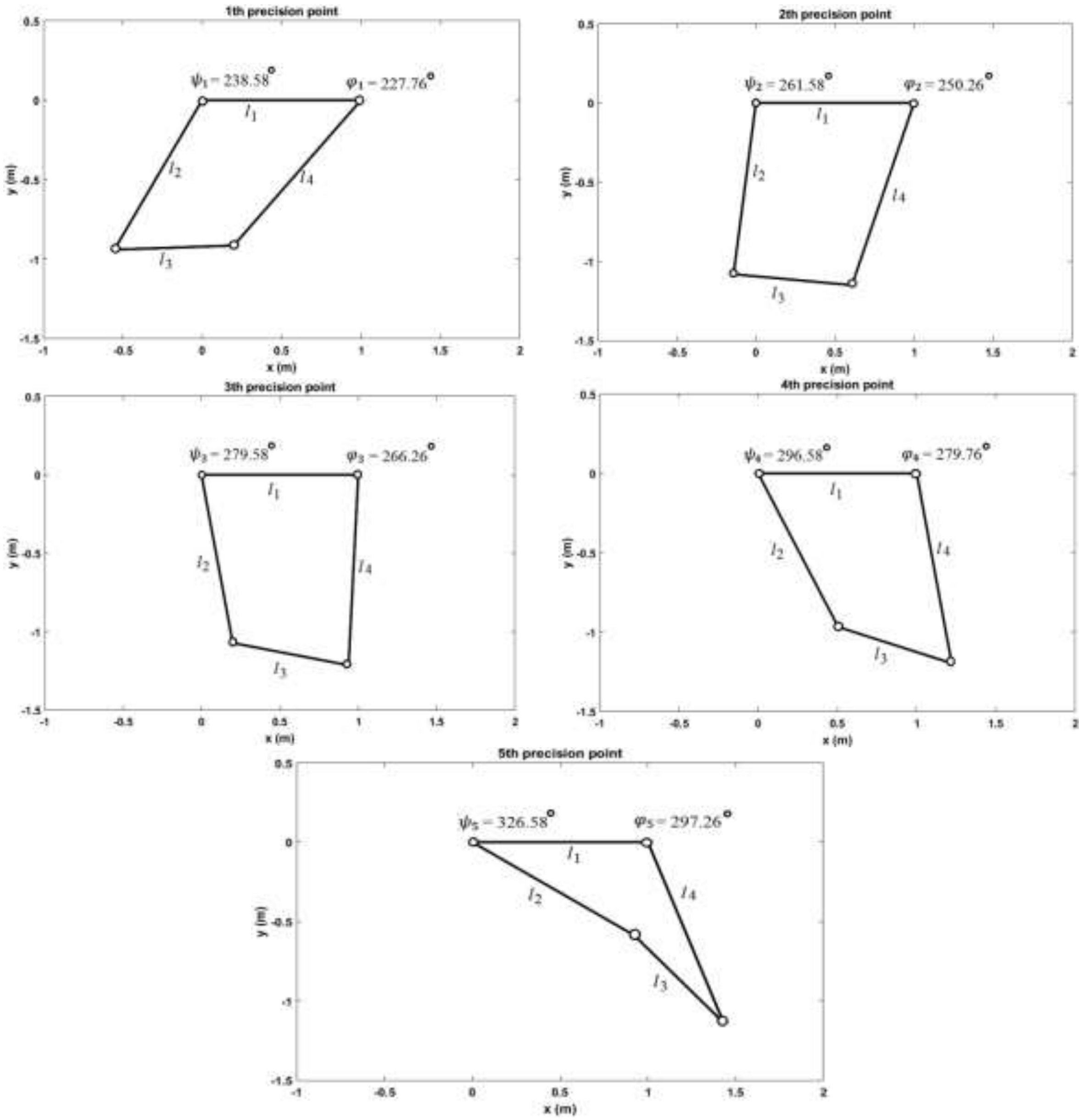


Fig. 5 Five precision points of the designed mechanism for the 5PP problem.

If the number of precision points is more than five, then the problem does not have any precise solution; this is because the number of the Equation is more than the unknowns (the maximum number of unknowns for this problem is five). Therefore, one must search for solutions with the minimum error by using the optimization algorithms.

6 CONCLUSIONS

This research uses a powerful approach to solve the nonlinear Equations arising in the function generation synthesis of the planar four-bar mechanism. The problem has been considered in two cases: with four and five precision points. The solution procedure is based on the Newton-HCM, combining numerical and semi-analytical methods.

The Equations reveal that the synthesis problem leads to a system of nonlinear Equations by four Equations and four unknown parameters for the 4PP problem and a system of nonlinear Equations by five Equations and five unknown parameters for the 5PP problem. Both cases have been solved and using the Numerical examples show that the considered algorithm is capable and highly accurate for the mechanism synthesis problem, and the designed mechanisms precisely cover the desired values.

REFERENCES

[1] Kachapi, S. H., Ganji, D. D., Davodi, A. G., and Varedi, S. M., Periodic Solution for Strongly Nonlinear Vibration Systems by He's Variational Iteration Method, Mathematical Methods in the Applied Sciences, Vol. 32, No. 18, 2009, pp. 2339-2349, DOI: 10.1002/mma.1135.

[2] Kosari, A., Jahanshahi, H., and Razavi, A. A., Design of Optimal PID, Fuzzy and New Fuzzy-PID Controller for CANSAT Carrier System Thrust Vector, ADMT Journal, Vol. 8, No. 2, 2015, pp. 1-9.

[3] Varedi-Koulaei, S. M., Rezagholizadeh, H., Synthesis of the Four-Bar Linkage as Path Generation by Choosing the Shape of The Connecting Rod, Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, Vol. 234, No. 13, 2020, pp. 2643-2652, DOI: 10.1177/0954406220908616.

[4] Chebyshev, P. L., Theorie Des Mecanismes Connaissent Comme Parallelogrames, Memoires Presenes à l'Academie Imperial Des Sciences De St.- Petersbourg Par Divers Savants, 1854, pp. 539-568.

[5] Freudenstein, F., An Analytical Approach to The Design of Four-Link Mechanisms, Trans. ASME, Vol. 76, 1954, pp. 483–492, DOI: 10.1115/1.4014881.

[6] Rao, A. C., Kinematic Design of Four-Bar Function Generators with Optimum Sensitivity, Mech Mach Theory, Vol. 10, 1975, pp. 531–535, DOI: 10.1016/0094-114X(75)90008-7.

[7] Rao, A. C., Optimum Design of Four-Bar Function Generators with Minimum Variance Criterion, J Optim Theory Appl, Vol. 29, No. 1, 1979, pp. 147–153, DOI: 10.1007/BF00932641.

[8] Sun, W., Optimum Design Method for Four-Bar Function Generators, J. Optim Theory Appl, Vol. 38, No. 2, 1982, pp. 287–293, DOI: 10.1007/BF00934089.

[9] Chen, F. Y., Chan, V. L., Dimensional Synthesis of Mechanisms for Function Generator Using Marguardt's compromise, J. Eng Ind, Vol. 96, 1974, pp. 131–137, DOI: 10.1115/1.3438287.

[10] Guj, G., Dong, Z. Y., and Giacinto, M. D., Dimensional Synthesis of Four Bar Linkage for Function Generation with Velocity and Acceleration Constraints. Meccanica, Vol. 16, No. 4, 1981, pp. 210–219, DOI: 10.1007/BF02128323.

[11] Söylemez, E., Freudenstein, F., Transmission Optimization of Spatial 4-Link Mechanisms, Mech Mach Theory, Vol. 17, No. 4, 1982, pp. 263–283, DOI: 10.1016/0094-114X(82)90050-7.

[12] Gosselin, C., Angeles, J., Optimization of Planar and Spherical Function Generators as Minimum-Defect Linkages, Mech Mach Theory, Vol. 24, No. 4, 1989, pp. 293–307, DOI: 10.1016/0094-114X(89)90049-9.

[13] Angeles, J., Alivizatos, A., and Akhras, R., An Unconstrained Nonlinear Least-Square Method of Optimization of RRRR Planar Path Generators, Mech Mach Theory, Vol. 23, No. 5, 1988, pp. 343–353, DOI: 10.1016/0094-114X(88)90048-1.

[14] Akhras, R., Angeles, J., Unconstrained Nonlinear Least-Square Optimization of Planar Linkages for Rigid-Body Guidance, Mech Mach Theory, Vol. 25, No. 1, 1990, pp. 97–118, DOI: 10.1016/0094-114X(90)90110-6.

[15] Shariati, M., Norouzi, M., Optimal Synthesis of Function Generator of Four-Bar Linkages Based on Distribution of Precision Points, Meccanica, Vol. 46, 2011, pp. 1007–1021, DOI: 10.1007/s11012-010-9357-1.

[16] Daniali, H. M., Varedi, S. M., Dardel, M., and Fathi, A., A Novel Algorithm for Kinematic and Dynamic Optimal Synthesis of Planar Four-bar Mechanisms with Joint Clearance, Journal of Mechanical Science and Technology, Vol. 29, No. 5, 2015, pp. 2059-2065, DOI: 10.1007/s12206-015-0426-1.

[17] Farajtabar, M., Daniali, H. M., and Varedi, S. M., Pick and Place Trajectory Planning of Planar 3-Rrr Parallel Manipulator in The Presence of Joint Clearance, Robotica, Vol. 35, No. 2, 2017, pp. 241-253. DOI: 10.1017/S0263574714002768.

[18] Sardashti, A., Daniali, H. M., and Varedi, S. M., Optimal Free-Defect Synthesis of Four-Bar Linkage with Joint Clearance Using PSO Algorithm, Meccanica, Vol. 48, No. 7, 2013, pp. 1681-1693, DOI: 10.1007/s11012-013-9699-6.

[19] Penunuri, F., Peon Escalante, R., Villanueva, C. and Pech Oy, D., Synthesis of Mechanism for Single- and Hybrid-Tasks Using Differential Evolution. Mech Mach Theory, Vol. 46, No. 10, 2011, pp. 1335-1349, DOI: 10.1016/j.mechmachtheory.2011.05.013.

[20] Shpli, O. A., Genetic Algorithms in Synthesis of Path Generator Four-Bar Mechanism with Maximum Mechanical Advantage. ASM 2007, The 16th IASTED International Conference on Applied Simulation and Modelling; 2007 Aug 29-31, Palma de Mallorca, Spain, California: ACTA Press, pp. 154-161.

[21] Fernández-Bustos, I., Aguirrebeitia, J., Avilés, R., and Angulo, C., Kinematical Synthesis of 1-Dof Mechanisms Using Finite Elements and Genetic Algorithm, Finite Elements in Analysis and Design, Vol. 41, No. 15, 2005, pp. 1441-1463, DOI: 10.1016/j.finel.2005.04.001.

[22] Cabrera, J. A., Simon, A., and Prado, M., Optimal Synthesis of Mechanisms with Genetic Algorithms, Mech Mach Theory, 37, 2002, pp. 1165–1177, DOI: 10.1016/S0094-114X(02)00051-4.

[23] Wampler, C. W., Solving the Kinematics of Planar Mechanisms, ASME Journal of Mechanical Design, Vol. 121, 1999, pp. 387-391, DOI: 10.1115/1.2829473.

[24] Wampler, C.W., Solving the Kinematics of Planar Mechanisms by Dixon Determinant and a Complex-Plane Formulation, ASME J. Mechanical Design, Vol. 123, 2001, pp. 3382–387, DOI: 10.1115/1.1372192.

[25] Wampler, C. W., Sommese, A. J., Numerical Algebraic Geometry and Algebraic Kinematics, Acta Numerica, 2011, pp. 469–567, DOI: 10.1017/S0962492911000067.

[26] Varedi, S. M., Daniali, H. M., and Ganji, D. D., Kinematics of an Offset 3-Upu Translational Parallel Manipulator by The Homotopy Continuation Method, Nonlinear Analysis: Real World Applications, Vol. 10, 2009, pp. 1767–1774, DOI: 10.1016/j.nonrwa.2008.02.014.

[27] Tari, H., Su, H. J., and Li, T. Y., A Constrained Homotopy Technique for Excluding Unwanted Solutions from Polynomial Equations Arising in Kinematics Problems, Mech Mach Theory, Vol. 45, No. 6, 2010, pp. 898–910, DOI: 10.1016/j.mechmachtheory.2010.01.002.

[28] Tari, H. Su, H. J., A Complex Solution Framework for The Kinetostatic Synthesis of a Compliant Four-Bar Mechanism, Mech Mach Theory, Vol. 46, No. 8, 2011, pp. 1137–1152, DOI: 10.1016/j.mechmachtheory.2011.03.003.

[29] Shafiee-Ashtiani, M., Yousefi-Koma, A., Keshavarz, H., and Varedi-Koulaei, S. M., Real Time Direct Kinematics Solution of 3-RPS Parallel Robot Using a Semi-Analytical Homotopy Method, Modares Mechanical Engineering, Vol. 17, No. 6, 2017, pp. 303-310 (in Persian).

[30] Varedi-Koulaei, S. M., Rahimi, M., Direct Kinematics Solution of 3-RCC Parallel Robot using a Semi-Analytical Homotopy Method, ADMT Journal, Vol. 12, No. 1, 2019, pp. 1-12.