A Comprehensive Study on the Effects of the Boundary Conditions on the Elastic Buckling Capacity of a Perforated Plate

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Received: 10 November 2020, Revised: 23 February 2021, Accepted: 14 March 2021

Abstract: Nowadays, different industries are using sheets, plates, and shells as important parts of their components. Because of their small thickness compare to other dimensions, their structural safety requires more attention. Therefore, increasing their strength and intensifying their resistance against any kind of failure type could be introduced as an important problem for enhancing the structural safety. Buckling is one of the most significant failure type that should be considered in the stability of any parts such as sheet metals. Thus, investigation of the buckling capacity of the sheet metals is remarkable. On the other hand, the existence of discontinuity like holes and notches in sheet metals can decrease their buckling capacity, significantly. In current study, based on Finite Element Method (FEM), ABAQUS/Explicit has been employed to determine the elastic buckling capacity in a perforated rectangular sheet metal with different boundary conditions on its edges. Afterward, the effect of the hole position and the plate aspect ratios (plate length/plate width) on the buckling capacity of sheet metal was studied. Finally, in order to enhance the sheet metal buckling capacity, two different types of stiffeners were used. The outcomes showed that the maximum buckling coefficient is related to the sheet metal which have four clamped edges. Moreover, For all boundary conditions, the buckling coefficient does not change significantly for the sheet metals with aspect ratio of more than 4. Also, stiffener type 2 increased the buckling capacity of sheet metal up to 83%.

Keywords: Boundary Conditions, Buckling Capacity, FEM, Perforated Rectangular Plates, Stiffener

How to cite this paper: Sadegh Ghorbanhosseini, Saeed Yaghoubi, and Mohammad Reza Bahrambeigi, "A Comprehensive Study on the Effects of the Boundary Conditions on the Elastic Buckling Capacity of a Perforated Plate", Int J of Advanced Design and Manufacturing Technology, Vol. 14/No. 3, 2021, pp. 45-53. DOI: 10.30495/admt.2021.1914605.1227

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1 INTRODUCTION

Slender metallic panels in structures like shells and plates have extensive applications in industries such as aerospace, building construction, pressure vessels and ship structures. Metal sheets or plates in these applications are often subjected to different in-plane loading. Therefore, designing high-performance structures and obtaining a high safety factor for these plates is a very important problem. Because of these topics, the investigation of different types of failure for metal plates seems to be important. Failure of these plates may occur by many different mechanical phenomena like fracture, fatigue, and buckling. The buckling of a rectangular plate subjected to a uniform uniaxial compressive load is one of the most important factors to estimate the stability of these components [1]. On the other hand, notches and holes in plates have remarkable role in reduction of the stability of the structures. When a geometrical discontinuity like hole appears in a plate, the probability of failure will increase. The presence of a notch or a hole in a plate may intensify the distribution of the membrane stresses around the hole. Therefore, it could reduce the stability of the structure against buckling significantly. Thus, studying the mechanical behavior of thin perforated plates under buckling can help engineers to improve the safety of many structures.

Several studies have been carried out to demonstrate the behavior of the plate under buckling and determine the buckling load in various conditions. The elastic buckling of a rectangular plate was studied by Timoshenko for the first time [2]. He suggested that the critical stress for a plate depends on length, width, thickness, elastic modulus, and poisson’s ratio of the plate. He introduced a parameter as the buckling coefficient. This coefficient is a dimensionless parameter that depends on the boundary condition of the plate’s edges, type of loading, and aspect ratio of the plate. Thin steel plates with holes have received much attention from researchers in the last years. Two methods have been used to assess the buckling capacity of plates when various types of loading such as uniaxial, biaxial, moment loading, and shear loadings subjected to the plate. These methods are the Conjugate Load/Displacement Method (CLDM) and the Finite-Element Method (FEM).

Brown et al. [3-5] have used CLDM to estimate the elastic buckling load of perforated square plates with rectangular holes located at the center of the plate, when it is subjected to various types of loadings. They concluded that as the hole size is increased, the buckling shear load would decrease significantly. However, other buckling loads such as uniaxial, biaxial, and bending have shown a smooth decrease for moderate hole sizes and it even increased in some cases. Shakerley and Brown [6] have used the CLDM to study the effect of eccentric holes in a square plate subjected to axial or shear loads on the buckling capacity. They found that for square plates subjected to uniaxial compression, the center of a small square hole should not be located near the center of the square plate. On the other hand, the center of a large square hole should be placed at the center of the plate to obtain better stability for the plate. Some researchers concentrated on the effect of cracks on the critical buckling load [7-10].

Sabir and Chow [11] and Azizian and Roberts [12] have used the FEM to study the buckling behavior of thin perforated square plates under uniform uniaxial compression load. Their studies showed that the critical buckling load may rise by increasing the hole size due to the redistribution of the membrane stresses. El-Sawy and Nazmy [13] investigated the effects of the plate's aspect ratio on the elastic buckling of uniaxially loaded plates with eccentric holes. Shanmugam et al. [14] have used the FEM to determine the uniaxial and biaxial load on the square plates with circular and square perforations located at the center of the plate. The results of their studies expressed that the buckling load of the square perforated plate is affected significantly by the hole size and the slenderness ratio of the plate. They also concluded that in general, plates with circular holes have higher buckling capacity in comparison with the plates with square holes.

El-Sawy and Martini [15] discussed the elastic buckling of thin rectangular perforated plates subjected to biaxial loadings for different circular hole locations. Their observations demonstrated that the larger hole diameter decreases the critical buckling load. They suggested a ratio between hole diameter and plate width as an optimum parameter to increase the plate stability under compression load. Shariati et al. [16] studied the buckling and post-buckling of cracked plates under axial compression load. In their research, the effects of mechanical and geometrical parameters of the workpiece were considered. Kumar et al. [17] investigated the buckling of perforated square and rectangular plates subjected to in-plane compressive edge loading. They found that the square plates are highly sensitive to buckling when the loading is at the center of the plate.

Prajapat et al. [18] used the finite element method to obtain elastic buckling loads and mode shapes of plates for predicting the buckling load in practical applications. Seifi et al. [19] performed some experiments to investigate the effect of hole radius, the width of the strip, and the thickness of the tube on global buckling of perforated plates reinforced with the circumferential strip. Azmy et al. [20] presented the ultimate load behavior of perforated plates girder with inclined stiffeners. Pham [21] provided a solution to determine the shear buckling load using the Spline Finite Strip Method (SFSM) for a hole in thin-walled channels. A comparison between the hole shapes, loading cases, and buckling modes are shown in his research work. Abolghasemi et al. [22-23] studied the buckling of plates with circular cutout, analytically. In their studies, the effects of parameters such as cutout size, uniaxial and biaxial loading profiles, and boundary conditions were investigated. Soltani and Kharrazi [24] researched the plastic buckling and post-buckling analysis of plates using three-dimensional standard and incompatible elements. Their outcomes showed a significant difference in post-buckling and bifurcation point behavior of the model with standard elements in comparison with incompatible ones.

It is clear from the above literature review that the effect of various sheet edges' boundary condition on the buckling behavior of perforated plates has not been studied. Also, the effect of the hole position on the sheet buckling capacity was not investigated. Hence, in the present research work, FEM analysis has been employed to study the effect of boundary conditions of plate's edges on the elastic buckling behavior of perforated rectangular Aluminum plates under uniaxial compression. Also, the effects of hole position on the plate buckling capacity were found. Finally, the effect of two different types of stiffeners on the plate's buckling capacity was investigated.

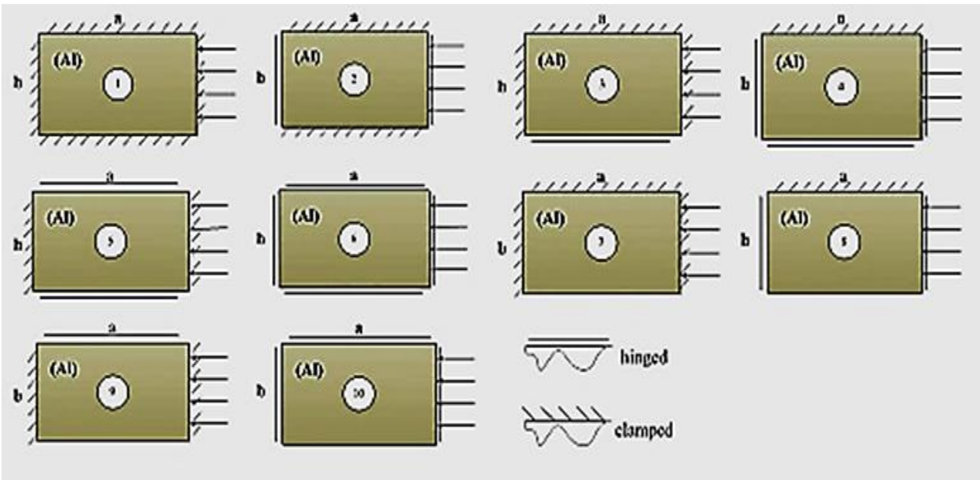


Fig. 1 Different boundary conditions for a perforated rectangular plate.

2 PROBLEM DEFINITION

In this paper, the elastic buckling of a perforated rectangular plate subjected to uniaxial uniform compression load along its ends is considered. The geometrical parameters of the plate are thickness (*t*), length (*a*) and width (*b*). A circular hole with the diameter (*d*) is assumed to exist in the center of the plate. The plate is subjected to a uniform compressive load along the x-direction (direction parallel to the side of the plate with length a). The following parameters have been considered:

1- Aspect ratio of plate (*a/b*): varies from 0.5 to 5.
2- Hole size (*d/b*): 0.1, 0.2, 0.3 and 0.5.
3- The angle of rotation for a circular hole around the center of plate (θ): varies from zero to 180 degree.

2.1. Boundary Conditions of the Rectangular Plate

The buckling stresses of biaxially loaded in aluminium plate are greatly affected by the boundary conditions. When the plate’s four edges are clamped, there will be no lateral edge displacements perpendicular to the plate’s plane and no rotations about the axes along the plate edges. When four edges of the plate are simply supported (S.S.), there will be no lateral edge displacements perpendicular to the plate’s plane, but rotation about the axis of each edge is allowed and about the free edges, both displacement and rotation are permissible. In current research work, different types of boundary conditions are considered which are shown in “Fig. 1” .

3 ANALYSIS METHOD

Researchers had used FEM and CLDM to determine the buckling load of perforated plates. Both methods are based on solving an eigenvalue problem that can define the behavior of a plate under buckling. The lowest eigenvalue is related to the critical buckling load, while the eigenvector describes the deformed shape of the plate as a mode shape. In current research work, numerical simulation has been employed to obtain the elastic buckling behavior of perforated plates and determine the critical buckling load. The stiffness matrix is composed of the conventional constant small deformation stiffness matrix (K_E) and another matrix (K_σ) which accounts for the effect of the existing stresses (σ_x). Therefore, the total stiffness matrix of the plate with stress level (σ_{x0}) can be written as:

$$K(\sigma_{x0}) = K_E + K_\sigma(\sigma_{x0}) \quad (1)$$

When the stresses reach to ($\lambda\sigma_0$), the stiffness matrix can be defined as:

$$K(\lambda\sigma_{x0}) = K_E + K_\sigma(\lambda\sigma_{x0}) = K_E + \lambda K_\sigma(\sigma_{x0}) \quad (2)$$

Now, the governing equations for the behavior of plate can be written as:

$$dF = [K_E + \lambda K_\sigma(\sigma_{x0})]du \quad (3)$$

Where *du* is the incremental displacement vector and *dF* is the corresponding incremental force vector.
When the buckling phenomenon occurs, the determinant of the stiffness matrix should be zero and the plate exhibits an increase in its displacement with no increase in the load. This can be shown by “Eq. (4)”.

$$\det[K_E + \lambda K_\sigma(\sigma_{x0})] = 0 \quad (4)$$

This equation represents an eigenvalue problem. Solving this problem provides the lowest eigenvalue (λ_1) that corresponds to the critical stress level ($\sigma_{x,cr} = \lambda_1 \sigma_{x0}$).

4 DESCRIPTION OF FE MODEL

The schematic of the problem is shown in “Fig. 2”.

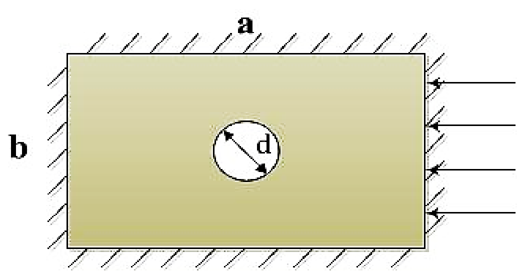


Fig. 2 Rectangular plate with a circular hole in the center of the plate under uniaxially compression load.

This figure demonstrates a rectangular thin plate with length (*a*), width (*b*), thickness (*t*) and a circular hole with diameter (*d*) at the center of a plate subjected to a uniform uniaxial compression load on the right edge of the plate (all the edges are clamped).

4.1. Eccentric Hole

To investigate the effect of circular hole locus around the center of the plate on buckling behavior, a critical buckling load was determined on different positions of the circular hole. The hole location has a constant distance of 0.2*b from the center of the plate (*r*) and is rotated around the plate center from 0 to 180 degree. The effect of hole rotation on the critical buckling load has been done for 6th boundary condition (see “Fig. 1”). This mode has been shown in “Fig. 3”.

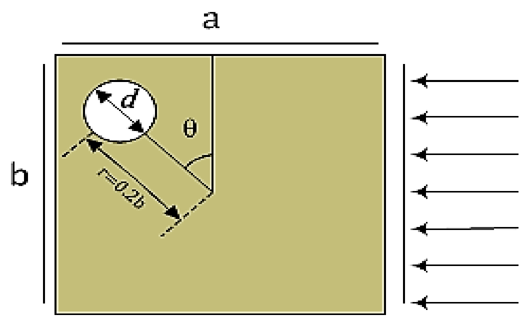


Fig. 3 Rectangular plate with an eccentric hole under uniaxially compression load.

4.2. Stiffeners

One of the important ways to secure plates against the buckling is using stiffeners. It can be placed on one side of the plane perpendicular to the plate plane next to the square hole. The length of these stiffeners can be equal to the distance between the hole and the plate edge. The stiffeners' width is assumed to be equal to 0.1*b. In this study, two types of stiffeners are employed. In the first type of stiffener, the plates are perpendicularly welded to the main plate and fill the gap between the hole and edges. In the second type, the plate is welded to connect the corners of the square hole perpendicular to the main plate, with the corners of the main plate. These two stiffener types are demonstrated in “Fig. 4”.

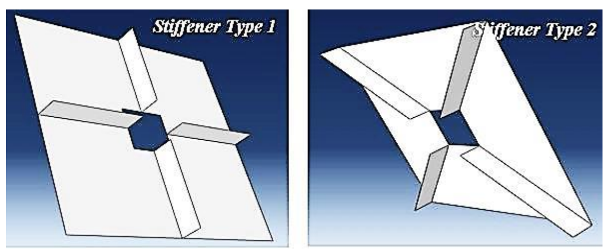


Fig. 4 Two types of Stiffeners used to improve the buckling capacity of a perforated plate.

4.3. Mesh type

S8R5 element is a common element type for sheet buckling problems. It is an 8-node shell element type with 5 degrees of freedom at each node. By using 1600 elements in each square meter, the final results converge to a constant value in each case. Therefore, the mesh convergence of the problem can be obtained by 1600 elements per square meter. Figure 5 shows an example of FE mesh for the square plate with a central hole.

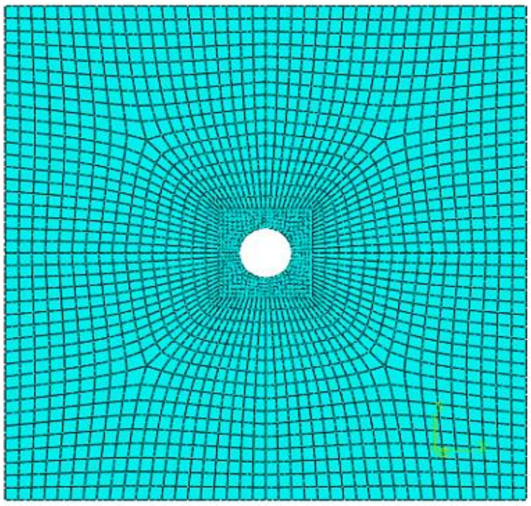


Fig. 5 FE model of the plate with circular hole.

5 VERIFICATION OF NUMERICAL SIMULATION

To verify the method of analysis used in current study, a comparison with existing results in the literature (Ref No. 13) on buckling of perforated plates has been performed ("Fig. 6"). The comparison shows good agreement for all normalized hole sizes and the general difference is less than 6%.

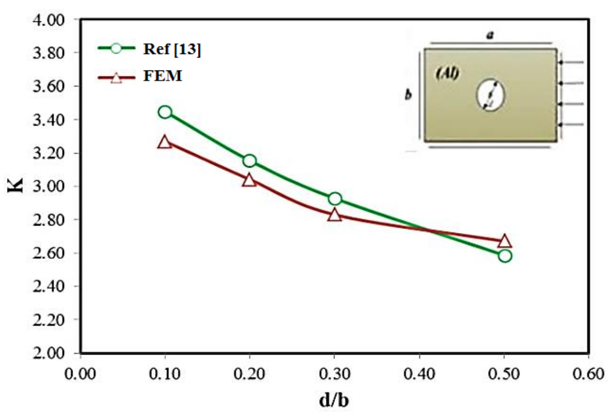


Fig. 6 Determination of buckling coefficients with FEM results and Ref [13].

6 RESULTS AND DISCUSSIONS

In the current section, the results from the analysis performed to study the effect of the boundary conditions and rotation angle (θ) on the buckling coefficient (k) for a rectangular perforated plate subjected to uniaxial uniform loading are presented and discussed in the following sections.

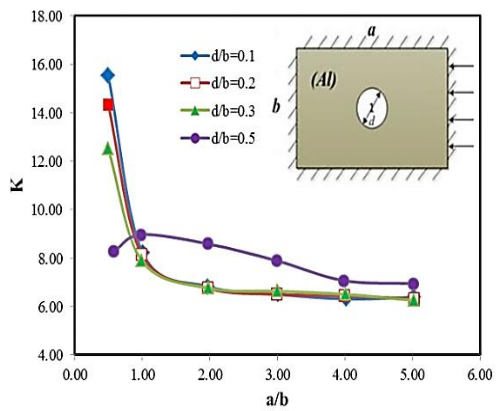
6.1. The Effects of Boundary Conditions

The curves shown in "Fig. 7" illustrate the change in the buckling coefficient (k) versus the plate aspect ratio for different values of hole diameters in the various boundary conditions. As seen in "Fig. 7" , it can be claimed that for boundary conditions No.3, No. 5, No. 7, No. 8, No. 9 and No. 10 ("Figs. 7c, e, g, h, i and j"), by increasing the aspect ratio, the diagram has a completely downward trend which means the buckling capacity of plate decreases. It can be said that existence of at least one free edge on boundary condition of plate can decrease the buckling capacity of it. This reduction continues up to aspect ratio of 4. After that, the diagram has a constant magnitude which for boundary condition No. 3, the minimum of the buckling coefficient is equal to 5. It means that the effect of a circular hole on the buckling behavior of the plate will be eliminated for a very long plate (aspect ratio more than 4).

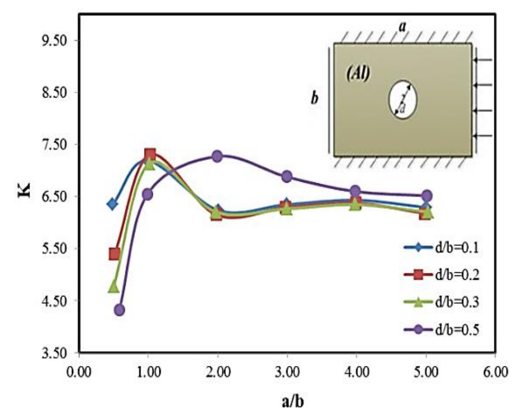
Figures 7a, 7b, 7d and 7f express that if the aspect ratio of plate increases, the buckling coefficient does not have a uniform trend and depending on the hole dimension for boundary conditions No. 1, 2, 4 and 6. It has a decreasing or increasing trend, but even in these boundary conditions for the aspect ratio of more than 4, no change can be seen for buckling coefficient. In some cases, when the hole diameter increases, it can be observed that the buckling capacity increases unexpectedly. It can be claimed that for boundary conditions No. 3, No. 5, No.7, No. 8, No. 9 and No. 10 ("Figs. 7c, e, g, h, i and j"), by increasing the aspect ratio, the diagram has a completely downward trend. This means that the buckling capacity of plate decreases, when the aspect ratio of plate increases.

It can be said that existence of at least one free edge on boundary condition of plate decreases the buckling capacity of the plate (by increasing the hole size). This reduction continues up to aspect ratio of 4. After that, diagram has a constant magnitude which for boundary condition No. 3, the minimum of the buckling coefficient is equal to 5 for all different diameters of the holes. It means that the effect of a circular hole on the buckling behavior of the plate will be eliminated for a very long plate (aspect ratio more than 4). In boundary conditions No. 1 and 2 ("Figs. 7a and b") for the aspect ratio of more than 1, the buckling coefficient increases, if a larger ratio of hole diameter to width of the plate is chosen.

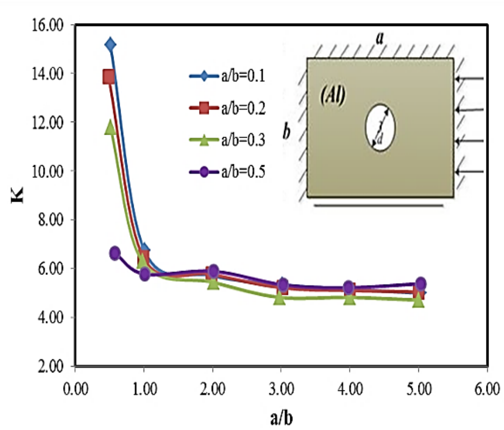
This state occurs for an aspect ratio of more than 1 in boundary condition No. 4 ("Fig. 7d") and an aspect ratio between 1.5 and 3 in boundary condition No. 6 ("Fig. 7f"). For this object, it can be said that when at least a rigid edge (like clamp edge) is added to boundary condition, the hole diameter rising put down the effect of plate's width and it acts as a column. Therefore, a two dimensional (2D) buckling problem changes into a one dimensional (1D) type. According to all of the figures, it can be obtained generally that the effect of circular hole diameter on the buckling coefficient is significant for aspect ratio of less than 2. For larger aspect ratios, there is no tangible change in buckling coefficient. This means that if there is a request to have a circular hole with large dimension in the plate, by choosing the minimum aspect ratio of 2, the decrease in stability of plate can be obviated.



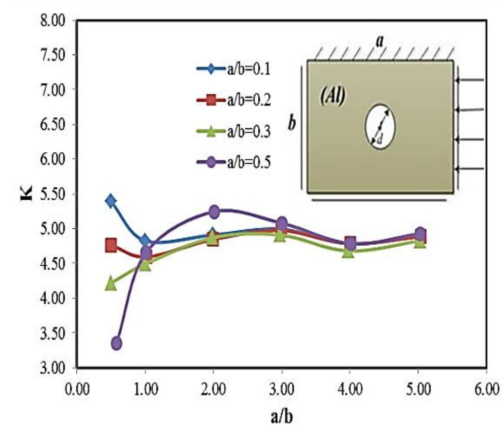
(a)



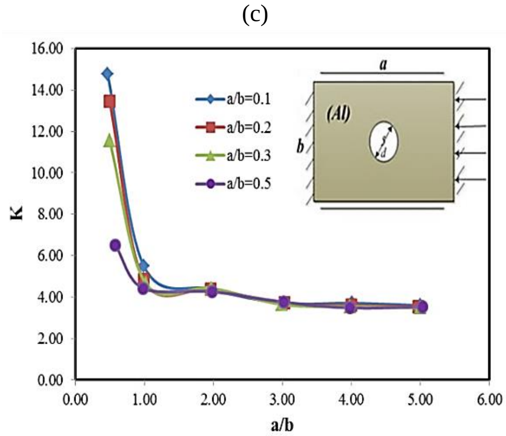
(b)



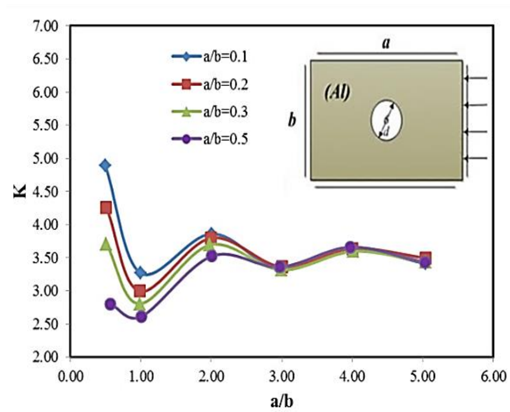
(c)



(d)



(e)



(f)

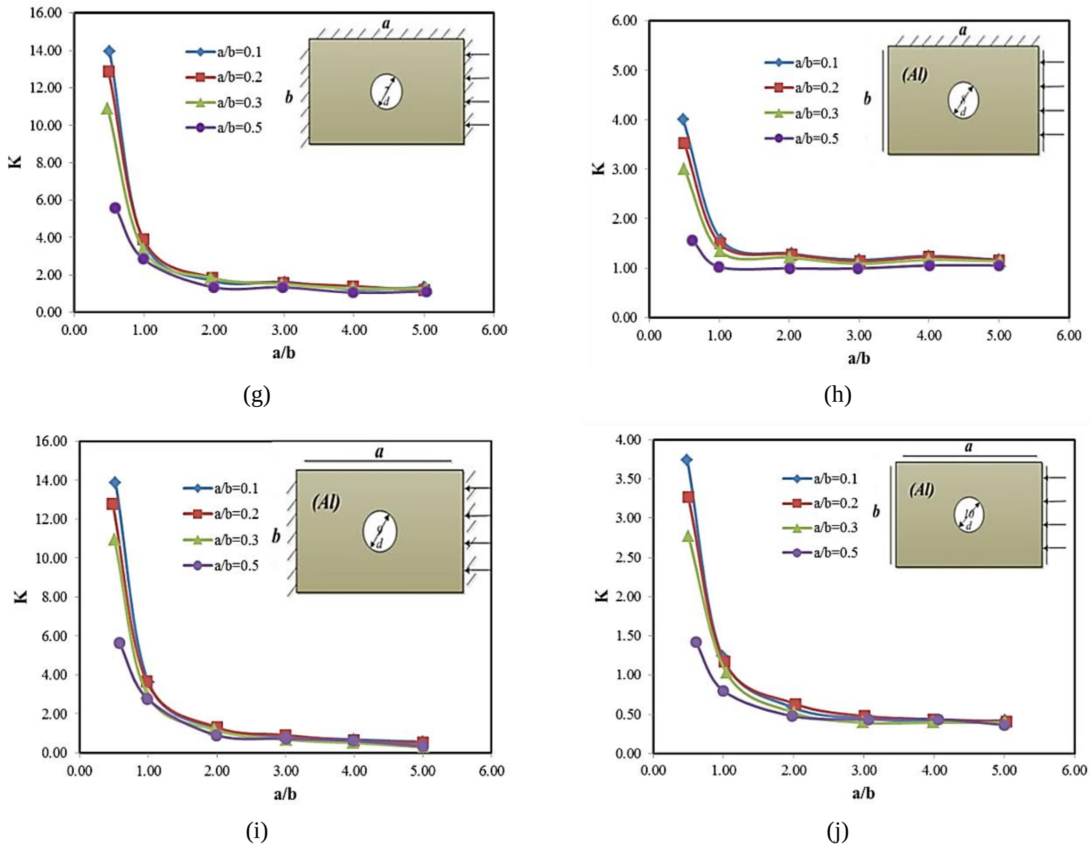


Fig. 7 Buckling coefficient versus the aspect ratio (*a/b*) of a rectangular plate with a central circular hole for different B.C.

6.2. The Effect of Hole Location

The diagram shown in "Fig. 8" illustrates the change in the buckling coefficient versus the angle θ for different values of normalized hole diameters.

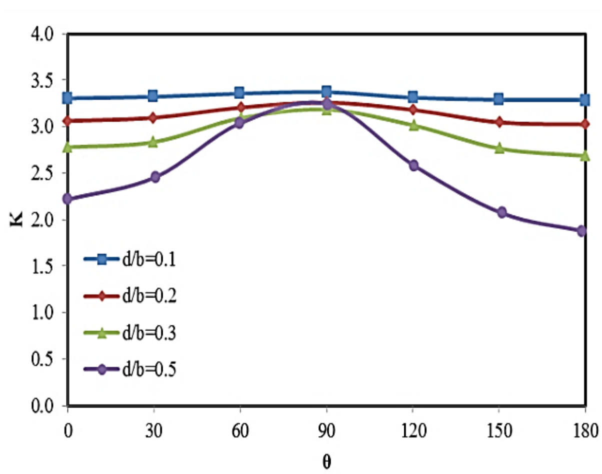


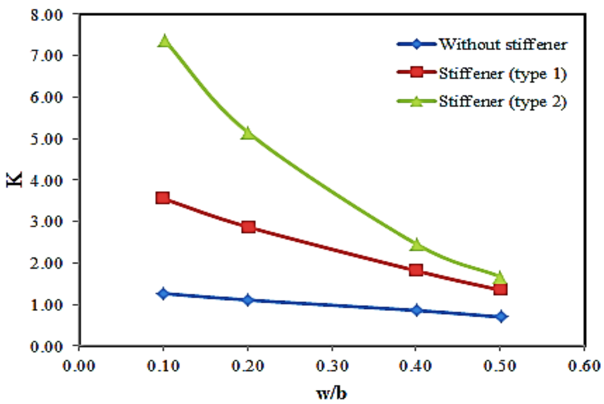
Fig. 8 Buckling coefficient versus the angle of a rectangular plate with an eccentric circular hole (all edges are S.S.).

The following observations can be obtained from this figure. The results shown in "Fig. 8" demonstrated that for normalized hole diameter (*d/b*) of less than 0.3, by increasing the angle (θ) from zero to 180 degree, the change in buckling coefficient is negligible which means for these ratios of (*d/b*) hole rotation has no significant effect on the buckling coefficient. For ratios of (*d/b*) of 0.5, by increasing the angle from zero up to 90 degree, it can be observed that the buckling capacity increases and for 90 to 180 degree, it decreases symmetrically (because of symmetrical boundary condition in 4 edges). In other words, as the hole location is near to the loading edge, a more buckling coefficient is expected. According to this point, it can be claimed that for square plates with simply-supported in all edges, to have a better and more optimum condition, the hole should be located near to loading edge than other ones. The convergence of different ratios of (d/b) occurs at θ=90 which buckling coefficient is 3.2.

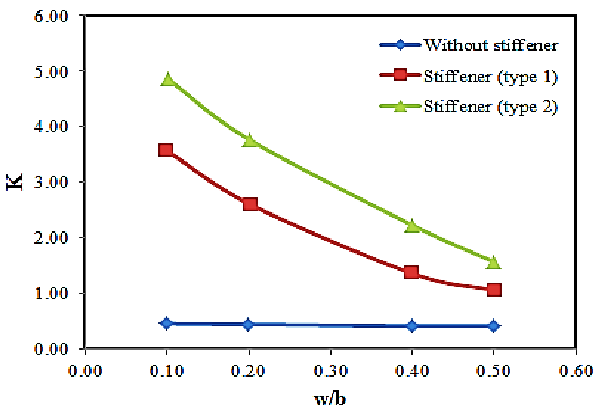
As can be shown for the case with d/b=0.5, the diagram is not completely symmetric and it is because of displacement constraint along y-direction for the top edge which is not applied for the bottom edge. This constraint is applied to increase the stability of the plate. According to this point, it can be obtained that if the hole location is near to the edge which has a displacement constraint along the y-direction, the stability of the plate increases.

6.3. The Effects of Stiffeners

The buckling coefficient comparisons between the plate without stiffener and the plate reinforced by them (type 1 and 2) are presented in "Fig. 9" . As is clear from these figures, the buckling coefficient for both aspect ratio a/b=1 and a/b=4 have a greater amount with stiffener type 2 than type 1. Buckling coefficient for the plate with stiffener type 1 is more than the plate without stiffener. That is why the stiffener type 2 has an angle with the plate axes, but the stiffener type 1 is parallel to it. This angle could decrease the load required for the plate buckling. Based on the equilibrium equations, the critical buckling load required for the plate buckling will be decomposed into two components, which will decrease the required force for plate buckling and thereby increase the buckling capacity of the plate.



(a)



(b)

Fig. 9 Buckling coefficient vs. hole width to plate width ratio(w/b) for boundary condition 10, a) for aspect ratio 1 (a/b=1), b) for aspect ratio 4 (a/b=4).

It can be seen that in both diagrams when the hole dimension increases, the effect of stiffener presence is decreased. In other words, the buckling coefficients will converge for three different conditions. Also, in the case where the plate has aspect ratio of 1, the buckling coefficient is much higher than the plate by aspect ratio of 4. For example, in a plate with a/b=1 and a 100 mm hole length, stiffener type 1 increases the buckling coefficient by 65% in comparison with a non-stiffened plate. If stiffener type 2 is used in the same condition, the buckling coefficient will increase by 83% relative to the non-stiffened plate. Therefore, it can be concluded that the optimum mode for increasing the buckling capacity of the plate against buckling is using stiffener type 2.

7 CONCLUSION

In current research work, the influence of the boundary conditions on the elastic buckling of perforated plate was investigated using numerical simulation. Based on observations made in the present study, the conclusions can be summarized as below:

- The buckling capacity of plates that have at least one free edge decreased when the aspect ratio of plate increased.
- When at least one clamped edge was added to boundary conditions, the plate acted as a column (changing the problem from 2D to 1D).
- For normalized hole diameter (d/b) less than 0.3, the hole rotation has no significant effect on the buckling coefficient.
- For normalized hole diameter (d/b) 0.5, by increasing the angle from zero to 90 degree, the buckling capacity increases and for 90 to 180 degree, it decreases symmetrically.
- The optimum mode for increasing the buckling capacity of the plate against buckling is using stiffener type 2 (increasing the buckling capacity more than 83%).

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