Design an Adaptive Sliding Mode Controller for a Class of Underactuated Systems

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Abstract: The majority of underactuated systems are nonholonomic, due to non-integrable differential constraints. Therefore, controlling an underactuated system is considered as a challenging problem. In this study, an adaptive controller based on super-twisting sliding mode controller is proposed for a class of robust underactuated systems subjected to uncertainties and external disturbances. The adaptive compensator was designed so that there would be no need to the upper bound of the external disturbance. The controller parameters of adaptive sliding mode control are tuned based on a multi-objective non-dominated sorting of genetic optimization algorithm. The results of simulation and the demonstration of the effectiveness and applicability of the proposed scheme are presented.

Keywords: Adaptive Sliding Mode Control, Multi-Objective Non-Dominated Sorting, Robust Controller, Stability Analysis, Underactuated Systems

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1 INTRODUCTION

In the past few decades, underactuated mechanical systems have attracted much attention. These systems have different practical applications in engineering and modern science [1]. Systems in which the number of control inputs is smaller than their degree of freedom are called underactuated systems [2]. Underactuated systems are adopted in many real-time applications such as aerospace [3], robotics [4-5], underwater vehicles [6-7], and flexible systems [8]. A system could be underactuated for different reasons, such as system's inherent dynamic [9], the actual design method [10], as well as malfunction of actuators. Also, under-actuations are applied to the system for research and scientific purposes or the mathematical model used for design procedure could cause under actuation [11]. Since underactuated systems need a smaller number of actuators, therefore their cost and complexity are less and they also consume less energy. Research for the study, analyse and control of underactuated systems has been continuing from a long time ago [12-17].

There are numerous control methods for fully actuated systems, i.e. systems in which the number of control inputs equals their degree of freedom. Nonetheless, these methods cannot be applied to underactuated mechanical systems, therefore controlling an underactuated system is considered as a challenging problem. Classical and modern control theories were successful for systems that were well-defined according to deterministic as well as stochastic descriptions. In robotics, similar to many engineering applications, achieving an accurate dynamic model is extremely challenging or impossible, and the factors contributing to this include high flexibility, Coulomb friction, payload variation, unknown disturbances, backlash, huge dynamic couplings among different links and time-varying parameters such as parameters concerning robot aging. Therefore, mathematical model of a robot could be an estimation of the real robot, in a best-case scenario; as a result, there shall always exist a modelling error. When dealing with time-invariant linear models, this error grows even larger [18-19]. Therefore, there is a need for control strategies characterized with robustness, adoptability, fast convergence, and simple structure. Robust controllers with variant structures using one Sliding Mode Control are of great interest, since they are capable of dealing with uncertainties, and achieve good transient response (i.e. have a low tracking error), and have a fast response. Adaptive sliding mode hybrid controllers have been studied [20-24] as a method for coping with the shortcomings of adaptive control and sliding mode control. The idea behind these controllers is that they use adaptive control to estimate the unknown parameters of the dynamic system, and apply sliding mode control to deal with the dynamics that are not modelled, as well as external disturbances.

After that, many researches have been presented with different ideas in the field of adaptive sliding mode controllers by numerical and experimental approach [25-29]. Deng et al [30] proposed a nonlinear multiple-input multiple-output controller based on adaptive sliding mode control for helicopter experimental system. System stability was evaluated based on Lyapunov function. Lee et al [31] have presented two types of nonlinear controllers that include a feedback linearization controller and a novel adaptive sliding mode controller for autonomous control of quadrotor. The adaptive sliding mode control acts very well under noisy conditions. Khalaji and Moosavian [32] introduced a Lyapunov kinematic controller based on an adaptive sliding mode control for controlling the experimental mobile robot with external disturbances and uncertainties. The adaptation law according to a Lyapunov analysis is utilized to stabilize tracking errors. Su et al [33] employed a sliding-mode controller based on an adaptive neural network with decoupled method for underactuated system. An overhead crane system has been used to demonstrate the robustness and rapidness of this method. Liu et al [34] designed an adaptive hierarchical sliding mode control with ActorCritic learning method to optimize the control parameters. Also, a novel PID sliding surface is proposed to create a sliding mode controller. Ovalle et al [35] designed a sliding mode controller with relative degree of one and two for the stabilization of a class of underactuated systems. Rangel et al [36] used a technique based on terminal sliding mode control for an unmanned underwater vehicle. The presented adaptation approach ensures which the gains stay bounded.

Motivated by above literature reviewing and discussion, in present study, an adaptive controller based on sliding mode control for a class of underactuated mechanical systems is presented. The control parameters of ASMC are tuned based on a multi-objective non-dominated sorting of genetic optimization algorithm. Eventually, the proposed controller is used on a double inverted pendulum system to verify the applicability and efficiency of this scheme. The important advantages of this research are as follows:

- The strategy challenge is investigated in the nonlinear underactuated benchmark systems.
- Design of an adaptive controller based on super-twisting sliding mode controller is proposed for a class of underactuated systems subjected to uncertainties and external disturbances.
- The Genetic multi-objective algorithm is used to reach the optimal controller gains.
- The presented control approach has some of superiorities including low Control effort, fast convergence speed, and low chattering.

The organization of this study is as follows: Section 2 introduces basic problem formulations. Controller ASMC will be introduced in Section 3. The strategy challenge is investigated in the nonlinear underactuated benchmark systems in Sections 4. Eventually, the work will be concluded in Section 5.

2 PROBLEM FORMULATION

The general dynamical equations of motion for a simple mechanical control system with n DOF are given by Euler-Lagrange equation as follows:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\mathbf{q}}} - \frac{\partial L}{\partial \mathbf{q}} = \mathbf{F}(\mathbf{q})\mathbf{u} \quad (1)$$

In "Eq. (1)" , $\mathbf{q}\epsilon R^n$ is the generalized configuration vector, $\mathbf{u} \in R^m$ is the control input vector and $\mathbf{F}(\mathbf{q}) = [f_1(\mathbf{q}), \dots\ f_m(\mathbf{q})]^T$ is the matrix of external forces. Where, L is the Lagrangian function which is defined by the difference of kinetic energy K and potential energy V. Its mathematical expression appears as follows:

$$L(\mathbf{q}, \dot{\mathbf{q}}) = K - V = \frac{1}{2}\dot{\mathbf{q}}^T\mathbf{M}(\mathbf{q})\dot{\mathbf{q}} - V(\mathbf{q}) \quad (2)$$

In vector form, the dynamics (1) can be written as:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\mathbf{q} + \mathbf{G}(\mathbf{q}) = \mathbf{F}(\mathbf{q})\mathbf{u} \quad (3)$$

For the general case $\mathbf{F}(\mathbf{q}) = [f_1(\mathbf{q}), f_2(\mathbf{q})\ ]^{\mathrm{T}}$ and partitioning $\mathbf{q} = [\mathbf{q_1}, \mathbf{q_2}\ ]^T$ according to $f(\mathbf{q})$, where $\mathbf{q_1} \in R^{n-m}$ and $\mathbf{q_2} \in R^m$, dynamics (3) can be written as:

$$\begin{aligned} m_{11}\ddot{\mathbf{q}}_1 + m_{12}\ddot{\mathbf{q}}_2 + \mathbf{c_1} + \mathbf{g_1} &= \mathbf{f_1}(\mathbf{q})\mathbf{u} \\ m_{21}\ddot{\mathbf{q}}_1 + m_{22}\ddot{\mathbf{q}}_2 + \mathbf{c_2} + \mathbf{g_2} &= \mathbf{f_2}(\mathbf{q})\mathbf{u} \end{aligned} \quad (4)$$

$\mathbf{M}(\mathbf{q}) = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$ is the symmetrical positive definite inertia matrix, $\mathbf{c_1}(\mathbf{q}, \dot{\mathbf{q}}) \in R^{n-m}$ and $\mathbf{c_2}(\mathbf{q}, \dot{\mathbf{q}}) \in R^m$ are the centrifugal and coriolis terms, $\mathbf{g_1}(\mathbf{q}) \in R^{n-m}$ and $\mathbf{g_2}(\mathbf{q}) \in R^m$ are the gravitiational terms. For $f_1(\mathbf{q}) = \mathbf{0}$ and $f_2(\mathbf{q}) = 1$, one may have:

$$\begin{aligned} m_{11}\ddot{\mathbf{q}}_1 + m_{12}\ddot{\mathbf{q}}_2 + \mathbf{c_1} + \mathbf{g_1} &= \mathbf{0} \\ m_{21}\ddot{\mathbf{q}}_1 + m_{22}\ddot{\mathbf{q}}_2 + \mathbf{c_2} + \mathbf{g_2} &= \mathbf{u} \end{aligned} \quad (5)$$

Substituting the result in the second equation, (5) can be written as:

$$\begin{aligned} \bar{m}_{11}\ddot{\mathbf{q}}_1 + \bar{\mathbf{c}}_1 + \bar{\mathbf{g}}_1 &= \mathbf{u} \\ \bar{m}_{22}\ddot{\mathbf{q}}_2 + \bar{\mathbf{c}}_2 + \bar{\mathbf{g}}_2 &= \mathbf{u} \end{aligned} \quad (6)$$

Where:

$$\begin{aligned} \bar{m}_{11}(\mathbf{q}) &= m_{21} - m_{22}m_{12}^{-1}m_{11} \\ \bar{\mathbf{c}}_1(\mathbf{q}, \dot{\mathbf{q}}) &= \mathbf{c_2} - m_{22}m_{12}^{-1}\mathbf{c_1} \\ \bar{\mathbf{g}}_1(\mathbf{q}) &= \mathbf{g_2} - m_{22}m_{12}^{-1}\mathbf{g_1} \\ \bar{m}_{22}(\mathbf{q}) &= m_{22} - m_{21}m_{11}^{-1}m_{12} \\ \bar{\mathbf{c}}_2(\mathbf{q}, \dot{\mathbf{q}}) &= \mathbf{c_2} - m_{21}m_{11}^{-1}\mathbf{c_1} \\ \bar{\mathbf{g}}_2(\mathbf{q}) &= \mathbf{g_2} - m_{21}m_{11}^{-1}\mathbf{g_1} \end{aligned} \quad (7)$$

Since $\mathbf{q_1} = R^{n-m}$ and $\mathbf{q_2} = R^m$, dynamics (6) is a set of two second order systems in state variables. A class of underactuated systems of (6) can be written as:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f_1(\mathbf{x}) + b_1(\mathbf{x})u(t) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= f_2(\mathbf{x}) + b_2(\mathbf{x})u(t) \\ &\vdots \\ \dot{x}_{2n-1} &= x_2 \\ \dot{x}_{2n} &= f_n(\mathbf{x}) + b_n(\mathbf{x})u(t) \end{aligned} \quad (8)$$

Here $\mathbf{x} = [x_1, x_2 \dots, x_{2n-1}, x_{2n}]^T$ is the state vector $f_i(\mathbf{x})$ and $b_i(\mathbf{x})$, $i = 1,2, \dots, n$ are the nonlinear functions of the states and u is the single control input.

3 ADAPTIVE CONTROLLERS BASED ON SLIDING MODE CONTROL

The sliding surface is as follows:

$$s(t) = \mathbf{c}r(t) - \mathbf{c}r(0)\exp(-\beta) \quad (9)$$

Where, $r(t) = \dot{x}(t) + \lambda x(t)$, $r(0) = \dot{x}(0) + \lambda x(0)$, $\mathbf{c} = [c_1\ c_2\ c_3\ c_4\ c_5\ c_6]$ and β is constant with positive sign. Then:

$$\mathbf{c}r(t) - \mathbf{c}r(0)\exp(-\beta) = 0 \quad (10)$$

We have:

$$\mathbf{c}r(t) = \mathbf{c}r(0)\exp(-\beta) \quad (11)$$

Where, "Eq. (4)" is solution of the first-order differential equation:

$$\mathbf{c}\dot{r}(t) + \beta\mathbf{c}r(t) = 0 \quad (12)$$

Consider an adaptive control signal for sliding surface "Eq. (9)" as follows:

$$\dot{u}(t) = -B(t)^{-1}(\Psi(t)u(t) + F(t) + G(t) + \hat{v}sgn(s(t)) + ls(t)) \quad (13)$$

l is constant with positive sign and $v \geq \max(D(t))$is a scalar, we have:

$$D(t) = c_2\dot{d}_1(t) + c_4\dot{d}_2(t) + c_6\dot{d}_3(t) + (c_1 + c_2\lambda)d_1(t) + (c_3 + c_4\lambda)d_2(t) + (c_5 + c_6\lambda)d_3(t) \quad (14)$$

$$B(t) = c_2b_1(\mathbf{x}, t) + c_4b_2(\mathbf{x}, t) + c_6b_3(\mathbf{x}, t) \quad (15)$$

$$G(t) = \beta\mathbf{c}r(0)\exp(-\beta t) \quad (16)$$

$$F(t) = c_2\dot{f}_1(\mathbf{x}, t) + c_4\dot{f}_2(\mathbf{x}, t) + c_6\dot{f}_3(\mathbf{x}, t) + (c_1 + c_2\lambda)f_1(\mathbf{x}, t) + (c_3 + c_4\lambda)f_2(\mathbf{x}, t) + (c_5 + c_6\lambda)f_3(\mathbf{x}, t) + (c_1x_2 + c_3x_4 + c_5x_6)\lambda \quad (17)$$

$$\Psi(t) = c_1b_1(\mathbf{x}, t) + c_2\dot{b}_1(\mathbf{x}, t) + \lambda b_1(\mathbf{x}, t) + c_3b_2(\mathbf{x}, t) + c_4\dot{b}_2(\mathbf{x}, t) + \lambda b_2(\mathbf{x}, t) + c_5b_3(\mathbf{x}, t) + c_6\dot{b}_3(\mathbf{x}, t) + \lambda b_3(\mathbf{x}, t) \quad (18)$$

By the adaptive law, we have:

$$\dot{\hat{v}} = \frac{1}{k}|s(t)| \quad (19)$$

Where estimation error is:

$$\tilde{v} = \hat{v} - v \quad (20)$$

From “Eqs. (19) and (20)” , it is resulted that:

$$\dot{\tilde{v}} = \frac{1}{k}|s(t)| \quad (21)$$

Lyapunov function can be considered as follows:

$$V(t) = \frac{1}{2}(s(t)^2 + k\tilde{v}^2) \quad (22)$$

The sliding dynamics by “Eqs. (14)-(18)” is obtained as:

$$\dot{s}(t) = B(t)\dot{u}(t) + \Psi(t)u(t) + G(t) + F(t) + D(t) \quad (23)$$

With derivative from “Eq. (22)” is obtained as:

$$\dot{V}(t) = s(t)[B(t)\dot{u}(t) + \Psi(t)u(t) + G(t) + F(t) + D(t)] + \tilde{v}|s(t)| \quad (24)$$

By substituting “Eq. (13) into Eq. (24)” , we gain:

$$\dot{V}(t) = s(t)\left[-ls(t) - \hat{v}sgn\big(s(t)\big) + D(t)\right] + \tilde{v}|s(t)| \le -ls(t)^2 - \hat{v}|s(t)| + |D(t)||s(t)| + (\hat{v} - v)|s(t)| \le -ls(t)^2 - (v - |D(t)|)|s(t)| \quad (25)$$

Due to $v > |D(t)|$, we have:

$$\dot{V}(t) \le -ls(t)^2 < 0 \quad (26)$$

Eventually, can be say that the sliding surfaces is asymptotically stable. The proposed control block diagram of the ASMC is shown in “Fig. 1” .

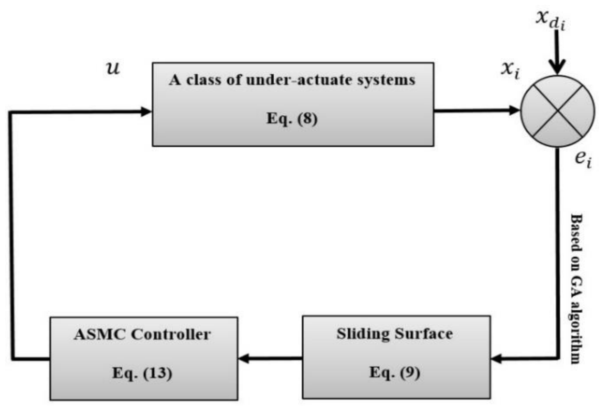


Fig. 1 A general view on the proposed control.

To minimize the integrals of absolute errors, the objective functions are considered as follows:

$$absolute\ errore = \int t|e_i(t)|\,dt \quad (27)$$

Where:

$$e_i(t) = x_i^d - x_i\,;\, i = 1,2,3 \quad (28)$$

Also, the defined constraints on the control effort are as follows:

$$\int t|u(t)|\,dt < \lambda_u \quad (29)$$

$$max|u| < \mu_u \quad (30)$$

Where, λ_u and μ_u are upper bounds for integral and maximum of absolute control effort.
The characteristics of non-dominated sorting genetic algorithm are shown in “Table 1” .

Table 1 The characteristics of NSGA

NSGA	Population size	Crossover fraction	Pareto front population fraction	Maximum generation
Parameter value	300	0.7	0.45	7000

5 SIMULATIONS

In this section, the effectiveness and good performance of the suggested controller is shown. The double inverted pendulum structure is made up of two links pendulums on a moving cart as is shown in "Fig. 2" . The control task is to move the cart on a rail origin by balancing both of the pendulums upright. Based on the vertical line, $\theta_1$and θ_2 are the angle of pendulum 1 and pendulum 2. Whereas, for origin z, be the cart position. u be the control force. Let $m_i, i = 1,2,3$ be the masses of the cart, the pendulum 1 and the pendulum 2. Let $L_i, i = 2,3$ be the respective length of the lower and upper pendulums and $l_i, i = 2,3$ be the lengths from their center of masses, respectively.

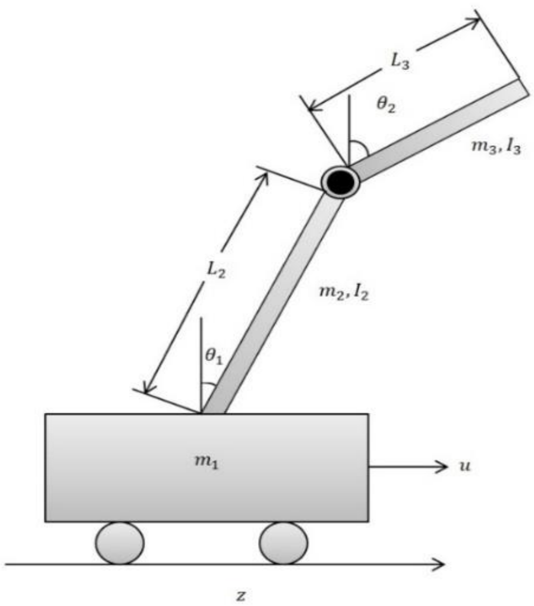


Fig. 2 Double inverted pendulum system.

For $n = 3$, "Eq. (8)" can be written as:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f_1(\mathbf{x}) + b_1(\mathbf{x})u(t) + d_1(t) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= f_2(\mathbf{x}) + b_2(\mathbf{x})u(t) + d_2(t) \\ \dot{x}_5 &= x_6 \\ \dot{x}_6 &= f_3(\mathbf{x}) + b_3(\mathbf{x})u(t) + d_3(t) \end{aligned} \quad (31)$$

The control objective is from the initial conditions $[\pi/6,0,\pi/9,0,0,0]^T$ of the inverted pendulum system to the desired state $[0,0,0,0,0,0]^T$ of the double inverted pendulum system. The mismatched uncertain terms of the system are assumed as follows:

$$d_1 = 0.1 + 0.5p\ , d_2 = 0.1 + 0.5p\ , d_3 = 0.5p$$

Here, p is a random number whose range is from -1 to 1. Also, the controller parameters are given in "Tables 2" , respectively. The physical parameters of the double inverted pendulum system are selected in "Table 3" .

Table 2 Optimal values of the controller parameters for the double inverted pendulum system

Parameter	Optimal values
c_1	14.8847
c_2	0.9702
c_3	-9.0875
c_4	0.9746
c_5	0.9944
c_6	2.4008
λ	0.0360
k	0.1440
ξ	5.2881
β	0.5762

Table 3 The physical parameters of the double inverted pendulum system

Parameter	m_1	m_2	m_3	l_2	l_3
value	1kg	$0.3kg$	$0.3kg$	$0.1m$	$0.1m$

Figure 3 shows the results of the sliding surface for a double inverted pendulum. As can be seen, the adaptive sliding mode control correctly acts and converges sliding surface to zero.

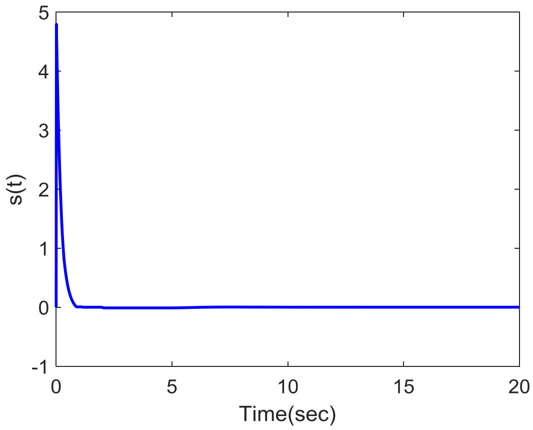


Fig. 3 Sliding surfaces of the double inverted pendulum system.

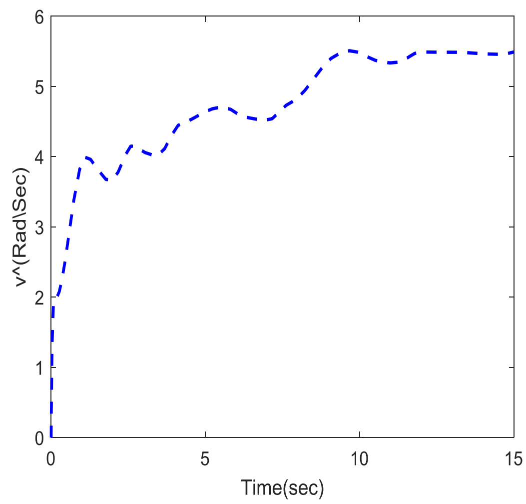
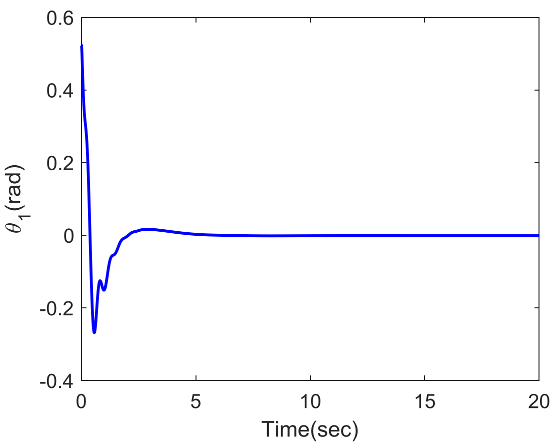
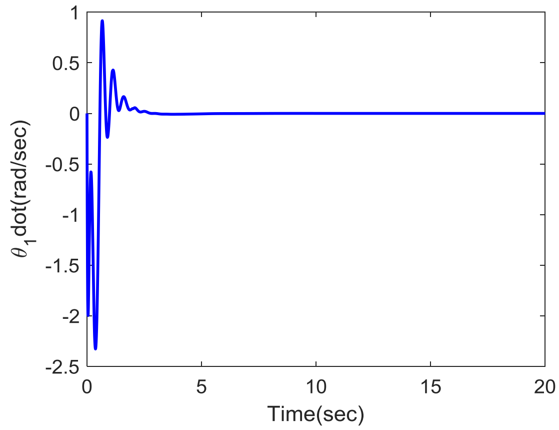


Fig. 4 Adaptive parameter of controller.

Figures 5-7 show the state trajectories of the controlled system and the ability of the proposed controller in improving the convergence rate.

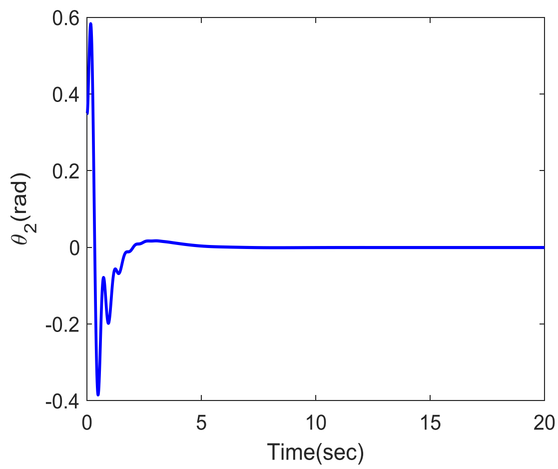


a

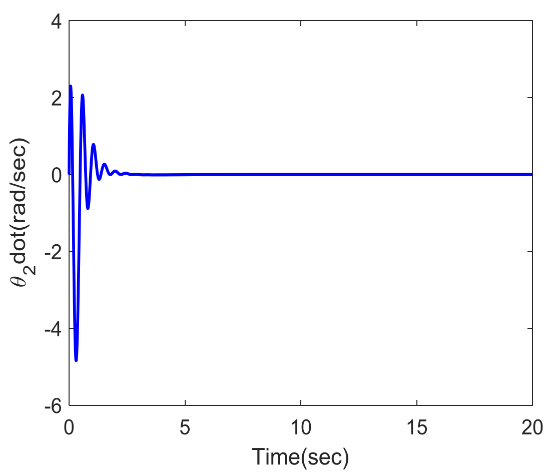


b

Fig. 5 State trajectories for the double inverted pendulum system: (a): State trajectory of θ_1, and (b): State trajectory of $\dot{\theta}_1$.



a



b

Fig. 6 State trajectories for the double inverted pendulum system: (a): State trajectory of θ_2, and (b): State trajectory of $\dot{\theta}_2$.

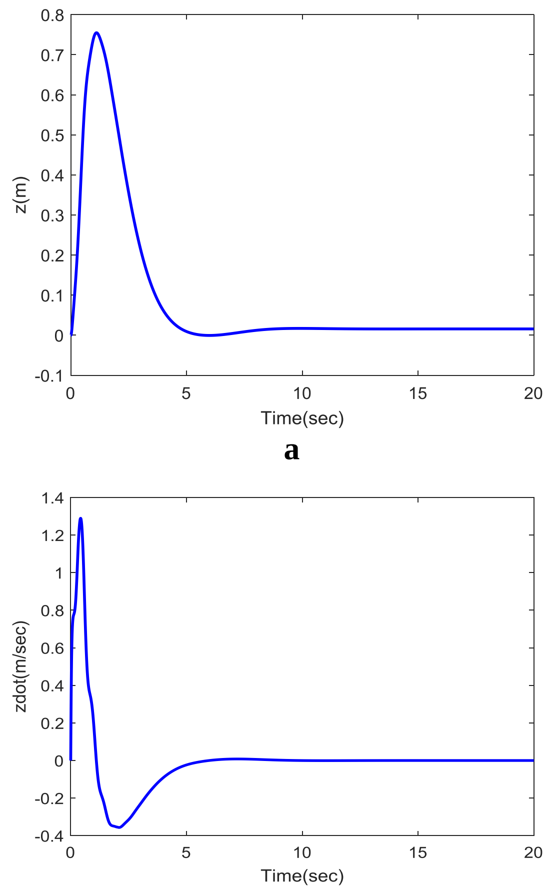


Fig. 7 State trajectories for the double inverted pendulum system: (a): State trajectory of z, and (b): State trajectory of $\dot{z}$.

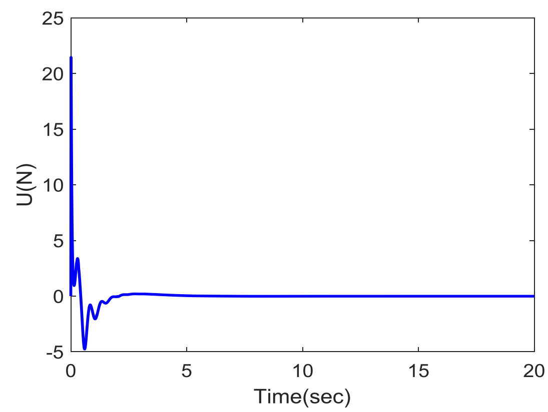


Fig. 8 Control effort for double inverted pendulum system.

The results clearly show that the controller in the presence of the existing uncertainties converges the states from the initial conditions to the desired value that converges to zero. The control input is illustrated in “Fig. 8” . From “Fig. 8” , we can conclude that the proposed controller can reduce chattering phenomenon and Control effort. One of the main advantages of the controller presented in this research is the lack of appearance of the chattering phenomenon, which is one of the basic problems of the chattering phenomenon that causes inability to implement on real systems, increases control effort consumption and stimulates high frequency modes systems. As the simulation results show, the controller has a very desirable performance.

6 CONCLUSIONS

This paper presented a simple stabilizing control algorithm for a class of underactuated systems with n degrees of freedom. The methodology is based on an ASMC. A non-dominated sorting genetic algorithm is proposed to reach the optimal controller gains. The presented control approach has some of superiorities including low control effort, fast convergence speed, and low chattering. The effectiveness of the proposed method is applied to double inverted pendulum. Numerical simulation verified the feasibility of the proposed control method.

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