

# The Inverse Method of Damage Detection using Swarm Life Cycle Algorithm (SLCA) via Modal Parameters in Beam Like Structures

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**Abstract:** The Non-destructive vibration based structural damage detection techniques have been developed in the recent decades. They are usually converted into a mathematical optimization problem that should be solved using optimization algorithms. In this paper, a new hybrid algorithm, using a particle swarm - genetic optimization, is proposed that is called Swarm Life Cycle Algorithm (SLCA). Additionally, Modified Total Modal Assurance Criterion (MTMAC) that is modal based and involved natural frequencies and mode shapes, is used as an objective function. A cantilever beam is modelled and simulated using finite element method as a numerical case study with several different damage scenarios. To compare the effectiveness of the proposed algorithm with GA and PSO, they are applied to detect the locations and severities of damages of numerical cases separately. To assess the robustness of them, the effects of environmental noise, coordinate and mode incompleteness on the accuracy of damage detection have investigated. For experimental validation of the proposed method, empirical studies of single and double crack aluminium cantilever beams were conducted. The numerical and experimental results show that the proposed algorithm has great potential in crack identification. It is observed that SLCA is able to detect the location and extent of damage irrespective of the noise level and perform well in the presence of mode and coordinate incompleteness.

**Keywords:** Damage Detection, Genetic Algorithm, Hybrid Algorithm, Modal Properties, Particle Swarm Optimization

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**Biographical notes:** **Alireza Arghavan** is PhD candidate of Mechanical Engineering at Semnan university. His field of research is Meta Heuristic Algorithms and Structural Damage Detection. **Ali Ghoddosian** is Associate Professor of Mechanical Engineering at Semnan university. His main research interests are Topology Structure, Meta Heuristic Algorithms, Structural Optimization and Numerical Optimization Methods. **Ehsan Jamshidi** is Assistant Professor of Mechanical Engineering at Semnan branch of Islamic Azad University and his research field of study focuses on Structural Model Updating, Vibration Absorber Design, Modal Analysis and Nonlinear Modal Analysis.

## 1 INTRODUCTION

In recent decades, structural damage detection techniques have been developed regarding their important role in structural health monitoring. Such damages will severely affect the structural durability and reduce the designed working life if it is not duly detected and controlled. Some cracks take place in available components and are sensible; so, the damage is detected by visual inspection. However, most of others are too small and not accessible. Therefore, they are identified by advanced methods, for instance, vibration-based damage techniques.

Many researchers have investigated the identification of vibration-based damage. A comprehensive study of vibration-based damage detection was presented by Doebling et al. [1]. The presence of damage can result in some vibrational parameters, mainly because the system loses its stiffness. Modal properties of a structure change when its components are damaged mainly due to a reduction in the stiffness [2]. Several review articles of vibration-based Structural Damage Detection (SDD) techniques have been undertaken [3–11].

SDD is usually converted into a mathematical optimization problem that should be solved using optimization algorithms. Due to the large search space, finding the global optimum in a reasonable time may not be possible using traditional optimization techniques. On the other hand, the heuristic algorithms can deal with finding global optimum for such complex problems with a significantly less computational cost. In one aspect, meta-heuristic optimization methods are generally classified into population-based and single-solution based categories, which are exploitation and exploration-oriented, respectively. Two main types of population-based optimization algorithms are Swarm Intelligence (SI) and Evolutionary Computation (EC). SIs are derived from the social behavior of swarm and ECs are inspired by Darwin's reproduction and survival of the fittest Theory. Among the SIs and ECs, Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) are superior to the others, respectively. Many researchers have presented novel improved methods for damage detection by GA [12–29] and many others have implemented damage identification techniques by PSO and improved it [30–46]. However, they both have their strengths and weaknesses. GAs use evolutionary operators like selection, mutation, and crossover can reach the global region and are quite robust. Nevertheless, the convergence speed of these methods is slow. Moreover, as in GAs the worst individuals are eliminated and not permitted to pass to the Next Generation, the experiences of the individuals are missed compared to PSO. On the other hand, PSOs have easier coding and are faster in convergence because of their mathematical operators. But

premature convergence may occur due to a lack of diversity. The lack of selection operator in PSOs leads to wasting resources on poor individuals. To boost the strengths of PSO and GA and to overcome their weaknesses, hybridizing them can be beneficial. In this way, a method with high diversity and fast convergence will be obtained [47–59].

In this paper, a new hybrid PSO-GA algorithm is proposed which is called the Swarm Life Cycle Algorithm (SLCA). It is adopted specially for structural damage detection problems. Numerical and experimental results illustrate the effectiveness of the proposed method in the identification of the location and severity of the damage. The accuracy of the method in predicting the damage location and severity, as well as its convergence speed, are investigated. Moreover, the effect of the number of Modes, coordinate incompleteness and environmental noise on obtained results is studied.

## 2 VIBRATION BASED STRUCTURAL DAMAGE DETECTION

### 2.1. Theoretical Background

The equation of motion for free vibration of an undamped multi-degree of freedom (MDOF) system is given by:

$$[M]\{\ddot{X}(t)\} + [K]\{X(t)\} = 0 \quad (1)$$

Where,  $[M]$  and  $[K]$  are respectively mass and stiffness matrices and  $\{\ddot{X}(t)\}$  and  $\{X(t)\}$  are acceleration and displacement vectors, respectively. For an N-DOF system, there are N natural frequencies and mode shapes that can be obtained by solving the following eigenvalue problem:

$$([K] - \lambda_i[M])\{\varphi_i\} = 0 \quad (2)$$

Where,  $\lambda_i$  is the  $i^{th}$  eigenvalue and  $\{\varphi_i\}$  is its corresponding eigenvector. Here,  $\lambda_i$  is the square of the  $i^{th}$  natural frequency. Natural frequencies and mode shapes are called modal parameters, which are functions of the physical properties of the system.

### 2.2. Modeling of Damage

Physical properties of a system such as mass and stiffness are affected by the crack occurrence. As the change of mass is negligible compared to the change of stiffness, cracks are usually modeled through the reduction in local stiffness of the structure, as formulated below:

$$[K_i^d] = (1 - \beta_i)[K_i], \quad (i = 1, 2, 3, \dots, e) \quad (3)$$

Where,  $[K_i^d]$  and  $[K_i]$  are the  $i^{th}$  element stiffness matrices of damaged and intact structure, respectively.  $\beta$  is the damage index, which is defined between 0 and 1. If  $\beta_i = 0$ , the  $i^{th}$  element is completely intact; on the contrary, if  $\beta_i = 1$ , the  $i^{th}$  element is destroyed. Although in this model, the damage severity does not exactly match the crack depth and also is affected by mesh density, Friswell et al. [60] showed that the crack is modeled correctly by this method. They also showed that if the crack is modeled elaborately by more details, it can substantially improve the damage assessment results.

### 3 OBJECTIVE FUNCTION

Choosing the appropriate objective function is a key success factor for any optimization problem. In vibration-based damage detection, the objective function is usually constructed from modal parameters such as natural frequency mode shapes or a combination of them.

Perera and Torres [17] presented an objective function based on the Modified Total Modal Assurance Criterion (MTMAC). MTMAC, in turn, is based on the Total Modal Assurance Criterion (TMAC) developed by Gao and Spencer [61]. These researchers, however, modified this criterion by introducing the frequency parameter as follows:

$$MTMAC = \prod_{i=1}^m \frac{MAC([\varphi_i^E], [\varphi_i^N])}{(1 + \frac{|\lambda_i^N - \lambda_i^E|}{|\lambda_i^N + \lambda_i^E|})} \quad (4)$$

Where,  $[\varphi_i^E]$  and  $[\varphi_i^N]$  are the  $i^{th}$  experimental and numerical mode shapes,  $\lambda_i^E$  and  $\lambda_i^N$  are the  $i^{th}$  experimental and numerical eigenvalues, respectively, and MAC is the modal assurance criterion defined by Evins 1984 [62], as follows:

$$MAC_{ij} = \frac{|\{\varphi_A\}_i^T \{\varphi_B\}_j|^2}{(\{\varphi_A\}_i^T \{\varphi_A\}_i)(\{\varphi_B\}_j^T \{\varphi_B\}_j)} \quad (5)$$

According to Equation (4), the MTMAC value is between 0 and 1, where 1 denotes a perfect correlation. They formulated the objective function as:

$$F = 1 - MTMAC \quad (6)$$

Since the objective function has shown its robustness in experimental damage detection [17], it was used in this paper as well.

## 4 OPTIMIZATION ALGORITHM

In this study, a new hybrid algorithm called Swarm Life Cycle Algorithm (SLCA) is proposed and adopted specially for structural damage detection problems. For more elucidating, a brief overview of GA, PSO, and the new hybrid method is provided in this section.

### 4.1. Genetic Algorithm

John Holland developed GA in 1975 based on the principles of genetics and natural selection. In this method, a population of individuals is repeatedly modified in every generation. At each step, children for the next generation are provided using parents that are selected randomly from the current population. There are three types of rules to create the next generation from the current one; i.e., selection, crossover, and mutation. Selection rules select parents that contribute to the next generation. Crossover rules form children by combining two parents. Mutation rules alter some of the individuals by changing their genes randomly to increase diversity. Some of the individuals that have the highest scores based on their fitness are chosen as elite and are passed on to the next generation.

### 4.2. Particle Swarm Optimization (PSO)

Edward and Kennedy (1995) formulated PSO in [63] by inspiration from the social behavior of animals such as insects swarming, fish schooling, or flocks of birds. PSO, like continuous GA, begins with a random population. However, PSO does not use evolution operators. That particles in PSO play the role of chromosomes in GA.

Initially, PSO creates particles and assigns their velocities randomly. After evaluating the fitness function of the particle's location, the method determines the best locations and their corresponding function value. New velocities are chosen in terms of the current velocities, individual best locations of the particle, and the best locations of their neighbors.

$$V^{new} = W * V^{old} + Y1 * U1 * (P - X) + Y2 * U2 * (G - X) \quad (7)$$

Where,  $V^{new}$  is the updated velocity,  $W$  is an inertia weight,  $V^{old}$  is the velocity of the previous step,  $Y1$  and  $Y2$  are self and social adjacent weights, respectively,  $U1$  and  $U2$  are independent uniformly distributed random numbers between 0 and 1,  $P$  is the best position of the particle,  $G$  is the best position in the current neighborhood, and  $X$  is the current position of the particle. The particle locations are updated using the following formula:

$$X^{new} = X^{old} + V^{new} \quad (8)$$

Where,  $X^{new}$  and  $X^{old}$  are the new and old position of the particle, respectively.

### 4.3. Swarm Life Cycle Algorithm (SLCA)

In this paper, a new hybrid technique PSO-GA is proposed to overcome the weaknesses of PSO and GA algorithms and incorporate their strengths. SLCA consists of six stages (“Fig. 1”).

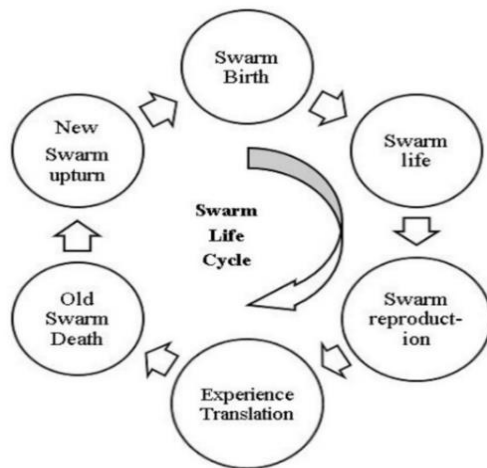


Fig. 1 SLCA consists of six stages.

At first, the parameters related to PSO and GA algorithms are determined by the user according to the flowchart in “Fig. 1”. Also, the new algorithm needs to define two parameters, i.e., the minimum variance ratio (VR-Min) and the contraction coefficient (Cc), which are suggested according to experience to be 0.1 and 100, respectively.

The first stage of the new algorithm is called swarm birth in which particles are generated according to the determined number, and the position and velocity of each will be determined randomly. The second stage is related to the life of the individuals, which are produced in the first step and will proceed using the PSO algorithm. The algorithm will continue until the variance of the fitness values is less than the specified minimum variance. In this way, this method prevents the PSO algorithm from being trapped in local minima. The termination conditions will be checked and the algorithm will end in case of satisfaction; otherwise, it will go on to the next step, which is creating the next generation by GA. The population size of the genetic algorithm must be equal to the size of the PSO particles obtained in the previous step and will be stored as PSO particles.

It is noteworthy that particles with their best own memory are transferred to this stage, which is particle reproduction. A GA was used in this step. In this step, at the time of choosing the bests, each particle will be chosen with their best personal memory. After reproduction and in the process of crossover, the

offspring receives the best personal memory from the parents. In the mutation, the particle’s best personal memory is also transferred to the mutated particle. Therefore, the operation of experience transform will be reproduced.

After a few generations by GA (usually 6 or 7 generations), the optimum solutions are achieved and the next stage begins, which is the death of the swarm. Particles that are formed after swarm life will be replaced by particles produced by GA, and this is the swarm upturn. At this stage, the particles are ready to start life. Before the modified particles can enter the life stage, the minimum ratio of variance must be reduced using the contraction coefficient, which is also shown in the flowchart of “Fig. 2”. The minimum ratio of variance is obtained from the following equation:

$$(VR-Min)_{new} = (VR-Min)_{old} / Cc \quad (9)$$

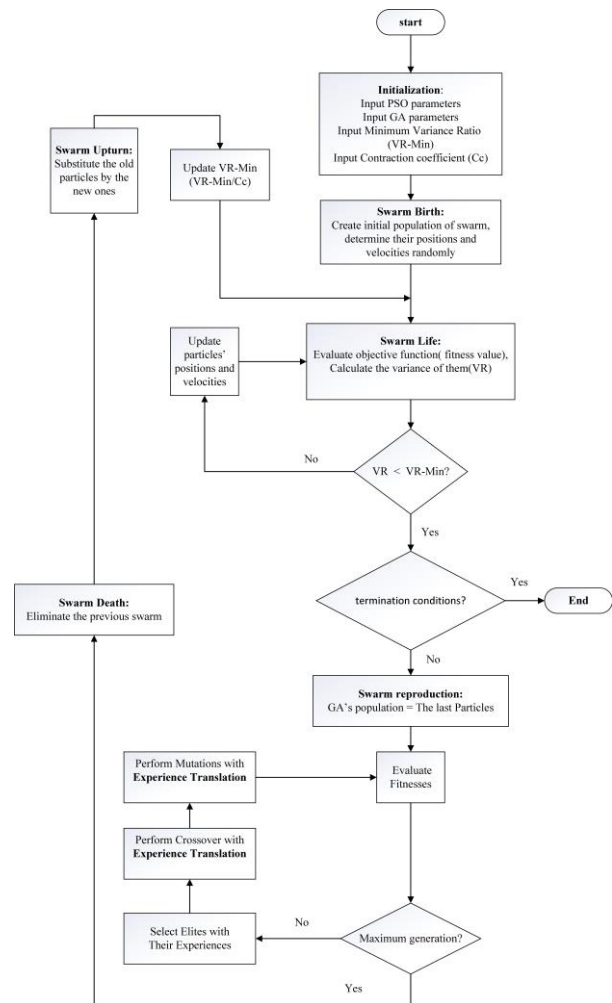


Fig. 2 Flowchart of SLCA.

The purpose of this section is to achieve solutions with less variance; i.e., the PSO algorithm can get closer to the optimal solutions. These steps will be repeated until the termination conditions of the algorithm are satisfied and reaching the optimal solution out of the swarm life stage.

**5 NUMERICAL IMPLEMENTATION AND COMPARATIVE STUDY**

A cantilever beam was considered here as a numerical case study to detect the location and severity of damage using the proposed method and then its results were compared with the solutions obtained from GA and PSO. To assess the robustness of the new algorithm, the effects of environmental noise, coordinate incompleteness and mode incompleteness on the accuracy of identification of damage were investigated. In order to extract natural frequencies and mode shapes of the beam, the Finite Element (FE) model and simulation were performed in MATLAB software. The material properties and dimensions of the beam are presented in “Table 1”.

**Table 1** The material properties and dimensions of the beam

Properties			Dimensions		
E(pa)	$\rho$ (kg/m <sup>3</sup> )	$\nu$	Length (mm)	Width(m)	Height(m)
70*10 <sup>9</sup>	2750	0.33	1000	46	9

The beam was discretized to 20 2-nodes 4DOF beam elements. Damage scenarios with different locations and severities of cracks are specified in “Table 2”.

**Table 2** Detail of damage scenarios

Scenario	Damaged Elements' Numbers	Damage Severity
1	5	0.25
2	2, 18	0.25, 0.1
3	2, 9, 18	0.1, 0.2, 0.3

The initial parameters required for each applied algorithm are presented in “Table 3”. It is noteworthy, since the optimization is a stochastic process, the average values of 10 individual runs are noticed as the damage detection results for each scenario.

**Table 3** Determination of the initial parameters of GA, PSO and SLCA in Initialization step

Algorithm	Parameters	Value
PSO	Swarm size	
	Maximum Iterations	200
	Self-Adjustment Weight	100
	Social-Adjustment Weight	2
	Social-Adjustment Weight	2
GA	Population size	200
	Maximum generations	150
	Crossover fraction	0.7 * Population size
	Elite count	0.05* Population size
SLCA	Swarm size	
	Maximum Iterations	
	Self-Adjustment Weight	150
	Social-Adjustment Weight	100
	Social-Adjustment Weight	1.5
	Population size	150
	Maximum generations	5
	Crossover fraction	0.7 * Population size
	Elite count	0.1 * Population size
	contraction coefficient	100
	minimum variance ratio	0.1

**5.1. Experimental Case Studies**

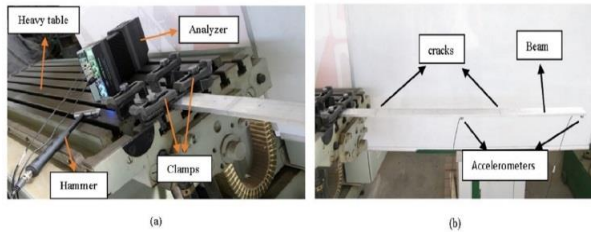
**5.1.1. Setup for Experimental Case Studies**

Single and double crack aluminum cantilever beams, which are fixed to a heavy table by clamps, were considered for the experimental validation of the proposed method. Their dimensions are the same as the numerical case. The properties of the cracks are listed in “Table 4”.

**Table 4** Location and depth of damage related to the clamped end of the beams

case	Location (mm)	Depth(mm) @ Location (mm)
1	Single crack	6@175
2	Double cracks	6@175, 4@675

The beams were excited by impact hammer DJB type IH-02 at ten equally spaced along the beam. Two Bruel and Kjaer (B&K) accelerometers type 4533-B were used to measure the responses. The analyzer used in this study was B&K Lan XI type 3160. The experimental setup is shown in “Fig. 3”.



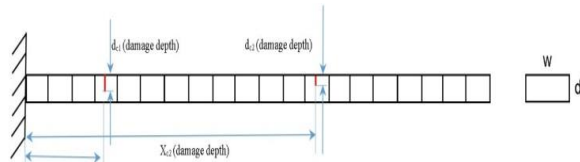
**Fig. 3** Experimental setup of cantilever beam: (a): setup components, and (b): the locations of the sensors and cracks.

### 5.2.2. Estimation of Damage Indices

To assess the ability of the proposed method in estimating the severity of the damage, first, it is necessary to obtain exact damage indices from the damage depths. To achieve this aim, the following formula can be used:

$$\text{damage severity \%} = \frac{EI(X)}{EI} = \frac{1}{1 + Ce^{\frac{-2\alpha|X-X_c|}{d}}} \quad (10)$$

Where,  $C = (I_0 - I_c) / I_c$  for  $I_0 = \frac{wd^3}{12}$  and  $I_c = \frac{w(d-d_c)^3}{12}$ . According to “Fig. 4”,  $X$  is any position along the beam and  $\alpha$  is a constant and equal to 0.667 [45].



**Fig. 4** The location of the cracks on the experimental beam.

Based on Equation (10), damage severity indices were calculated and listed in “Table 5”.

**Table 5** damage severity indices on FEM with 20 elements regarded to single and double crack beams

case	Depth(mm) @ Location (mm)	damage severity (%)	number of elements
Single	6@175	3.01, 59.02, 3.01	3, 4, 5
double	6@175 4@675	3.01, 59.02, 3.01 0.30, 19.36, 0.30	3, 4, 5 13, 14, 15

## 6 RESULT AND DISCUSSIONS

### 6.1. Numerical Results

In practical structural cases, vibration-based damage detection is sensitive to several parameters such as environmental noise, coordinate incompleteness, and mode shape incompleteness. In this study, the effect of these parameters on the accuracy of the method in the

prediction of location and severity of damage is investigated

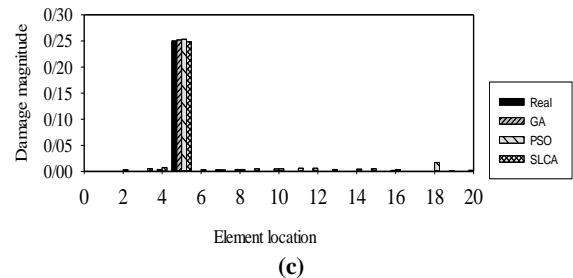
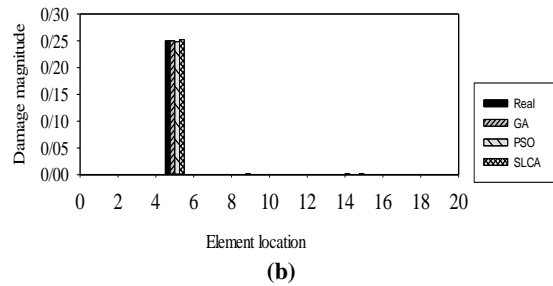
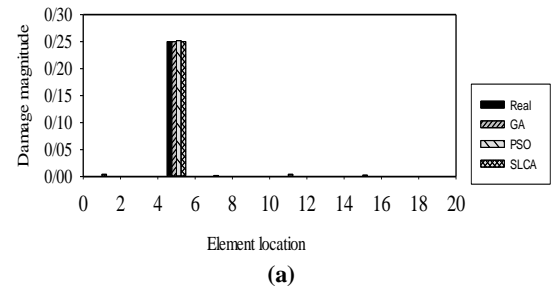
#### 6.1.1. Sensitivity to Noise

Real measurement data are generally contaminated by environmental noise. The effect of noise is taken into consideration for eigenvalues and eigenvectors with the following formulas:

$$\varphi_{ij}^k = \varphi_{ij}(1 - \eta \xi_{ij}^k) \quad (11)$$

$$\lambda_{ij}^k = \lambda_{ij}(1 - \eta \xi_{ij}^k), \quad i = j \quad (12)$$

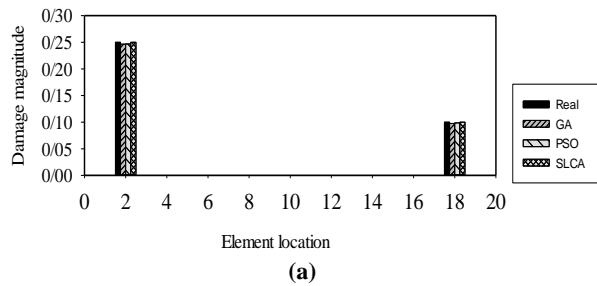
Where,  $\varphi_{ij}^k$  is the  $j^{\text{th}}$  component of the  $i^{\text{th}}$  noise-contaminated mode for  $k^{\text{th}}$  measurement.  $\lambda_{ij}^k$  is the  $i^{\text{th}}$  eigenvalue,  $\eta$  is the noise level, and  $\xi_{ij}^k$  is a random number in the range of [0, 1].



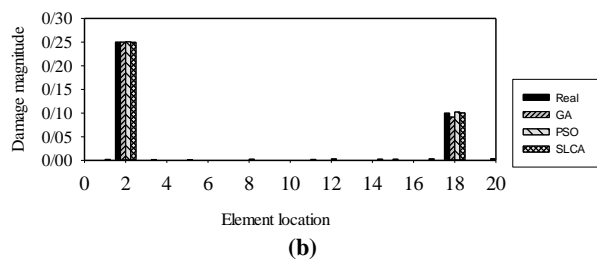
**Fig. 5** damage indices for three levels of noise: (a): 0%, (b): 2.5%, and (c): 5%, regarding Scenario-1.

Figures 5 to 7 show the effect of noise on the accuracy of identification of the location and severity of the damage in PSO, GA, and SLCA methods for damage

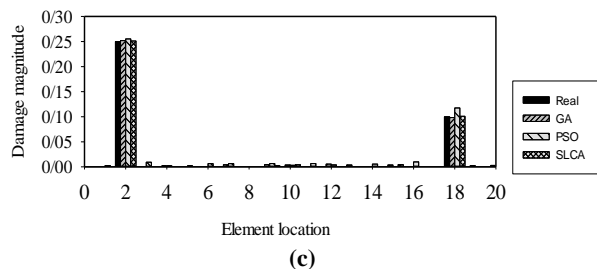
scenarios 1, 2, and 3, respectively. As can be seen, all algorithms can detect the damage locations, but GA and PSO are not able to determine the damage extent exactly when noise levels are increased. Three noise levels applied in this process are 0, 2.5, and 5%. Moreover, some extent of damage is assigned to intact elements when the noise level increases. Interestingly, it is observed that SLCA can detect the location and extent of damage irrespective of the noise level.



(a)

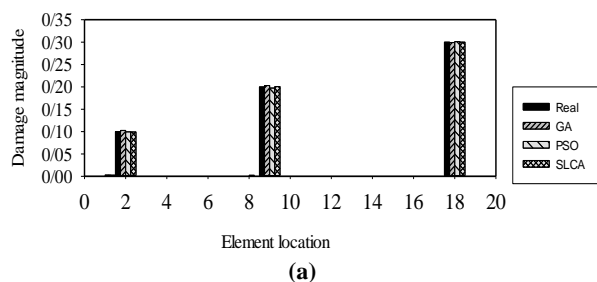


(b)

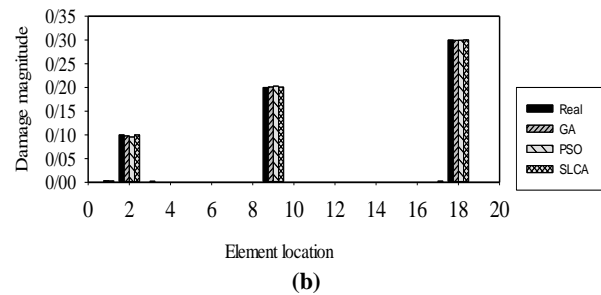


(c)

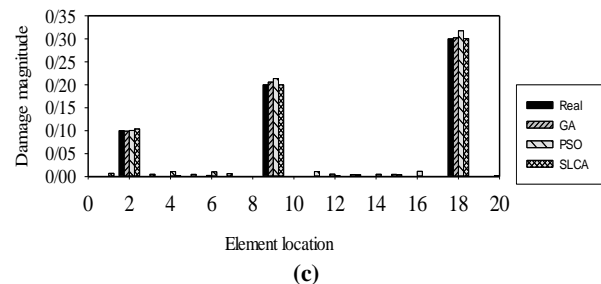
Fig. 6 damage indices for three levels of noise: (a): 0%, (b): 2.5%, and (c): 5%, regarding Scenario-2.



(a)



(b)



(c)

Fig. 7 damage indices for three levels of noise: (a): 0%, (b): 2.5%, and (c): 5%, regarding Scenario-3.

To investigate the accuracy of the solutions, the error values are presented in “Table 6”. These values are obtained by the following equation:

$$\text{Error \%} = \frac{|\text{real value} - \text{numerical value}|}{|\text{real value}|} * 100 \quad (13)$$

Table 6 Average error of damage severity regarded to each scenario

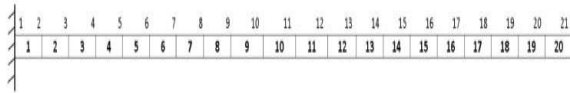
scenario	algorithm	noise (%)		
		0	2.5	5
Scenario 1	GA	0	1.3532	5.5514
	PSO	2.9411	1.1638	9.0453
	SLCA	0.8152	1.2092	2.2479
Scenario 2	GA	1.6037	3.7075	4.4104
	PSO	1.3501	2.6483	10.3291
	SLCA	0.3516	1.394	2.6642
Scenario 3	GA	1.2567	1.23	3.6013
	PSO	1.4097	1.9482	9.1526
	SLCA	0.9223	0.2548	1.3155

According to “Table 6”, the proposed algorithm achieves better results and shows its robustness to noise for single-crack and multi-crack beams.

**6.1.2. Sensitivity to Coordinate Incompleteness**

In practice, it is not possible to measure all DOFs in a structure, either due to the physical inaccessibility, difficulties encountered in the measurement of rotational DOFs, lack of the number of accelerometers and analyzer channels, or the cost of time of the experiment. Thus, in this research, the effect of coordinate incompleteness on results is investigated.

According to “Fig. 8”, three sets of coordinate incompleteness are used for comparing the performance of the methods. Due to experimental limitations, only the displacement coordinates can be measured, so the odd numbers correspond to the coordinates of each node are considered. First, the responses are measured only at nodes 2, 9, and 17. Next, the coordinates of nodes 2, 5, 10, 13, 17, and 20 are considered. Lastly, it is assumed that all nodes are measured and the measurements are done at odd coordinate numbers shown in “Table 7”.

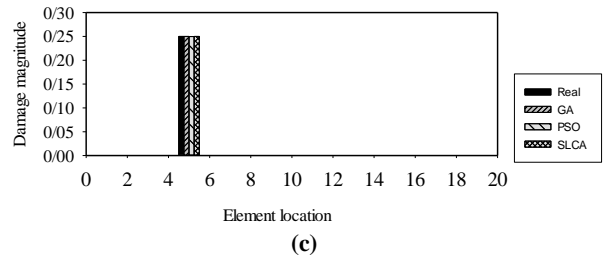
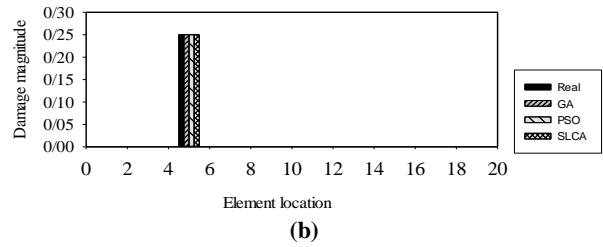
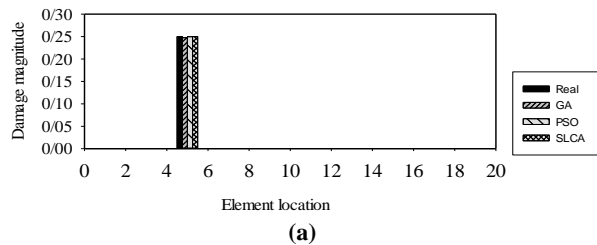


**Fig. 8** Finite element model of cantilevered beam with 20 elements, 21 nodes and 42 DOFs.

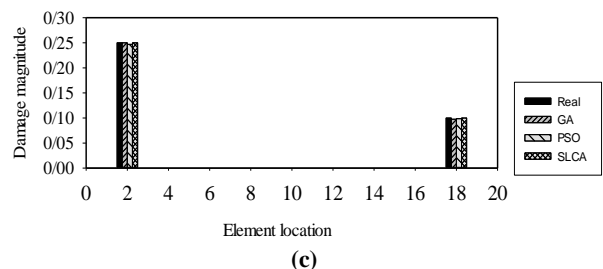
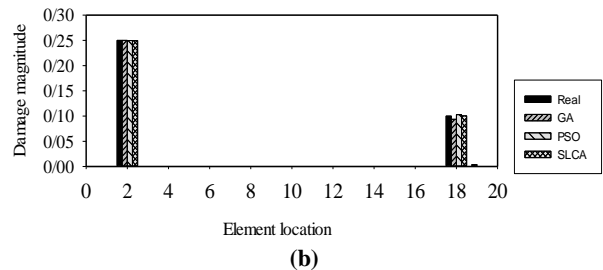
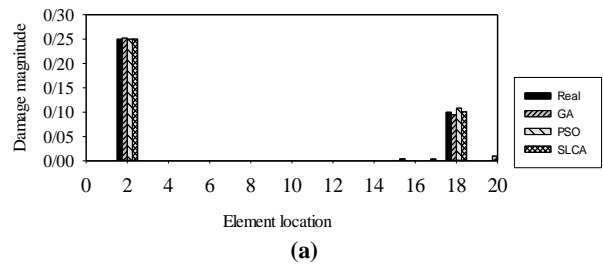
**Table 7** Three sets of coordinate incompleteness related to displacement coordinates

Set	Number of coordinates	Coordinates' numbers
1	3	3, 17, 33
2	6	3, 11, 19, 27, 33, 41
3	20	3, 5, 7, . . . , 37, 39, 41

Figures 9 to 11 present the effect of coordinate incompleteness on the accuracy of identification of the location and severity of the damage by PSO, GA, and new hybrid method for damage scenarios 1 to 3, respectively.



**Fig. 9** Damage identification for three sets of coordinate incompleteness: (a): set-1, (b): set-2, and (c): set-3, regarding Scenario 1.



**Fig. 10** Damage identification for three sets of coordinate incompleteness: (a): set-1, (b): set-2, and (c): set-3, regarding Scenario 2.



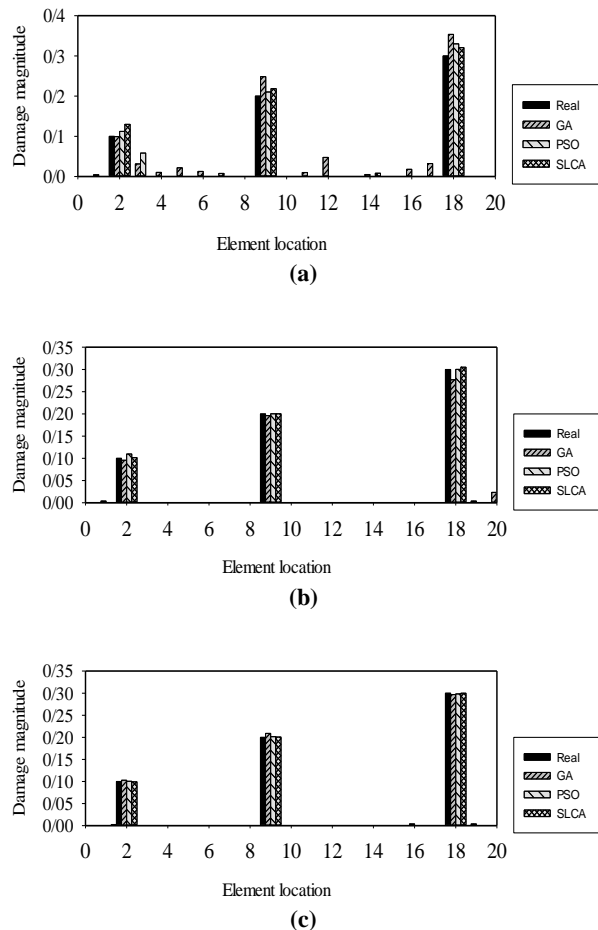


Fig. 11 Damage identification for three sets of coordinate incompleteness: (a) set-1, (b): set-2, and (c): set-3, regarding Scenario 3.

Table 8 Error average of damage severity for each scenario vs. three sets of coordinate incompleteness

Scenario	Set	GA	PSO	SLCA
Scenario 1	1	1.1785	0.291	0
	2	0	0.04	0.2715
	3	0	0.8152	0.4426
Scenario 2	1	4.591	3.161	1.9888
	2	2.7849	1.0888	0.6187
	3	1.0144	1.3501	0.3516
Scenario 3	1	27.7762	18.066	11.183
	2	9.089	2.603	1.3922
	3	3.0637	0.5996	0.9223

As can be seen from “Table 8”, the algorithms in the first and second scenario have presented good results in the presence of the coordinate incompleteness.

However, in the third scenario, in the first set, a large error is seen. Although the proposed algorithm presented better results, it needs more points to get better results.

### 6.1.3. Sensitivity to the Number of Measured Modes

It is known that measuring all the natural frequencies and their corresponding mode shapes is not possible due to the limits of the frequency range of accelerometers, force transducers, and analyzer channels as well as the restrictions ahead of the frequency range of exciters like hammer or shakers, the mode shape incompleteness.

To survey mode incompleteness accurately, damage identification was carried out on the cantilever beam applying three frequency levels. Levels 1, 2, and 3 include the first 3, 6, and 10 bending natural frequencies along with their mode shapes, respectively. It is evident from the results shown in “Figs. 12-14” and “Table 9” that PSO is severely sensitive to number of measured modes.

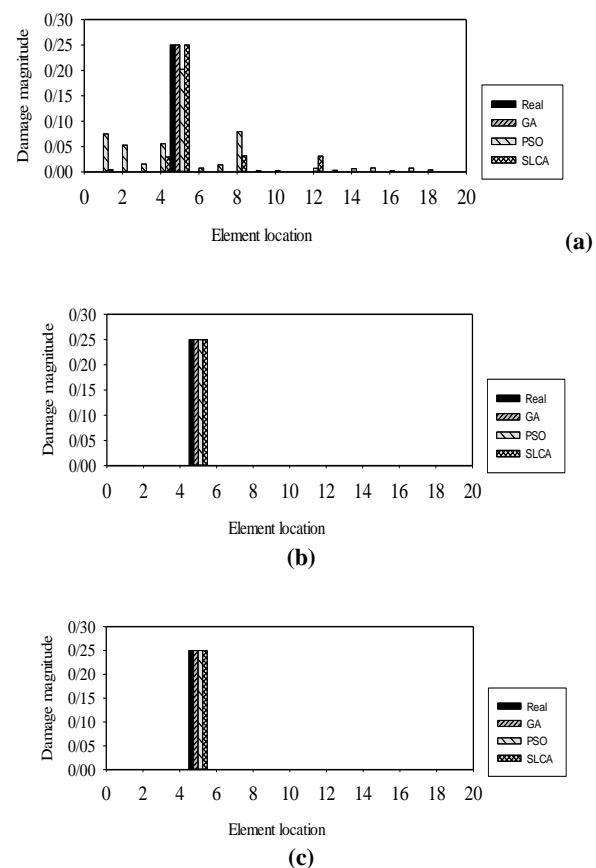


Fig. 12 Damage indices for mode incompleteness: (a): the three first, (b): the six first, and (c): the ten first frequencies and mode shapes - Scenario 1.

As a result, it is not recommended when there are a few numbers of modes measured. On the contrary, GA performed better than the other two algorithms in the case of single damage, but does not present good

accuracy in multi crack beams. (specially in level 3 of three crack beam). SLCA presents better results compared to the other two methods specially in multi crack beams except for level 1 for single crack beam. However, it shows a level of sensitivity to mode incompleteness.

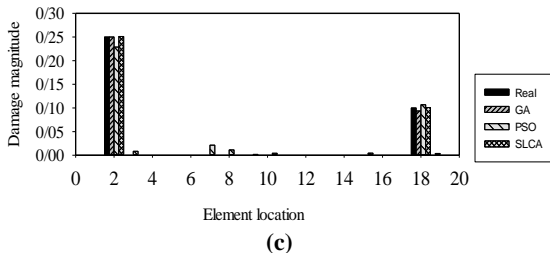
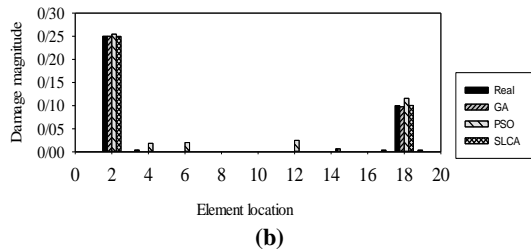
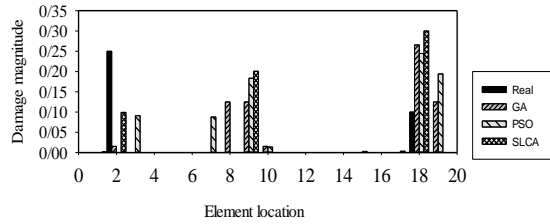


Fig. 13 Damage indices for mode incompleteness: (a): the three first, (b): the six first, and (c): the ten first frequencies and mode shapes - Scenario 2.

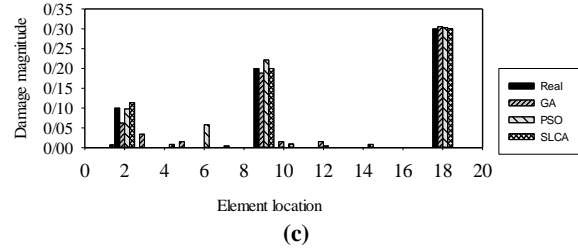
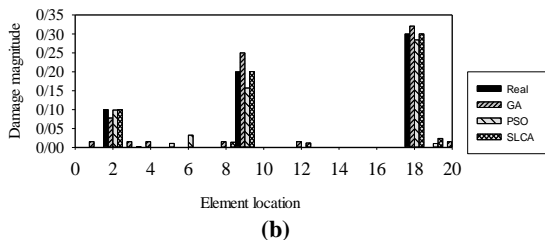
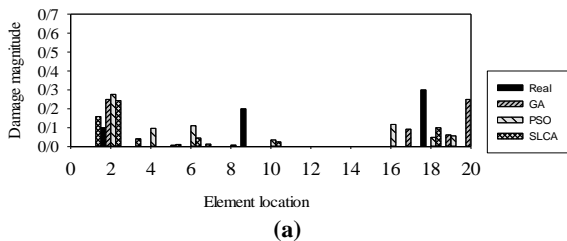


Fig. 14 Damage indices for mode incompleteness: (a): the three first, (b): the six first, and (c): the ten first frequencies and mode shapes - Scenario 3.

Table 9 Average error of damage severity for each scenario with three sets of mode incompleteness

scenario	level	GA	PSO	SLCA
Scenario 1	1	0.0002	57.886	21.5404
	2	0.0011	0.0938	0.0172
	3	0.0004	0.0022	0.0001
Scenario 2	1	133.64	154.86	119.19
	2	2.2287	15.1114	2.9431
	3	2.7373	12.7274	2.6642
Scenario 3	1	127.51	199.33	123.9
	2	18.6614	15.5132	8.7587
	3	15.7532	16.6678	5.917

6.2. Experimental Results

As mentioned in section 5.2, single-crack and double-crack beams were tested and the results are obtained in terms of Frequency Response Functions (FRFs). As the beam was excited by hammer at ten equally spaced nodes and the responses were measured at two nodes on the beams, twenty FRFs were measured for each of single and double cracked beams. Figure 15 shows magnitude and phase of one of the measured accelerances (point FRF for measuring force and response at point free end of the beams) for intact, single-crack, and double-crack beams. As can be seen from “Fig. 15”, the existence of the single crack and double crack has a considerable effect on the decrease of natural frequencies due to reduction of the local stiffness of the beam, which is more observable on higher modes.

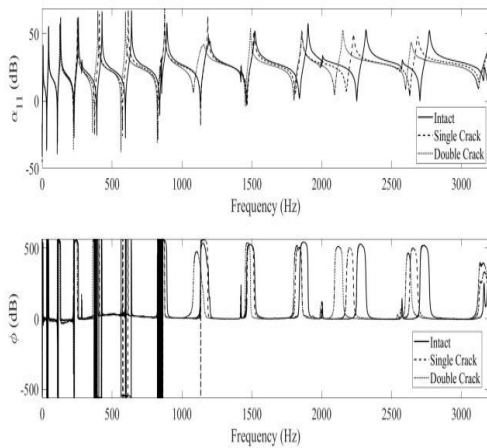


Fig. 15 Magnitude and phase of one of the measured FRFs for intact, single-crack, and double-crack beams.

Natural frequencies and mode shapes of the beams were calculated from obtained FRFs using the rational fraction Polynomial-Z (RFP-Z) method by BK Connect software. The results are presented in “Table 10”.

Table 10 The first ten natural frequencies (Hz)

Mode	Intact	Single	Double
1	7.48978	6.9002	6.88947
2	46.93581	46.6315	44.93315
3	132.1978	132.0289	126.4366
4	259.719	252.3577	252.1405
5	428.7169	409.4698	398.5931
6	639.9772	616.2975	596.7422
7	893.1108	876.6919	876.4676
8	1190.111	1184.987	1156.168
9	1522.263	1515.491	1490.863
10	1902.634	1858.843	1857.81

A beam was discretized to a twenty 2-noded 4DOF elements in order to use by the proposed method in the numerical identification process. First, SLCA algorithm was applied to the undamaged beam to determine the global Young’s modulus and the density of the beam for updating the FE model. These values were obtained to be 68.7 Gpa and 2590 kg/m<sup>3</sup>, respectively. Comparison of the first 10 natural frequencies by experiments and FE before and after updating are listed in “Table 11”.

Table 11 The first ten natural frequencies comparison experiment and FEM before and after updating

Modes	Experiment (Hz)	Before updating (Hz)	After updating (Hz)
1	7.49	7.34	7.49
2	46.93	45.97	46.93
3	132.2	128.75	131.39
4	259.72	252.47	257.49
5	428.72	418	425.7
6	639.98	626.21	636.05
7	893.11	878.6	888.67
8	1190.11	1177.08	1183.78
9	1522.26	1521.95	1521.71
10	1902.63	1892	1902.92

As it is evident from “Fig. 16”, the proposed method can estimate the damage location and severity in both cases successfully. It should be noted that the method was able to detect the damage locations exactly and estimate the severity of damage by 2.4% and 4.8% error for single-crack and double-crack beams, respectively.

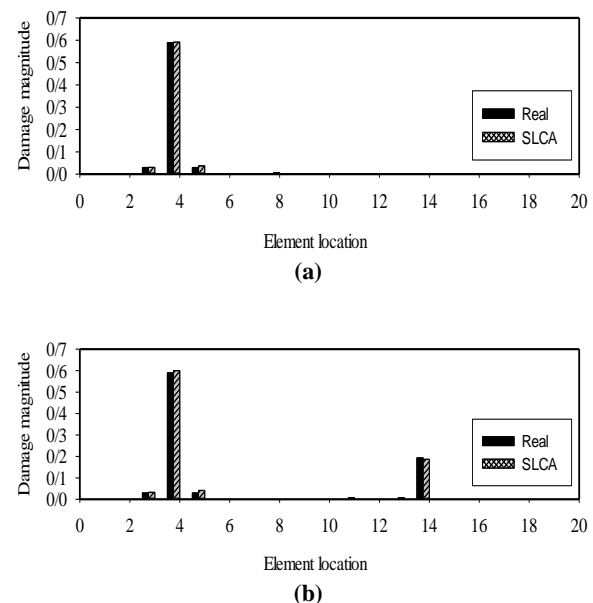


Fig. 16 damage indices for: (a): single, and (b): double crack beams.

## 7 CONCLUSIONS

In this paper, a novel hybrid algorithm (SLCA) approach based on PSO and GA was presented for estimating

damage locations and severity in beam like structures. The objective function was based on modal parameters (natural frequencies and mode shapes). The effectiveness of the proposed method has been evaluated by numerical and experimental cantilever beams. The effects of several parameters such as environmental noise, coordinate and mode incompleteness on the accuracy of the method in identification of location and severity of damage have been investigated. Some conclusions can be reached as follows:

1. In contrast to PSO and GA, which were not able to estimate the severity of damages accurately, the proposed algorithm achieved acceptable outcomes and showed its robustness to noise for single and multi-crack beams.
2. All algorithms have presented good results for the single and double crack beams in the presence of the coordinate incompleteness. Although, the proposed algorithm presented better results compared two other algorithms for the case of multi crack beams, with only three measured coordinates (sever coordinate incompleteness) a large error was seen. So, it is

recommended that the method needs more points to get better results.

3. It is evident from the obtained results that PSO is severely sensitive to number of measured modes. As a result, it is not recommended when only a few modes have been measured. On the contrary, GA performed better than the other two algorithms in the case of single damage, but does not present good accuracy in multi crack beams. (specially in level 3 of three crack beam). SLCA presents better results compared to the other two methods specially in multi crack beams except for level 1 for single crack beam.
4. In experimental studies, the proposed method was able to detect the damage locations exactly and estimate the severity of damage by 2.4% and 4.8% error for single and double crack beams. respectively. Therefore, it can be mentioned that SLCA is successful as a damage detection technique on experimental cases. "Table 12" presents a qualitative glance at the results of numerical studies in which the symbols ++, +, \* and – stand for up to 3%, 3% to 6%, 6% to 10% and over 10% error, respectively.

**Table 12** A qualitative glance at the results of numerical studies

		Environmental			The coordinate incompleteness levels			The mode		
		Noise levels						Incompleteness levels		
		1	2	3	1	2	3	1	2	3
GA	Scenario 1	++	++	+	++	++	++	++	++	++
	Scenario 2	++	+	+	+	++	++	-	++	++
	Scenario 3	++	++	++	-	*	+	-	-	-
PSO	Scenario 1	++	++	*	++	++	++	-	++	++
	Scenario 2	++	++	-	+	++	++	-	-	-
	Scenario 3	++	++	*	-	++	++	-	-	-
SLCA	Scenario 1	++	++	++	++	++	++	-	++	++
	Scenario 2	++	++	++	++	++	++	-	++	++
	Scenario 3	++	++	++	-	++	++	-	*	+

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